OR 505 HW 2

Blake Schwartz

(a)

(1.1) $\min\ 4x_1 + \sqrt{2}\,x_2 - .35x_3$

s.t. $.001x_1 + 200x_2 - x_4 = 7\sqrt{261}$ } slack & excess var's

$7.07x_2 - 2.62x_3 + x_5 = 4$

But x_2 is still unbounded, so:

$\min\ 4x_1 + \sqrt{2}\,x_2^+ - \sqrt{2}\,x_2^- - .35x_3$

s.t. $.001x_1 + 200x_2^+ - 200x_2^- - x_4 = 7\sqrt{261}$

$7.07x_2^+ - 7.07x_2^- - 2.62x_3 + x_5 = 4$

$x_1, x_2^+, x_2^-, x_3, x_4, x_5 \geq 0$

(b) $\bar{x}_1 = x_1 - 20\ ;\ \bar{x}_3 = x_3 + 15$

(-)minimize $3.1\bar{x}_1 - 2\sqrt{2}\,x_2 - \bar{x}_3$

s.t. $100\bar{x}_1 - 20x_2 = -13$

$-11\bar{x}_1 - 7\pi x_2 - 2\bar{x}_3 + x_4 = 150$

$\bar{x}_1, x_2, \bar{x}_3, x_4 \geq 0$

(c) (-)minimize $x_1^+ - x_1^- + 3x_2 - 2x_3^+ + 2x_3^-$

s.t. $3x_1^+ - 3x_1^- - 5x_2 + x_4 = 15$

$3x_1^+ - 3x_1^- - 5x_2 - x_5 = -2$

$-5x_1^+ + 5x_1^- + 20x_2 + x_6 = 40$

$-5x_1^+ + 5x_1^- + 20x_2 - x_7 = 11$

$x_3^+ - x_3^- - x_8 = 0$

$x_1^+, x_1^-, x_2, x_3^+, x_3^-, x_4, x_5, x_6, x_7, x_8 \geq 0$

1.2 (a) minimize $2x_1^+ - 2x_1^- + 6x_2 + 8x_3$

s.t. $x_1^+ - x_1^- + 2x_2 + x_3 = 5$

$4x_1^+ - 4x_1^- + 6x_2 + 2x_3 = 12$

$x_1^+, x_1^-, x_2, x_3 \geq 0$

(b) $x_1 = 5 - 2x_2 - x_3$, so:

minimize $10 - 4x_2 - 2x_3 + 6x_2 + 8x_3$

$= 2x_2 + 6x_3 + 10$

s.t. $2x_2 + 2x_3 = 8$

$x_2, x_3 \geq 0$

(c) minimize $2x_2 + 6x_3$

s.t. $2x_2 + 2x_3 = 8$

$x_2, x_3 \geq 0$

(d) To solve, take x_3 to lowest value $= 0$

Then $2x_2 = 8$

$x_2 = 4$, $x_3 = 0$ and $x_1 = 5 - 4 - 0 = 1$

(1.3) (a) No. There is an x^2 term.

(b) Yes. Let $x_1^2 = x_2$, and you have a linear programming problem:

Minimize $2x_2 + 4x_3$

s.t. $2x_2 + 4x_3 \geq 4$

$x_2 \geq 2, \quad x_3 \geq 0$

(c) minimize $2\bar{x}_2 + 4x_3$ where $\bar{x}_2 = x_2 - 2$

s.t. $2\bar{x}_2 + 4x_3 + x_4 = 2$

$\bar{x}_2, x_3, x_4 \geq 0$

(d) The optimal solution to the linear problem is

$\bar{x}_2 = 0, \quad x_3 = 0, \quad x_4 = 2, \quad f(\bar{x}_2, x_3) = 0$

So the original problem has solution:

