ISE 505

James Durst
200215249

1.1a) Min $Z = 4x_1 + \sqrt{2}(x_2^+ - x_2^-) - 0.35x_3$

s.t.

$$
\begin{aligned}
-0.001x_1 + 200(x_2^+ - x_2^-) - e_1 &= 7\sqrt{281} \\
-7.07(x_2^+ - x_2^-) + 2.62x_3 - e_2 &= 4 \\
x_1, x_2^+, x_2^-, x_3, e_1, e_2 &\geq 0
\end{aligned}
$$

1.1b) Max $z = -3.1x_1 + 2\sqrt{2}x_2 - (x_3^+ - x_3^-)$

s.t.

$$
\begin{aligned}
100x_1 - 20x_2 &= 7 \\
-11x_1 - 7\pi x_2 - 2(x_3^+ - x_3^-) + s_2 &= 400 \\
(x_3^- - x_3^+) + s_3 &= 15 \\
x_1 \geq 0, x_2 \geq 0, x_3^+ \geq 0, x_3^- \geq 0, s_2 \geq 0, s_3 &\geq 0
\end{aligned}
$$

1.1c) Max $(x_1^+ - x_1^-) + 3x_2 - 2(x_3^+ - x_3^-)$

s.t.

$$
\begin{aligned}
-3(x_1^+ - x_1^-) + 5x_2 + s_1 &= 2 \\
3(x_1^+ - x_1^-) - 5x_2 + s_2 &= 15 \\
-5(x_1^+ - x_1^-) + 20x_2 - e_3 &= 11 \\
-5(x_1^+ - x_1^-) + 20x_2 + s_4 &= 40 \\
(x_3^+ - x_3^-) + s_5 &= 10 \\
x_1^+, x_1^-, x_2, x_3^+, x_3^-, s_1, s_2, e_3, s_4, s_5 &\geq 0
\end{aligned}
$$

1.2) Min $2x_1 + 6x_2 + 8x_3 = z$

s.t.

$$
\begin{aligned}
x_1 + 2x_2 + x_3 &= 5 \\
4x_1 + 6x_2 + 2x_3 &= 12 \\
x_2, x_3 &\geq 0
\end{aligned}
$$

a) Min $2(x_1^+ - x_1^-) + 6x_2 + 8x_3 = z$

s.t.

$$
\begin{aligned}
(x_1^+ - x_1^-) + 2x_2 + x_3 &= 5 \\
4(x_1^+ - x_1^-) + 6x_2 + 2x_3 &= 12 \\
x_1^+, x_1^-, x_2, x_3 &\geq 0
\end{aligned}
$$

$10 + 2x_2 + 6x_3$

$10 - 4x_2 - 2x_3$

b) Min $2(5 - 2x_2 - x_3) + 6x_2 + 8x_3 = z$

s.t. $\cancel{(5 - 2x_2 - x_3) + 2x_2 + x_3 = 5}$ $\quad s = s$

$4(5 - 2x_2 - x_3) + 6x_2 + 2x_3 = 12$ $\quad 20 - 2x_2 - 2x_3 = 12$

$20 - 8x_2 - 4x_3$

$x_2, x_3 \geq 0$

Min $2x_2 + 6x_3 + 10 = z$

s.t.

$-2x_2 - 2x_3 = -8$

$x_2, x_3 \geq 0$

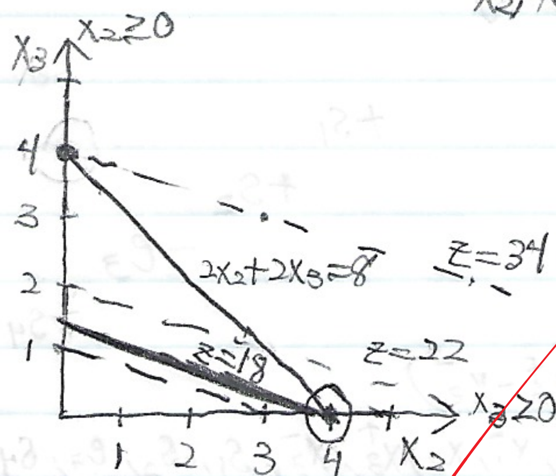
c) Min $z = 2x_2 + 6x_3 + 10$

s.t.

