

OR 505 - HX12

1.1

$$\text{a. Min: } 4x_1 + \sqrt{2}x_2 - 0.35x_3$$

Subject to

$$\begin{aligned} -0.001x_1 + 200x_2 &\geq 7\sqrt{261} & (1) \\ 7.07x_2 - 2.62x_3 &\leq -4 & (2) \\ x_1, x_3 &\geq 0 \end{aligned}$$

Converting to standard form.

i Eliminating inequalities by introducing surplus variable in equation (1) & slack variable in equation (2).

$$\Rightarrow -0.001x_1 + 200x_2 - e_1 = 7\sqrt{261}$$

$$7.07x_2 - 2.62x_3 + s_1 = -4.$$

ii Converting unrestricted variables x_2 to non negative variable

$$\Rightarrow \text{let } x_2 = \bar{x}_2 - \hat{x}_2, \text{ where } \bar{x}_2 \geq 0, \hat{x}_2 \geq 0$$

$$\Rightarrow -0.001x_1 + 200(\bar{x}_2 - \hat{x}_2) - e_1 = 7\sqrt{261}$$

$$7.07(\bar{x}_2 - \hat{x}_2) - 2.62x_3 + s_1 = -4.$$

$$\Rightarrow -0.001x_1 + 200\bar{x}_2 - 200\hat{x}_2 - e_1 = 7\sqrt{261}$$

$$7.07\bar{x}_2 - 7.07\hat{x}_2 - 2.62x_3 + s_1 = -4$$

standard form

$$\text{Min } Z = 4x_1 + \sqrt{2}\bar{x}_2 - \sqrt{2}\hat{x}_2 - 0.35x_3$$

subject to:

$$-0.001x_1 + 200\bar{x}_2 - 200\hat{x}_2 - e_1 = 7\sqrt{261}$$

$$7.07\bar{x}_2 - 7.07\hat{x}_2 - 2.62x_3 + s_1 = -4$$

where $x_1, \bar{x}_2, \hat{x}_2, x_3, e_1, s_1 \geq 0$

b, Maximize $Z = -3.1x_1 + 2\sqrt{2}x_2 - x_3$

subject to

$$100x_1 - 20x_2 = 7 \quad \text{---(1)}$$

$$-11x_1 - 7x_2 - 2x_3 \leq 400 \quad \text{---(2)}$$

where $x_1 \geq 20, x_2 \geq 0, x_3 \geq -15$

Convert to standard form.

i) eliminate inequality in eq (2)

$$\Rightarrow -11x_1 - 7x_2 - 2x_3 + s_1 = 400$$

ii) converting restricted variables to non-negative variables.

$$x_1 \geq 20 \rightarrow x_1 = \bar{x}_1 + 20 \text{ where } \bar{x}_1 \geq 0$$

$$x_3 \geq -15 \rightarrow x_3 = \bar{x}_3 - 15 \text{ where } \bar{x}_3 \geq 0$$

the new constraints will be.

$$100(\bar{x}_1 + 20) - 20x_2 = 7$$

$$\Rightarrow 100\bar{x}_1 + 2000 - 20x_2 = 7$$

$$\Rightarrow 100\bar{x}_1 - 20x_2 = -1993$$

$$-11(\bar{x}_1 + 20) - 7x_2 - 2(\bar{x}_3 - 15) + s_1 = 400$$

$$\Rightarrow -11\bar{x}_1 - 220 - 7x_2 - 2\bar{x}_3 + 30 + s_1 = 400$$

$$-11\bar{x}_1 - 7x_2 - 2\bar{x}_3 + s_1 = 590$$

The objective function will be.

$$\max z = -3 \cdot 1 (\bar{x}_1 + 20) + 2\sqrt{2} x_2 + (\bar{x}_3 - 15)$$

$$\hat{=} -3 \cdot 1 \bar{x}_1 - 62 + 2\sqrt{2} x_2 - \bar{x}_3 + 15$$

$$= -3 \cdot 1 \bar{x}_1 + 2\sqrt{2} x_2 - \bar{x}_3 - 47.$$

Converting max to min.

