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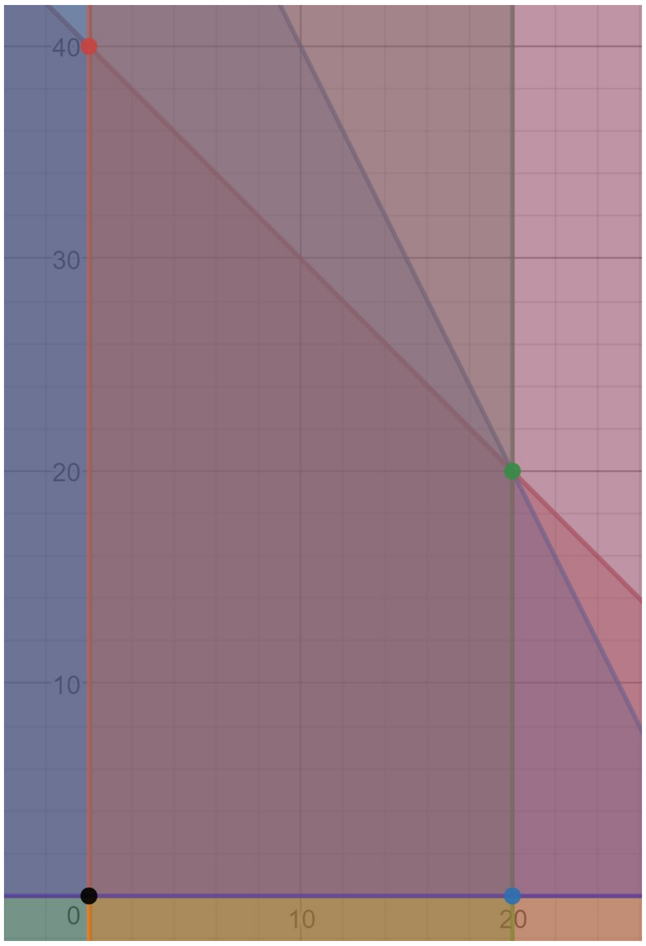
MA 505 HW #4

2.7 (a)



(b) $P = \{x_1, x_2 \varepsilon R^2 |$

$x_1 + x_2 + s_1 = 40,$

$2x_1 + x_2 + s_2 = 60,$

$x_1 + s_3 = 20,$

$x_1, x_2, s_1, s_2, s_3 \geq 0\}$

(c) There are 10 possibilities. Take $s_2, s_3 = 0$

$x_1 = 20, x_2 = 20, s_1 = 0, s_2 = 0, s_3 = 0$

Notice that taking any combination of s variables will produce this result. Taking $x_1, x_2 = 0$, we have $x_1 = 0, x_2 = 0, s_1 = 40, s_2 = 60, s_3 = 20$. If we take $x_1, s_1 = 0$, we have $x_1 = 0, x_2 = 40, s_1 = 0, s_2 = 20, s_3 = 20$. If we take $x_2, s_1 = 0$, we have $x_1 = 40, x_2 = 0, s_1 = 0, s_2 =- 20, s_3 =- 20$. If we take $x_1, s_2 = 0$, we have $x_1 = 0, x_2 = 60, s_1 =- 20, s_2 = 0, s_3 = 20$. If we take $x_2, s_2 = 0$, we have $x_1 = 30, x_2 = 0, s_1 10, s_2 = 0, s_3 =- 10$. We can't take $x_1, s_3 = 0$, since this would be an inconsistency. If we take $x_2, s_3 = 0$, we have $x_1 = 20, x_2 = 0, s_1 = 20, s_2 = 20, s_3 = 0$. These are our basic solutions.

