



Please be sure to mark all answers for **all** problems on this final on your scantron sheet!! Every single question! There is not one single long answer question!

1. [5 Points] An observer sees a system moving past her at a relativistic speed that consists of a mass attached to the end of a spring, oscillating. She measures a period  $T_1$ . A second observer, who is moving with the spring-mass system, measures a period  $T_2$ . Which of the following is true:

- A.  $T_1 > T_2$   
 B.  $T_1 < T_2$   
 C.  $T_1 = T_2$   
 D. Not enough information to tell.
- $T_2$  - proper time.  
 $\therefore T_1 > T_2$*

2. [5 Points] In frame S, event B occurs  $5.5 \mu\text{s}$  after event A ( $5.5 \times 10^{-6} \text{ s}$ ). Event A occurs at the origin, and event B occurs along the positive  $x$  axis at position L. An observer traveling at a speed  $v = 0.9c$  observes the two events to be simultaneous. Determine the value of L in frame S.

- A.  $1.3 \times 10^{-8} \text{ km}$   
 B.  $0.80 \text{ km}$   
 C.  $1.8 \text{ km}$   
 D.  $4.2 \text{ km}$   
 E.  $5.5 \times 10^8 \text{ km}$
- $t'_1 = 0(t_1 + \frac{x_1 v}{c^2})$   
 $t'_2 = \gamma(t_2 + \frac{x_2 v}{c^2})$   
 $t'_1 = t'_2$   
 $t_1 + \frac{x_1 v}{c^2} = t_2 + \frac{x_2 v}{c^2}$   
 $0 + \frac{0 \cdot v}{c^2} = t_2 + \frac{L v}{c^2}$   
 $0 = t_2 + \frac{L v}{c^2}$   
 $t_2 = -\frac{L v}{c^2}$   
 $L = \frac{c^2}{v} \Delta t = \frac{c}{0.9} \Delta t = 1.8 \text{ km}$*

3. [5 Points] At what speed is a particle moving when its kinetic energy is 2.5 times its rest energy?

- A.  $0.85 c$   
 B.  $0.91 c$   
 C.  $0.92 c$   
 D.  $0.96 c$   
 E. Not enough information to determine.
- $KE = \frac{m_0 c^2}{\sqrt{1 - u^2/c^2}} - m_0 c^2$   
 $RE = m_0 c^2$   
 $\frac{KE}{RE} = 2.5 = \frac{1}{\sqrt{1 - u^2/c^2}} - 1$   
 $3.5 = \frac{1}{\sqrt{1 - u^2/c^2}}$   
 $1 - u^2/c^2 = \frac{1}{3.5^2}$   
 $u^2/c^2 = 1 - \frac{1}{3.5^2} \Rightarrow \frac{u}{c} = 0.96$*

4. [4 Points] Homer remains on earth while Ulysses travels 6 light years to planet P and turns around and comes back. Assume Ulysses spends almost all his time traveling at  $0.6c$ . How much does Homer predict that his clock will advance while Ulysses fly's to planet P?

- A. 1 yr.  
 B. 5 yr.  
 C. 10 yr.  
 D. 15 yr.  
 E. 20 yr.
- $L = vt \Rightarrow t = 10 \text{ yr}$   
 There is no ref frame issue.*

5. [4 Points] Ulysses, in the spaceship reference frame, calculates how much Homer's clock will advance while he makes the trip from earth to planet P. How long does he think Homer will think it takes?

- A. 6.4 yr.  
 B. 8.0 yr.  
 C. 8.5 yr.  
 D. 10 yr.  
 E. 13 yr.
- U is still - so expects Homer's clock to go slower  
 $\Delta t_H = \gamma \Delta t_U$   
 $\Delta t_U$  is how long it takes U.  
 Homer knows that  
 $\Delta t_U = \frac{\Delta t_{Homer}}{\gamma} = (10 \text{ yr})(\sqrt{1 - 0.6^2}) = 8 \text{ yr}$   
 $\Delta t_H = (8 \text{ yr})(0.8) = 6.4 \text{ yr}$*

last

first



6. [4 Points] Which of the following features correctly explains how the Photoelectric Effect experiment/demo done in class makes the case that light is quantized (as photons)?

