1.1 #11 (a)

Minimize $4x_1 + \sqrt{2}x_2 - 0.35x_3$

s.t. $-0.001x_1 + 200x_2 \geq 7\sqrt{261}$

$7.07x_2 - 2.62x_3 \leq -4$

$x_1, x_3 \geq 0$

surplus variables: e_1, e_2

substitution variable: $\bar{x}_2 - \hat{x}_2 = x_2$

Standard form

Minimize $4x_1 + \sqrt{2}\bar{x}_2 - \sqrt{2}\hat{x}_2 - 0.35x_3$

s.t. $-0.001x_1 + 200\bar{x}_2 - 200\hat{x}_2 - e_1 = 7\sqrt{261}$

$-7.07\bar{x}_2 + 7.07\hat{x}_2 + 2.62x_3 - e_2 = 4$

$x_1, \bar{x}_2, \hat{x}_2, x_3, e_1, e_2 \geq 0$

(b)

Maximize $-3.1x_1 + 2\sqrt{2}x_2 - x_3$

s.t. $100x_1 - 20x_2 = 7$

$-11x_1 - 7\pi x_2 - 2x_3 \leq 400$

$x_1 \geq 20, x_2 \geq 0, x_3 \geq -15$

slack variable: s_1

substitutions: $\bar{x}_1 + 20 = x_1, \bar{x}_3 - 15 = x_3$

Standard form

Mimimize $3.1\bar{x}_1 - 2\sqrt{2}x_2 + \bar{x}_3 + 45$

s.t. $-100\bar{x}_1 + 20x_2 = 1993$

$-11\bar{x}_1 - 7\pi x_2 - 2\bar{x}_3 + s_1 = 590$

$\bar{x}_1, x_2, \bar{x}_3, s_1 \geq 0$

[Courier 16 pt]

(c) Maximize $x_1 + 3x_2 - 2x_3$

s.t. $-2 \le 3x_1 - 5x_2 \le 15$

$11 \le -5x_1 + 20x_2 \le 40$

$x_2 \ge 0, \quad x_3 \le 10$

Split inequalities: 1) $-3x_1 + 5x_2 \le 2$

$3x_1 - 5x_2 \le 15$

2) $-5x_1 + 20x_2 \ge 11$

$-5x_1 + 20x_2 \le 40$

slack variables: s_1, s_2, s_3, s_4

surplus variable: e_1

substitutions: $x_1 = \bar{x}_1 - \hat{x}_1$

$x_3 = 10 - \bar{x}_3$

Standard form:

Minimize $-\bar{x}_1 + \hat{x}_1 - 3x_2 - 2\bar{x}_3 + 20$

s.t. $-3\bar{x}_1 + 3\hat{x}_1 + 5x_2 + s_1 = 2$

$3\bar{x}_1 - 3\hat{x}_1 - 5x_2 + s_2 = 15$

$-5\bar{x}_1 + 5\hat{x}_1 + 20x_2 - e_1 = 11$

$-5\bar{x}_1 + 5\hat{x}_1 + 20x_2 + s_3 = 40$

$\bar{x}_3 + s_4 = 10$

$\bar{x}_1, \hat{x}_1, x_2, \bar{x}_3, \hat{x}_3, s_1, s_2, s_3, s_4, e_1 \ge 0$

1.2 (a) convert to standard form

$$\begin{aligned} \text{Minimize} \quad & 2x_1 + 6x_2 + 8x_3 \\ \text{s.t.} \quad & x_1 + 2x_2 + x_3 = 5 \\ & 4x_1 + 6x_2 + 2x_3 = 12 \\ & x_2 \geq 0, \; x_3 \geq 0 \end{aligned}$$

substitution: $x_1 = \bar{x}_1 - \hat{x}_1$

standard form:

$$\begin{aligned} \text{Minimize} \quad & 2\bar{x}_1 - 2\hat{x}_1 + 6x_2 + 8x_3 \\ \text{s.t.} \quad & \bar{x}_1 - \hat{x}_1 + 2x_2 + x_3 = 5 \\ & 4\bar{x}_1 - 4\hat{x}_1 + 6x_2 + 2x_3 = 12 \\ & \bar{x}_1, \hat{x}_1, x_2, x_3 \geq 0 \end{aligned}$$

