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MA 505 HW #2

1.1 I'm using * as a substitute for bars (the book's notation) for this problem.

A. Minimize $4x_1 + \sqrt{2}(x_2^+ - x_2^-) - 0.35x_3$

Subject to $-0.001x_1 + 200(x_2^+ - x_2^-) - e_1 = 7\sqrt{261}$

$7.07(x_2^+ - x_2^-) - 2.62x_3 + s_1 = -4$

$x_1, x_2^+, x_2^-, x_3, e_1, s_1 \ge 0$

B. Minimize $3.1x_1^* - \sqrt{2}x_2 + 0.35x_3^*$

Subject to $100(x_1^* + 20) - 20x_2 = 7 \Rightarrow 100x_1^* - 20x_2 = -1993$

$-11(x_1^* + 20) - 7\pi x_2 - 2(x_3^* - 15) + s_1 = 400 \Rightarrow$

$-11x_1^* - 7\pi x_2 - 2x_3^* + s_1 = 590$

$x_1^*, x_2, x_3^*, s_1 \ge 0$

C. For an expression between inequalities, we can simply separate it into two equations. I'll do the problem in two steps. Step 1:

Minimize $-x_1 - 3x_2 + 2x_3$

Subject to $3x_1 - 5x_2 + s_1 = 15$

$3x_1 - 5x_2 - e_1 = -2$

$-5x_1 + 20x_2 + s_2 = 40$

$-5x_1 + 20x_2 - e_2 = 11$

$x_2, s_1, s_2, e_1, e_2 \ge 0, x_3 \le 10$

The next step is to properly restrain the variables. $x_1 = x_1^+ - x_1^-$ and $x_3 = 10 - x_3^*$:

Minimize $-(x_1^+ - x_1^-) - 3x_2 - 2x_3^*$

Subject to $3(x_1^+ - x_1^-) - 5x_2 + s_1 = 15$

$3(x_1^+ - x_1^-) - 5x_2 - e_1 = -2$

$-5(x_1^+ - x_1^-) + 20x_2 + s_2 = 40$

$-5(x_1^+ - x_1^-) + 20x_2 - e_2 = 11$

$x_1^+, x_1^-, x_2, x_3^*, s_1, s_2, e_1, e_2 \ge 0$

1.2 A. Minimize $2(x_1^+ - x_1^-) + 6x_2 + 8x_3$

Subject to $x_1^+ - x_1^- + 2x_2 + x_3 = 5$

$4(x_1^+ - x_1^-) + 6x_2 + 2x_3 = 12$

B. Minimize $2x_2 + 6x_3$ −1

Subject to $-2x_2 - 2x_3 = -8$

$x_2, x_3 \ge 0$

C. If we eliminate x_1 in this way, the remaining problem is in standard form.

D. There are vertices at (0,4) and (4,0). The second produces smaller result (8 vs 24). Note this would make $x_1 = -3$ or $x_1^+ = 0,\ x_1^- = 3$.

1.3 A. No. x_1^2 is a nonlinear term.

B. Yes, we can eliminate x_1^2 by substituting it for x_2.

Minimize $2x_2 + 4x_3$

Subject to $2x_2 + 4x_3 \geq 4$

$x_2, x_3 \geq 0$

C. Yes, we simply subtract a dummy variable from the constraint.

Minimize $2x_2 + 4x_3$

Subject to $2x_2 + 4x_3 - e_1 = 4$

$x_2, x_3, e_1 \geq 0$

D. We can solve for the minimum value but not the variables: $2x_2 + 4x_3 = 4 + e_1$. This is minimized when $e_1 = 0$, but the values of the variables fall on a line segment. $x_1^2 = x_2$, so the original system also has infinitely many solutions, but the minimum is still 4.

1.4 A. No. Absolute values are nonlinear.

B. Minimize $x_1^+ + x_1^{-1} + 2x_2^+ + 2x_2^- - x_3^+ - x_3^-$

Subject to $x_1^+ - x_1^{-1} + x_2^+ - x_2^- - x_3^+ + x_3^- + s_1 = 10$

$x_1^+ - x_1^{-1} - 3x_2^+ + 3x_2^- + 2x_3^+ - 2x_3^- = 12$

$x_1^+, x_1^{-1}, x_2^+, x_2^-, x_3^+, x_3^- \geq 0$

C. The first step is to fix the constraints:

Minimize $|x_1 - 5| + |x_2 + 4|$

Subject to $x_1 + x_2 + s_1 = 10$

$x_1 - 3x_2 - e_1 = 2$

From here, we can substitute in new variables: $\bar{x}_1 = x_1 - 5, \bar{x}_2 = x_2 + 4$

Minimize $|\bar{x}_1| + |\bar{x}_2|$

Subject to $\bar{x}_1 + \bar{x}_2 + s_1 = 9$

$\bar{x}_1 - 3\bar{x}_2 - e_1 = -15$

And restrain variables:

Minimize $\bar{x}_1^+ + \bar{x}_1^- + \bar{x}_2^+ + \bar{x}_2^-$

Subject to $\bar{x}_1^+ - \bar{x}_1^- + \bar{x}_2^+ - \bar{x}_2^- + s_1 = 9$

$\bar{x}_1^+ - \bar{x}_1^- - 3(\bar{x}_2^+ - \bar{x}_2^-) - e_1 = -15$

$\bar{x}_1^+, \bar{x}_1^-, \bar{x}_2^+, \bar{x}_2^-, s_1, e_1 \geq 0$

1.5 Presumably, we'd like to maximize the total selling price. So we want to

Maximize $15x_1 + 25x_2$, where x_1 is the number of Chip-1 and x_2 the number of Chip-2. The constraints have to do mostly with manpower and raw materials.

