

Methods of determining T_1 and T_2 relaxation times

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Methods for measuring T_1 and T_2 relaxation times are used in the medical field for magnetic resonance imaging to diagnose diseases and improve research. The objective in this experiment is to measure the T_1 and T_2 relaxation times of mineral oil. Here, we present two methods for determining the T_1 relaxation time via inversion recovery and saturation recovery sequences for mineral oil. For determining the T_2 relaxation time of mineral oil, we present three methods via spin echo, Carr-Purcell, and Carr-Purcell-Meiboom-Gill sequences. Our experiment yields $T_1 = 31.8 \pm 2.5$ ms and $T_1 = 13.9 \pm 2.5$ ms via inversion recovery and saturation recovery sequences respectively. Whereas the results of the spin echo, Carr-Purcell, and Carr-Purcell-Meiboom-Gill yield $T_2 = 35.1 \pm 2.5$ ms, $T_2 = 31 \pm 2$ ms, and $T_2 = 23 \pm 2$ ms respectively. These measurements contribute to existing data for identifying and characterizing mineral oil in magnetic resonance imaging.

Nuclear magnetic resonance (NMR) is a phenomena where nuclei inside a magnetic field become magnetized and precess. Radio frequency (RF) pulses can then be applied to these nuclei to alter this precession and magnetization. After each RF pulse, the magnetization of these nuclei will begin to return to thermal equilibrium. This relaxation to thermal equilibrium along the longitudinal direction (along the magnetic field) is characterized by a time constant T_1 . In the transverse direction (perpendicular to the magnetic field) this relaxation is characterized by a time constant T_2 . There are various methods for measuring T_1 and T_2 , which are important for applications in magnetic resonance imaging (MRI). The methods explored for measuring T_1 are inversion recovery and saturation recovery, while the methods for measuring T_2 are spin echo, Carr-Purcell, and Meiboom-Gill. Previous experiments obtained values of $T_1 = 25.4 \pm 0.7$ ms and $T_2 = 15.7 \pm 0.1$ ms for mineral oil[1].

When radio frequency pulses are applied to nuclei possessing spin inside a magnetic field, a number of these spins absorb energy from these RF pulses that are radiated. This energy absorption results in these spins occupying a higher energy level. These excess of spins in a higher energy level result in a change in the net magnetization M_0 . Eventually these spins will return to thermal equilibrium by a relaxation process. This relaxation process can be in the form of longitudinal relaxation, with time T_1 , and transversal relaxation, with time T_2 . Where the longitudinal direction is defined to be along the magnetic field \vec{B} in the \vec{z} direction, while the transverse plane is the plane spanned by \vec{x} and \vec{y} .

The RF pulses were generated with the apparatus shown in FIG 1. The frequency and length of these pulses are controlled and sent to the transmitter coil to produce the desired π and $\pi/2$ pulses. The receiver coil is then used to measure the induced electromotive force (emf) as a result of the precession of the magnetic moments in the samples measured. This induced emf is measured as a voltage, which is proportional to the magnetization of the sample.