$$\max(z) = -\min(z)$$

$$\Rightarrow -(-3 \cdot 1 \bar{x}_1 + 2\sqrt{2} x_2 - \bar{x}_3 - 47)$$

$$\Rightarrow 3 \cdot 1 \bar{x}_1 - 2\sqrt{2} x_2 + \bar{x}_3 + 47.$$

standard form is

$$\min z = 3 \cdot 1 \bar{x}_1 - 2\sqrt{2} x_2 + \bar{x}_3 + 47$$

$$\text{subject to } 100\bar{x}_1 - 20x_2 = -1993$$

$$-11\bar{x}_1 - 7x_2 - 2\bar{x}_3 + s_1 = 590$$

where $\bar{x}_1, x_2, \bar{x}_3, s_1 \geq 0$

$$c, \max z = x_1 + 3x_2 - 2x_3$$

subject to:

$$-2 \leq 3x_1 - 5x_2 \leq 15 \quad (1)$$

$$11 \leq -5x_1 + 20x_2 \leq 40 \quad (2)$$

$$x_2 \geq 0, x_3 \leq 10$$

converting to standard form.

lets break constraints (1) & (2)

$$\Rightarrow -2 \leq 3x_1 - 5x_2 \leq 15$$

$$\Rightarrow 3x_1 - 5x_2 \geq -2 \quad 4$$

$$3x_1 - 5x_2 \leq 15$$

adding slack & surplus variables.

$$3x_1 - 5x_2 - e_1 = -2$$

$$3x_1 - 5x_2 + s_1 = 15$$

$$\Rightarrow 11 \leq -5x_1 + 20x_2 \leq 40$$

$$\Rightarrow -5x_1 + 20x_2 \geq 11$$

$$4 - 5x_1 + 20x_2 \leq 40$$

adding slack & surplus variables.

$$-5x_1 + 20x_2 + e_2 = 11$$

$$-5x_1 + 20x_2 + s_2 = 40$$

Converting unrestricted & unrestricted variables
to non negative variables.

let $x_1 = \bar{x}_1 - \hat{x}_1$, where $\bar{x}_1, \hat{x}_1 \geq 0$

$x_3 \leq 10 \Rightarrow$ let $x_3 = 10 - \bar{x}_3$, where $\bar{x}_3 \geq 0$

The new constraints would be.

$$3(\bar{x}_1 - \hat{x}_1) - 5x_2 - e_1 = -2 \\ \Rightarrow 3\bar{x}_1 - 3\hat{x}_1 - 5x_2 - e_1 = -2$$

$$3(\bar{x}_1 - \hat{x}_1) - 5x_2 + s_1 = 15 \\ \Rightarrow 3\bar{x}_1 - 3\hat{x}_1 - 5x_2 + s_1 = 15$$

$$-5(\bar{x}_1 - \hat{x}_1) + 20x_2 - e_2 = 11 \\ \Rightarrow -5\bar{x}_1 + 5\hat{x}_1 + 20x_2 - e_2 = 11$$

$$-5(\bar{x}_1 - \hat{x}_1) + 20x_2 + s_2 = 40 \\ \Rightarrow -5\bar{x}_1 + 5\hat{x}_1 + 20x_2 + s_2 = 40$$

The new objective function would be.

$$\max z = x_1 + 3x_2 - 2(10 - \bar{x}_3) \\ = x_1 + 3x_2 - 20 + 2\bar{x}_3 \\ = x_1 + 3x_2 + 2\bar{x}_3 - 20$$

converting $\max(z)$

$$\max(z) = -\min(-z)$$

$$\Rightarrow -x_1 - 3x_2 - 2\bar{x}_3 + 20$$

standard form is

$$\min z = -x_1 - 3x_2 - 2x_3 + 20$$

subject to

$$3\bar{x}_1 - 3\hat{x}_1 - 5x_2 - e_1 = -2$$

$$3\bar{x}_1 - 3\hat{x}_1 - 5x_2 + s_1 = 15$$

$$-5\bar{x}_1 + 5\hat{x}_1 + 20x_2 - e_2 = 11$$

$$-5\bar{x}_1 + 5\hat{x}_1 + 20x_2 + s_2 = 40$$

where

$$\bar{x}_1, \hat{x}_1, x_2, \bar{x}_3, e_1, e_2, s_1, s_2 \geq 0$$

1-2)

$$\min z = 2x_1 + 6x_2 + 8x_3$$

subject to $x_1 + 2x_2 + x_3 = 5 \quad (1)$
 $4x_1 + 6x_2 + 2x_3 = 12 \quad (2)$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

