Homework 9 Solutions

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Problem 1

(a) We list the key results at each iteration.

Iteration k	$d_y^k = -X_k r^k$	Step length α_k	x^{k+1}	$c^T x^k - c^T x^{k+1}$
0	$(-0.0682, 0.0227, 0.0227)$	14.52	$(0.0025, 0.6650, 0.3325)$	0.0825
1	$(-0.8333, 0.0021, 0.0021) \times 10^{-3}$	1.1880×10^3	$(0.0000, 0.6666, 0.3333)$	8.2500×10^{-4}
2	$(-0.8333, 0.0000, 0.0000) \times 10^{-5}$	1.1880×10^5	$(0.0000, 0.6667, 0.3333)$	8.2500×10^{-6}

Results:

```
%Solution:
solu =
     0.0000
     0.6667
     0.3333

%Number of iterations
itnum =
      6
%Objective value
obj =
     1.6667
```

Iteration k	0	1	2
d_w^k	$(-0.5714, 1.6327)$	$(0.0221, 0.0223)$	$(0.0888, 0.1853) \times 10^{-3}$
d_s^k	$(-1.0612, -0.4898, -2.2041)$	$(-0.0444, -0.0665, -0.0003)$	$(-0.0965, -0.0077, -0.2741) \times 10^{-3}$
β_k	1.3475	5.0597	
w^k	$(0, 0)$	$(-0.7700, 2.2000)$	$(-0.6582, 2.3131)$
s^k	$(2, 1, 3)$	$(0.5700, 0.3400, 0.0300)$	$(0.3452, 0.0034, 0.0287)$

(b) Results:

```
%Solution:
solu_w =
```

```
    -0.6582
     2.3131
solu_s =
     0.3452
     0.0034
     0.0287

%Number of iterations
itnum =
      2
%Objective value
obj =
     1.6548
```

Problem 2

It is on HW8.

Problem 3

(a) At the vertex $x = (0, 0, 0.5, 0.5)$, the reduced costs are $r_1 = -1$ and $r_2 = 0$. So the moving direction is $d_1 = (1, 0, -0.5, -0.5)$.

(b) $x = (0.01, 0.01, 0.49, 0.49)$. Use Karmarkar's algorithm. $d_y^k = (0.0075, -0.0025, -0.0025, -0.0025)$. Since the transformation between x and y is nonlinear, we don't have a moving "direction" in $x-$space.

(c) Use the primal affine scaling algorithm. Moving direction is $(0.9998, -0.0002, -0.4998, -0.4998) \times 10^{-4}$.

(d) Set $\mu = 0.01$. The central force is $(0.0098, 0.098, -0.0098, -0.0098)$.

Moving direction is $(0.0198, 0.0098, -0.0148, -0.0148)$.

(e) Compare the moving directions given by different algorithms. Think about how they are defined. Consider their effects on the performance of the algorithms. Think about the meaning of μ.

Problem 4

(a) Dual problem is

$$\begin{aligned} \max \quad & w_2 + 1 \\ \text{s.t.} \quad & w_2 \leq -1 \\ & w_2 \leq 0 \\ & w_1 + w_2 \leq 0 \\ & -w_1 + w_2 \leq 0 \end{aligned}$$

(b) Obviously.

(c) At $w = (1, -2)$, we find $s = (1, 2, 1, 3)$. $d_w^k = (-0.4848, 0.6061)$. $d_s^k = (-0.6061, -0.6061, -0.1212, -1$

(d) This moving direction is pointing to one of the dual optimal solutions.

(e) Set $\mu = 0.01$. Centering force is $(-0.5152, 1.3939)$. The moving direction is $(-47.9697, 59.2121)$.

(f) Whether the direction in (e) is better than in (c) depends on the choice of μ. Please try different μ. Some directions fail to point to an optimal solution.

Problem 5

(a) Based on the representation of $x = y + q$, we may convert the original LP problem to the following LP problem in terms of the new variable y:

$$\begin{aligned} \min \quad & c^T y + c^T q \\ \text{s.t.} \quad & Ay = b - Aq \\ & y \geq 0. \end{aligned}$$

Subtracting the constant $c^T q$ in the objective, it becomes the following standard form LP problem:

$$\begin{aligned} \min \quad & c^T y \\ \text{s.t.} \quad & Ay = b - Aq \\ & y \geq 0. \end{aligned}$$

(b) For the above standard form LP problem, its dual problem is given by

$$\begin{aligned} \min \quad & (b - Aq)^T w \\ \text{s.t.} \quad & A^T w \leq c. \end{aligned}$$

When $q = 0$, the above dual problem becomes

$$\begin{aligned} \min \quad & b^T w \\ \text{s.t.} \quad & A^T w \leq c, \end{aligned}$$

which is actually a regular dual problem for the original LP problem with $q = 0$.