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Exercise (1)

We list the key results at each iteration.

Iteration k	$d_y^k = -X_k r^k$	Step length α_k	x^{k+1}	$c^T x^k - c^T x^{k+1}$
0	(-0.0682, 0.0227, 0.0227)	14.52	(0.0025, 0.6650, 0.3325)	0.0825
1	$(-0.8333, 0.0021, 0.0021) \times 10^{-3}$	1.1880×10^3	(0.0000, 0.6666, 0.3333)	8.2500×10^{-4}
2	$(-0.8333, 0.0000, 0.0000) \times 10^{-5}$	1.1880×10^5	(0.0000, 0.6667, 0.3333)	8.2500×10^{-6}

Textbook Problem 7.2

- (a) At the vertex x = (0, 0, 0.5, 0.5), the reduced costs are $r_1 = -1$ and $r_2 = 0$. So the moving direction is $d_1 = (1, 0, -0.5, -0.5)$.
- (b) x = (0.01, 0.01, 0.49, 0.49). Use Karmarkar's algorithm. $d_y^k = (0.0075, -0.0025, -0.0025, -0.0025)$. Since the transformation between x and y is nonlinear, we don't have a moving "direction" in x-space.
- (c) Use the primal affine scaling algorithm. Moving direction is $(0.9998, -0.0002, -0.4998, -0.4998) \times 10^{-4}$.
- (d) Set $\mu = 0.01$. The central force is (0.0098, 0.098, -0.0098, -0.0098). Moving direction is (0.0198, 0.0098, -0.0148, -0.0148).
- (e) Compare the moving directions given by different algorithms. Think about how they are defined. Consider their effects on the performance of the algorithms. Think about the meaning of μ .

Textbook Problem 7.3

(a) Dual problem is

$$\begin{array}{ll} \max & w_2 + 1 \\ \text{s.t.} & w_2 \leq -1 \\ & w_2 \leq 0 \\ & w_1 + w_2 \leq 0 \\ & -w_1 + w_2 \leq 0 \end{array}$$

- (b) Obviously.
- (c) At w = (1, -2), we find s = (1, 2, 1, 3). $d_w^k = (-0.4848, 0.6061)$. $d_s^k = (-0.6061, -0.6061, -0.1212, -1.0909)$.
- (d) This moving direction is pointing to one of the dual optimal solutions.
- (e) Set $\mu = 0.01$. Centering force is (-0.5152, 1.3939). The moving direction is (-47.9697, 59.2121).
- (f) Whether the direction in (e) is better than in (c) depends on the choice of μ . Please try different μ . Some directions fail to point to an optimal solution.

Textbook Problem 7.6

(a) Based on the representation of x = y + q, we may convert the original LP problem to the following LP problem in terms of the new variable y:

$$\begin{aligned} & \text{min} & c^T y + c^T q \\ & \text{s.t.} & Ay = b - Aq \\ & y \geq 0. \end{aligned}$$

Subtracting the constant c^Tq in the objective, it becomes the following standard form LP problem:

$$\begin{aligned} & \text{min} & c^T y \\ & \text{s.t.} & & Ay = b - Aq \\ & & y \ge 0. \end{aligned}$$

(b) For the above standard form LP problem, its dual problem is given by

$$\min \quad (b - Aq)^T w$$
s.t. $A^T w < c$.

When q = 0, the above dual problem becomes

which is actually a regular dual problem for the original LP problem with q = 0.