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Shilo Chaffee Lawrence
ISE 505
Fall 2017
Homework # 2

1.1:

a) Minimize $z = 4x_1 + \sqrt{2}(x_2^+ - x_2^-) - 0.35x_3$

subject to $-0.001x_1 + 200(x_2^+ - x_2^-) - x_4 = 7\sqrt{261}$

$7.07(x_2^+ - x_2^-) - 2.62x_3 + x_5 = -4$

$x_1, x_2^+, x_2^-, x_3, x_4, x_5 \geq 0$

b) Minimize $z = 3.1x_1 - 2\sqrt{2}x_2 + (x_3^+ - x_3^-)$

subject to $100x_1 - 20x_2 = 7$

$-11x_1 - 7\pi x_2 - 2(x_3^+ - x_3^-) + x_4 = 400$

$x_1 - x_5 = 20$

$(x_3^+ - x_3^-) - x_6 = -15$

$x_1, x_2, x_3^+, x_3^-, x_4, x_5, x_6 \geq 0$

1.1: c)

Minimize $z = -(x_1^+ - x_1^-) - 3x_2 + 2(x_3^+ - x_3^-)$

subject to $3(x_1^+ - x_1^-) - 5x_2 - x_4 = -2$

$3(x_1^+ - x_1^-) - 5x_2 + x_5 = 15$

$-5(x_1^+ - x_1^-) + 20x_2 - x_6 = 11$

$-5(x_1^+ - x_1^-) + 20x_2 + x_7 = 40$

$x_3^+ - x_3^- + x_8 = 10$

$x_1^+, x_1^-, x_2, x_3^+, x_3^-, x_4, x_5, x_6, x_7, x_8 \geq 0$

1.2: a)

Minimize $z = 2(x_1^+ - x_1^-) + 6x_2 + 8x_3$

Subject to $x_1^+ - x_1^- + 2x_2 + x_3 = 5$

$4(x_1^+ - x_1^-) + 6x_2 + 2x_3 = 12$

$x_1^+, x_1^-, x_2, x_3 \geq 0$

b)

$x_1 = 5 - 2x_2 - x_3$

Minimize $2(5 - 2x_2 - x_3) + 6x_2 + 8x_3$

subject to $5 - 2x_2 - x_3 + 2x_2 + x_3 = 5$

$4(5 - 2x_2 - x_3) + 6x_2 + 2x_3 = 12$

$x_2, x_3 \geq 0$

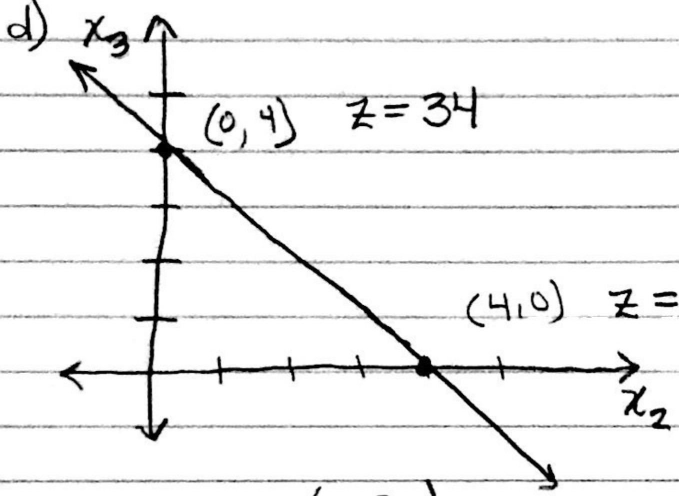
1.2:

c) Minimize $z = 2x_2 + 6x_3 + 10$

subject to $-2x_2 - 2x_3 = -8$

$x_2, x_3 \geq 0$

d)



$$x_1 = 5 - 2x_2 - x_3$$
$$= 5 - 2(4) - 0$$
$$= -3$$

$$\mathbf{x} = \begin{pmatrix} -3 \\ 4 \\ 0 \end{pmatrix}$$

1.3:

a) Minimize $x_1^2 + x_2 + 4x_3$

Subject to $x_1^2 - x_2 = 0$

$2x_2 + 4x_3 \geq 4$

$x_1 \geq 0, \quad x_2 \geq 2, \quad x_3 \geq 0$

As it is currently written, this is <u>not</u> a linear programming problem because of the x_1^2.