(d) Our basic feasible solutions are:

$x_1 = 20, x_2 = 20, s_1 = 0, s_2 = 0, s_3 = 0$ (degenerate) (1,2,3)

$x_1 = 0, x_2 = 0, s_1 = 40, s_2 = 60, s_3 = 20$ (4)

$x_1 = 0, x_2 = 40, s_1 = 0, s_2 = 20, s_3 = 20$ (5)

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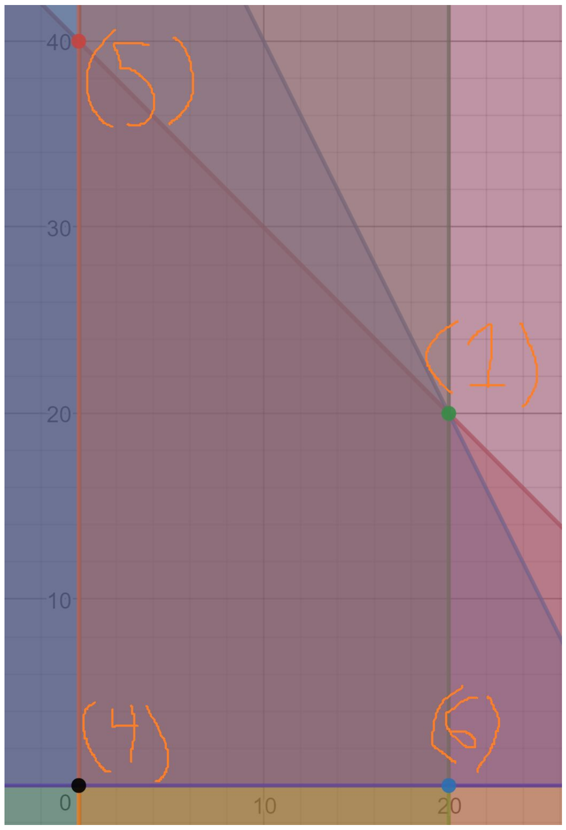
$$x_1 = 20, x_2 = 0, s_1 = 20, s_2 = 20, s_3 = 0 \qquad (6)$$

(e)



(f) The point (20,20) corresponds to a degenerate bfs.

2.10 Take a degenerate bfs. A bfs occurs at the intersection of lines where the respective $n-p$ are zero. Normally we have $C(n,m)$ ways to choose or $(n-m)$ non-basic variables to be zero. In the degenerate case, the difference is we pick $(n-m)$ variables to be zero, and the remaining $(m-p)$ happen to go to zero at the intersection (see 2.7c). Because of this, we only still of choose $(n-m)$ from $(n-p)$ that can go to zero, so we have $C(n-p, n-m)$ possibilities.

2.11 (a) Take an arbitrary vector $(a,b)^T$ with $a,b \geq 0$ in the first orthant. $(a,b)^T = a(1,0)^T + b(1,0)^T$. This is an arbitrary element in the first orthant. Since a convex cone requires positive coefficients, we can only generate elements in the first orthant. Therefore, the convex cone generated by M **is** the first orthant.

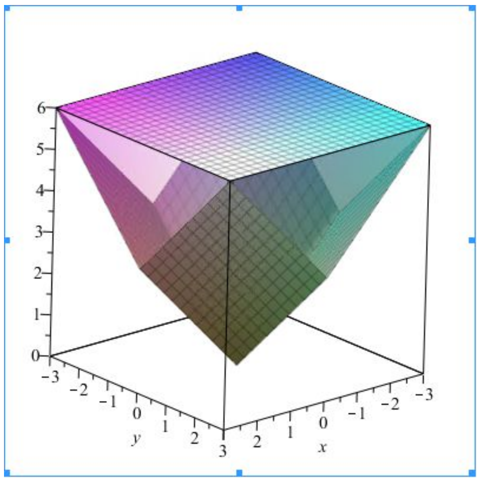
Explain more please

(b) Suppose we had a convex cone, S, a subset of the first octant such that $R^2_+ - S \neq \{\}$. This means we can take a point $(x_1, x_2)^T$ such that $(x_1,x_2)^T \varepsilon R^2_+$ and $(x_1,x_2)^T \not\in S$. This point can be generated by $x_1(1,0)^T + x_2(0,1)^T$, which implies $(1,0)^T, (0,1)^T$ are not contained in S. Therefore, R^2_+ is the smallest convex cone that can contain both.

