ISE 505
H.W #4
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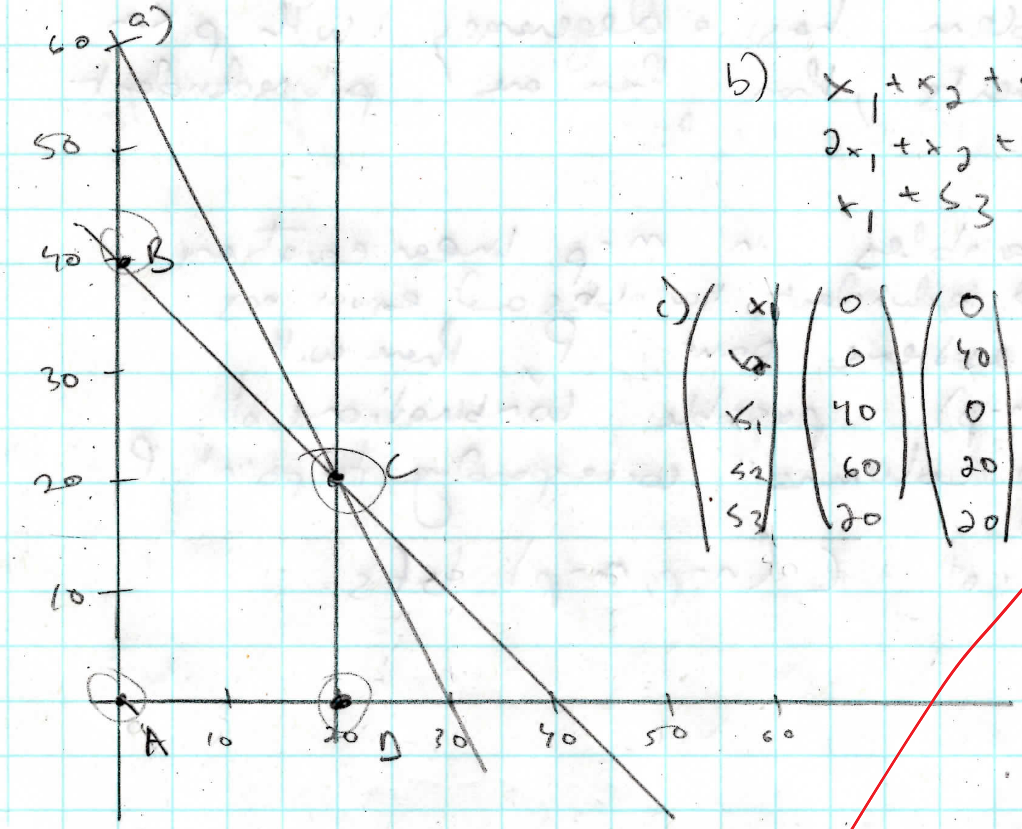
2.7) Let $P = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1 + x_2 \leq 40,\ 2x_1 + x_2 \leq 60,\ x_1 \leq 20,\ x_1, x_2 \geq 0\}$

a)



b) $x_1 + x_2 + s_1 = 40$
$2x_1 + x_2 + s_2 = 60$
$x_1 + s_3 = 20$

c)
$$\begin{array}{c} x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{array} \begin{pmatrix} 0 \\ 0 \\ 40 \\ 60 \\ 20 \end{pmatrix} \begin{pmatrix} 0 \\ 40 \\ 0 \\ 20 \\ 20 \end{pmatrix} \begin{pmatrix} 0 \\ 60 \\ -20 \\ 0 \\ 20 \end{pmatrix} \begin{pmatrix} 20 \\ 20 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 40 \\ 0 \\ 0 \\ -20 \\ -20 \end{pmatrix} \begin{pmatrix} 30 \\ 0 \\ 10 \\ 0 \\ -10 \end{pmatrix} \begin{pmatrix} 20 \\ 0 \\ 20 \\ 20 \\ 0 \end{pmatrix}$$

d) basic feasible solutions

$$\begin{pmatrix} 0 \\ 0 \\ 40 \\ 60 \\ 20 \end{pmatrix} \begin{pmatrix} 0 \\ 40 \\ 0 \\ 20 \\ 20 \end{pmatrix} \begin{pmatrix} 20 \\ 20 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 20 \\ 0 \\ 20 \\ 20 \\ 0 \end{pmatrix}$$

$\downarrow$ point A $\quad \downarrow$ point B $\quad \downarrow$ point C $\quad \downarrow$ point D

F) point C corresponds to a degenerate bfs

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2.10) for any linear programming problem with n variables in m linear equations there is / a maximum of $C(n,m)$ possible vertices / basic solutions

If the problem has a degeneracy with $p < m$ positive elements, then there are p redundant constraints

Leaving $n-p$ variables in $m-p$ linear equations +subtracting the p redundant variables and equations. So, at that extreme point P, there will be $C(n-p, m-p)$ possible combinations of the remaining columns corresponding to point P with a max of $C(n-p, m-p)$ bfs.