$2x_2 + 2x_3 = 8$

$x_2, x_3 \geq 0$

d.)



$z = 18$

$x_2 = 4$

$x_3 = 0$

1.3)

a) No

b) Yes, plug in x_2 for x_1^2.

Min $2x_2 + 4x_3 = z$

s.t.

$2x_2 + 4x_3 \geq 4$

$x_2 \geq 2, x_3 \geq 0$

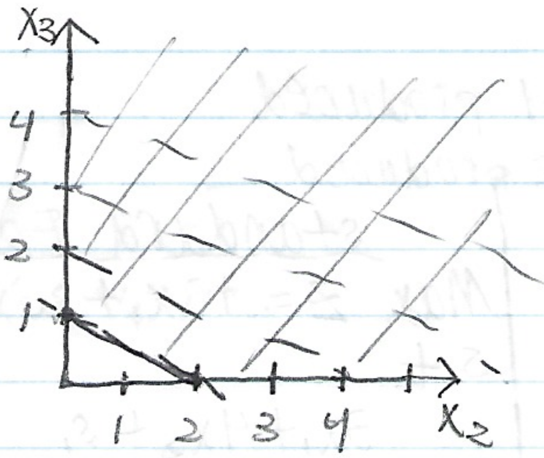
c.) Standard

Min $2x_2 + 4x_3 = z$

s.t.

$2x_2 + 4x_3 - e_1 = 4$

$x_2 \geq 2, x_3 \geq 0, e_1 \geq 0$

d.) 

Unbounded

1.4)

a.) Yes

−2

b.) Min $(x_1^+ + x_1^-) + 2(x_2^+ + x_2^-) - (x_3^+ + x_3^-)$

s.t.

$$(x_1^+ - x_1^-) + (x_2^+ - x_2^-) - (x_3^+ - x_3^-) + s_1 = 10$$
$$(x_1^+ - x_1^-) - 3(x_2^+ - x_2^-) + 2(x_3^+ - x_3^-) = 12$$

c.) $x_3 = x_1 - 5,\ x_4 = x_2 + 4$

$$x_1 - 5 - 3x_2 \geq -3$$
$$x_3 - 3(x_2 + 4) \geq -15$$
$$x_3 - 3x_4 \geq -15$$

Min $|x_3| + |x_4|$

s.t.

$$x_3 + x_4 \leq 9$$
$$x_3 - 3x_4 \geq -15$$

↓

Standard Form

Min $x_3^+ + x_3^- + x_4^+ + x_4^-$

s.t.

$$(x_3^+ - x_3^-) + (x_4^+ - x_4^-) + s_1 = 9$$
$$-(x_3^+ - x_3^-) + 3(x_4^+ - x_4^-) + s_2 = 15$$
$$x_3^+, x_3^-, x_4^+, x_4^-, s_1, s_2 \geq 0$$

1.5) x_1: Quantity of Chip-1 produced
x_2: Quantity of Chip-2 produced

$$\text{Max } Z = 15x_1 + 25x_2$$
s.t.
$$3x_1 + 4x_2 \leq 100$$
$$2x_1 + 3x_2 \leq 70$$
$$x_1 + 2x_2 \leq 30$$
$$x_1 \geq 0,\ x_2 \geq 3$$

<u>standard Form</u>

$$\text{Max } Z = 15x_1 + 25x_2$$
s.t.
$$\begin{aligned} 3x_1 + 4x_2 + s_1 \quad\quad &= 100 \\ 2x_1 + 3x_2 \quad + s_2 \quad &= 70 \\ x_1 + 2x_2 \quad\quad + s_3 &= 30 \end{aligned}$$
$$x_1, s_1, s_2, s_3 \geq 0$$
$$x_2 \geq 3$$

