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ISE 505

Fall 2017

Homework # 2

1.1:

a) Minimize $Z = 4x_1 + \sqrt{2}(x_2^+ - x_2^-) - 0.35x_3$

Subject to $-0.001x_1 + 200(x_2^+ - x_2^-) - x_4 = 7\sqrt{261}$

$7.07(x_2^+ - x_2^-) - 2.62x_3 + x_5 = -4$

$$x_1, x_2^+, x_2^-, x_3, x_4, x_5 \geq 0$$

b)

Minimize $Z = 3.1x_1 - 2\sqrt{2}x_2 + (x_3^+ - x_3^-)$

Subject to $100x_1 - 20x_2 = 7$

$-11x_1 - 7\pi x_2 - 2(x_3^+ - x_3^-) + x_4 = 400$

$x_1 - x_5 = 20$

$(x_3^+ - x_3^-) - x_6 = -15$

$$x_1, x_2, x_3^+, x_3^-, x_4, x_5, x_6 \geq 0$$

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1.1:

c)

$$\text{Minimize } z = -(x_1^+ - x_1^-) - 3x_2 + 2(x_3^+ - x_3^-)$$

$$\text{subject to } 3(x_1^+ - x_1^-) - 5x_2 - x_4 = -2$$

$$3(x_1^+ - x_1^-) - 5x_2 + x_5 = 15$$

$$-5(x_1^+ - x_1^-) + 20x_2 - x_6 = 11$$

$$-5(x_1^+ - x_1^-) + 20x_2 + x_7 = 40$$

$$x_3^+ - x_3^- + x_8 = 10$$

$$x_1^+, x_1^-, x_2, x_3^+, x_3^-, x_4, x_5, x_6, x_7, x_8 \geq 0$$

1.2:

a)

$$\text{Minimize } z = 2(x_1^+ - x_1^-) + 6x_2 + 8x_3$$

$$\text{Subject to } x_1^+ - x_1^- + 2x_2 + x_3 = 5$$

$$4(x_1^+ - x_1^-) + 6x_2 + 2x_3 = 12$$

$$x_1^+, x_1^-, x_2, x_3 \geq 0$$

b)

$$x_1 = 5 - 2x_2 - x_3$$

$$\text{Minimize } 2(5 - 2x_2 - x_3) + 6x_2 + 8x_3$$

$$\text{Subject to } 5 - 2x_2 - x_3 + 2x_2 + x_3 = 5$$

$$4(5 - 2x_2 - x_3) + 6x_2 + 2x_3 = 12$$

$$x_2, x_3 \geq 0$$

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1.2:

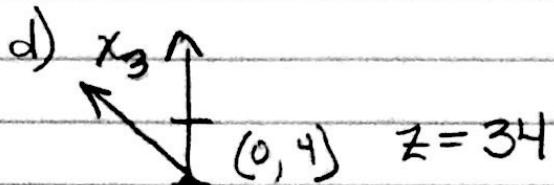
c)

$$\text{Minimize } z = 2x_2 + 6x_3 + 10$$

$$\text{subject to } -2x_2 - 2x_3 = -8$$

$$x_2, x_3 \geq 0$$

d)



$$(4,0) z = 18 *$$

$$x = \begin{pmatrix} -3 \\ 4 \\ 0 \end{pmatrix}$$

$$\begin{aligned} x_1 &= 5 - 2x_2 - x_3 \\ &= 5 - 2(4) - 0 \\ &= -3 \end{aligned}$$

1.3:

a)

$$\text{Minimize } x_1^2 + x_2 + 4x_3$$

$$\text{Subject to } x_1^2 - x_2 = 0$$

$$2x_2 + 4x_3 \geq 4$$

$$x_1 \geq 0, x_2 \geq 2, x_3 \geq 0$$

AS it is currently written, this is
not a linear programming problem
because of the x_1^2 .