2.11 $M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

M_c - convex cone of M → for all $\lambda \geq 0$
and $x \in M$
$\lambda x \in M$

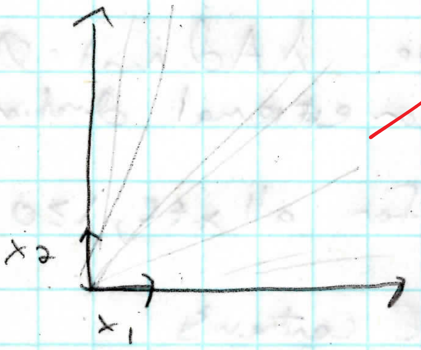
M has two column vectors x_1 $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ x_2

and M_c will contain all λx for $\lambda \geq 0$

Which will span all of the first orthant

$M_c \in \mathbb{R}^2$

The definition is on Page 19 of textbook



for any $\lambda \geq 0$

$\lambda_i x_i \rightarrow \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \in M_c$

for all $\lambda \geq 0$ will span first orthant

b) M_c is the smallest convex cone to contain the columns

Say there is a smaller convex cone than M_c → M_D

Since M_D is a convex cone that contains
all $\lambda_1 x_1, \lambda_2 x_2$ for $x_1, x_2 \in M_D$ $\lambda \geq 0$

It will have to be the same cone to contain

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ Yes But need to prove it

4) Let E be formed by all extremal directions of the nonempty feasible domain $P = \{x \in R^n \mid Ax = b, x \geq 0\}$

By definition, an extremal direction of a set P is a non zero vector $d \in R^n$ such that for each $x^\circ \in P$ the ray $\{x \in R^n \mid x = x^\circ + \lambda d, \lambda \geq 0\}$ is contained in d

$d \geq 0$?

~~if~~ $x = x^\circ + \lambda d$

$Ax = Ax^\circ + A\lambda d$

$= Ax^\circ + \lambda A d$ And also, "if and only if"?

If $x \in P$ and $x^\circ \in P$, for this to be true $\lambda A d$ must $= \emptyset$ hence Ad must $= \emptyset$ for d to be an extremal direction

b) for E to be a cone, it must contain λx for all $x \in E, \lambda \geq 0$

since we know that $Ad = \emptyset$ we know that E contains all λx for $x \in E$ and $\lambda \geq 0$

for any $x \in E$ $Ax = 0$ so $A \lambda x$ will $= \emptyset$ so all $\lambda x \in E$

$d \geq 0$?

c) To be a convex subset, for any $x_1, x_2 \in P$ and $\lambda_1 + \lambda_2 = 1$ $\lambda_1, \lambda_2 \geq 0$ $\lambda_1 x_1 + \lambda_2 x_2 \in P$

since we know for all $x_i \in E$ $Ax_i = \emptyset$

we know that $A\lambda_1 x_1 + A\lambda_2 x_2 = \emptyset$ hence E is an convex combination

- also all cones are convex

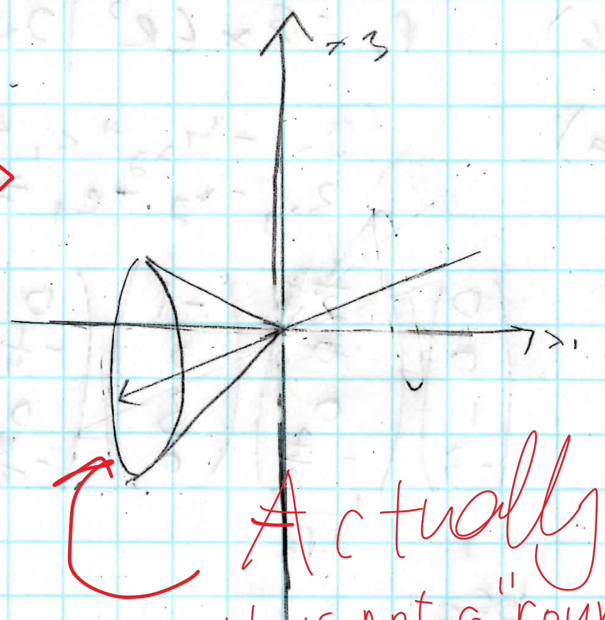
$d \geq 0$?