A. Coherent light shown through two slits shows an interference pattern.  
 B. White light shown through a diffraction grating shows as a rainbow on a screen.  
 C. Charge is lost from the experiment's plate faster when the light is brighter  
 D. Light longer than a certain wavelength will not discharge the plate  
 E. Light shorter than a certain wavelength will not discharge the plate.

off to make  
 $\lambda$  short enough!

7. [5 Points] For a particle in a box, the energy levels are denoted  $E_n$ . Approximately, what is  $(E_{n+1} - E_n)/E_n$  dependence on  $n$  for very large  $n$ ?

A.  $n^2$   
 B.  $n$   
 C. 1  
 D.  $n^{-1}$   
 E.  $n^{-2}$

$$E_n = n^2 E_1 \quad \frac{E_{n+1} - E_n}{E_n} = \frac{(n+1)^2 - n^2}{n^2} = \frac{n^2 + 2n + 1 - n^2}{n^2} = \frac{2}{n} + \frac{1}{n^2}$$

as  $n \rightarrow \infty \sim \frac{2}{n} \therefore$  goes as  $1/n$

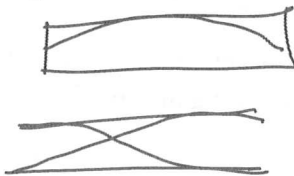
8. [4 Points] Standing waves result from the superposition of two traveling waves that have:

A. The same amplitude, frequency, and direction of propagation  
 B. The same amplitude, frequency, and opposite directions of propagation  
 C. The same amplitude, slightly different frequencies, and the same propagation direction  
 D. The same amplitude, slightly different frequencies, and the opposite directions of propagations  
 E. None of the above



9. [5 Points] The fundamental frequency of a pipe that has both ends closed is 230 Hz. The speed of sound for this problem is  $v_s = 340$  m/s. When both ends of the same pipe are opened, the fundamental frequency is:

A. 58 Hz  
 B. 230 Hz  
 C. 460 Hz  
 D. 920 Hz  
 E. 1400 Hz



$$\frac{1}{2}\lambda = L$$

$$\frac{1}{2}\lambda = L$$

$\Rightarrow$  freq is same

10. [5 Points] Tritium is an unstable isotope of hydrogen with two neutrons and one proton. It has a half-life of 12.3 years. Suppose  $1.0 \times 10^{10}$  atoms of tritium were stored in your freshman year at UW. What is the decay rate per minute four years later?

A. 856 per minute  
 B. 1070 per minute  
 C. 1120 per minute  
 D. 15200 per minute  
 E. 813000000 per minute

$$R = R_0 e^{-\lambda t}$$

$$R_0 = \lambda N_0 \quad N_0 = 10^{10}$$

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{12.3 \text{ yr}} = 0.05633$$

$$R_0 = \lambda N_0 = 563539293$$

$$\therefore R = R_0 e^{-\lambda \cdot 4 \text{ yr}} = 449805192 / \text{yr}$$

convert to /min  $\Rightarrow 855.8 / \text{min}$





16. [5 Points] Three plane polarizers are arranged in sequence. The first and third are rotated 90 degrees apart from each other, the first polarizer being lined up with the vertical. The second polarizer is rotated 30 degrees from the vertical. What is the intensity of the light as it leaves the third polarizer relative to unpolarized light that is shone on the first polarizer ( $I_0$ )?

☒ A.  $0.09 I_0$       $I_1 = \frac{1}{2} I_0$   
☐ B.  $0.11 I_0$   
☐ C.  $0.19 I_0$       $I_2 = I_1 \cos^2(30^\circ)$   
☐ D.  $0.25 I_0$       $I_3 = I_2 \cos^2(90^\circ - 30^\circ)$   
☐ E.  $0.50 I_0$       $\Rightarrow I_3 = \frac{1}{2} I_0 \cos^2(30^\circ) \cos^2(60^\circ) = 0.09 I_0$

$I_0 \quad I_1 \quad I_2 \quad I_3$

17. [5 Points] The expression for the electric field of a certain EM wave is  $E(y, t) = E_0 \sin(ky + \omega t) \hat{z}$ . What is the proper expression for the magnetic field?