4. (1) We know P is convex from the previous homework. d is an extremal direction iff $\forall x \varepsilon P, \lambda \geq 0,\ x + \lambda d \varepsilon P$. If d is an extremal direction, for an arbitrary $x \varepsilon P$, $A(x+\lambda d) = b = Ax + \lambda Ad \Rightarrow \lambda Ad = 0 \Rightarrow Ad = 0$. We also restrict $d \geq 0$ since any element in P must be positive. Now, assume $d \geq 0, Ad = 0$. Take $x \varepsilon P$. Then $x + \lambda d \geq 0$ and $A(x+\lambda d) = Ax = b$.

(2) From the definition in part (a), simply take $a > 0$. $A(ad) = a(Ad) = 0 \Rightarrow ad \varepsilon E$.

(3) Take $d_1, d_2 \varepsilon E$ and $a_1, a_2 > 0$ $s.t.$ $a_1 + a_2 = 1$.
$A(a_1 d_1 + a_2 d_2) = a_1(Ad_1) + a_2(Ad_2) = 0$.

5. (1) 

Where $x_1 = x,\ x_3 = y,\ x_2 = z$.

(2) $int(F_3) = \{x \varepsilon R^3 \mid |x_1| + |x_3| < x_2\}$

(3) $bdry(F_3) = \{x \varepsilon R^3 \mid |x_1| + |x_3| = x_2 \text{ or } x_2 = 0\}$

(4) The only real extreme point is at (0,0,0). In reality, this is the intersection of 4 edges, which are $x_2 = x_1$, where $x_1 > 0, x_3 = 0$, $x_2 = -x_1$, where $x_1 < 0, x_3 = 0$, $x_2 = x_3$, where $x_3 > 0, x_1 = 0$, and $x_2 = -x_3$, where $x_3 < 0, x_1 = 0$. These are vertices of the polyhedron.

(5) Suppose $(y_1, y_2, y_3)^T, (z_1, z_2, z_3)^T \varepsilon R^3$ and $a_1, a_2 \varepsilon R$ $s.t.$ $a_1 + a_2 = 1$, $a_1, a_2 \geq 0$. $a_1|y_1| + a_1|y_3| \leq a_1 y_2$ and $a_2|z_1| + a_2|z_3| \leq a_2 z_2$, since we can just multiply through by a positive constant. Note that the constants can also go inside the absolute value. We know $|w_1 + w_2| \leq |w_1| + |w_2|$. With these two facts together, we can say: $|a_1 y_1 + a_2 z_1| + |a_1 y_3 + a_2 z_3| \leq a_1 y_2 + a_2 z_2$. Therefore, F_3 is convex. Note that the second line above is also true if we don't restrict the constant to be less than or equal to one, so F_3 is also a cone.

(6) R^3_+ can be generated by vectors in F_3. The vectors $(0, 1, 0)^T$, $(1, 2, -1)^T$, and $(1, 2, 1)^T$. I can't really think of another relationship.

6. (a) $P = \{x_1, x_2 \varepsilon R^2 \mid$

$2x_1 - 4x_2 + s_1 = 1,$

$3x_1 - x_2 - e_1 = -3,$

$x_1, x_2, s_1, e_1 \geq 0\}$

Take $s_1, e_1 = 0$. Then $x_1 = \frac{-13}{10}, x_2 = \frac{-9}{10}, s_1 = 0, e_1 = 0$. Take $x_1, x_2 = 0$. Then $x_1 = 0, x_2 = 0, s_1 = 1, e_1 = 3$. Take $x_1, s_1 = 0$. Then $x_1 = 0, x_2 = \frac{-1}{4}, s_1 = 0, e_1 = \frac{-11}{4}$. Take $x_1 = 0, e_1 = 0$. Then $x_1 = 0, x_2 = 3, s_1 = 13, e_1 = 0$. Take $x_2, s_1 = 0$. $x_1 = \frac{1}{2}, x_2 = 0, s_1 = 0, e_1 = \frac{9}{2}$. Take $x_2, e_1 = 0$. Then $x_1 = -1, x_2 = 0, s_1 = 3, e_1 = 0$.

