SEMIDEFINITE OPTIMIZATION (MA 591) — HW 4

Path-following IPM (in lieu of the October 23 class)

1. (A toy LP.) Consider the convex polyhedron $P = \{(x,y) \in \mathbb{R}^2 \,|\, 2x + y \leq 8, x \geq 0, y \geq 0\}$.

(a) Find the analytic center of P.

(b) Describe (in closed form) the (primal) central path corresponding to the LP in which we minimize $-2x - y$ over P.

2. (Every point is on a central path.) Consider a polytope in standard form and a point $\bar{\mathbf{x}}$ in its interior. Show that there exists a linear objective for which $\bar{\mathbf{x}}$ is the point on the central path corresponding to $\mu = 1$. (Motivation: this can be used to initialize the primal-only path following method, without the homogeneous self-dual embedding. Just follow this central path towards the analytic center, then switch to the path corresponding to the actual objective we wish to optimize.)

3. (We have stated this in class, but did not prove it.) Let $B\colon \operatorname{int}(S) \mapsto \mathbb{R}$ be a *nonnegative* continuous barrier function for the closed and bounded set S, and $f\colon S \mapsto \mathbb{R}$ be a continuous function. Finally, let $\mathbf{x}^*$ be a minimizer of f over S, and for every $\mu > 0$, let $\mathbf{x}_\mu$ be a minimizer of the function $f + \mu B$ over S. (These minimizers exist under the above assumptions. It should also be clear that $f(\mathbf{x}_\mu) + \mu B(\mathbf{x}_\mu)$ is nondecreasing in μ.)

(a) Prove that $f(\mathbf{x}_\mu)$ is nondecreasing and $B(\mathbf{x}_\mu)$ is nonincreasing in μ.

(b) Prove that $\lim_{\mu\to 0+} f(\mathbf{x}_\mu) + \mu B(\mathbf{x}_\mu) = \lim_{\mu\to 0+} f(\mathbf{x}_\mu) = f(\mathbf{x}^*)$.

Due on November 1 (Wednesday), by the start of the class. (You may turn in typeset solutions by email.)