Q1, convert to standard form.

let $x_i = \bar{x}_i - \hat{x}_i$, where $\bar{x}_i, \hat{x}_i \geq 0$

$$\min z = 2(\bar{x}_1 - \hat{x}_1) + 6x_2 + 8x_3$$

$$= 2\bar{x}_1 - 2\hat{x}_1 + 6x_2 + 8x_3$$

new constraints

$$\bar{x}_1 - \hat{x}_1 + 2x_2 + x_3 = 5.$$

$$4(\bar{x}_1 - \hat{x}_1) + 6x_2 + 2x_3 = 12.$$

$$\Rightarrow 4\bar{x}_1 - 4\hat{x}_1 + 6x_2 + 2x_3 = 12.$$

∴ standard form is

$$\min z = 2\bar{x}_1 - 2\hat{x}_1 + 6x_2 + 8x_3$$

subject to

$$\bar{x}_1 - \hat{x}_1 + 2x_2 + x_3 = 5$$

$$4\bar{x}_1 - 4\hat{x}_1 + 6x_2 + 2x_3 = 12$$

where $\bar{x}_1, \hat{x}_1, x_2, x_3 \geq 0$

P.

let

from

①

$$\text{let } x_1 = 5 - 2x_2 - x_3$$

substituting x_1 in ② a objective function

Eq ②

$$\Rightarrow 4(5 - 2x_2 - x_3) + 6x_2 + 2x_3 = 12$$

$$20 - 8x_2 - 4x_3 + 6x_2 + 2x_3 = 12$$

$$-2x_2 - 2x_3 = -8$$

$$\Rightarrow 2x_2 + 2x_3 = 8$$

$$x_2 + x_3 = 4$$

3.

$$\Rightarrow 2(5 - 2x_2 - x_3) + 6x_2 + 8x_3$$

$$= 10 - 4x_2 - 2x_3 + 6x_2 + 8x_3$$

$$\min. Z. = 2x_2 + 6x_3 + 10$$

where

x_1, x_2

The new linear programming would be.

$$\min z = 2x_2 + 6x_3 + 10$$

subject to

$$x_2 + x_3 = 4$$

$$\text{where } x_2 \geq 0, x_3 \geq 0$$

above linear program.

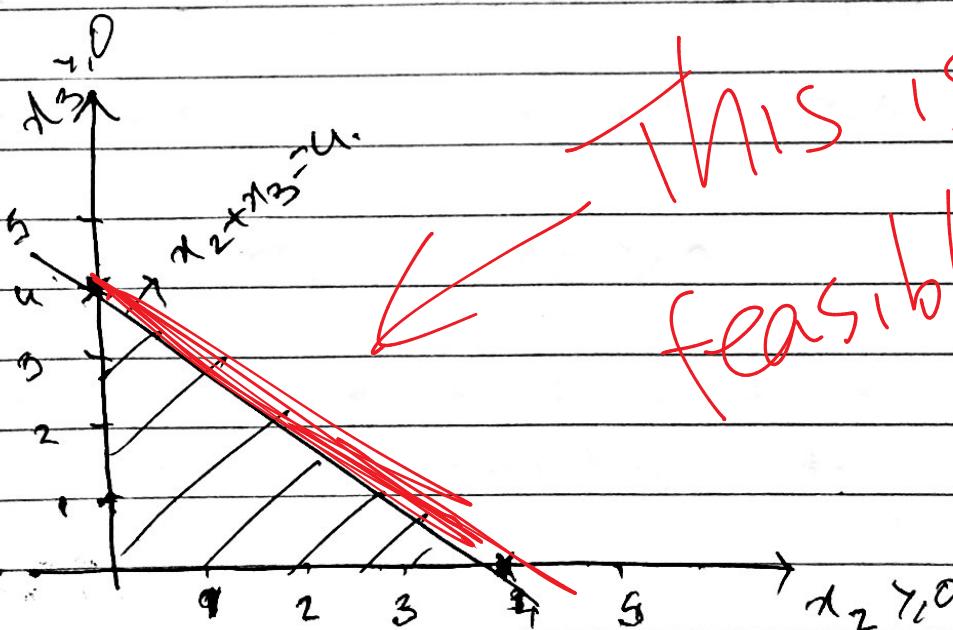
c, the converted form is in standard form.

