

Homework 2

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September 5, 2017

OR 505

Dr. Fong

- 1.1 Convert the following linear programming problems into standard form.

a) Min $4x_1 + \sqrt{2}x_2 - 0.35x_3$

s.t. $-0.001x_1 + 200x_2 - 7\sqrt{261} \geq 0$

$7.07x_2 - 2.62x_3 \leq -4$

$x_1, x_3 \geq 0$

x_2 unrestricted, so replace with $x_2^+ - x_2^-$ and add constraints $x_2^+, x_2^- \geq 0$

C₁ - subtract a surplus variable $e_1, e_1 \geq 0$

C₂ - add a slack variable $s_2, s_2 \geq 0$

Min $4x_1 + \sqrt{2}(x_2^+ - x_2^-) - 0.35x_3$

s.t. $-0.001x_1 + 200(x_2^+ - x_2^-) - e_1 = 7\sqrt{261}$

$7.07(x_2^+ - x_2^-) - 2.62x_3 + s_2 = -4$

$x_1, x_2^+, x_2^-, x_3, e_1, s_2 \geq 0$

b) Max $-3.1x_1 + 2\sqrt{2}x_2 - x_3$

s.t. $100x_1 - 20x_2 = 7$

$-11x_1 - 7\pi x_2 - 2x_3 \leq 400$

$x_1 \geq 20, x_2 \geq 0, x_3 \geq -15$

Objective is a max function so multiply by -1 to convert to a min function.

C₂ → add a slack variable $s_2, s_2 \geq 0$

$x_1 \geq 20 \rightarrow$ replace x_1 with $\bar{x}_1 + 20$ and replace $x_1 \geq 20$ with $\bar{x}_1 \geq 0$

$x_3 \geq -15 \rightarrow$ replace x_3 with $\bar{x}_3 - 15$ and replace $x_3 \geq -15$ with $\bar{x}_3 \geq 0$

Min $3.1(\bar{x}_1 + 20) - 2\sqrt{2}x_2 + (\bar{x}_3 - 15)$

s.t. $100(\bar{x}_1 + 20) - 20x_2 = 7$

$-11(\bar{x}_1 + 20) - 7\pi x_2 - 2(\bar{x}_3 - 15) + s_2 = 400$

$\bar{x}_1, x_2, \bar{x}_3, s_2 \geq 0$

c) Max $x_1 + 3x_2 - 2x_3$
 s.t. $-2 \leq 3x_1 - 5x_2 \leq 15$
 $11 \leq -5x_1 + 20x_2 \leq 40$
 $x_2 \geq 0, x_3 \leq 10$

Simplify Constraints

Max $x_1 + 3x_2 - 2x_3$ → multiply by -1 to convert to a min function
 s.t. $3x_1 - 5x_2 \geq -2$ → subtract a surplus variable $e_1, e_1 \geq 0$
 $3x_1 - 5x_2 \leq 15$ → add a slack variable $s_2, s_2 \geq 0$
 $-5x_1 + 20x_2 \geq 11$ → subtract a surplus variable $e_3, e_3 \geq 0$
 $-5x_1 + 20x_2 \leq 40$ → add a slack variable $s_4, s_4 \geq 0$
 $x_2 \geq 0, x_3 \leq 10$ → x_1 unrestricted, so replace with $x_1^+ - x_1^-$ and add $x_1^+, x_1^- \geq 0$
 $\rightarrow x_3 \leq 10 \rightarrow$ replace x_3 with $10 - \bar{x}_3$ and $x_3 \leq 10$ with $\bar{x}_3 \geq 0$

Min	$-(x_1^+ - x_1^-) - 3x_2 + 2(10 - \bar{x}_3)$
s.t.	$3(x_1^+ - x_1^-) - 5x_2 - e_1 = -2$
	$3(x_1^+ - x_1^-) - 5x_2 + s_2 = 15$
	$-5(x_1^+ - x_1^-) + 20x_2 - e_3 = 11$
	$-5(x_1^+ - x_1^-) + 20x_2 + s_4 = 40$
	$x_1^+, x_1^-, x_2, \bar{x}_3, e_1, s_2, e_3, s_4 \geq 0$

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- 1.2. Consider a linear programming problem:

$$\text{Min } 2x_1 + 6x_2 + 8x_3$$

$$\text{s.t. } x_1 + 2x_2 + x_3 = 5$$

$$4x_1 + 6x_2 + 2x_3 = 12$$

$$x_2, x_3 \geq 0$$

- a) Convert this problem to standard form.

