Soham Joshi OR 505 HWA #2. due : 09/05/17.

200179423

1.1.

Given problem:

a. minimize $4x_1 + \sqrt{2}\, x_2 - 0.35 x_3$

s.t. $-0.001 x_1 + 200 x_2 \geq 7\sqrt{261}$

$7.07 x_2 - 2.62 x_3 \leq -4$

$x_1, x_3 \geq 0$

Converting to standard form,

we get

$$\boxed{\begin{array}{ll} \text{minimize} & 4x_1 + \sqrt{2}\, x_2^+ - \sqrt{2}\, x_2^- - 0.35 x_3 \\ \text{s.t.} & -0.01 x_1 + 200 x_2^+ - 200 x_2^- - e_1 = 7\sqrt{261} \\ & 7.07 x_2^+ - 7.07 x_2^- - 2.62 x_3 + s_1 = -4 \\ \text{and} & x_1, x_2^+, x_2^-, x^3, e_1, s_1 \geq 0 \end{array}}$$

where, $e_1 \rightarrow$ excess variable

$s_1 \rightarrow$ slack variable

$x_2^+, x_2^- \rightarrow$ non negative real numbers.

inequalities and unrestricted variables have been removed. Above LP is in standard form.

b. Given problem:

maximize $-3.1x_1 + 2\sqrt{2}x_2 - x_3$

s.t. $100x_1 - 20x_2 = 7$

$-11x_1 - 7\pi x_2 - 2x_3 \leq 400$

$x_1 \geq 20,\ x_2 \geq 0,\ x_3 \geq -15$

Converting to standard form,

$x_1 \geq 20 \quad \therefore \quad x_1 = \bar{x}_1 + 20 \quad \bar{x}_1 = x_1 - 20$

and $\bar{x}_1 \geq 0$

$x_3 \geq -15 \quad \therefore \quad x_3 = \bar{x}_3 - 15$ and $\bar{x}_3 = x_3 + 15$

and $\bar{x}_3 \geq 0$.

substituting these values in constraints, →

$100(\bar{x}_1 + 20) - 20x_2 = 7$

ie $100\bar{x}_1 - 20x_2 = -1993$

and $-11(\bar{x}_1 + 20) - 7\pi x_2 - 2(\bar{x}_3 - 15) + s_1 = 400$

ie $-11\bar{x}_1 - 7\pi x_2 - 2\bar{x}_3 + s_1 = 590$.

substituting into objective function,

$-3.1(\bar{x}_1 + 20) + 2\sqrt{2}x_2 - (\bar{x}_3 - 15)$

ie $-3.1\bar{x}_1 + 2\sqrt{2}x_2 - \bar{x}_3 - 47$.

$\therefore$ The LP can be written in standard form as follows →

minimize $3.1\bar{x}_1 - 2\sqrt{2}x_2 + \bar{x}_3 + 47$ (3)

s.t. $100\bar{x}_1 - 20x_2 = -1993$

$-11\bar{x}_1 - 7\pi x_2 - 2\bar{x}_3 + s_1 = 590$

and $\bar{x}_1, x_2, \bar{x}_3 \geq 0$

c. Given problem:

maximize $x_1 + 3x_2 - 2x_3$

s.t. $-2 \leq 3x_1 - 5x_2 \leq 15$

$11 \leq -5x_1 + 20x_2 \leq 40$

$x_2 \geq 0, \; x_3 \leq 10$

Consider the constraints

1] $-2 \leq 3x_1 - 5x_2 \leq 15$

$3x_1 - 5x_2 \geq -2$

$3x_1 - 5x_2 \leq 15$

2] $11 \leq -5x_1 + 20x_2 \leq 40$

$-5x_1 + 20x_2 \geq 11$

$-5x_1 + 20x_2 \leq 40$

Let $x_1 = x_1^+ - x_1^-$ $\quad x_3 = 10 - \bar{x}_3$

where $x_1^+, x_1^-, \bar{x}_3 \to$ non negetive real numbers.