1.3:

b)

This problem can be converted to
a linear programming problem by substituting

$$x_1^2 = x_2.$$

$$\text{Minimize } x_2 + x_2 + 4x_3$$

$$\text{subject to } x_2 - x_2 = 0$$

$$2x_2 + 4x_3 \geq 4$$

$$x_2 \geq 2, x_3 \geq 0$$

c)

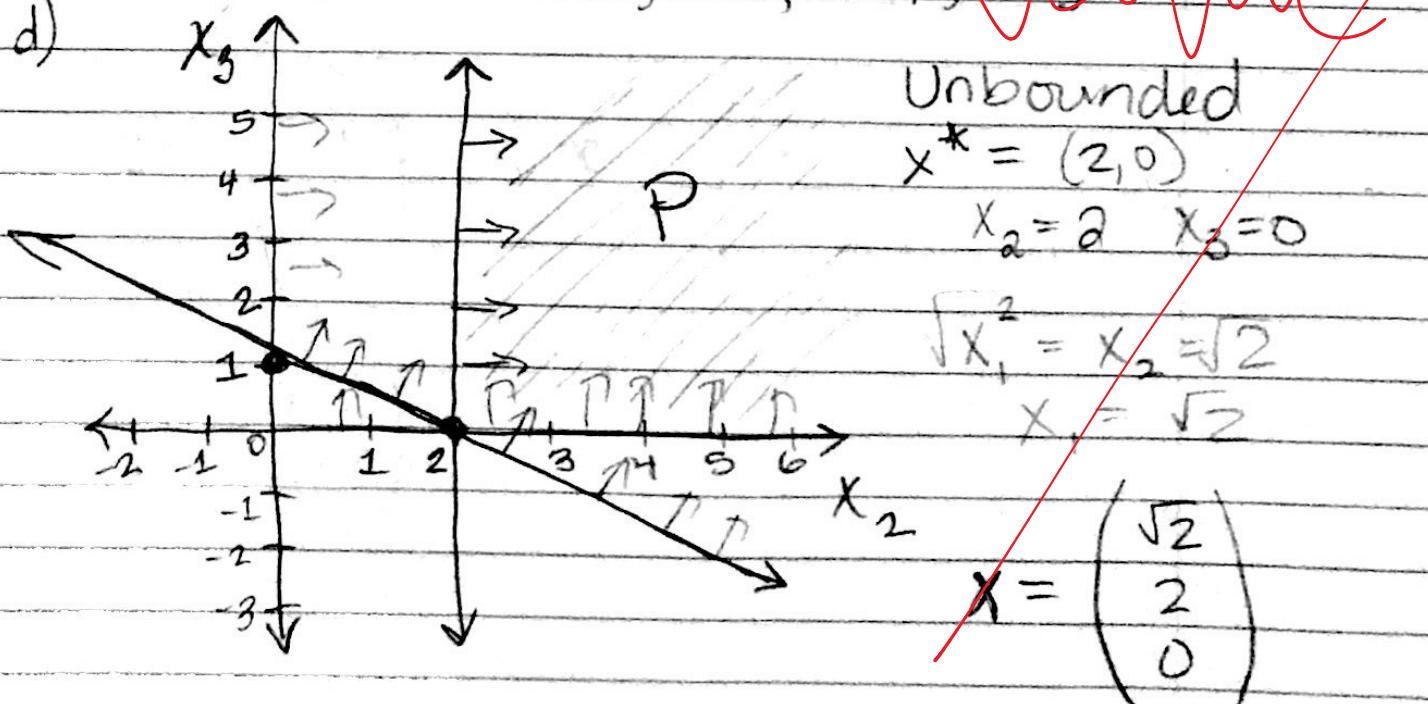
$$\text{Minimize } z = 2x_2 + 4x_3$$

$$\text{subject to } 2x_2 + 4x_3 - x_4 = 4$$

$$x_2 - x_5 = 2$$

$$x_2, x_3, x_4, x_5 \geq 0$$

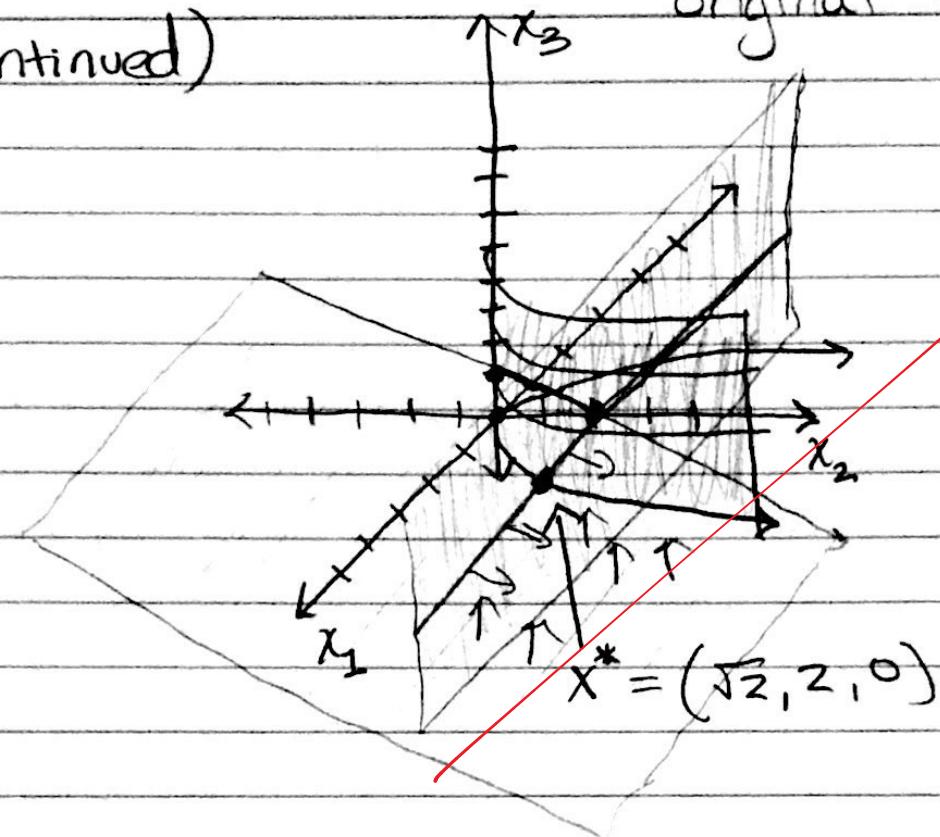
d)



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1.3 : d) (continued)

Original Non-linear Problem



It is a mess, and hard to see from my graph, but $(\sqrt{2}, 2, 0)$ is a vertex of the feasible domain. The linear method is much better.

1.4 : a) Minimize $|x_1| + 2|x_2| - |x_3|$

subject to $x_1 + x_2 - x_3 \leq 10$

$$x_1 - 3x_2 + 2x_3 = 12$$

As it is currently written, this is not a linear programming problem because of the absolute values.

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1.4:

b) Minimize $Z = x_1^+ + x_1^- + 2(x_2^+ + x_2^-) - (x_3^+ + x_3^-)$

Subject to $x_1^+ - x_1^- + x_2^+ - x_2^- - (x_3^+ - x_3^-) + x_4 = 10$

~~$x_1^+ - x_1^- - 3(x_2^+ - x_2^-) + 2(x_3^+ - x_3^-) = 12$~~

$$x_1^+, x_1^-, x_2^+, x_2^-, x_3^+, x_3^-, x_4 \geq 0$$

c)

Minimize $Z = x_3 + x_4$

Subject to $x_1 + x_3 - x_5 = 5$

~~$x_1 - x_3 - x_6 = 5$~~

~~$x_2 + x_4 - x_7 = -4$~~

~~$x_2 - x_4 - x_8 = -4$~~

~~$x_1 + x_2 + x_9 = 10$~~

~~$x_1 - 3x_2 - x_{10} = 2$~~

~~$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} \geq 0$~~

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$$1.5: \quad x_1 = \text{Chip-1} \quad x_2 = \text{Chip-2}$$