• x_1 unrestricted, replace with $x_1^+ - x_1^-$ and add $x_1^+, x_1^- \geq 0$

$$\text{Min } 2(x_1^+ - x_1^-) + 6x_2 + 8x_3$$

$$\text{s.t. } (x_1^+ - x_1^-) + 2x_2 + x_3 = 5$$

$$4(x_1^+ - x_1^-) + 6x_2 + 2x_3 = 12$$

$$x_1^+, x_1^-, x_2, x_3 \geq 0$$

- b) Can you find an equivalent linear programming problem with only 2 variables?

$$x_1 + 2x_2 + x_3 = 5 \quad 4x_1 + 6x_2 + 2x_3 = 12$$

$$x_1 = 5 - 2x_2 - x_3 \quad x_1 = 3 - 1.5x_2 - 0.5x_3 \Rightarrow 6 - 3x_2 - x_3 = 2x_1$$

$$\text{Min } 2(5 - 2x_2 - x_3) + 6x_2 + 8x_3 \quad \text{Min } 10 + 2x_2 + 6x_3$$

$$\text{s.t. } 3 - 1.5x_2 - 0.5x_3 + 2x_2 + x_3 = 5 \Rightarrow \text{s.t. } 0.5x_2 + 0.5x_3 = 2$$

$$20 - 8x_2 - 4x_3 + 6x_2 + 2x_3 = 12 \quad -2x_2 - 2x_3 = -8$$

$$x_2, x_3 \geq 0$$

- c) Convert to standard form.

(Already in standard form)

$$\text{Min } 10 + 2x_2 + 6x_3$$

$$\text{s.t. } x_2 + x_3 = 4$$

$$-2x_2 - 2x_3 = -8$$

$$x_2, x_3 \geq 0$$

- d) Solve the problem.

$$x_2 + x_3 = 4 \quad -2(4 - x_3) - 2x_3 = 8$$

$$x_2 = 4 - x_3 \quad -8 - 2x_3 - 2x_3 = 8$$

$$x_2 = 4 - 0 \quad -4x_3 = 0$$

$$\underline{x_2 = 4} \quad \underline{x_3 = 0}$$

$$x_1 = 5 - 2(4) - 0 = -3$$

Objective function value of new problem

$$= 10 + 2(4) + 6(0) = 18$$

Objective function value of original problem

$$= 2(-3) + 6(4) + 8(0) = 18$$

1.3 Consider the following problem:

$$\text{Min } x_1^2 + x_2 + 4x_3$$

$$\text{s.t. } x_1^2 - x_2 = 0$$

$$2x_2 + 4x_3 \geq 4$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

a) No, x_1^2 is not a linear function of x_1 .

b) $x_1^2 - x_2 = 0 \Rightarrow x_1^2 = x_2 \rightarrow$

$$\text{Min } x_2 + x_2 + 4x_3$$

$$\text{s.t. } x_2 - x_2 = 0$$

$$2x_2 + 4x_3 \geq 4$$

$$x_2 \geq 2, x_3 \geq 0$$

$$\text{Min } 2x_2 + 4x_3$$

$$\text{s.t. } 2x_2 + 4x_3 \geq 4$$

$$x_2 \geq 2, x_3 \geq 0$$

c) Convert to standard form.

• $2x_2 + 4x_3 \geq 4$: subtract a surplus variable e_1 .

• $x_2 \geq 2$: replace x_2 with $\bar{x}_2 + 2$ and $x_2 \geq 2$ with $\bar{x}_2 \geq 0$

$$\text{Min } 2(\bar{x}_2 + 2) + 4x_3 \rightarrow \text{Min } 4 + 2\bar{x}_2 + 4x_3$$