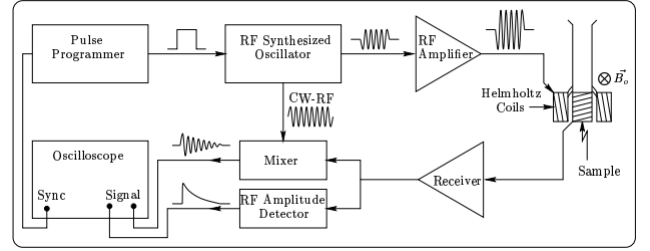


Fig. 1: Diagram of experimental apparatus [4]. The sample of mineral oil is placed inside 3 magnetic coils. The first of which is a Helmholtz coil oriented in the \vec{z} direction to generate the magnetic field. Next is the receiver coil which is oriented in the \vec{y} direction to measure the magnetization in the xy plane. Thirdly is the transmitter coil also oriented in the \vec{y} direction to emit the RF pulses. The transmitter coil is aligned perpendicular to the Helmholtz coil to apply a RF pulse perpendicular to the magnetic field caused by the Helmholtz coil. Whereas the receiver coil is perpendicular to both the transmitter coil and the Helmholtz coil. This is oriented as such to measure the induced emf caused by the spin precession of the sample. The RF pulses are generated through a series of instruments. Starting at the pulse programmer, a square wave is generated which is passed to the RF synthesized oscillator to control the frequency of the pulse. The amplitude of this signal is then increased through the RF amplifier, where the pulse is then passed into the transmitter coils. The receiver picks up the signal from the receiver coils which passes the signal along with the input RF pulse from the RF synthesized oscillator into the mixer which is then passed to an oscilloscope. The signal from the receiver is also passed through an RF amplitude detector which outputs the magnitude of the signal to the oscilloscope.

By varying the RF pulses, different relaxation processes can be observed. For measuring T_1 relaxation, both saturation recovery and inversion recovery can be used. The inversion recovery process contains the following RF pulse sequence: $\pi - \tau - \pi/2$ [2]. Where τ represents the time delay between the RF pulses. Thus for inversion recovery, we start with a net magnetization along the \vec{z} direction, $M_z = M_0$. The first π pulse flips

the magnetization to $-M_z$ where it starts to relax back to M_0 . After time τ , the $\pi/2$ pulse is applied and rotates the magnetization to the (x, y) plane, where the magnetization starts to relax back to M_0 . This relaxation process follows the following equation as a result of the Bloch equations [3]:

$$M_z = M_0(1 - 2e^{-\tau/T_1}) \quad (1)$$

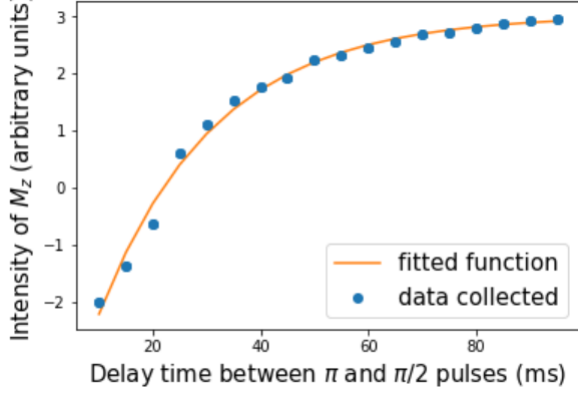


FIG. 2: Plot of the intensity of the signal of M_z after the $\pi/2$ pulse for various τ , the delay times between the two RF pulses (π and $\pi/2$), for the inversion recovery sequence. The intensity of the signal of M_z corresponds to the magnitude of the signal measured from the receiver coil, which is proportional to the magnetization of the mineral oil sample. Fitted to the function: $M_z = M_0(1 - A \exp(-\tau/T_1))$. M_0 represents the initial magnetization before the two RF pulses, T_1 represents the longitudinal relaxation time of mineral oil, and A is a constant. A is expected to be equal to 2 for the inversion recovery sequence, as can be seen in eqn (1). The resulting fit obtained a $T_1 = 22 \pm 1$ ms, $M_0 = 3.02 \pm 0.04$, and $A = 2.75 \pm 0.04$. The uncertainty in T_1 was obtained from $1/t_{dead}$, where t_{dead} is the receiver dead time, which is the time delay between the start of signal acquisition and the end of the applied RF pulse. The uncertainty in M_0 was determined from the standard deviation of the noise in M_z . The fit had a reduced chi-squared value of $\chi^2_{13} \sim 0.01$. A $\chi^2 < 1$ indicates an over fit to the data. This is evident as we are over fitting to our function as the constant A deviates from the theoretical value of 2.

Another method for measuring T_1 relaxation time is through the saturation recovery process. This consists of the following RF pulse sequence: $\pi/2 - \tau - \pi/2$ [1]. Which follows the following equation:

$$M_z = M_0(1 - e^{-\tau/T_1}) \quad (2)$$

Both the saturation recovery and inversion recovery measure the time for the magnetization M_z to return to M_0 .