1.3:

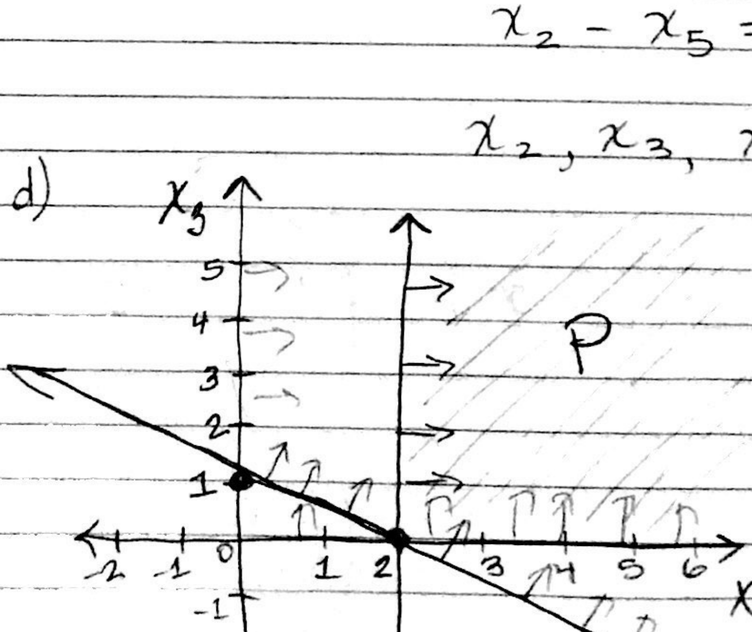
b) This problem can be converted to a linear programming problem by substituting $x_1^2 = x_2$.

$$\begin{aligned} \text{Minimize} \quad & x_2 + x_2 + 4x_3 \\ \text{subject to} \quad & x_2 - x_2 = 0 \\ & 2x_2 + 4x_3 \geq 4 \\ & x_2 \geq 2, \quad x_3 \geq 0 \end{aligned}$$

c)

$$\begin{aligned} \text{Minimize} \quad & z = 2x_2 + 4x_3 \\ \text{subject to} \quad & 2x_2 + 4x_3 - x_4 = 4 \\ & x_2 - x_5 = 2 \\ & x_2, x_3, x_4, x_5 \geq 0 \end{aligned}$$

d)



Unbounded

$x^* = (2,0)$

$x_2 = 2 \quad x_3 = 0$

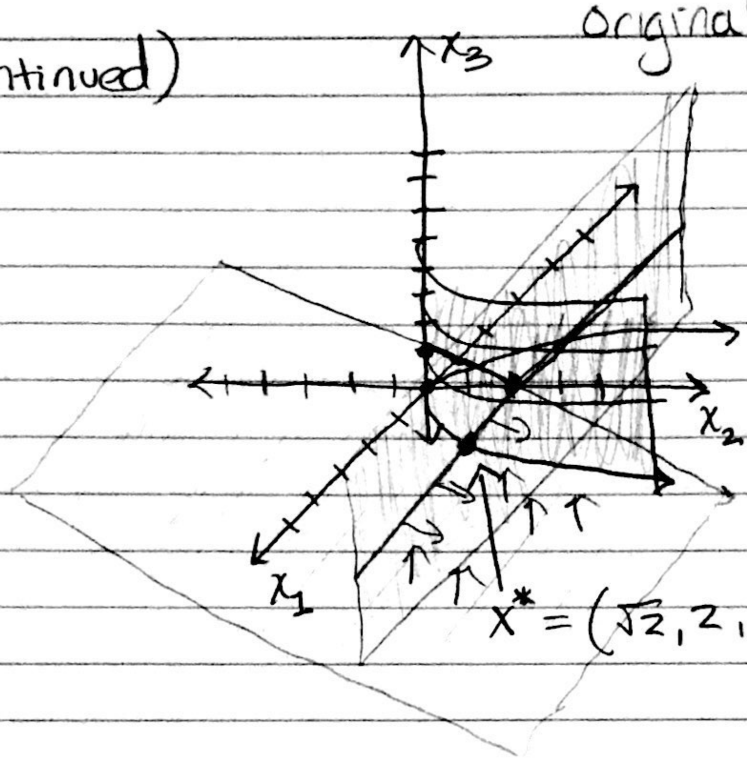
$\sqrt{x_1^2} = x_2 = \sqrt{2}$

$x_1 = \sqrt{2}$

$$x = \begin{pmatrix} \sqrt{2} \\ 2 \\ 0 \end{pmatrix}$$

1.3: d) (continued)

Original Non-linear Problem



It is a mess, and hard to see from my graph, but $(\sqrt{2}, 2, 0)$ is a vertex of the feasible domain. The linear method is <u>much</u> better.

1.4: a)

Minimize $|x_1| + 2|x_2| - |x_3|$

subject to $x_1 + x_2 - x_3 \leq 10$

$x_1 - 3x_2 + 2x_3 = 12$

As it is currently written, this is <u>not</u> a linear programming problem because of the absolute values.

