ISE 505
Hw # 2
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1.1 a) $\underline{\text{Min}}$ $z = 4x_1 + \sqrt{2}x_2 - 0.35x_3$

S.T $-0.001x_1 + 200x_2 \geq 7\sqrt{261}$

$7.07x_2 - 2.62x_3 \leq -4$

$x_1, x_3 \geq 0$

Min $z = 4x_1 + \sqrt{5}(x_2^+ - x_2^-) - 0.35x_3$

S.T $-0.01x_1 + 200(x_2^+ - x_2^-) - e_1 = 7\sqrt{261}$

$7.07(x_2^+ - x_2^-) - 2.62x_3 + s_1 = -4$

$x_2 = x_2^+ - x_2^-$ $\quad x_1, x_2^+, x_2^-, x_3 \geq 0$

b) $\underline{\text{Max}}$ $-3.1x_1 + 2\sqrt{2}x_2 - x_3$

S.T.

$100x_1 - 20x_2 = 7$

$-11x_1 - 7\pi x_2 - 2x_3 \leq 400$ $\quad x_1 \geq 20, x_2 \geq 0, x_3 \geq -15$

Min $z = 3.1x - 2\sqrt{2}x_2 - y_3 + 15$ $\quad y_3 = x_3 + 15$

$x_3 = y_3 - 15$

S.T $100x_1 - 20x_2 = 7$

$-11x_1 - 7\pi x_2 - 2y_3 + s_1 = 370$

$x_1 \geq 20$ $x_2, y_3 \geq 0$

c) Max $z = x_1 + 3x_2 - 2x_3$

S.T

$-2 \leq 3x_1 - 5x_2 \leq 15$

$11 \leq -5x_1 + 20x_2 \leq 40$

$x_2 \geq 0$ $\quad x_3 \leq 10$

Min $z = x_1^+ + x_1^- - 3x_2 - 2y_3$

S.T. $3(x_1^+ - x_1^-) - 5x_2 - e_1 = -2$

$3(x_1^+ - x_1^-) - 5x_2 + s_1 = 15$

$-5(x_1^+ - x_1^-) + 20x_2 - e_2 = 11$

$-5(x_1^+ - x_1^-) + 20x_2 + s_2 = 4$

$\therefore$ $x_1^+, x_1^-, x_2 \geq 0$, $\quad y_3 \geq 10$

1.2

$$\text{Min } z = 2x_1 + 6x_2 + 8x_3$$

S.T.

$$x_1 + 2x_2 + x_3 = 5$$
$$4x_1 + 6x_2 + 2x_3 = 12$$

$$x_2 \geq 0 \quad x_3 \geq 0$$

$$\text{Min } z = 2(x_1^+ - x_1^-) + 6x_2 + 8x_3$$

S.T.

$$(x_1^+ - x_1^-) + 2x_2 + x_3 = 5$$
$$4(x_1^+ - x_1^-) + 6x_2 + 2x_3 = 12 \qquad x_1^+, x_1^-, x_2, x_3 \geq 0$$

$$x_1 = 5 - 2x_2 - x_3$$

$$\text{Min } z = 10 - 2x_2 + 6x_3$$

S.T. $2x_2 + 4x_3 = 8$

$x_2 = 4$, $x_3 = 0$ $\quad z = 2$

$x_1 = -3$

1.3 Minimize $x_1^2 + x_2 + 4x_3 = z$

S.T $x_1^2 - x_2 = 0$

$2x_2 + 4x_3 \geq 4$

$x_1 \geq 0, x_2 \geq 2 \quad x_3 \geq 0$

a) no - x_1^2 is nonlinear

b) set $x_1^2 = x_2$

c) Minimize $z = 2x_2 + 4x_3$

S.T. $2x_2 + 4x_3 - e_1 = 4$

$x_2 \geq 2, \quad x_3, e_1 \geq 0$

d) $x_2 = 2 \quad x_3 = 0 \quad z = 4$

$x_1 = \sqrt{2}$

1.4 Minimize $z = |x_1| + 2|x_2| - |x_3|$

S.T $x_1 + x_2 - x_3 \leq 10$

$x_1 - 3x_2 + 2x_3 = 12$

a) no.

b) $x_i = x_i^+ - x_i^- \quad |x_i| = x_i^+ + x_i^-$

Minimize $z = x_1^+ + x_1^- + 2(x_2^+ + x_2^-) - x_3^+ - x_3^-$

S.T.

$x_1^+ - x_1^- + x_2^+ - x_2^- - x_3^+ + x_3^- + s_1 = 10$

$x_1^+ - x_1^- - 3(x_2^+ - x_2^-) + 2(x_3^+ - x_3^-) = 12$

$x_1^+, x_1^-, x_2^+, x_2^-, x_3^+, x_3^-, s_1 \geq 0$

c) $y_1 = x_1 - 5$

$y_2 = x_2 + 4$

$x_1 = y_1 + 5$

$x_2 = y_2 - 4$

Minimize $z = y_1^+ + y_1^- + y_2^+ + y_2^-$

S.T.