d, Using geometrical method.

$$x_2 + x_3 = 4.$$

$$\text{let } x_2 = 0 \Rightarrow x_3 = 4.$$

$$\text{let } x_3 = 0 \Rightarrow x_2 = 4.$$



This is the
feasible region

checking for the extreme points in the feasible region with the objective function

the extreme points being

$$(0,0), (4,0), (0,4)$$

$$Z = 2x_2 + 6x_3 + 10$$

$$(0,0)$$

$$Z = 10$$

$$(4,0)$$

$$Z = 8 + 10 = 18$$

$$(0,4)$$

$$Z = 6(4) + 10$$

$$= 34$$

the optimal solution is $x_2 = 0, x_3 = 0$.

1.3

$$\min z = x_1^2 + x_2 + 4x_3$$

subject to

$$x_1^2 - x_2 = 0 \quad (1)$$

$$2x_2 + 4x_3 \geq 4 \quad (2)$$

$$x_1 > 0, x_2 \geq 2, x_3 \geq 0$$

a, No, this is not a linear program in this form.

b, Yes, we can convert into a linear program.

let $x_1^2 = x_2$ from constraint (1)

$$\begin{aligned} z &= x_2 + x_2 + 4x_3 \\ &= 2x_2 + 4x_3. \end{aligned}$$

The equivalent linear program is

$$\min z = 2x_2 + 4x_3$$

subject to $2x_2 + 4x_3 \geq 4$.

$$x_2 \geq 2, x_3 \geq 0.$$

c, Yes, to convert to standard form, we need to convert restricted variables to non-negative variables.

$$x_2 \geq 2 \Rightarrow \text{let } x_2 = \bar{x}_2 + 2.$$

the new objective function would be

$$\begin{aligned} \min z &= 2(\bar{x}_2 + 2) + 4x_3 \\ &= 2\bar{x}_2 + 4x_3 + 4. \end{aligned}$$