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1.4:

b) Minimize $z = x_1^+ + x_1^- + 2(x_2^+ + x_2^-) - (x_3^+ + x_3^-)$

subject to $x_1^+ - x_1^- + x_2^+ - x_2^- - (x_3^+ - x_3^-) + x_4 = 10$

$x_1^+ - x_1^- - 3(x_2^+ - x_2^-) + 2(x_3^+ - x_3^-) = 12$

$x_1^+, x_1^-, x_2^+, x_2^-, x_3^+, x_3^-, x_4 \geq 0$

c) Minimize $z = x_3 + x_4$

subject to $x_1 + x_3 - x_5 = 5$

$x_1 - x_3 - x_6 = 5$

$x_2 + x_4 - x_7 = -4$

$x_2 - x_4 - x_8 = -4$

$x_1 + x_2 + x_9 = 10$

$x_1 - 3x_2 - x_{10} = 2$

$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} \geq 0$

1.5: $x_1 = \text{Chip-1}$ $\quad x_2 = \text{Chip-2}$

Minimize $\quad z = -15x_1 - 25x_2$

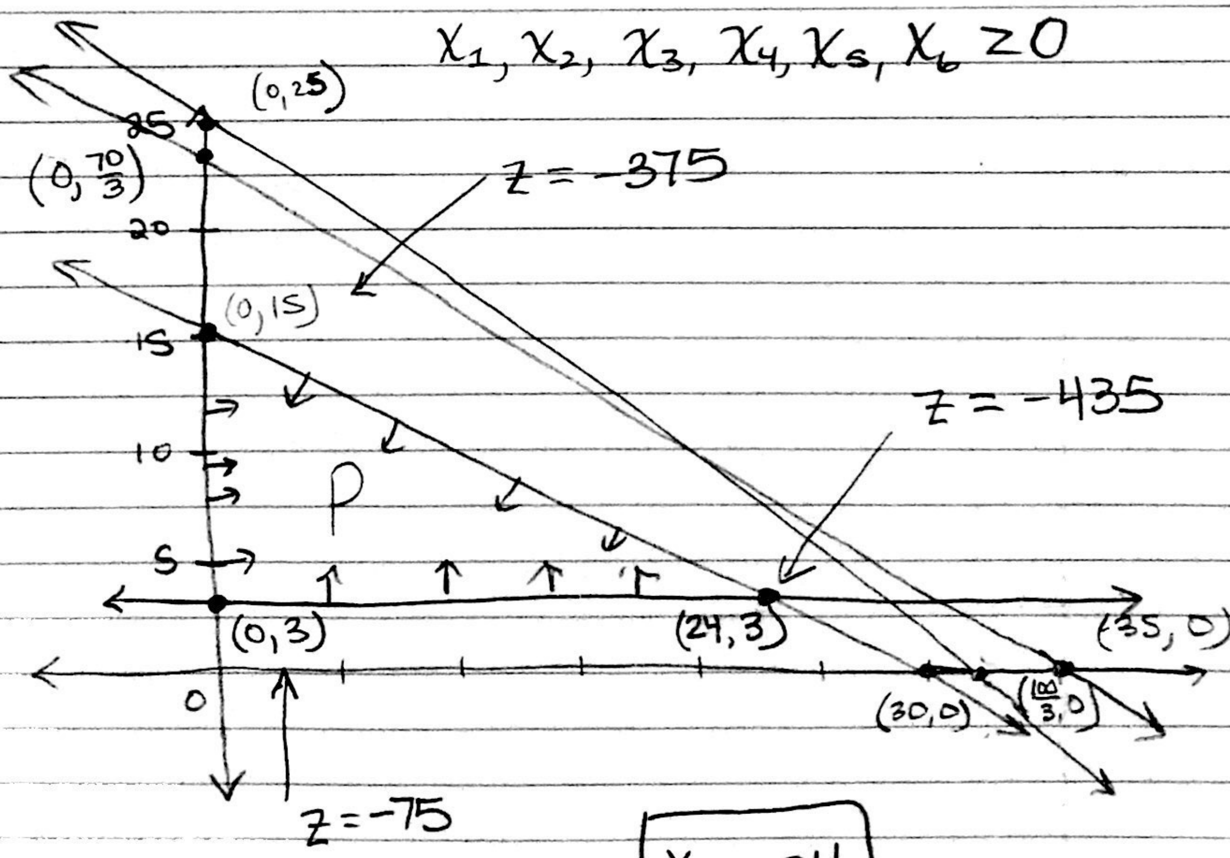
subject to $\quad 3x_1 + 4x_2 + x_3 = 100$

$$2x_1 + 3x_2 + x_4 = 70$$

$$x_1 + 2x_2 + x_5 = 30$$

$$x_2 - x_6 = 3$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$



$$\boxed{\begin{array}{c} x_1 = 24 \\ x_2 = 3 \end{array}}$$

1.6: $x_{ij} = \begin{cases} 1 & \text{if job } j \text{ is performed by worker } i \\ 0 & \text{otherwise} \end{cases}$