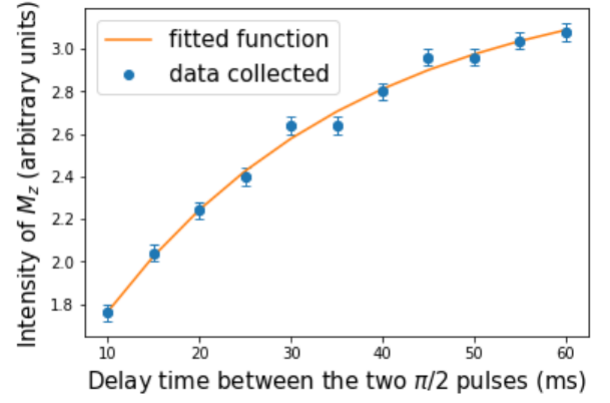


FIG. 3: Plot of the intensity of the signal obtained for M_z after the first $\pi/2$ pulse for various τ . Where τ are the different delay times between two RF pulses of $\pi/2$, for the saturation recovery sequence performed on mineral oil. The intensity of the signal of M_z corresponds to the magnitude of the signal measured from the receiver coil, which is proportional to the magnetization of the mineral oil sample. Fitted to: $M_z = M_0(1 - Ae^{(-\tau/T_1)})$. M_0 represents the initial magnetization before the second RF pulse, T_1 represents the longitudinal relaxation time of mineral oil and A is a constant which should be equal to 1 for saturation recovery. The resulting fit obtained a $T_1 = 14 \pm 1$ ms, $M_0 = 3.02 \pm 0.04$, and $A = 0.68 \pm 0.04$. The uncertainty in T_1 was obtained from $1/t_{dead}$, where t_{dead} is the receiver dead time, which is the time delay between the start of signal acquisition and the end of the applied RF pulse. The uncertainty in M_0 was determined from the standard deviation of the noise in M_z . The fit had a reduced chi-squared value of $\chi^2_7 \sim 0.01$. A $\chi^2 < 1$ indicates an over fit to the data. This is evident as we are over fitting to our function as the constant A deviates from the theoretical value of 1.

As for measuring T_2 relaxation, spin echo, Carr-Purcell, and Meiboom-Gill processes can be used. The spin echo RF pulse sequence consists of: $\pi/2 - \tau - \pi$. The initial $\pi/2$ pulse dephases the spins and some magnetization is lost, both due to the inhomogeneities of the magnetic field, while the π pulse rephases these spins afterwards. After the π pulse the spins begin to dephase again, and refocus at time 2τ where an echo of the magnetization will be observed. The decay of this magnetization follows:

$$M_{xy} = M_0 e^{-2\tau/T_2} \quad (3)$$

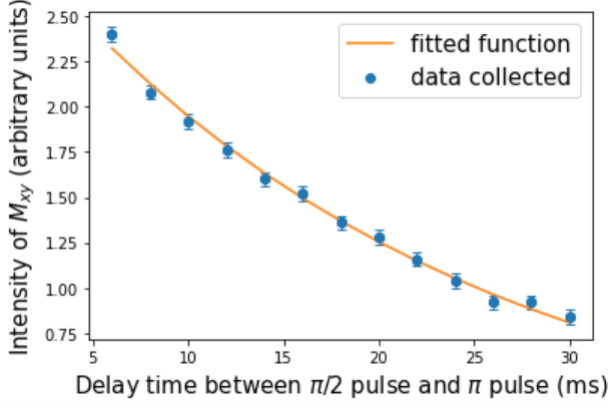


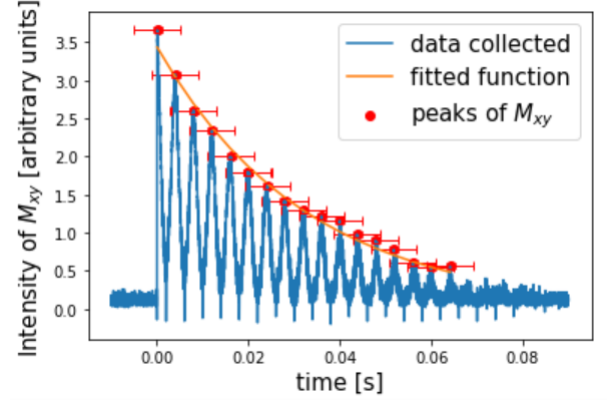
FIG. 4: Plot of the intensity of the signal for M_{xy} after the $\pi/2$ pulse for various τ , the delay times between the two RF pulses (π and $\pi/2$), for the spin echo sequence. The intensity of the signal of M_{xy} corresponds to the magnitude of the signal measured from the receiver coil, which is proportional to the magnetization of the mineral oil sample. Fitted to eqn (3): $M_{xy} = M_0(e^{-2\tau/T_2})$. M_0 represents the initial magnetization before the $\pi/2$ RF pulse, whereas T_2 represents the longitudinal relaxation time of mineral oil. The resulting fit obtained a $T_2 = 35 \pm 1$ ms, and a $M_0 = 3.78 \pm 0.04$. The uncertainty in T_2 was obtained from $1/t_{dead}$, where t_{dead} is the receiver dead time, which is the time delay between the start of signal acquisition and the end of the applied RF pulse. The uncertainty in M_0 was determined from the standard deviation of the noise in M_{xy} . The fit had a reduced chi-squared value of $\chi^2_{11} \sim 0.12$. A $\chi^2 < 1$ indicates an over fit to the data.

Next the Carr-Purcell process consists of the following pulse sequence: $\pi/2 - \tau - \pi - \tau - \pi - \dots$, where the π pulses are repeated indefinitely. This is similar to the spin echo sequence except that the π pulses are repeated. Each time the π pulse is applied the spins are rephased but the net magnetization decreases due to loss of magnetization by the inhomogenous magnetic field. This decay in the net magnetization follows:

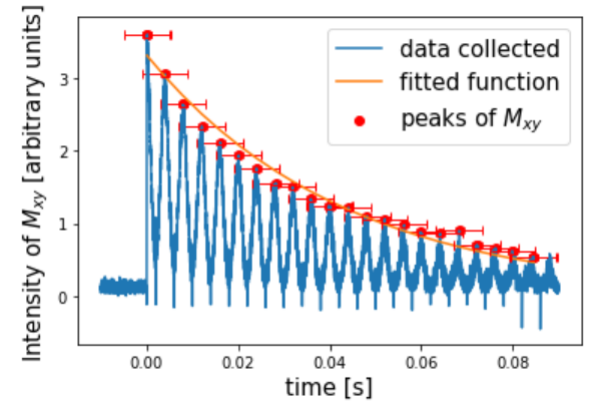
$$M_z = M_0 e^{-\tau/T_2} \quad (4)$$

Lastly, the Carr-Purcell-Meiboom-Gill (CPMG) follows a similar pulse sequence as the Carr-Purcell sequence, with the difference being the time delay between π pulses. The CPMG follows the following pulse sequence: $\pi/2 - \tau_0 - \pi - \tau_1 - \pi - \dots$, where $\tau_n = (2n+1)\tau_{n-1}$, ($n = 0, 1, 2, \dots$), with a predetermined τ_0 . This results in a coherence of successive π pulses, which reduces the dependence on the accuracy of the π pulses[6].