$x_2 = \bar{x}_2 + 2 = 2, \quad x_3 = 0; \quad x_1 = \sqrt{x_2} = \sqrt{2}$

$f(x_1, x_2, x_3) = 2 + 2 + 0 = 4$

(1.4) (a) yes. All constraints are linear

(b) minimize $x_1^+ + x_1^- + 2x_2^+ + 2x_2^- - x_3^+ - x_3^-$

s.t. $x_1^+ - x_1^- + x_2^+ - x_2^- - x_3^+ + x_3^- - x_4 = 10$

$x_1^+ - x_1^- - 3x_2^+ + 3x_2^- + 2x_3^+ - 2x_3^- = 12$

$x_1^+, x_1^-, x_2^+, x_2^-, x_3^+, x_3^-, x_4 \geq 0$

(c) Let $u = x_1 - 5$ and $v = x_2 + 4$

minimize $u^+ + u^- + v^+ + v^-$

s.t. $u^+ - u^- + v^+ - v^- - x_3 = 9$

$u^+ - u^- - 3v^+ + 3v^- + x_4 = 9$

$u^+, u^-, v^+, v^-, x_3, x_4 \geq 0$

(1.5) chip type = X_1 (Chip 1), X_2 (Chip 2)

maximize revenue = $15X_1 + 25X_2$

s.t. resource constraints: skilled labor = 100
unskilled labor = 70
raw material = 30

$$3X_1 + 4X_2 \leq 100$$
$$2X_1 + 3X_2 \leq 70$$
$$X_1 + 2X_2 \leq 30$$
$$X_1 \geq 0, \quad X_2 \geq 3$$

(1.6) minimize $60 \cdot \sum_{i=1}^{5} \sum_{j=1}^{5} X_{ij} C_{ij}$, where i = projects 1...5
j = Person A...E

C_{ij} is time for person j to finish project i

$X = \begin{cases} 1 & \text{if } j \text{ is assigned to } i \\ 0 & \text{otherwise.} \end{cases}$

s.t. $\sum_{i=1}^{5} X_{ij} = 1$ for each j

$\sum_{j=1}^{5} X_{ij} = 1$ for each i

for example, $X_{A1} + X_{A2} + X_{A3} + X_{A4} + X_{A5} = 1$
and $X_{A1} + X_{B1} + X_{C1} + X_{D1} + X_{E1} = 1$

So each project is assigned exactly one person at minimum cost.

	China	India	Philippines	USA	France
Hong Kong	X_1	X_2	X_3	X_4	X_5
Taiwan	Y_1	y_2	y_3	y_4	y_5

(1.7) (a) where the variables represent flow between nodes in the table above,

minimize $50x_1 + 90x_2 + 70x_3 + 150x_4 + 180x_5$
$+ 60y_1 + 95y_2 + 50y_3 + 130y_4 + 200y_5$

s.t. $x_1 + y_1 = 60$
$x_2 + y_2 = 45$
$x_3 + y_3 = 30$
$x_4 + y_4 = 80$
$x_5 + y_5 = 55$
} Production & demand

$x_1 + x_2 + x_3 - x_4 - x_5 = 0$
$y_1 + y_2 + y_3 - y_4 - y_5 = 0$
} Total flow in & out of Hong Kong and Taiwan

$x_1, \ldots, x_5, y_1, \ldots, y_5 \geq 0$

(b) Add the constraint $x_1 + x_2 + x_3 \leq 60$

(c) In addition to $x_1 + x_2 + x_3 \leq 60$, add the constraint $y_1 + y_2 + y_3 \leq 50$

But now total supply processing capacity = 110 < the total demand at USA and France = 135.

The LP will be infeasible.

(d) Hong Kong: minimize $50x_1 + 90x_2 + 70x_3 + 150x_4 + 180x_5$

s.t. $x_1 + x_2 + x_3 = 60$
$x_1 + x_2 + x_3 - x_4 - x_5 = 0$
$x_1, x_2, x_3, x_4, x_5 \geq 0$
$x_4 \leq 80$
$x_5 \leq 55$

Taiwan: minimize $60y_1 + 95y_2 + 50y_3 + 130y_4 + 200y_5$

s.t. $y_1 + y_2 + y_3 = 50$
$y_1 + y_2 + y_3 - y_4 - y_5 = 0$
$y_1, y_2, y_3, y_4, y_5 \geq 0$
$y_4 \leq 80$ $y_5 \leq 55$