5) 1) $f_3 \ \{x \in \mathbb{R}^3 \mid |x_1| + |x_3| \le x_2\}$

2) $B = \{x \in \mathbb{R}^3 \mid |x_1| + |x_3| = x_2\}$

3) $I = \{x \in \mathbb{R}^3 \mid |x_1| + |x_3| < x_2\}$

4) $\{0,0,0\}$ extremal point and vertex



Actually, It is not a "round" one

5) for x to be a convex cone, it must contain all $\lambda_i x_i$ for $x_i \in f_3$ $\lambda \ge 0$

if $x_1, x_3 \in f_3$

then $|x_1| + |x_3| \le x_2$

~~for any $\lambda_1 |x_1| + \lambda_2 |x_3|$ $\lambda_1, \lambda_2 \ge 0$ there will be a value $\lambda_3 x_2$ for which~~

~~$\lambda_1 |x_1| + \lambda_2 |x_3| \le \lambda_3 x_2$ hence f_3 is convex cone~~

6) F_3 is a quarter cone in $\mathbb{R}^3_+$ centred around the j^2 direction

6) $P_1 = \{x \in \mathbb{R}^2 \mid 2x_1 - 4x_2 - 1 \le 0,\ 3x_1 - x_2 + 3 \ge 0,\ x_1 \ge 0\ \ x_2 \ge 0\}$

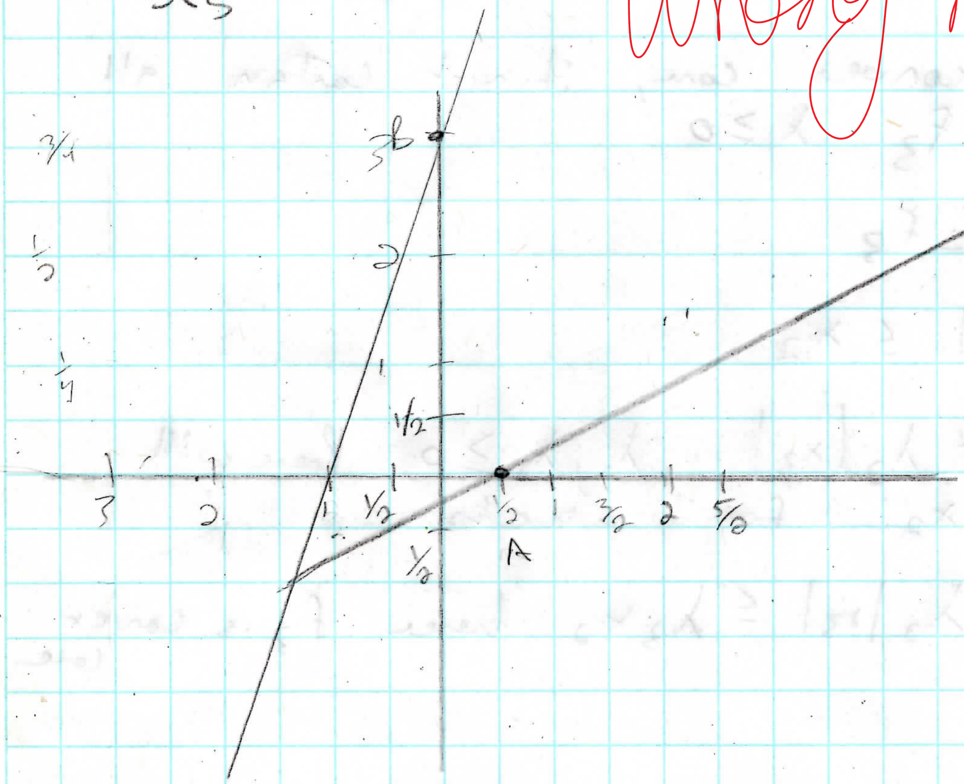
a) $2x_1 - 4x_2 + s_1 = 1$

$3x_1 - x_2 - e_1 = -3$ $\qquad x_1, x_2, s_1, e_1 \ge 0$

b) $\underbrace{\begin{pmatrix} 0 \\ 0 \\ 1 \\ -3 \end{pmatrix}}_{\text{bfs}} \underbrace{\begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \\ \frac{7}{2} \end{pmatrix}}_{\text{bfs}} \begin{pmatrix} -1 \\ 0 \\ 3 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ -\frac{1}{4} \\ 0 \\ -\frac{11}{4} \end{pmatrix} \underbrace{\begin{pmatrix} 0 \\ 3 \\ 13 \\ 0 \end{pmatrix}}_{\text{bfs}} \begin{pmatrix} \\ \\ \\ \end{pmatrix}$