(b) let $x_1 = 5 - 2x_2 - x_3$

$$\begin{aligned} \text{Minimize} \quad & 2(5 - 2x_2 - x_3) + 6x_2 + 8x_3 \\ \text{s.t.} \quad & 0 = 0 \\ & 4(5 - 2x_2 - x_3) + 6x_2 + 2x_3 = 12 \\ & x_2, x_3 \geq 0 \end{aligned}$$

(c) standard form

$$\begin{aligned} \text{Minimize} \quad & 2x_2 + 6x_3 + 10 \\ \text{s.t.} \quad & 2x_2 + 2x_3 = 8 \\ & x_2, x_3 \geq 0 \end{aligned}$$

(d) minimum $= 2(4) + 6(0) + 10 = 18$

at $x_2 = 4, \; x_3 = 0$

1.3

$$\begin{aligned} \text{Minimize} \quad & x_1^2 + x_2 + 4x_3 \\ \text{s.t.} \quad & x_1^2 - x_2 = 0 \\ & 2x_2 + 4x_3 \geq 4 \\ & x_1 \geq 0,\ x_2 \geq 2,\ x_3 \geq 0 \end{aligned}$$

(a) no, it is not a linear programming problem as x_1^2 is not linear

(b) yes, substitute $\bar{x}_1 = x_1^2 \rightarrow \bar{x}_1 \geq 0$ because $x_1 \geq 0$

(c) slack variables: e_1
substitution: $\bar{x}_2 + 2 = x_2$
standard form

$$\begin{aligned} \text{Minimize} \quad & \bar{x}_1 + \bar{x}_2 + 4x_3 + 4 \\ \text{s.t.} \quad & \bar{x}_1 - \bar{x}_2 = 2 \\ & 2\bar{x}_2 + 4x_3 - e_1 = 0 \\ & \bar{x}_1, \bar{x}_2, x_3, e_1 \geq 0 \end{aligned}$$

(d) linear problem — yes

original problem — no except if $\bar{x}_1$ is a square of an integer

1.4

Minimize $|x_1| + 2|x_2| - |x_3|$

s.t. $x_1 + x_2 - x_3 \leq 10$

$x_1 - 3x_2 + 2x_3 = 12$

(a) no, the absolute value function is non-linear

(b) yes, set new non-negative decision variables v_1, v_2, v_3 and u_1, u_2, u_3 such that $|x_1| = u_1 + v_1$ and $x_1 = u_1 - v_1$

$|x_2| = u_2 + v_2$ and $x_2 = u_2 - v_2$

$|x_3| = u_3 + v_3$ and $x_3 = u_3 - v_3$

then convert existing equations

(c) Minimize $|x_1 - 5| + |x_2 + 4|$

s.t. $x_1 + x_2 \leq 10$

$x_1 - 3x_2 \geq 2$

substitutions: $y_1 + 5 = x_1$, $y_2 - 4 = x_2$

$|y_1| = \bar{y}_1 + \hat{y}_1$ $|y_2| = \bar{y}_2 + \hat{y}_2$

$y_1 = \bar{y}_1 - \hat{y}_1$ $y_2 = \bar{y}_2 - \hat{y}_2$

Minimize $\bar{y}_1 + \hat{y}_1 + \bar{y}_2 + \hat{y}_2$

s.t. $\bar{y}_1 - \hat{y}_1 + \bar{y}_2 - \hat{y}_2 + s_1 = 9$

$-\bar{y}_1 + \hat{y}_1 + 3\bar{y}_2 - 3\hat{y}_2 + s_2 = 15$

$\bar{y}_1, \hat{y}_1, \bar{y}_2, \hat{y}_2, s_1, s_2 \geq 0$

1.5

$$\begin{aligned} \text{Maximize} \quad & 15x_1 + 25x_2 \\ \text{s.t.} \quad & 3x_1 + 4x_2 \leq 100 \\ & 2x_1 + 3x_2 \leq 70 \\ & x_1 + 2x_2 \leq 30 \\ & x_1 \geq 0, \; x_2 \geq 3 \end{aligned}$$