$\therefore$ Constraints become

$$3x_1^+ - 3x_1^- - 5x_2 - e_1 = -2$$

$$3x_1^+ - 3x_1^- - 5x_2 + s_1 = 15$$

$$-5x_1^+ + 5x_1^- + 20x_2 - e_2 = 11$$

$$-5x_1^+ + 5x_1^- + 20x_2 + s_2 = 40$$

$\therefore$ The LP can be written in its standard form as $\rightarrow$

~~minimize~~ $\cancel{-x_1 - 3x_2 - 2\bar{x}_3 + 20}$

$$\text{minimize} \quad -x_1^+ + x_1^- - 3x_2 - 2\bar{x}_3 + 20$$

$$\begin{aligned}
\text{s.t.} \quad & 3x_1^+ - 3x_1^- - 5x_2 - e_1 &&= -2 \\
& 3x_1^+ - 3x_1^- - 5x_2 + s_1 &&= 15 \\
& -5x_1^+ + 5x_1^- + 20x_2 - e_2 &&= 11 \\
& -5x_1^+ + 5x_1^- + 2x_2 + s_2 &&= 40
\end{aligned}$$

$$\text{and} \quad x_1^+, x_1^-, x_2, \bar{x}_3 \geq 0$$

1.2 Given problem

$$\text{minimize} \quad 2x_1 + 6x_2 + 8x_3$$

$$\text{s.t.} \quad x_1 + 2x_2 + x_3 = 5$$

$$4x_1 + 6x_2 + 2x_3 = 12$$

$$x_2 \geq 0 \quad x_3 \geq 0$$

a. converting to standard form

Let $x_1 = x_1^+ - x_1^-$

$\therefore$ standard form $\rightarrow$

$$\text{minimize} \quad 2x_1^+ - 2x_1^- + 6x_2 + 8x_3$$

$$\text{s.t.} \quad x_1^+ - x_1^- + 2x_2 + x_3 = 5$$

$$4x_1^+ - 4x_1^- + 6x_2 + 2x_3 = 12$$

$$\text{and} \quad x_1^+, x_1^-, x_2, x_3 \geq 0$$

b. Let $x_1 = 5 - 2x_2 - x_3$

$\therefore$ objective func^n becomes $\rightarrow$

$$2(5 - 2x_2 - x_3) + 6x_2 + 8x_3$$

ie $10 - 4x_2 - 2x_3 + 6x_2 + 8x_3$

ie $2x_2 + 6x_3 + 10$

2nd constraint becomes $\rightarrow$

$$4(5 - 2x_2 - x_3) + 6x_2 + 2x_3 = 12$$

ie $20 - 8x_2 - 4x_3 + 6x_2 + 2x_3 = 12$

ie $x_2 + x_3 = 4$

$\therefore$ equivalent LP $\rightarrow$

minimize $2x_2 + 6x_3 + 10$

st $x_2 + x_3 = 4$

and $x_2, x_3 \geq 0$

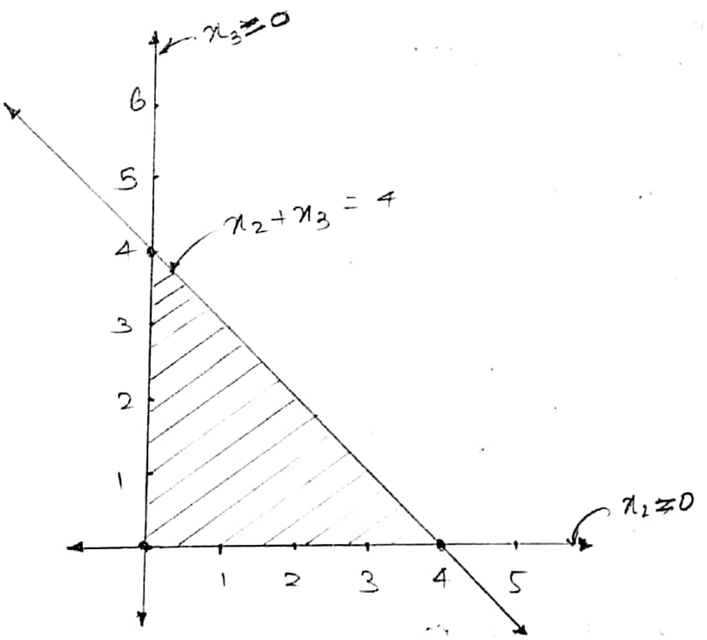
c. The equivalent LP is already in its standard form.

d. The problem may be solved geometrically.