$y_1^+ - y_1^- + y_2^+ - y_2^- + s_1 = 9$

$y_1^+ - y_1^- - 3(y_2^+ - y_2^-) - e_1 = 9$

$y_1^+, y_1^-, y_2^+, y_2^-, s_1, e_1 \geq 0$

1.5

2 chips Chip-1 Chip-2 Chip 1 - 15 $ Chip 2 - 25

Minimize $z = -15x_1 - 25x_2$

S.T
$3x_1 + 4x_2 + s_1 = 100$ - skilled labor
$2x_1 + 3x_2 + s_2 = 70$ - unskilled labor
$x_1 + 2x_2 + s_3 = 30$ - raw material

$x_1 \geq 0 \quad x_2 \geq 3 \quad s_1, s_2, s_3 \geq 0$

1.6

$$\text{Min. } z = 300A_1 + 300A_2 + 420A_3 + 240A_4 + 480A_5 + 360B_1 + 300B_2 + 480B_3 + 180B_4 + 420B_5 + 360C_1 + 480C_2 + 540C_3 + 300C_4 + 600C_5 + 420D_1 + 360D_2 + 360D_3 + 180D_4 + 360D_5 + 360E_1 + 420E_2 + 600E_3 + 360E_4 + 660E_5$$

S.T.
$A_1 + B_1 + C_1 + D_1 + E_1 = 1$
$A_2 + B_2 + C_2 + D_2 + E_2 = 1$
$A_3 + B_3 + C_3 + D_3 + E_3 = 1$
$A_4 + B_4 + C_4 + D_4 + E_4 = 1$
$A_5 + B_5 + C_5 + D_5 + E_5 = 1$

$$\sum_{i=1}^{5} A_i = 1, \quad \sum_{i=1}^{5} B_i = 1, \quad \sum_{i=1}^{5} C_i = 1, \quad \sum_{i=1}^{5} D_i = 1, \quad \sum_{i=1}^{5} E_i = 1$$

All variables $= 0, 1$
≥ 0, binary

Or

cost $\rightarrow C_{ij}$ $\rightarrow$ from cost matrix

$$\text{Min } z = \sum_{i=1}^{5} \sum_{j=1}^{5} x_{ij} C_{ij}$$

S.T. $\sum_{i=1}^{5} x_{ij} = 1 \quad \forall j$

$\sum_{j=1}^{5} x_{ij} = 1 \quad \forall i$

$x_{ij} \geq 0$ $\rightarrow$ binary

1.7

i ↓ \ j →	China	India	Philippines	USA	France
1 Hong Kong	50	90	70	150	180
2 Taiwan	60	95	50	130	200

k →	1	2

Minimize $\sum_{i=1}^{2}\sum_{j=1}^{3} c_{ij}x_{ij} + \sum_{i=1}^{2}\sum_{k=1}^{2} c_{ik}x_{ik}$

S.T. $\sum_{i=1}^{2} x_{ij}\ (j=1) = 60$

$\sum_{i=1}^{2} x_{ij}\ \ j=2 = 45$

$\sum_{i=1}^{2} x_{ij}\ \ j=3 = 30$

$\sum_{i=1}^{2} x_{ik}\ \ k=1 = 80$

$\sum_{i=1}^{2} x_{ik}\ \ k=2 = 55$

b) add constraint

$\sum_{j=1}^{3} x_{ij}\ \ i=1 \le 60$

c) add constraint

$\sum_{j=1}^{3} x_{ij}\ \ i=2 \le 50$ and add $\le$ to all equalities constraints

cannot meet demand in US+France

d) Min $\sum_{j=1}^{2}\sum_{j=1}^{3} c_{ij}x_{ij}$ ~ Min $\sum_{i=1}^{2}\sum_{k=1}^{2} c_{ik}x_{ik}$

S.T $\sum_{i=1}^{2} x_{ij}\ \ j=1 \le 60$ S.T $\sum_{i=1}^{2} x_{ik}\ \ k=1 \le 80$

$\sum x_{ij}\ \ j=2 \le 45$ $\sum_{i=1}^{2} x_{ik}\ \ k=2 \le 55$

$\sum_{i=1}^{2} x_{ij}\ \ j=3 \le 30$ $\sum_{k=1}^{2} x_{ik}\ \ i=1 \le 60$

$\sum_{j=1}^{3} x_{ij}\ \ i=1 \le 60$ $\sum_{j=1}^{3} x_{ij}\ \ i=2 \le 50$ $\sum_{k=1}^{2} x_{ik}\ \ i=2 \le 50$