$$\text{Minimize} \quad Z = -15x_1 - 25x_2$$

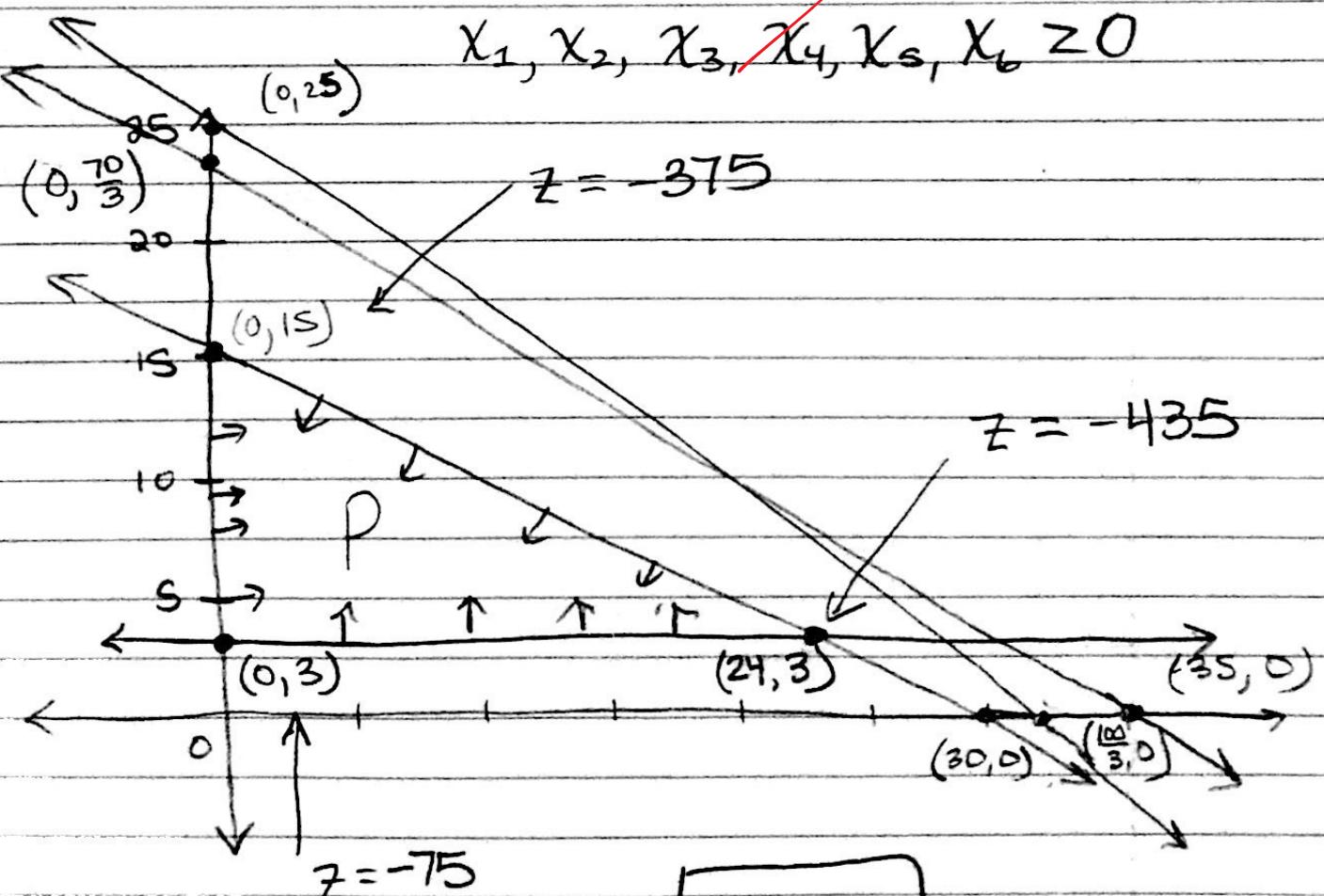
$$\text{Subject to} \quad 3x_1 + 4x_2 + x_3 = 100$$

$$2x_1 + 3x_2 + x_4 = 70$$

$$x_1 + 2x_2 + x_5 = 30$$

$$x_2 - x_6 = 3$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$



$x_1 = 24$
$x_2 = 3$

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1.6 : $x_{ij} = \begin{cases} 1 & \text{if job } j \text{ is performed by worker } i \\ 0 & \text{otherwise} \end{cases}$

Mimimize $Z = 300x_{11} + 300x_{12} + 420x_{13} + 240x_{14} + 480x_{15}$
 $+ 360x_{21} + 300x_{22} + 480x_{23} + 180x_{24} + 420x_{25}$
 $+ 360x_{31} + 480x_{32} + 540x_{33} + 300x_{34} + 600x_{35}$
 $+ 420x_{41} + 360x_{42} + 360x_{43} + 180x_{44} + 360x_{45}$
 $+ 360x_{51} + 420x_{52} + 600x_{53} + 360x_{54} + 600x_{55}$