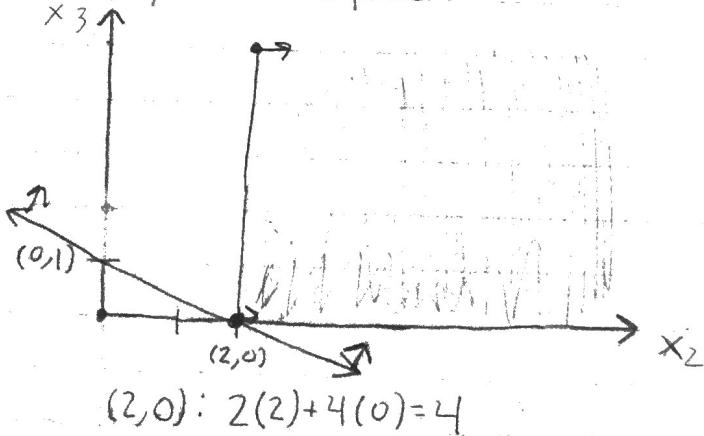
$$\text{s.t. } 2(\bar{x}_2 + 2) + 4x_3 - e_1 = 4$$

$$e_1, \bar{x}_2, x_3 \geq 0$$

$$\text{s.t. } 2\bar{x}_2 + 4x_3 - e_1 = 0$$

$$\bar{x}_2, x_3, e_1 \geq 0$$

d) Try to solve the problem



$$\boxed{\begin{aligned} x_2 &= 2 \\ x_3 &= 0 \\ Z &= 4 = 2(2) + 4(0) \end{aligned}}$$

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- 1.4 Consider the following optimization problem:

$$\text{Min } |x_1| + 2|x_2| - |x_3|$$

$$\text{s.t. } x_1 + x_2 - x_3 \leq 10$$

$$x_1 - 3x_2 + 2x_3 = 12$$

a) No, the absolute value function of unrestricted variables prevents this from being a linear programming problem.

b) Can you convert it into a linear program in standard form?

$$|x_1|: \bar{x}_1 + \hat{x}_1 = |x_1|; \bar{x}_1 - \hat{x}_1 = x_1; \bar{x}_1, \hat{x}_1 \geq 0$$

$$|x_2|: \bar{x}_2 + \hat{x}_2 = |x_2|; \bar{x}_2 - \hat{x}_2 = x_2; \bar{x}_2, \hat{x}_2 \geq 0$$

$$|x_3|: \bar{x}_3 + \hat{x}_3 = |x_3|; \bar{x}_3 - \hat{x}_3 = x_3; \bar{x}_3, \hat{x}_3 \geq 0$$

$x_1 + x_2 - x_3 \leq 10$: add a slack variable s_1 and constraint $s_1 \geq 0$

$$\text{Min } (\bar{x}_1 + \hat{x}_1) + 2(\bar{x}_2 + \hat{x}_2) - (\bar{x}_3 + \hat{x}_3)$$

$$\text{s.t. } (\bar{x}_1 - \hat{x}_1) + (\bar{x}_2 - \hat{x}_2) - (\bar{x}_3 - \hat{x}_3) + s_1 = 10$$

$$(\bar{x}_1 - \hat{x}_1) - 3(\bar{x}_2 - \hat{x}_2) + 2(\bar{x}_3 - \hat{x}_3) = 12$$

$$\bar{x}_1, \hat{x}_1, \bar{x}_2, \hat{x}_2, \bar{x}_3, \hat{x}_3, s_1 \geq 0$$

c) Convert the following problem into a linear program in standard form:

$$\text{Min } |x_1 - 5| + |x_2 + 4|$$

$$\text{s.t. } x_1 + x_2 \leq 10$$

$$x_1 - 3x_2 \geq 2$$

$$\cdot |x_1 - 5|: |x_1 - 5| = (\bar{x}_1 - 5) + (\hat{x}_1 - 5), \bar{x}_1 - 5 \geq 0 \Rightarrow \bar{x}_1 \geq 5, \hat{x}_1 - 5 \geq 0 \Rightarrow \hat{x}_1 \geq 5$$

$$\cdot |x_2 + 4|: |x_2 + 4| = (\bar{x}_2 + 4) + (\hat{x}_2 + 4), \bar{x}_2 + 4 \geq 0 \Rightarrow \bar{x}_2 \geq -4, \hat{x}_2 + 4 \geq 0 \Rightarrow \hat{x}_2 \geq -4$$

$$\cdot x_1: x_1 = \bar{x}_1 - \hat{x}_1$$

$$\cdot x_2: x_2 = \bar{x}_2 - \hat{x}_2$$

$$\text{Min } (\bar{x}_1 - 5) + (\hat{x}_1 - 5) + (\bar{x}_2 + 4) + (\hat{x}_2 + 4)$$

