Issued: Sep. 12, 2017 **IE/MA/OR505 HW 4** **Due: Sep. 21, 2017**

I. Reading Assignment:

1. Textbook Chapter 3

2. Learn more about "Mathematical Proofs" at $< en.wikipedia.org/wiki/Mathematical\_proof >$ and $< Supplemental\ Reading\ -\ Introduction\ to\ Mathematical\ Arguments >$.

II. Exercises:

Textbook Chapter 2

1. (20 points) 2.7

2. (10 points) 2.10

3. (10 points) 2.11

4. (10 points) Let the set E be formed by all extremal directions of the nonempty feasible domain $P = \{\mathbf{x} \in R^n \mid A\mathbf{x} = \mathbf{b},\ \ \mathbf{x} \geq \mathbf{0}\}$ of a standard form linear program.

   (1) Show that, for any vector $\mathbf{d} \in R^n$, $\mathbf{d} \in E$ if and only if "$A\mathbf{d} = \mathbf{0}$ *and* $\mathbf{d} \geq \mathbf{0}$".

   (2) Prove that E is a cone in R^n.

   (3) Prove that E is a convex subset of R^n.

5. (18 points) In R^3, define the set $F_3 = \{\mathbf{x} \in R^3 \mid |x_1| + |x_3| \leq x_2\}$.

   (1) Draw the graph of F_3.

   (2) Let the set B be formed by all boundary points of F_3. Write down the mathematical expression of set B.

   (3) Let the set I be formed by all interior points of F_3. Write down the mathematical expression of set I.

   (4) Find all extremal points of F_3. Also find all vertices of F_3.

   (5) Prove that F_3 is a convex cone.

   (6) What's the relation between F_3 and the nonnegative orthant $R^3_+ = \{\mathbf{x} \in R^3 \mid x_j \geq 0,\ for\ j = 1, 2, 3\}$?

6. (16 pts) Let $P_1 = \{\mathbf{x} \in R^2 \mid 2x_1 - 4x_2 - 1 \leq 0, \;\; 3x_1 - x_2 + 3 \geq 0, \;\; x_1 \geq 0, \;\; x_2 \geq 0\}$.
   (a) Find all basic solutions of P_1.
   (b) Find all basic feasible solutions of P_1.
   (c) Find all extremal directions of P_1.
   (d) From the vertex $(0, 0)^T$, find the moving directions to its adjacent vertices.

7. (16 pts) Let $P_2 = \{\mathbf{x} \in R^2 \mid 2x_1 - 2x_2 - 3 \leq 0, \;\; 8x_1 - x_2 + 4 \geq 0, \;\; x_1 \geq 0\}$.
   (a) Find all basic solutions of P_2.
   (b) Find all basic feasible solutions of P_2.
   (c) Find all extremal directions of P_2.
   (d) From the vertex $(0, 4)^T$, find the moving directions to its adjacent vertices.