Skilled labor: $3x_1 + 4x_2 \leq 100$

Unskilled labor: $2x_1 + 3x_2 \leq 70$

Raw Material: $x_1 + 2x_2 \leq 30$

The number of chips made can't be negative, and 3 units of Chip-2 have to be made, so

$x_1 \geq 0, x_2 \geq 3$

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1.6 Presumably, we'd like to minimize hours worked. This is a bit of a peculiar problem. Basically, we can assign variables to each element in the table. For example, let $x_{i,j}$ correspond to person i and project j. $x_{i,j}$ is 1 if that person is assigned to that job and 0 otherwise. Our problem becomes : Minimize $\sum_{i=A}^{E}\sum_{j=1}^{5} c_{i,j}x_{i,j}$ where c_{ij} corresponds to the table entries. The constraints, then, are fairly straightforward.

$\forall j,\ \sum_{i=A}^{E} x_{i,j} = 1$ (each project has one person)

$\forall i,\ \sum_{j=1}^{5} x_{i,j} = 1$ (each person has one project)

Additionally, $x_{i,j} \in \{0,1\}\ \forall i,j$

1.7 A. We want to minimize shipping costs. We just need to make variables for the number of tons shipped at each rate. These are $x_{i,j}$ where i=1 is Hong Kong and i=2 Taiwan, and j's 1,2,3,4, and 5 correspond to China, India, the Philippines, the US, and France, respectively. Let $c_{i,j}$ correspond to the matrix elements formed by the table. We want to

Minimize $\sum_{i=1}^{2}\sum_{j=1}^{5} c_{i,j}x_{i,j}$. The system is

Subject to
$$x_{1,1}+x_{2,1}=60$$
$$x_{1,2}+x_{2,2}=45$$
$$x_{1,3}+x_{2,3}=30$$
$$x_{1,4}+x_{2,4}=80$$
$$x_{1,5}+x_{2,5}=55$$
$$x_{1,1},x_{2,1},x_{1,2},x_{2,2},x_{1,3},x_{2,3},x_{1,4},x_{2,4},x_{1,5},x_{2,5}\geq 0$$

B. If Hong Kong can only process 60, we gain an additional constraint:

$$x_{1,4}+x_{1,5}\leq 60$$

This is perfectly consistent as long as the Taiwan site has unlimited capacity.

C. This makes fairly substantial changes to the problem. The company order 135 tons of material with a combined market demand of 135 tons, but they only have 110 tons worth of processing capacity. Since "purchased" is in past tense, it can be assumed that all 135 tons of material ordered must be shipped. However, it's impossible to meet both market demands, so we must change the selling constraints from equalities to inequalities. We then have to make a decision as to whether or not the company actually processes all that's possible from each site. The result of not making this the case would be that the optimal solution would be to not ship processed products, so I'm going to make those equation equalities. This adds the assumption that at least 60 units make it to Hong Kong and 50 to Taiwan, but this isn't inconsistent with what we're trying to accomplish. The problem is now:

$$\text{Minimize } \sum_{i=1}^{2}\sum_{j=1}^{5} c_{i,j} x_{i,j}$$

Subject to
$$x_{1,1} + x_{2,1} = 60$$
$$x_{1,2} + x_{2,2} = 45$$
$$x_{1,3} + x_{2,3} = 30$$
$$x_{1,1} + x_{1,2} + x_{1,3} \geq 60$$
$$x_{2,1} + x_{2,2} + x_{2,3} \geq 50$$
$$x_{1,4} + x_{1,5} = 60$$
$$x_{2,4} + x_{2,5} = 50$$
$$x_{1,4} + x_{2,4} \leq 80$$
$$x_{1,5} + x_{2,5} \leq 55$$
$$x_{1,1}, x_{2,1}, x_{1,2}, x_{2,2}, x_{1,3}, x_{2,3}, x_{1,4}, x_{2,4}, x_{1,5}, x_{2,5} \geq 0$$

D. What we can do is separate the problem into two shipping problems. For shipping to the processing sites:

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$$\text{Minimize } \sum_{i=1}^{2}\sum_{j=1}^{3} c_{i,j} x_{i,j}$$

Subject to
$$x_{1,1} + x_{2,1} = 60$$
$$x_{1,2} + x_{2,2} = 45$$
$$x_{1,3} + x_{2,3} = 30$$
$$x_{1,1} + x_{1,2} + x_{1,3} \geq 60$$
$$x_{2,1} + x_{2,2} + x_{2,3} \geq 50$$
$$x_{1,1}, x_{2,1}, x_{1,2}, x_{2,2}, x_{1,3}, x_{2,3} \geq 0$$

Actually, you need $\leq$

This ensures that each site can reach full processing capacity. For sales-related shipping, we have:

$$\text{Minimize } \sum_{i=1}^{2}\sum_{j=4}^{5} c_{i,j} x_{i,j}$$

Subject to
$$x_{1,4} + x_{2,4} \leq 80$$
$$x_{1,5} + x_{2,5} \leq 55$$
$$x_{1,4} + x_{1,5} = 60$$
$$x_{2,4} + x_{2,5} = 50$$
$$x_{1,4}, x_{2,4}, x_{1,5}, x_{2,5} \geq 0$$

This will ensure that both sites reach full processing capacity and that market demand is not exceeded.

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