Subject to $x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 1$
 ~~$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 1$~~
 ~~$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = 1$~~
 ~~$x_{41} + x_{42} + x_{43} + x_{44} + x_{45} = 1$~~
 ~~$x_{51} + x_{52} + x_{53} + x_{54} + x_{55} = 1$~~
 ~~$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} = 1$~~
 ~~$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} = 1$~~
 ~~$x_{13} + x_{23} + x_{33} + x_{43} + x_{53} = 1$~~
 ~~$x_{14} + x_{24} + x_{34} + x_{44} + x_{54} = 1$~~
 ~~$x_{15} + x_{25} + x_{35} + x_{45} + x_{55} = 1$~~

$x_{ij} = 0 \text{ or } x_{ij} = 1 \text{ for all } i \text{ and } j$

Q

1.7:

a)

$$\text{Minimize } Z = 50x_{11} + 90x_{12} + 70x_{13} + 150x_{14} + 180x_{15} \\ + 60x_{21} + 95x_{22} + 50x_{23} + 130x_{24} + 200x_{25}$$

$$\text{subject to } x_{11} + x_{21} = 60$$

$$x_{12} + x_{22} = 45$$

$$x_{13} + x_{23} = 30$$

$$x_{14} + x_{24} = 80$$

$$x_{15} + x_{25} = 55$$

$$x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25} \geq 0$$

b)

Include restriction $x_{11} + x_{12} + x_{13} \leq 60$,
but write it in standard form a)

$$x_{11} + x_{12} + x_{13} + S_1 = 60, S_1 \geq 0$$

c)

In addition to the new restrictions
in part b), I would also include
the restriction $x_{21} + x_{22} + x_{23} \leq 50$,
but write it in standard form a)

$$x_{21} + x_{22} + x_{23} + S_2 = 50, S_2 \geq 0$$

We also cannot meet use up all of
the supply, or meet all of the
demand in one run, so all of the
original restrictions will need slack
variables. The new system
will be:

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c) (continued)

1.7:

$$\text{Minimize: } Z = 50x_{11} + 90x_{12} + 70x_{13} + 150x_{14} + 180x_{15} \\ + 60x_{21} + 95x_{22} + 50x_{23} + 130x_{24} + 200x_{25}$$

$$\text{Subject to } x_{11} + x_{21} + s_3 = 60$$

$$x_{12} + x_{22} + s_4 = 45$$

$$x_{13} + x_{23} + s_5 = 30$$

$$x_{14} + x_{24} + s_6 = 80$$

$$x_{15} + x_{25} + s_7 = 55$$

$$x_{21} + x_{22} + x_{23} + s_1 = 60$$

$$x_{21} + x_{22} + x_{23} + s_2 = 50$$

 $x_{ij} \geq 0 \text{ for all } i \text{ and } j, s_k \geq 0 \text{ for all } k$

d)

Supply:

$$\text{Minimize: } Z = 50x_{11} + 90x_{12} + 70x_{13} \\ + 60x_{21} + 95x_{22} + 50x_{23}$$

~~$$\text{Subject to } x_{11} + x_{21} + s_3 = 60$$~~

~~$$x_{12} + x_{22} + s_4 = 45$$~~

~~$$x_{13} + x_{23} + s_5 = 30$$~~

~~$$x_{11} + x_{12} + x_{13} + s_1 = 60$$~~

~~$$x_{21} + x_{22} + x_{23} + s_2 = 50$$~~

X Please check the

 $x_{ij} \geq 0 \text{ for all } i \text{ and } j, s_k \geq 0 \text{ for all } k$

Solution

Demand:

~~$$\text{Minimize: } Z = 150x_{14} + 180x_{15} + 130x_{24} + 200x_{25}$$~~

~~$$\text{Subject to } x_{14} + x_{24} + s_6 = 80$$~~

~~$$x_{15} + x_{25} + s_7 = 55$$~~

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 $x_{ij} \geq 0 \text{ for all } i \text{ and } j, s_k \geq 0 \text{ for all } k$