Project Report

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1 Max Cut problem

The max cut problem can be reformulate as the following form

$$\begin{array}{lll} \max & \frac{1}{4}L \bullet X & \\ \text{s.t.} & (e_j e_j^T) \bullet X & = 1, \quad \forall j = 1, \ldots, n \\ & X & \succeq 0. \end{array} \tag{1}$$

where $X \in \mathbb{S}_+^n$.

If we denote the optimal value as b and scale X by $1/n$, then we would like to check the feasibility of the constraints:

$$\begin{array}{ll} \frac{n}{4b}L \bullet X & \geqslant 1 \\ n(e_j e_j^T) \bullet X & = 1, \quad \forall j = 1, \ldots, n \\ X & \succeq 0. \end{array}$$

Finally, if we take $A_j = n(e_j e_j^T) - I, (j = 1, \ldots, n)$, and $A_{n+1} = \frac{n}{4b}L - I$, then the feasibility problem can be reformed as

$$\begin{array}{lll} A_j \bullet X & \geqslant 0 & \forall j = 1, \ldots, n+1 \\ \mathrm{Tr}(X) & = 1 & \\ X & \succeq 0. & \end{array} \tag{2}$$

on which the Matrix-MW algorithm can be applied. To apply Matrix-MW, we would like to solve the large-margin problem with an error parameter $\delta > 0$. Denote $K = \{X \succeq 0 | \mathrm{Tr}(X) = 1\}$, we either solve the problem to an additive error of δ, i.e., find $X \in K$ such that $A_j \bullet X \geqslant -\delta, \forall j = 1, \ldots, n+1$, or prove the system (2) is infeasible.

Initialize $X^{(1)} = 1/nI$, and set parameters ϵ, $\eta = -\ln(1-\epsilon)$, we apply Matrix-MW on three different examples. For each example, we list the result and errors for different parameters.

2 Results

2.1 Comments

1. "# or rounds" is the number of iterations for Matrix-MW algorithm. It stops when $A_j \bullet X \geqslant -\delta$ is satisfied for any $j = 1, \ldots, n+1$.

ϵ	0.1	0.01	0.001	0.0001
δ	0.1	0.01	0.001	0.0001
# of rounds	20	328	3465	34827
Rank(X)	4	4	4	4
$\|X - X^*\|_2$	2.7700	2.7557	2.7556	2.7545
$\|X - X^*\|_2/\|X^*\|_2$	0.6925	0.6889	0.6887	0.6886
$\|b^* - b\|$	0.4595	0.0482	0.0039	0.0002

Table 1: toy example

ϵ	0.1	0.01	0.001	0.0001
δ	0.1	0.01	0.001	0.0001
# of rounds	19	633	7230	73228
Rank(X)	10	10	10	10
$\|X - X^*\|_2$	8.9298	8.9171	8.9169	8.9168
$\|X - X^*\|_2/\|X^*\|_2$	0.8930	0.8917	0.8917	0.8917
$\|b^* - b\|$	1.9107	0.1908	0.0204	0.0011

Table 2: 10 nodes example

ϵ	0.1	0.01	0.001	0.0001
δ	0.1	0.01	0.001	0.0001
# of rounds	2777	49971		
$\|b^* - b\|$	3.2430	0.7318		

Table 3: 100 nodes example

2. Solution X and the value of $b = \frac{1}{4}L \bullet X$ are obtained from iteration. X^* and b^* are optimal solution and optimal value to problem (1). We list the error of solution $\|X - X^*\|_2$ and the error of object value $|b - b^*|$.

3. Notice the solution absolute error in 2-norm is very large, so we also calculate the relative error in 2-norm for the toy example and the 10-node example.

4. The 100-node example has blank units in table since the algorithm converges very slow when ϵ and δ are smaller than 10^{-2}. (# of iteration will exceed 10^5).

5. Rank of the iteration solution is not 1. So the Rank = 1 constraint of original MAX CUT problem cannot be eliminated.

6. Notice that for each result, the ϵ and δ are in the same order of magnitude. The reason is, if we set ϵ and δ in different orders of magnitude, e.g., $\epsilon = 0.01$ and $\delta = 0.001$, or $\epsilon = 0.001$ and $\delta = 0.01$, the algorithm will not converge or converge very slow. From this we conclude that Matrix-MW is very parameter sensitive.