$$\text{s.t. } (\bar{x}_1 - \hat{x}_1) + (\bar{x}_2 - \hat{x}_2) \leq 10 \quad \text{add a slack variable } s_1$$

$$(\bar{x}_1 - \hat{x}_1) - 3(\bar{x}_2 - \hat{x}_2) \geq 2 \quad \text{subtract a surplus variable } e_1$$

$$\bar{x}_1, \hat{x}_1 \geq 5 \quad \bar{x}_2, \hat{x}_2 \geq -4$$

$\bar{x}_1, \hat{x}_1 \geq 5$: replace \bar{x}_1 w/ $\bar{x}_1 + 5$ and $\hat{x}_1 \geq 5$ with $\bar{x}_1 \geq 0$, replace \hat{x}_1 w/ $\hat{x}_1 + 5$ and $\hat{x}_1 \geq 5$ w/ $\hat{x}_1 \geq 0$

$\bar{x}_2, \hat{x}_2 \geq -4$: replace \bar{x}_2 w/ $\bar{x}_2 - 4$ and $\hat{x}_2 \geq -4$ with $\bar{x}_2 \geq 0$, replace \hat{x}_2 w/ $\hat{x}_2 - 4$ and $\hat{x}_2 \geq -4$ w/ $\hat{x}_2 \geq 0$

$$\begin{aligned}
 \text{Min} \quad & (\bar{x}_1 + 5 - 5) + (\hat{x}_1 + 5 - 5) + (\bar{x}_2 - 4 + 4) + (\hat{x}_2 - 4 + 4) \\
 \text{s.t.} \quad & (\bar{x}_1 + 5) - (\hat{x}_1 + 5) + (\bar{x}_2 - 4) - (\hat{x}_2 - 4) + s_1 = 10 \\
 & (\bar{x}_1 + 5) - (\hat{x}_1 + 5) - 3(\bar{x}_2 - 4) - (\hat{x}_2 - 4) + e_2 = 2 \\
 & \bar{x}_1, \hat{x}_1, \bar{x}_2, \hat{x}_2, s_1, e_2 \geq 0
 \end{aligned}$$

↓

$$\begin{aligned}
 \text{Min} \quad & \bar{x}_1 + \hat{x}_1 + \bar{x}_2 + \hat{x}_2 \\
 \text{s.t.} \quad & \bar{x}_1 - \hat{x}_1 + \bar{x}_2 - \hat{x}_2 + s_1 = 10 \\
 & \bar{x}_1 - \hat{x}_1 - 3\bar{x}_2 + 3\hat{x}_2 + e_2 = 2 \\
 & \bar{x}_1, \hat{x}_1, \bar{x}_2, \hat{x}_2, s_1, e_2 \geq 0
 \end{aligned}$$

1.5 x_1 : # chip 1 x_2 : # chip 2

x_1 cost \$15 x_2 cost \$25

$$x_1 \begin{cases} 3 \text{ hr skilled labor} \\ 2 \text{ hr unskilled labor} \\ 1 \text{ unit raw material} \end{cases} \quad x_2 \begin{cases} 4 \text{ hr skilled labor} \\ 3 \text{ hr unskilled labor} \\ 2 \text{ units raw material} \end{cases}$$

Total Resources

100 hrs skilled labor

70 hrs unskilled labor

30 units raw material

Requirements

At least $3x_2$ are made, fractional quantities

acceptable.

Formulate a linear program.

$$\begin{aligned}
 \text{max} \quad & 15x_1 + 25x_2 \\
 \text{s.t.} \quad & 3x_1 + 4x_2 \leq 100 \\
 & 2x_1 + 3x_2 \leq 70 \\
 & x_1 + 2x_2 \leq 30 \\
 & x_1 \geq 0 \\
 & x_2 \geq 3
 \end{aligned}$$

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1.6 Project #

	1	2	3	4	5	
1=A	5	5	7	4	8	1 person per project
2=B	6	5	8	3	7	1 project per person
3=C	6	8	9	5	10	\$60 per day
4=D	7	6	6	3	6	
5=E	6	7	10	6	11	

Formulate a integer linear program.