the new constraints would be

$$2(\bar{x}_2 + 2) + 4x_3 \geq 4.$$

$$\Rightarrow 2\bar{x}_2 + 4x_3 \geq 0.$$

where

$$\bar{x}_2, x_3 \geq 0$$

standard form would be

$$\min z = 2\bar{x}_2 + 4x_3 + 4.$$

$$\text{subject to: } 2\bar{x}_2 + 4x_3 + 4 \geq 0$$

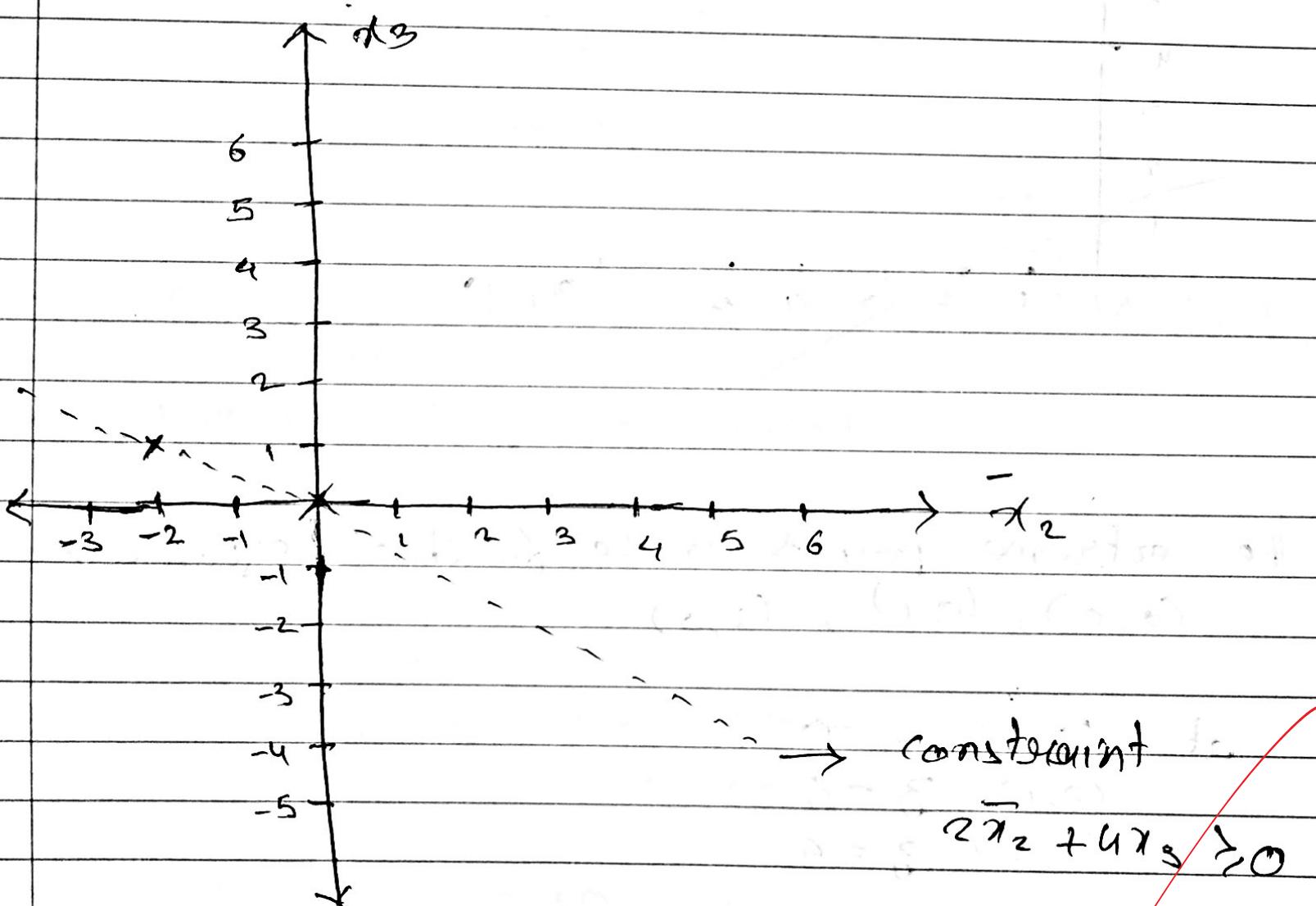
$$\text{where } \bar{x}_2, x_3 \geq 0$$

a) Solving using geometrical method.

$$2\bar{x}_2 + 4\bar{x}_3 - e_1 = 0$$

$$\text{let } \bar{x}_2 = 0 \Rightarrow \bar{x}_3 = 0$$

$$\text{let } \bar{x}_3 = 1 \Rightarrow \bar{x}_2 = -2$$



there is only one point in the feasible region

\therefore the solution is $\bar{x}_2 = 0, \bar{x}_3 = 0$

1-5

let x_1 be no. of chip-1 chips.
let x_2 be no. of chip-2 chips.

$$\text{max } Z = 15x_1 + 25x_2$$

① -

$$3x_1 + 4x_2 \leq 100$$

- skilled labor constraint

② -

$$2x_1 + 3x_2 \leq 70$$

- unskilled labor constraint

③ -

$$x_1 + 2x_2 \leq 30$$

- space material constraint

$$x_2 \geq 3$$

$$x_1, x_2 \geq 0$$

1.6.

	1	2	3	4	5
A	5	5	7	4	3
B	6	5	8	3	7
C	6	8	9	5	10
D	7	6	6	3	6
E	6	7	10	6	11

standard wage

=\$60/person/day.