$x_2 + x_3 = 4$

when $x_2 = 0$ $x_3 = 4$

when $x_3 = 0$ $x_2 = 4$.



The extreme points of the feasible region are (0,0), (0,4), and (4,0).

values of objective function

$Z = 2x_2 + 6x_3 + 10$ at the points are →

$(0,0) \rightarrow Z = 10$

$(4,0) \rightarrow Z = 18$

$(0,4) \rightarrow Z = 34.$

∴ O.F. is minimum at $(0,0)$

ie when $\underline{x_2 = 0}$ & $\underline{x_3 = 0}$

1.3. Given problem

$$\text{minimize} \quad x_1^2 + x_2 + 4x_3$$

$$\text{s.t.} \quad x_1^2 - x_2 = 0$$

$$2x_2 + 4x_3 \geq 4$$

$$x_1, x_3 \geq 0$$

$$x_1 \geq 0, x_2 \geq 2, x_3 \geq 0.$$

a. This problem has a quadratic term. Thus, it is not a linear program in this form.

b. Consider the first constraint

$x_1^2 - x_2 = 0 \quad \therefore x_1^2 = x_2.$

substituting this into the objective function gives us →

$$2x_2 + 4x_3$$

$\therefore$ The equivalent LP problem is $\rightarrow$

$$\text{minimize } 2x_2 + 4x_3$$
$$\text{s.t. } 2x_2 + 4x_3 \geq 4$$
$$x_2 \geq 2 \quad x_3 \geq 0$$

c. The above LP can be written in its standard form as follows:

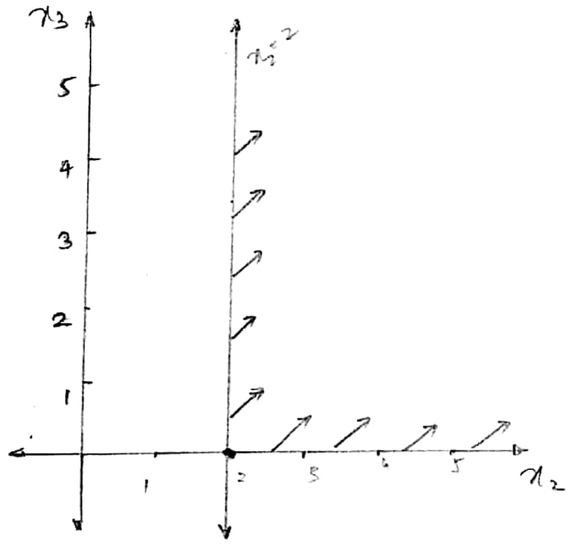
$$x_2 = \bar{x}_2 + 2$$

$\therefore$ standard form $\rightarrow$

$$\text{minimize} \quad 2\bar{x}_2 + 4x_3 + 4$$
$$\text{s.t.} \quad 2x_2 + 4x_3 - e_1 = 0$$
$$\text{and} \quad \bar{x}_2, \bar{x}_3, e_1 \geq 0$$

d. The problem can be solved geometrically in its equivalent form.

when $x_2 = 2$, $x_3 = 0$



There is only one point in feasible region ie $(2,0)$.

$\therefore$ optimal solution $\boxed{Z = 4.}$

The optimal solution for
$Z = 2x_2 + 4x_3$ is at →

(0,0) $z = 0$

(0,1) $z = 4$

(0,2) $z = 4$.

∴ OF is minimum at $x_2 = 0$ & $x_3 = 0$.

1.4.