Minimize
$$\begin{aligned} Z = {} & 300x_{11} + 300x_{12} + 420x_{13} + 240x_{14} + 480x_{15} \\ & + 360x_{21} + 300x_{22} + 480x_{23} + 180x_{24} + 420x_{25} \\ & + 360x_{31} + 480x_{32} + 540x_{33} + 300x_{34} + 600x_{35} \\ & + 420x_{41} + 360x_{42} + 360x_{43} + 180x_{44} + 360x_{45} \\ & + 360x_{51} + 420x_{52} + 600x_{53} + 360x_{54} + 660x_{55} \end{aligned}$$

subject to
$$\begin{aligned} x_{11} + x_{12} + x_{13} + x_{14} + x_{15} &= 1 \\ x_{21} + x_{22} + x_{23} + x_{24} + x_{25} &= 1 \\ x_{31} + x_{32} + x_{33} + x_{34} + x_{35} &= 1 \\ x_{41} + x_{42} + x_{43} + x_{44} + x_{45} &= 1 \\ x_{51} + x_{52} + x_{53} + x_{54} + x_{55} &= 1 \\ x_{11} + x_{21} + x_{31} + x_{41} + x_{51} &= 1 \\ x_{12} + x_{22} + x_{32} + x_{42} + x_{52} &= 1 \\ x_{13} + x_{23} + x_{33} + x_{43} + x_{53} &= 1 \\ x_{14} + x_{24} + x_{34} + x_{44} + x_{54} &= 1 \\ x_{15} + x_{25} + x_{35} + x_{45} + x_{55} &= 1 \end{aligned}$$

$x_{ij} = 0$ or $x_{ij} = 1$ for all i and j

1.7:

a)

Minimize $Z = 50x_{11} + 90x_{12} + 70x_{13} + 150x_{14} + 180x_{15} + 60x_{21} + 95x_{22} + 50x_{23} + 130x_{24} + 200x_{25}$

subject to
$$x_{11} + x_{21} = 60$$
$$x_{12} + x_{22} = 45$$
$$x_{13} + x_{23} = 30$$
$$x_{14} + x_{24} = 80$$
$$x_{15} + x_{25} = 55$$

$$x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25} \geq 0$$

b)

Include restriction $x_{11} + x_{12} + x_{13} \leq 60$, but write it in standard form as

$$x_{11} + x_{12} + x_{13} + S_1 = 60, \quad S_1 \geq 0$$

c)

In addition to the new restrictions in part b), I would also include the restriction $x_{21} + x_{22} + x_{23} \leq 50$, but write it in standard form as

$$x_{21} + x_{22} + x_{23} + S_2 = 50, \quad S_2 \geq 0$$

We also cannot ~~meet~~ use up all of the supply, or meet all of the demand in one run, so all of the original restrictions will need slack variables. The new system will be:

c) (continued)

1.7:

Minimize: $Z = 50x_{11} + 90x_{12} + 70x_{13} + 150x_{14} + 180x_{15} + 60x_{21} + 95x_{22} + 50x_{23} + 130x_{24} + 200x_{25}$

subject to
$$x_{11} + x_{21} + S_3 = 60$$
$$x_{12} + x_{22} + S_4 = 45$$
$$x_{13} + x_{23} + S_5 = 30$$
$$x_{14} + x_{24} + S_6 = 80$$
$$x_{15} + x_{25} + S_7 = 55$$
$$x_{11} + x_{12} + x_{13} + S_1 = 60$$
$$x_{21} + x_{22} + x_{23} + S_2 = 50$$

$x_{ij} \geq 0$ for all i and j, $S_K \geq 0$ for all K

d)

Supply:

Minimize: $Z = 50x_{11} + 90x_{12} + 70x_{13} + 60x_{21} + 95x_{22} + 50x_{23}$

subject to
$$x_{11} + x_{21} + S_3 = 60$$
$$x_{12} + x_{22} + S_4 = 45$$
$$x_{13} + x_{23} + S_5 = 30$$
$$x_{11} + x_{12} + x_{13} + S_1 = 60$$
$$x_{21} + x_{22} + x_{23} + S_2 = 50$$

$x_{ij} \geq 0$ for all i and j, $S_K \geq 0$ for all K.

Demand:

Minimize: $Z = 150x_{14} + 180x_{15} + 130x_{24} + 200x_{25}$

subject to
$$x_{14} + x_{24} + S_6 = 80$$
$$x_{15} + x_{25} + S_7 = 55$$

$x_{ij} \geq 0$ for all i and j, $S_K \geq 0$ for all K