	China	India	Philippines	USA	France
Hong Kong	x_1	x_2	x_3	x_4	x_5
Taiwan	y_1	y_2	y_3	y_4	y_5

1.7 (a) where the variables represent flow between nodes in the table above,

$$\text{minimize} \quad 50x_1 + 90x_2 + 70x_3 + 150x_4 + 180x_5 + 60y_1 + 95y_2 + 50y_3 + 130y_4 + 200y_5$$

s.t.

$$\left.\begin{aligned} x_1 + y_1 &= 60 \\ x_2 + y_2 &= 45 \\ x_3 + y_3 &= 30 \\ x_4 + y_4 &= 80 \\ x_5 + y_5 &= 55 \end{aligned}\right\} \text{Production \& demand}$$

$$\left.\begin{aligned} x_1 + x_2 + x_3 - x_4 - x_5 &= 0 \\ y_1 + y_2 + y_3 - y_4 - y_5 &= 0 \end{aligned}\right\} \text{Total flow in \& out of Hong Kong and Taiwan}$$

$$x_1, \dots, x_5, y_1, \dots, y_5 \geq 0$$

(b)

Add the constraint $x_1 + x_2 + x_3 \leq 60$

(c) In addition to $x_1 + x_2 + x_3 \leq 60$, add the constraint $y_1 + y_2 + y_3 \leq 50$

But now total supply processing capacity $= 110 <$ the total demand at USA and France $= 135$.

The LP will be infeasible.

(d) Hong Kong: minimize $50x_1 + 90x_2 + 70x_3 + 150x_4 + 180x_5$

s.t. $x_1 + x_2 + x_3 = 60$

$x_1 + x_2 + x_3 - x_4 - x_5 = 0$ $\quad$ $x_4 \leq 80$

$x_1, x_2, x_3, x_4, x_5 \geq 0$ $\quad$ $x_5 \leq 55$

Taiwan: minimize $60y_1 + 95y_2 + 50y_3 + 130y_4 + 200y_5$

s.t. $y_1 + y_2 + y_3 = 50$

$y_1 + y_2 + y_3 - y_4 - y_5 = 0$ $\quad$ $y_4 \leq 80$ $\quad$ $y_5 \leq 55$

$y_1, y_2, y_3, y_4, y_5 \geq 0$

OR 505 HW 2

Blake Schwartz

1.1 (a)

$$\min\ 4x_1 + \sqrt{2}\,x_2 - .35x_3$$

s.t.

$$-.001x_1 + 200x_2 - x_4 = 7\sqrt{261}$$

$$7.07x_2 - 2.62x_3 + x_5 = 4$$

slack & excess var's

But x_2 is still unbounded, so:

$$\min\ 4x_1 + \sqrt{2}\,x_2^+ - \sqrt{2}\,x_2^- - .35x_3$$

s.t.

$$-.001x_1 + 200x_2^+ - 200x_2^- - x_4 = 7\sqrt{261}$$

$$7.07x_2^+ - 7.07x_2^- - 2.62x_3 + x_5 = 4$$

$$x_1, x_2^+, x_2^-, x_3, x_4, x_5 \geq 0$$

(b) $\bar{x}_1 = x_1 + 20$; $\bar{x}_3 = x_3 - 15$

$$(-)\text{minimize}\quad 3.1\bar{x}_1 - 2\sqrt{2}\,x_2 - \bar{x}_3$$

s.t.

$$100\bar{x}_1 - 20x_2 = 2007$$

$$-11\bar{x}_1 - 7\pi x_2 - 2\bar{x}_3 + x_4 = 210$$

$$\bar{x}_1, x_2, \bar{x}_3, x_4 \geq 0$$

(c)

$$(-)\text{minimize}\quad x_1^+ - x_1^- + 3x_2 - 2x_3^+ + 2x_3^-$$

s.t.