Given problem :

minimize $|x_1| + 2|x_2| - x_3$

s.t. $x_1 + x_2 - x_3 \leq 10$

$x_1 - 3x_2 \geq 2$

a. Yes the given problem is a linear programming problem.

b. It can be written in the standard form as below:

minimize $(x_1^+ + x_1^-) + 2(x_2^+ + x_2^-) - (x_3^+ + x_3^-)$

s.t. $(x_1^+ - x_1^-) + (x_2^+ - x_2^-) - (x_3^+ - x_3^-) + s_1 = 10$

$(x_1^+ - x_1^-) - 3(x_2^+ - x_2^-) - e_2 = 2.$

$x_1^+, x_1^-, x_2^+, x_2^-, x_3^+, x_3^- \geq 0.$

c. Given probem:

minimize $|x_1 - 5| + |x_2 + 4|$

s.t. $x_1 + x_2 \leq 10$

$x_1 - 3x_2 \geq 2$

Consider the objective function. 11

Let $x_1 - 5 = x_3$ and $x_2 + 4 = x_4$.

$\therefore$ $x_1 = x_3 + 5$ and $x_2 = x_4 - 4$.

The problem can be rewritten in terms of x_3 and x_4 as follows.

minimize $|x_3| + |x_4|$

$$\text{s.t. } (x_3+5) + (x_4-4) \leq 10$$
$$(x_3+5) - 3(x_4-4) \geq 2.$$

constraints can be rewritten again as $\rightarrow$

$$\text{s.t. } x_3 + x_4 \leq 9$$
$$x_3 - 3x_4 \geq 9.$$

$\therefore$ Converting to standard form $\rightarrow$

minimize $x_3^+ + x_3^- + x_4^+ + x_4^-$

$$\text{s.t. } x_3^+ - x_3^- + x_4^+ - x_4^- + s_1 = 9$$

$$x_3^+ - x_3^- - 3x_4^+ + 3x_4^- - e_1 = 9$$

and $x_3^+, x_3^-, x_4^+, x_4^-, s_1, e_1 \geq 0$

1.5 Let x_1 and x_2 be the number of units ~~of~~ sold for Chip1 and Chip2 respectively.

The linear programming problem for an optimum mix can be formulated as below:

$$
\begin{aligned}
\text{maximize} \quad & 15x_1 + 25x_2 \\
\text{s.t.} \quad & 3x_1 + 4x_2 \leq 100 \quad \text{.... skilled labor} \\
& 2x_1 + 3x_2 \leq 70 \quad \text{... unskilled labor} \\
& x_1 + 2x_2 \leq 30 \quad \text{... raw material} \\
& x_2 \geq 3 \quad \text{... sales contract.} \\
& x_1, x_2 \geq 0
\end{aligned}
$$

slack variables & excess variables can be used to convert it into a standard form as follows:

$$
\begin{aligned}
& \cancel{\text{maximize}} \quad \cancel{15x_1 + 25x_2} \\
\cancel{\text{is}} \quad \text{minimize} \quad & -15x_1 - 25x_2 \\
\text{s.t.} \quad & 3x_1 + 4x_2 + s_1 = 100 \\
& 2x_1 + 3x_2 + s_2 = 70 \\
& x_1 + 2x_2 + s_3 = 30 \\
& x_2 - e_1 = 3 \\
& x_1, x_2, s_1, s_2, s_3, e_1 \geq 0.
\end{aligned}
$$

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1.6

	1	2	3	4	5
A	5	5	7	4	8
B	6	5	8	3	7
C	6	8	9	5	10
D	7	6	6	3	6
E	6	7	10	6	11

standard wage = \$60/person/day.

To formulate this question as an integer linear programming problem:

Let x_{ij} = Assignment of person 'i' to project 'j'

$\therefore$ $i \to A, B, C, D, E$ $\quad j \to 1, 2, 3, 4, 5$.