- ☒ A.  $B(y, t) = +B_0 \sin(ky + \omega t) \hat{x}$   
☐ B.  $B(y, t) = B_0 \sin(ky - \omega t) \hat{x}$   
☐ C.  $B(y, t) = -B_0 \sin(ky + \omega t) \hat{y}$   
☐ D.  $B(y, t) = B_0 \sin(ky - \omega t) \hat{y}$   
☐ E.  $B(y, t) = B_0 \sin(ky - \omega t) \hat{z}$

direc of prop =  $\vec{E} \times \vec{B}$   
 prop is  $-\hat{y}$  (+wt)  
 $\vec{E}$  is  $+\hat{z}$   $\therefore \vec{B} = -\hat{y}$



Consider the reflecting optical telescope shown at right for the next two questions. The concave mirror at the bottom has  $f = +1.5$  m with a diameter of 0.6 m as its objective. The eyepiece lens has a focal length of  $f = +20$  cm, and it is at a distance  $d$  from the objective mirror. It is mainly used to observe blue light with  $\lambda = 475$  nm.

18. [5 Points] Which is  $d$ , the approximate location of the eyepiece from the objective mirror?

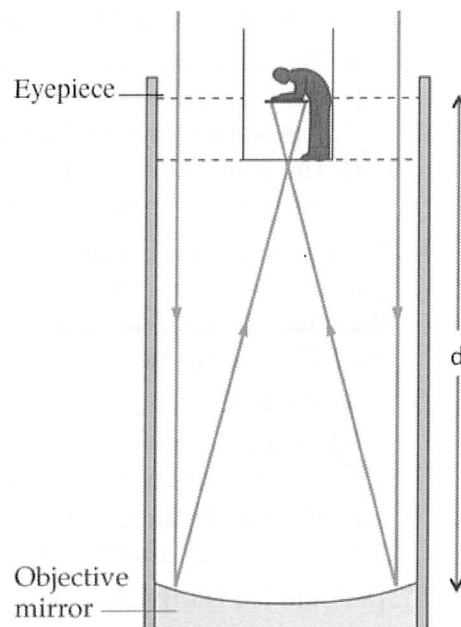
- ☐ A. 0.2 m  
☐ B. 1.3 m  
☐ C. 1.5 m  
☒ D. 1.7 m  
☐ E. 3.0 m

$f_1 + f_2 = 1.7 \text{ m}$

19. [5 Points] What is the smallest angle between two distant stars that can be resolved by the telescope (diffraction limit)?

- ☐ A.  $3.86 \times 10^{-7}$  rad  
☐ B.  $7.92 \times 10^{-7}$  rad  
☐ C.  $8.12 \times 10^{-7}$  rad  
☒ D.  $9.66 \times 10^{-7}$  rad  
☐ E.  $2.90 \times 10^{-6}$  rad

$R = \text{first min - for circular aperture}$   
 $R = 1.22 \frac{\lambda}{D} = 1.22 \frac{475 \times 10^{-9} \text{ m}}{0.6 \text{ m}} = 9.66 \times 10^{-7} \text{ m}$

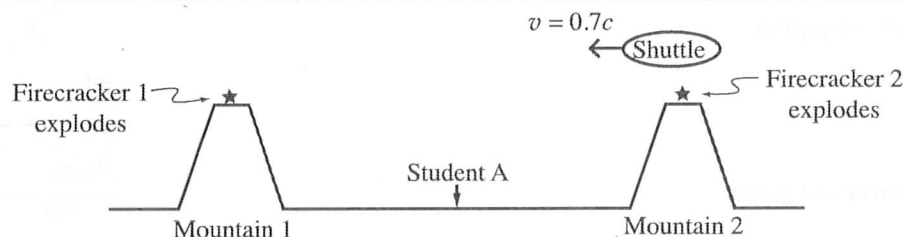






III. [30 points total]

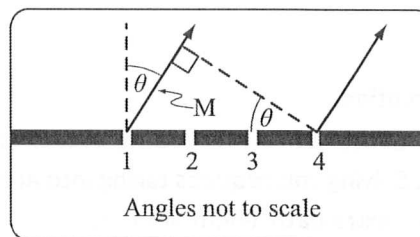
Students A and B stand at rest on the ground. Student A is exactly halfway between mountain 1 and mountain 2; student B's location on the ground is not known. A shuttle flies from right to left with speed  $v = 0.7c$  relative to the ground. Assume all motion is along one dimension (*i.e.*, ignore vertical distances), and that all observers are intelligent observers.