x_1 = # chips 1 produced
x_2 = # chips 2 produced

1.6

$$\begin{aligned} \text{Minimize} \quad & 5x_{A1} + 5x_{A2} + 7x_{A3} + 4x_{A4} + 8x_{A5} \\ & + 6x_{B1} + 5x_{B2} + 8x_{B3} + 3x_{B4} + 7x_{B5} \\ & + 6x_{C1} + 8x_{C2} + 9x_{C3} + 5x_{C4} + 10x_{C5} \\ & + 7x_{D1} + 6x_{D2} + 6x_{D3} + 3x_{D4} + 6x_{D5} \\ & + 6x_{E1} + 7x_{E2} + 10x_{E3} + 6x_{E4} + 11x_{E5} \end{aligned}$$

$$\begin{aligned} \text{s.t.} \quad & x_{A1} + x_{A2} + x_{A3} + x_{A4} + x_{A5} = 1 \\ & x_{B1} + x_{B2} + x_{B3} + x_{B4} + x_{B5} = 1 \\ & x_{C1} + x_{C2} + x_{C3} + x_{C4} + x_{C5} = 1 \\ & x_{D1} + x_{D2} + x_{D3} + x_{D4} + x_{D5} = 1 \\ & x_{E1} + x_{E2} + x_{E3} + x_{E4} + x_{E5} = 1 \\ & x_{A1} + x_{B1} + x_{C1} + x_{D1} + x_{E1} = 1 \\ & x_{A2} + x_{B2} + x_{C2} + x_{D2} + x_{E2} = 1 \\ & x_{A3} + x_{B3} + x_{C3} + x_{D3} + x_{E3} = 1 \\ & x_{A4} + x_{B4} + x_{C4} + x_{D4} + x_{E4} = 1 \\ & x_{A5} + x_{B5} + x_{C5} + x_{D5} + x_{E5} = 1 \end{aligned}$$

for $i = A, E$, $j = 1:5$ $x_{ij} \geq 0$, x_{ij} integer

1.7 for x_{ij},

$i=1 \Rightarrow$ Hong Kong, $i=2 \Rightarrow$ Taiwan

$j=1 \Rightarrow$ China, $2 \Rightarrow$ India, $3 \Rightarrow$ Philippines, $4 \Rightarrow$ US, $5 \Rightarrow$ France

(a) Minimize $50x_{11} + 90x_{12} + 70x_{13} + 150x_{14} + 180x_{15}$
$+ 60x_{21} + 95x_{22} + 50x_{23} + 130x_{24} + 200x_{25}$

s.t.
$$x_{11} + x_{21} = 60$$
$$x_{12} + x_{22} = 45$$
$$x_{13} + x_{23} = 30$$
$$x_{14} + x_{24} = 80$$
$$x_{15} + x_{25} = 55$$
$$x_{11} + x_{12} + x_{13} - x_{14} - x_{15} = 0$$
$$x_{21} + x_{22} + x_{23} - x_{24} - x_{25} = 0$$
all $x_{ij} \geq 0$

(b) add constraints: $x_{11} + x_{12} + x_{13} \leq 60$

(c) add constraints: $x_{11} + x_{12} + x_{13} = 60$
$x_{21} + x_{22} + x_{23} = 50$

also

modify constraints
$$x_{11} + x_{21} \leq 60$$
$$x_{12} + x_{22} \leq 45$$
$$x_{13} + x_{23} \leq 30$$
$$x_{14} + x_{24} \leq 80$$
$$x_{15} + x_{25} \leq 55$$