$$3x_1^+ - 3x_1^- - 5x_2 + x_4 = 15$$

$$3x_1^+ - 3x_1^- - 5x_2 - x_5 = -2$$

$$-5x_1^+ + 5x_1^- + 20x_2 + x_6 = 40$$

$$-5x_1^+ + 5x_1^- + 20x_2 - x_7 = 11$$

$$x_3^+ - x_3^- - x_8 = 0$$

$$x_1^+, x_1^-, x_2, x_3^+, x_3^-, x_4, x_5, x_6, x_7, x_8 \geq 0$$

1.2 (a) minimize $2x_1^+ - 2x_1^- + 6x_2 + 8x_3$

s.t. $x_1^+ - x_1^- + 2x_2 + x_3 = 5$

$4x_1^+ - 4x_1^- + 6x_2 + 2x_3 = 12$

$x_1^+, x_1^-, x_2, x_3 \geq 0$

(b) $x_1 = 5 - 2x_2 - x_3$, so:

minimize $10 - 4x_2 - 2x_3 + 6x_2 + 8x_3$

$= 2x_2 + 6x_3 \cancel{+10}$

s.t. $2x_2 + 2x_3 = 8$

$x_2, x_3 \geq 0$

(c) minimize $2x_2 + 6x_3$

s.t. $2x_2 + 2x_3 = 8$

$x_2, x_3 \geq 0$

(d) To solve, take x_3 to lowest value $= 0$

Then $2x_2 = 8$

$x_2 = 4, \ x_3 = 0$ and $x_1 = 5 - 4 - 0 = 1$

(1.3) (a) No. There is an x^2 term.

(b) Yes. Let $x_1^2 = x_2$, and you have a linear programming problem:

$$\text{Minimize } 2x_2 + 4x_3$$
$$\text{s.t. } 2x_2 + 4x_3 \geq 4$$
$$x_2 \geq 2, \quad x_3 \geq 0$$

(c) minimize $2\bar{x}_2 + 4x_3$ where $\bar{x}_2 = x_2 + 2$

$$\text{s.t. } 2\bar{x}_2 + 4x_3 - x_4 = 0$$
$$\bar{x}_2, x_3, x_4 \geq 0$$

(d) The optimal solution to the linear problem is

$\bar{x}_2 = 0, \quad x_3 = 0, \quad x_4 = 0, \quad f(\bar{x}_2, x_3) = 0$

So the original problem has solution:

$x_2 = \bar{x}_2 + 2 = 2, \quad x_3 = 0, \quad x_1 = \sqrt{x_2} = \sqrt{2}$

$f(x_1, x_2, x_3) = 2 + 2 + 0 = 4$

(1.4) (a) yes. All constraints are linear

(b) minimize $x_1^+ + x_1^- + 2x_2^+ + 2x_2^- - x_3^+ - x_3^-$

$$\text{s.t. } x_1^+ - x_1^- + x_2^+ - x_2^- - x_3^+ + x_3^- - x_4 = 10$$
$$x_1^+ - x_1^- - 3x_2^+ + 3x_2^- + 2x_3^+ - 2x_3^- = 12$$
$$x_1^+, x_1^-, x_2^+, x_2^-, x_3^+, x_3^-, x_4 \geq 0$$

(c) Let $u = x_1 - 5$ and $v = x_2 + 4$

$$\text{minimize } u^+ + u^- + v^+ + v^-$$
$$\text{s.t. } u^+ - u^- + v^+ - v^- - x_3 = 9$$
$$u^+ - u^- - 3v^+ + 3v^- + x_4 = 9$$
$$u^+, u^-, v^+, v^-, x_3, x_4 \geq 0$$

chip type = $\overset{\text{Chip 1}}{X_1}$, $\overset{\text{Chip 2}}{X_2}$

(1.5) maximize revenue $= 15X_1 + 25X_2$

s.t. resource constraints: skilled labor = 100
unskilled labor = 70
raw material = 30

$$3X_1 + 4X_2 \le 100$$
$$2X_1 + 3X_2 \le 70$$
$$X_1 + 2X_2 \le 30$$
$$X_1 \ge 0, \quad X_2 \ge 3$$

(1.6) minimize $60 \cdot \sum_{i=1}^{5} \sum_{j=1}^{5} X_{ij} C_{ij}$, where i = projects 1…5
j = Person A…E

C_{ij} is time for person j to finish project i

$$X = \begin{cases} 1 & \text{if } j \text{ is assigned to } i \\ 0 & \text{otherwise.} \end{cases}$$

s.t. $\sum_{i=1}^{5} X_{ij} = 1$ for each j

$\sum_{j=1}^{5} X_{ij} = 1$ for each i

for example, $X_{A1} + X_{A2} + X_{A3} + X_{A4} + X_{A5} = 1$

and $X_{A1} + X_{B1} + X_{C1} + X_{D1} + X_{E1} = 1$

So each project is assigned exactly one person at minimum cost.