Firecrackers 1 and 2 explode at the tops of mountains 1 and 2. Each emits a flash of light. The firecrackers explode *simultaneously* in student A's reference frame.

The figure shows the locations of student A and the shuttle in the student A's reference frame at the instant both firecrackers explode. Call event 1 "firecracker 1 explodes" and event 2 "firecracker 2 explodes."

25. [5 pts] In student B's reference frame:
- A. Event 1 occurs *before* event 2.
  - B. Event 1 occurs *after* event 2.
  - C. Event 1 occurs *at the same time* that event 2 occurs.
  - D. It is impossible to determine the order of events without knowing student B's location.
26. [5 pts] Suppose student C stands in the middle of the shuttle (moving from right to left with a speed of  $v = 0.7c$  relative to the ground). In student C's frame:
- A. Event 1 occurs *before* event 2.
  - B. Event 1 occurs *after* event 2.
  - C. Event 1 occurs *at the same time* that event 2 occurs.
  - D. More information is needed to answer.
27. [5 pts] The location of event 1 in student C's frame is correctly determined to be  $x_1$ . In student C's frame, the wavefront of the light from event 1 is:
- A. spherical and centered at location  $x_1$  (which is stationary in student C's frame)
  - B. spherical and centered at firecracker 1 (which moves to the left in student C's frame)
  - C. non-spherical and centered at location  $x_1$  (which is stationary in student C's frame)
  - D. non-spherical and centered at firecracker 1 (which moves to the left in student C's frame).
28. [5 pts] Let  $\Delta x_A$  be the distance between events in student A's reference frame, and  $\Delta x_C$  be the distance between events in student C's reference frame. How does  $\Delta x_A$  compare to  $\Delta x_C$ ?
- A.  $\Delta x_C$  is not constant, and so cannot be compared to  $\Delta x_A$
  - B.  $\Delta x_A > \Delta x_C$
  - C.  $\Delta x_A < \Delta x_C$
  - D.  $\Delta x_A = \Delta x_C$



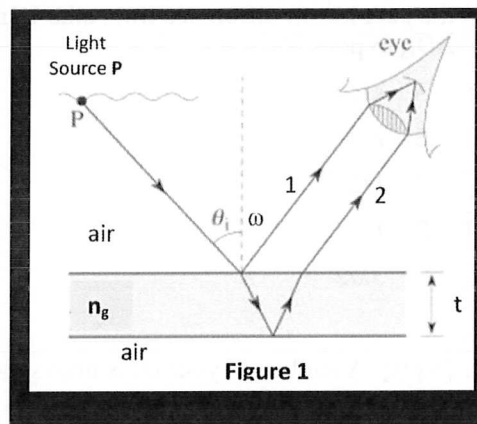


### Problem 22 solution

Light from a source P travels through air ( $n_{\text{air}}=1.0$ ) and reflects from a glass plate ( $n_g=1.5$ ), as shown in figure 1.

22 [10 points] The wave length of the light from source P is 550 nm. The observer sees constructive interference and concludes that the thickness  $t$  is equal to:

- (a) 183 nm
- (b) 92 nm
- (c) 370 nm
- (d) 122 nm
- (e) 137.5 nm



### Solution

1. Constructive interference occurs when the phase difference between rays 1 and 2 is a multiple of  $2\pi$ .
2. Solving this requires taking into account the phase change on reflection plus the phase change due to the extra path length of ray 2.
3. Ray 1 has a phase change of  $\pi$  because the light is reflecting from a higher index material than it is traveling in
4. Ray 2 reflects from the bottom surface and has zero phase change because it is "bouncing off" a lower index material than it is traveling in.
5. The path length in the glass plate is given by  $2\pi/\lambda'$  where  $\lambda' = \lambda/n_g = \lambda/1.5$ .
6. The total phase difference between two rays "bouncing off" the front and the back of the thin glass plate of thickness  $t$  and index of refraction  $n_g=1.5$  is given by equation 3, page 7-7 of the lab manual

$$\phi = \pi + 2t[2\pi/\lambda']$$

7. So  $\phi = 2\pi$  when  $2t[2\pi/\lambda'] = \pi$  and solving for  $t$  gives  $t = \lambda'/4 = \lambda/6 = 92 \text{ nm}$
8. If  $\lambda$  rather than the optical path  $\lambda'$  is used one gets 137.5 nm, which is incorrect.