LP objective function →

minimize

$$\begin{aligned} 60 \times [&5x_{A1} + 5x_{A2} + 7x_{A3} + 4x_{A4} + 8x_{A5} \\ &+ 6x_{B1} + 5x_{B2} + 8x_{B3} + 3x_{B4} + 7x_{B5} \\ &+ 6x_{C1} + 8x_{C2} + 9x_{C3} + 5x_{C4} + 10x_{C5} \\ &+ 7x_{D1} + 6x_{D2} + 6x_{D3} + 3x_{D4} + 6x_{D5} \\ &+ 6x_{E1} + 7x_{E2} + 10x_{E3} + 6x_{E4} + 11x_{E5}] \end{aligned}$$

s.t.

$$\begin{aligned} x_{A1} + x_{B1} + x_{C1} + x_{D1} + x_{E1} &= 1 \\ x_{A2} + x_{B2} + x_{C2} + x_{D2} + x_{E2} &= 1 \\ x_{A3} + x_{B3} + x_{C3} + x_{D3} + x_{E3} &= 1 \\ x_{A4} + x_{B4} + x_{C4} + x_{D4} + x_{E4} &= 1 \\ x_{A5} + x_{B5} + x_{C5} + x_{D5} + x_{E5} &= 1 \end{aligned}$$

and $x_{ij} = \begin{cases} 0 \\ 1 \end{cases}$ ... integers

1.7

	China	India	Philippines	USA	France
Hongkong	\$50/ton	\$90/ton	\$70/ton	\$150/ton	\$180/ton
Taiwan	\$60/ton	\$95/ton	\$50/ton	\$130/ton	\$200/ton

x_{11} = [tons of material shipped from] China to Hong Kong
x_{12} = [...] China to Taiwan
x_{21} = [..] India to Hong Kong
x_{22} = [..] India to Taiwan
x_{31} = [..] Philippines to Hong kong
x_{32} = [...] Philippines to Taiwan
x_{41} = [...] Hong Kong to USA
x_{51} = [..] Hongkong to France
x_{42} = [--] Taiwan to USA
x_{52} = [.] Taiwan to France.

Here, direction of shipping has changed but notation has been kept same to avoid confusion.

a. Linear program to minimize shipping cost.

$$\text{minimize } 50x_{11} + 60x_{12} + 90x_{21} + 95x_{22} + 70x_{31} + 50x_{32} + 150x_{41} + 130x_{42} + 180x_{51} + 200x_{52}$$

s.t.

$x_{11} + x_{12} = 60$ ... supply constraints China
$x_{21} + x_{22} = 45$ India
$x_{31} + x_{32} = 30$ Philippines
$x_{41} + x_{42} = 80$ demand constraints USA
$x_{51} + x_{52} = 55$ France.

and $x_{11}, x_{12}, x_{21}, x_{22}, x_{31}, x_{32}, x_{41}, x_{42}, x_{51}, x_{52} \geq 0$

(b) The already existing formulation will remain the same except for the addition of one constraint. This constraint can be aded either to the supply side or demand side →

$\boxed{x_{11} + x_{21} + x_{31} \leq 60}$ OR $\boxed{x_{41} + x_{51} \leq 60}$

(c) Similar to above, 2 constraints will be added to formulation in (a). →

1.] $\boxed{x_{11} + x_{21} + x_{31} \leq 60}$ OR $\boxed{x_{41} + x_{51} \leq 60}$

2.] $\boxed{x_{12} + x_{22} + x_{32} \leq 50}$ OR $\boxed{x_{42} + x_{52} \leq 50.}$

(d.) Two independent transportation problems can be formulated as below:

1] Suppliers to processors

minimize $50x_{11} + 60x_{12} + 90x_{21} + 95x_{22} + 70x_{31} + 50x_{32}$

s.t. $x_{11} + x_{12} = 60$

$x_{21} + x_{22} = 45$

$x_{31} + x_{32} = 30$

$x_{11} + x_{21} + x_{31} \leq 60$

$x_{12} + x_{22} + x_{32} \leq 50$

and $x_{11}, x_{12}, x_{21}, x_{22}, x_{31}, x_{32} \geq 0.$

2] Processors to markets.

minimize $150x_{41} + 130x_{42} + 180x_{51} + 200x_{52}$

s.t. $x_{41} + x_{42} = 80$

$x_{51} + x_{52} = 55$

$x_{41} + x_{51} \leq 60$

$x_{42} + x_{52} \leq 50$

and $x_{41}, x_{42}, x_{51}, x_{52} \geq 0$