$$x_{ij} = \begin{cases} 1, & \text{if job } i \text{ assigned to worker } j \\ 0, & \text{otherwise} \end{cases} \quad \begin{matrix} i=1,2,3,4,5 \\ j=1,2,3,4,5 \end{matrix}$$

$$\begin{array}{ll} \min & 60 \sum_{i=1}^5 \sum_{j=1}^5 x_{ij} \\ \text{s.t.} & \sum_{i=1}^5 x_{ij} = 1, \quad \text{for } j=1, \dots, 5 \\ & \sum_{i=1}^5 x_{ij} = 5, \quad \text{for } i=1, \dots, 5 \\ & x_{ij} = 0 \text{ or } 1 \quad \text{for } i=1, \dots, 5 \text{ and } j=1, \dots, 5 \\ & \text{China(1) India(2) Philippines(3) USA(4) France(5)} \end{array}$$

1.7 (1) Hong Kong	50/ton	90/ton	70/ton	150/ton	180/ton
i (2) Taiwan	60/ton	95/ton	50/ton	130/ton	200/ton
60 tons from China			80 tons to U.S.		
45 tons from India			55 tons to France		
30 tons from Phillipines					

x_{ij} = # tons moved between distribution center i and supplier/market j

c_{ij} = cost/ton moved between distribution center i and supplier/market j

$$\begin{array}{ll} \text{a) min} & \sum_{i=1}^2 \sum_{j=1}^5 c_{ij} x_{ij} \quad \begin{matrix} i=1,2 \\ j=1,2,3,4,5 \end{matrix} \\ \text{s.t.} & \sum_{i=1}^2 x_{i1} = 60 \\ & \sum_{i=1}^2 x_{i2} = 45 \\ & \sum_{i=1}^2 x_{i3} = 30 \\ & \sum_{i=1}^2 x_{i4} = 80 \\ & \sum_{i=1}^2 x_{i5} = 55 \\ & x_{ij} \geq 0 \quad \text{for } i=1,2, j=1,2,3,4,5 \end{array}$$

b) Add the constraint

$$\sum_{j=1}^3 X_{ij} \leq 60$$

c) The problem becomes unbalance. We will have more incoming goods than can be processed and will not be able to provide enough processed goods to meet demand.

d) Problem 1: Supply to Distribution

X_{ij} = # tons from supplier i to center j C_{ij} = cost/ton from i to j

Supply = 135 Demand = 110 , introduce a dummy center j

	$j=1$ Hong Kong	$j=2$ Taiwan	$j=3$ Dummy	Supply
$i=1$ China	50/ton	60/ton	-	60
$i=2$ India	90/ton	95/ton	-	45
$i=3$ Philippines	70/ton	50/ton	-	30
Demand	60	50	25	

$$\min \sum_{i=1}^3 \sum_{j=1}^3 C_{ij} X_{ij} \quad i=1, 2, 3$$

$$\text{s.t. } \sum_{j=1}^3 X_{1j} = 60 \quad j=1, 2, 3$$

$$\sum_{j=1}^3 X_{2j} = 45$$

$$\sum_{j=1}^3 X_{3j} = 30$$

$$\sum_{i=1}^3 X_{i1} = 60$$

$$\sum_{i=1}^3 X_{i2} = 50$$

$$\sum_{i=1}^3 X_{i3} = 25$$

$$X_{ij} \geq 0 \text{ for all } i \text{ and } j$$

Problem 2 : Distribution to Demand

Demand > Supply $(135 > 110)$

	$j=1$ U.S.	$j=2$ France	Supply	
$i=1$	Hong Kong	150/ton	180/ton	60
$i=2$	Taiwan	130/ton	200/ton	50
$i=3$	Dummy	-	-	25
	Demand	80	55	

X_{ij} = # tons from center i to buyer j

c_{ij} = cost/ton from i to j

$$\min \sum_{i=1}^3 \sum_{j=1}^2 c_{ij} X_{ij}$$

$$\text{s.t. } \sum_{i=1}^3 X_{i1} = 80$$

$$\sum_{i=1}^3 X_{i2} = 55$$

$$\sum_{j=1}^2 X_{1j} = 60$$

$$\sum_{j=1}^2 X_{2j} = 50$$

$$\sum_{j=1}^2 X_{3j} = 25$$

$$X_{ij} \geq 0 \text{ for each } i \text{ and } j$$