wrong number

-1



c) $\begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix}$?

-1

d) from $(0,0)^T$ to A $\rightarrow \begin{pmatrix} \frac{1}{2} \\ 0 \\ -1 \\ \frac{13}{2} \end{pmatrix}$, to B $\begin{pmatrix} 0 \\ 3 \\ 12 \\ 3 \end{pmatrix}$

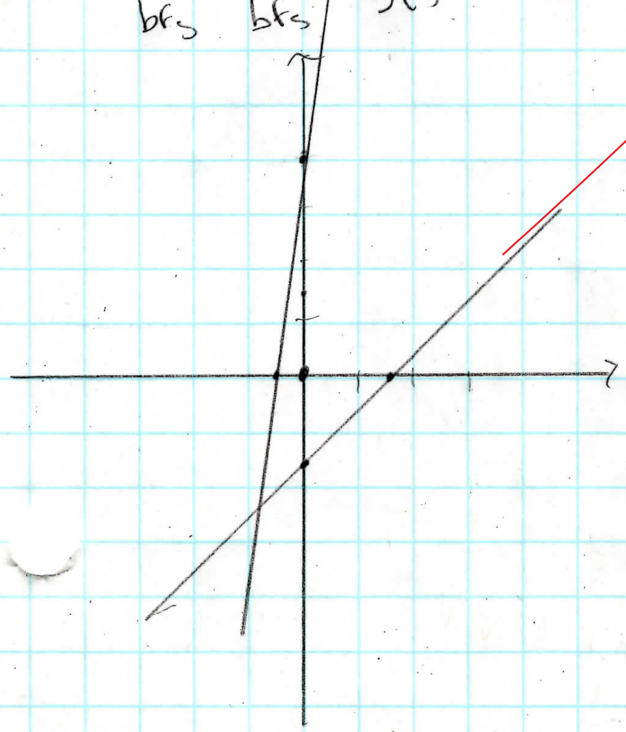
$\frac{1}{2}\begin{pmatrix} 1 \\ 0 \\ -2 \\ \cancel{13} \end{pmatrix} \qquad 3\begin{pmatrix} 0 \\ 1 \\ 4 \\ 1 \end{pmatrix}$

7) $P_2 = \{x \in \mathbb{R}^2 \mid 2x_1 - 2x_2 - 3 \le 0,\ 8x_1 - x_2 + 4 \ge 0,\ x_1 \ge 0\}$

$$2x_1 - 2x_2^+ + 2x_2^- + s_1 = 3$$
$$8x_1 - x_2^+ + x_2^- - e_1 = -4 \qquad x_1, x_2^+, x_2^-, s_1, e_1 \ge 0$$

a) $\begin{pmatrix}0\\0\\0\\3\\4\end{pmatrix} \begin{pmatrix}\frac{3}{2}\\0\\0\\0\\16\end{pmatrix} \begin{pmatrix}0\\0\\\frac{3}{2}\\0\\\frac{11}{2}\end{pmatrix} \begin{pmatrix}0\\-\frac{3}{2}\\0\\0\\\frac{11}{2}\end{pmatrix} \begin{pmatrix}-\frac{1}{2}\\0\\0\\4\\0\end{pmatrix} \begin{pmatrix}0\\4\\0\\11\\0\end{pmatrix} \begin{pmatrix}0\\0\\-4\\11\\0\end{pmatrix}$

bfs bfs bfs bfs



c) $\begin{pmatrix}1\\1\end{pmatrix} \begin{pmatrix}1\\6\end{pmatrix}$? X -2

Check the solution

d) from $(0,4)^T$ only one adjacent vertex $\rightarrow \begin{pmatrix}0\\-4\\0\\-8\\4\end{pmatrix} \rightarrow 4\begin{pmatrix}0\\-1\\0\\-2\\1\end{pmatrix}$