FIG. 5: Plot (a) shows the intensity of M_{xy} as π pulses are applied in intervals of 4 ms after an initial $\pi/2$ pulse at $t = 0$ s, representing the Carr-Purcell sequence for mineral oil. The intensity of the signal of M_{xy} corresponds to the magnitude of the signal measured from the receiver coil, which is proportional to the magnetization of the mineral oil sample. The peaks of the intensities were then fitted to equation (4): $M_{xy} = M_0 e^{-t/T_2}$, where M_{xy} represents the transverse magnetization along the xy plane, and M_0 is the initial magnetization after the initial $\pi/2$ pulse. Which resulted in $M_0 = 3.44 \pm 0.04$, and $T_2 = 31 \pm 1$ ms. The fit had a reduced chi-squared value of $\chi^2_{16} \sim 0.0966$. A $\chi^2 < 1$ indicates an over fit to the data. Plot (b) shows a similar pulse sequence, except for π pulses of increasing delay times., representing the Carr-Purcell-Meiboom-Gill sequence for mineral oil. Which resulted in $M_0 = 3.32 \pm 0.04$, and $T_2 = 23 \pm 1$ ms. The fit had a reduced chi-squared value of $\chi^2_{16} \sim 0.0966$. A $\chi^2 < 1$ indicates an over fit to the data.



(a)



(b)

The noise of the signal was obtained from the oscilloscope by observing the signal before the first π pulse in the Carr-Purcell sequence. The signal was expected to be 0, as there should be no magnetization along the (x,y) plane before the first π pulse as the magnetization is completely aligned in the \hat{z} direction. However the signal observed oscillated with a standard deviation of 0.04 V. This represents an uncertainty in the signal intensity of

the magnetization. However on the order of about 1 to 3 V, this error is negligible and the resulting error bars are too small to be seen on some of the plots. Next there is a noticeable receiver dead time, which is the time delay between the start of signal acquisition and the end of the applied RF pulse[7]. During the Carr-Purcell sequence, the oscilloscope was set to record the signal after the first π pulse. However, there is a 0.01 s delay from the start of acquisition of the signal and after this pulse had been applied. This represents a systematic error in the timing of RF pulses and the signal measured.

The results of the inversion recovery and saturation recovery sequences yield $T_1 = 32 \pm 1$ ms and $T_1 = 14 \pm 1$ ms respectively. Whereas the results of the spin echo, Carr-Purcell, and Carr-Purcell-Meiboom-Gill yield $T_2 = 35.1 \pm 2.5$ ms, $T_2 = 31 \pm 2$ ms, and $T_2 = 23 \pm 2$ ms respectively. There are noticeable differences in the fitted T_1 relaxation times, which could be the result of our sources of error. For our fitted models, we assumed that our π pulses in the inversion recovery sequence lead to perfect inversion. This is difficult in practice, which results in a significant systematic error [5]. There is a major discrepancy in the measurement of T_1 relaxation among the saturation and inversion recovery sequences, as the saturation recovery sequence did not have this reliance on a π pulse inversion whereas the inversion recovery sequence did. On the other hand, there is a noticeably lower discrepancy between our measured T_2 relaxation times between our spin echo and Carr-Purcell sequences as they both rely on π pulse inversion. This results in both methods inhibiting the same systematic error. The Carr-Purcell-Meiboom-Gill sequence differs in T_2 relaxation from the aforementioned sequences as it corrects for the π inversion pulse inaccuracies. Next inhomogeneities in the samples of mineral oil used may result in a source of random error. The mineral oil nuclei are shielded by its surrounding electrons, which creates a small magnetic field in the samples. This local magnetic field varies with the inhomogeneities in the sample, and directly interacts with the applied and measured magnetic fields from the Helmholtz and receiver coils.

In conclusion, fitted values for T_1 obtained for mineral oil were 31.8 ± 2.5 ms and 13.9 ± 2.5 ms via inver-

sion recovery and saturation recovery sequences respectively. While fitted values for T_2 were 35.1 ± 2.5 ms, 31 ± 2 ms, and 23 ± 2 ms via spin echo, Carr-Purcell, and Carr-Purcell-Meiboom-Gill respectively. These values represent the time required for the net magnetization in the nuclei of mineral oil to relax back to thermal equilibrium in the longitudinal direction (for T_1) and transversal direction (for T_2) after excited by RF pulses. These values differ from literature values mainly due to systematic errors in the RF pulses and signal acquisition. Future research could be done to improve the accuracy of these methods for determining relaxation times, as well as perform other experiments for determining the relaxation times for other substances.

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