1.6) $$\text{Min } \sum_{i=1}^{5}\sum_{j=1}^{5} c_{ij} x_{ij}$$

$$C = \begin{bmatrix} 5 & 5 & 7 & 4 & 8 \\ 6 & 5 & 8 & 3 & 7 \\ 6 & 8 & 9 & 5 & 10 \\ 7 & 6 & 6 & 3 & 6 \\ 6 & 7 & 10 & 6 & 11 \end{bmatrix}$$

s.t.
$$\sum_{j=1}^{5} x_{ij} = 1 \quad (i = 1,2,3,4,5)$$
$$\sum_{i=1}^{5} x_{ij} = 1 \quad (j = 1,2,3,4,5)$$

$x_{ij} = 0$ or 1
$(i = 1,2,3,4,5)$
$(j = 1,2,3,4,5)$

1.7) a.) $a = \begin{bmatrix} 60 \\ 45 \\ 30 \end{bmatrix}$ $b = \begin{bmatrix} 80 & 55 \end{bmatrix}$ $c = \begin{bmatrix} 190 & 230 \\ 225 & 270 \\ 180 & 250 \end{bmatrix}$

Take each lesser cost route from source to destination:

China → USA = 200 or 190

China → France = 230 or 260

India → USA = 240 or 225

India → France = 270 or 295

Phillipines → USA = 220 or 180

Phillipines → France = 250

$$\text{Min } z = \sum_{i=1}^{3} \sum_{j=1}^{2} c_{ij} x_{ij}$$

s.t.

x_{ij}: Quantity shipped from each source to each destination

$$\sum_{j=1}^{2} x_{ij} = a_i \quad (i = 1,2,3)$$

$$\sum_{i=1}^{3} x_{ij} = b_j \quad (j = 1,2)$$

$$x_{ij} \geq 0 \; ; (i = 1,2,3 \; ; \; j = 1,2)$$

b.) Nothing, based on previous formulation, Hong Kong only packages textiles shipping to France, which is already limited to 55 tons.

c.) It will force the model to split into multiple because the transportation model can't account for intermediate locations. The destination countries (USA and France) may also not receive their full demands since total demand is 135 tons and packaging capacity is 110 tons.

d) Source to Packaging Facilities

$a = \begin{bmatrix} 60 \\ 45 \\ 30 \end{bmatrix}$ $b = \begin{bmatrix} 60 & 50 \end{bmatrix}$ $C = \begin{bmatrix} 50 & 60 \\ 90 & 95 \\ 70 & 50 \end{bmatrix}$

$$\text{Min } z = \sum_{i=1}^{3} \sum_{j=1}^{2} c_{ij} x_{ij}$$

s.t.

$$\sum_{j=1}^{2} x_{ij} = a_{ij} \quad (i = 1,2,3)$$

$$\sum_{i=1}^{3} x_{ij} = b_j \quad (j = 1,2)$$

$$x_{ij} \geq 0 \,; (i = 1,2,3 \,; j = 1,2)$$

Packing Facilities to Destinations

$a = \begin{bmatrix} 60 \\ 50 \end{bmatrix}$ $b = \begin{bmatrix} 80 & 55 \end{bmatrix}$ $C = \begin{bmatrix} 150 & 180 \\ 130 & 200 \end{bmatrix}$

$$\text{Min } z = \sum_{i=1}^{2} \sum_{j=1}^{2} c_{ij} x_{ij}$$

s.t.

$$\sum_{j=1}^{2} x_{ij} = a_{ij} \quad (i = 1,2)$$

$$\sum_{i=1}^{2} x_{ij} \leq b_j \quad (j = 1,2)$$

$$x_{ij} \geq 0 \,; (i = 1,2 \,; j = 1,2)$$

please check the solution

96