To formulate

let x_{ij} = assignment of person 'i' to project 'j'

op

objective function would be

$$\min Z = 60(5x_{A1} + 5x_{A2} + 7x_{A3} + 4x_{A4} + 8x_{A5} + 6x_{B1} + 5x_{B2} + 8x_{B3} + 3x_{B4} + 7x_{C5} + 6x_{C1} + 8x_{C2} + 9x_{C3} + 5x_{C4} + 10x_{D5} + 7x_{D1} + 6x_{D2} + 6x_{D3} + 3x_{D4} + 6x_{D5} + 6x_{E1} + 7x_{E2} + 10x_{E3} + 6x_{E4} + 11x_{E5})$$

constraints would be.

Not enough!

$$x_{A1} + x_{B1} + x_{C1} + x_{D1} + x_{E1} = 1$$

$$x_{A2} + x_{B2} + x_{C2} + x_{D2} + x_{E2} = 1$$

$$x_{A3} + x_{B3} + x_{C3} + x_{D3} + x_{E3} = 1$$

$$x_{A4} + x_{B4} + x_{C4} + x_{D4} + x_{E4} = 1$$

$$x_{A5} + x_{B5} + x_{C5} + x_{D5} + x_{E5} = 1$$

please check
the solution

where $x_{ij} = \begin{cases} 0 & \text{when project } j \text{ not assigned to} \\ 1 & \text{when project } j \text{ assigned to person } i \end{cases}$

	China	India	Philippines	USA	France
Hongkong	50	90	70	150	(80)
Taiwan	60	95	50	130	200

- $x_{11} =$ tone shipped from China to Hongkong
 $x_{21} =$ " " India to "
 $x_{31} =$ " " Philippines to "
 $x_{41} =$ " " Hongkong to USA
 $x_{51} =$ " " " to France
 $x_{12} =$ " " " China to Taiwan
 $x_{22} =$ " " " India to Taiwan
 $x_{32} =$ " " " Philippines to "
 $x_{42} =$ " " " Taiwan to USA
 $x_{52} =$ " " " Taiwan to France

a. Min shipping cost

$$\begin{aligned}
 \text{min } Z = & 50x_{11} + 60x_{12} + 90x_{21} + 95x_{22} + 70x_{31} + 50x_{32} \\
 & + 150x_{41} + 130x_{42} + 180x_{51} + 200x_{52}
 \end{aligned}$$

S.t.

$$x_{11} + x_{12} = 60$$

$$x_{21} + x_{22} = 45$$

$$x_{31} + x_{32} = 30$$

$$x_{41} + x_{42} = 80$$

$$x_{51} + x_{52} = 55$$

where $x_{11}, x_{12}, x_{21}, x_{22}, x_{31}, x_{32}, x_{41}, x_{42}, x_{51}, x_{52} \geq 0$

b, only one additional constraint should be added.

to supply side as $x_{11} + x_{21} + x_{31} \leq 60$

or to demand side as $x_{41} + x_{51} \leq 60$.

c, Similarly, 2 constraints will be added to LP.

$$i \quad x_{11} + x_{21} + x_{31} \leq 60 \quad \text{or} \quad x_{41} + x_{51} \leq 60$$

$$ii \quad x_{12} + x_{22} + x_{32} \leq 50 \quad \text{or} \quad x_{42} + x_{52} \leq 50$$

∴ 2 transportation problems can be formed as

$$\text{i) } \min Z = 50x_{11} + 60x_{12} + 90x_{21} + 95x_{22} + 70x_{31} + 50x_{32}$$

$$\text{s.t. } x_{11} + x_{12} \leq 60$$

$$x_{21} + x_{22} \leq 45$$

$$x_{31} + x_{32} \leq 30$$

$$x_{11} + x_{21} + x_{31} \leq 60$$

$$x_{12} + x_{22} + x_{32} \leq 50$$

$$\text{and } x_{11}, x_{12}, x_{21}, x_{22}, x_{31}, x_{32} \geq 0$$

Please check

$$\text{ii) } \min Z = 150x_{41} + 130x_{42} + 180x_{51} + 200x_{52}$$

$$\text{s.t. } x_{41} + x_{42} \leq 80$$

$$x_{51} + x_{52} \leq 55$$

$$x_{41} + x_{51} \leq 60$$

$$x_{42} + x_{52} \leq 50$$

where

$$x_{41}, x_{42}, x_{51}, x_{52} \geq 0$$

The Solution

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