The basic solutions are:

$x_1 = \frac{-13}{10}, x_2 = \frac{-9}{10}, s_1 = 0, e_1 = 0$

$x_1 = 0, x_2 = 0, s_1 = 1, e_1 = 3$

$x_1 = 0, x_2 = \frac{-1}{4}, s_1 = 0, e_1 = \frac{-11}{4}$
$x_1 = 0, x_2 = 3, s_1 = 13, e_1 = 0$
$x_1 = \frac{1}{2}, x_2 = 0, s_1 = 0, e_1 = \frac{9}{2}$
$x_1 = -1, x_2 = 0, s_1 = 3, e_1 = 0$

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(b) The basic feasible solutions are:
$x_1 = 0, x_2 = 0, s_1 = 1, e_1 = 3$
$x_1 = 0, x_2 = 3, s_1 = 13, e_1 = 0$
$x_1 = \frac{1}{2}, x_2 = 0, s_1 = 0, e_1 = \frac{9}{2}$

please write it as a set

(c) This is, in fact, ~~an unbounded set.~~ The vector pointing from (0.5,0)+(0,3) is (-0.5,3). We need a vector perpendicular to this. (6,1) works.

(d) With just P, the directions are (0,1) and (1,0). In the standard form, these are (0,3,12,-3) and (0.5,0,-1,1.5).

7. (a) $P = \{x_1, x_2 \varepsilon R^2 |$
$2x_1 - 2x_2^+ + 2x_2^- + s_1 = 3,$
$8x_1 - x_2^+ + x_2^- - e_1 = -4,$
$x_1, x_2^+, x_2^-, s_1, e_1 \geq 0\}$

Take $x_1, x_2^+, x_2^- = 0$. Then $x_1 = 0, x_2^+ = 0, x_2^- = 0, s_1 = 3, e_1 = 4$. Take $x_1, s_1, e_1 = 0$. The equations are inconsistent, so it doesn't produce a bfs. Take $x_1, x_2^+, s_1 = 0$. Then $x_1 = 0, x_2^+ = 0, x_2^- = \frac{3}{2}, s_1 = 0, e_1 = \frac{11}{2}$. Take $x_1, x_2^-, s_1 = 0$. Then $x_1 = 0, x_2^+ = \frac{-3}{2}, x_2^- = 0, s_1 = 0, e_1 = \frac{11}{2}$. Take $x_1, x_2^-, e_1 = 0$. Then $x_1 = 0, x_2^+ = 4, x_2^- = 0, s_1 = 11, e_1 = 0$. Take $x_1, x_2^+, e_1 = 0$. Then $x_1 = 0, x_2^+ = 0, x_2^- = -4, s_1 = 11, e_1 = 0$. Take $x_2^+, x_2^-, s_1 = 0$. Then $x_1 = \frac{3}{2}, x_2^+ = 0, x_2^- = 0, s_1 = 0, e_1 = 16$. Take $x_2^+, x_2^-, e_1 = 0$. Then $x_1 = \frac{-1}{2}, x_2^+ = 0, x_2^- = 0, s_1 = 4, e_1 = 0$. Take $x_2^+, e_1, s_1 = 0$. Then $x_1 = \frac{-11}{14}, x_2^+ = 0, x_2^- = \frac{16}{7}, s_1 = 0, e_1 = 0$. Take $x_2^-, e_1, s_1 = 0$. Then $x_1 = \frac{-11}{14}, x_2^+ = \frac{-16}{7}, x_2^- = 0, s_1 = 0, e_1 = 0$. These are the basic solutions.

(b) The basic feasible solutions are
$x_1 = 0, x_2^+ = 0, x_2^- = 0, s_1 = 3, e_1 = 4$
$x_1 = 0, x_2^+ = 0, x_2^- = \frac{3}{2}, s_1 = 0, e_1 = \frac{11}{2}$
$x_1 = 0, x_2^+ = 4, x_2^- = 0, s_1 = 11, e_1 = 0$
$x_1 = \frac{3}{2}, x_2^+ = 0, x_2^- = 0, s_1 = 0, e_1 = 16$

The same

(c) We have to pick a direction perpendicular to (-1.5,4). (4,1.5) will do.

(d) There's only one edge to travel along. In R^2, the direction is $(0, -1)^T$. In R^5, it's $(0, -4, 0, -8, 4)^T$.

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