1.1 #4 (a)

Minimize $4x_1 + \sqrt{2}x_2 - 0.35x_3$
s.t. $-0.001x_1 + 200x_2 \geq 7\sqrt{261}$
$7.07x_2 - 2.62x_3 \leq -4$
$x_1, x_3 \geq 0$

surplus variables : e_1, e_2
substitution variables : $\bar{x}_2 - \hat{x}_2 = x_2$

Standard form

Minimize $4x_1 + \sqrt{2}\bar{x}_2 - \sqrt{2}\hat{x}_2 - 0.35x_3$
s.t. $-0.001x_1 + 200\bar{x}_2 - 200\hat{x}_2 - e_1 = 7\sqrt{261}$
$-7.07\bar{x}_2 + 7.07\hat{x}_2 + 2.62x_3 - e_2 = 4$
$x_1, \bar{x}_2, \hat{x}_2, x_3, e_1, e_2 \geq 0$

(b) Maximize $-3.1x_1 + 2\sqrt{2}x_2 - x_3$
s.t. $100x_1 - 20x_2 = 7$
$-11x_1 - 7\pi x_2 - 2x_3 \leq 400$
$x_1 \geq 20, x_2 \geq 0, x_3 \geq -15$

slack variable : s_1
substitutions : $\bar{x}_1 + 20 = x_1$, $\bar{x}_3 - 15 = x_3$

Standard form

Minimize $3.1\bar{x}_1 - 2\sqrt{2}x_2 + \bar{x}_3 + 45$
s.t. $-100\bar{x}_1 + 20x_2 = 1993$
$-11\bar{x}_1 - 7\pi x_2 - 2\bar{x}_3 + s_1 = 590$
$\bar{x}_1, x_2, \bar{x}_3, s_1 \geq 0$

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(c) Maximize $x_1 + 3x_2 - 2x_3$

s.t. $-2 \le 3x_1 - 5x_2 \le 15$

$11 \le -5x_1 + 20x_2 \le 40$

$x_2 \ge 0, \quad x_3 \le 10$

Split inequalities:

1) $-3x_1 + 5x_2 \le 2$

$3x_1 - 5x_2 \le 15$

2) $-5x_1 + 20x_2 \ge 11$

$-5x_1 + 20x_2 \le 40$

slack variables: s_1, s_2, s_3, s_4

surplus variable: e_1

substitutions: $x_1 = \bar{x}_1 - \hat{x}_1$

$x_3 = 10 - \bar{x}_3$

Standard form:

Minimize $-\bar{x}_1 + \hat{x}_1 - 3x_2 - 2\bar{x}_3 + 20$

s.t. $-3\bar{x}_1 + 3\hat{x}_1 + 5x_2 + s_1 = 2$

$3\bar{x}_1 - 3\hat{x}_1 - 5x_2 + s_2 = 15$

$-5\bar{x}_1 + 5\hat{x}_1 + 20x_2 - e_1 = 11$

$-5\bar{x}_1 + 5\hat{x}_1 + 20x_2 + s_3 = 40$

$\bar{x}_3 + s_4 = 10$

$\bar{x}_1, \hat{x}_1, x_2, \bar{x}_3, \hat{x}_3, s_1, s_2, s_3, s_4, e_1 \ge 0$

1.2 (a) convert to standard form

$$\text{Minimize } 2x_1 + 6x_2 + 8x_3$$
$$\text{s.t. } x_1 + 2x_2 + x_3 = 5$$
$$4x_1 + 6x_2 + 2x_3 = 12$$
$$x_2 \geq 0,\ x_3 \geq 0$$

substitution : $x_1 = \bar{x}_1 - \hat{x}_1$

standard form :

$$\text{Minimize } 2\bar{x}_1 - 2\hat{x}_1 + 6x_2 + 8x_3$$
$$\text{s.t. } \bar{x}_1 - \hat{x}_1 + 2x_2 + x_3 = 5$$
$$4\bar{x}_1 - 4\hat{x}_1 + 6x_2 + 2x_3 = 12$$
$$\bar{x}_1, \hat{x}_1, x_2, x_3 \geq 0$$

(b) let $x_1 = 5 - 2x_2 - x_3$

$$\text{Minimize } 2(5 - 2x_2 - x_3) + 6x_2 + 8x_3$$
$$\text{s.t. } 0 = 0$$
$$4(5 - 2x_2 - x_3) + 6x_2 + 2x_3 = 12$$
$$x_2, x_3 \geq 0$$

(c) standard form

$$\text{Minimize } 2x_2 + 6x_3 + 10$$
$$\text{s.t. } 2x_2 + 2x_3 = 8$$
$$x_2, x_3 \geq 0$$

(d) minimum $= 2(4) + 6(0) + 10 = 18$

at $x_2 = 4$, $x_3 = 0$

1.3

Minimize $x_1^2 + x_2 + 4x_3$

s.t. $x_1^2 - x_2 = 0$

$2x_2 + 4x_3 \geq 4$

$x_1 \geq 0,\ x_2 \geq 2,\ x_3 \geq 0$

(a) no, it is not a linear programming problem as x_1^2 is not linear

(b) yes, substitute $\bar{x}_1 = x_1^2 \rightarrow \bar{x}_1 \geq 0$
because $x_1 \geq 0$

(c) surplus variables: e_1
substitution: $\bar{x}_2 + 2 = x_2$
standard form

Minimize $\bar{x}_1 + \bar{x}_2 + 4x_3 + 4$

s.t. $\bar{x}_1 - \bar{x}_2 = 2$

$2\bar{x}_2 + 4x_3 - e_1 = 0$

$\bar{x}_1,\ \bar{x}_2,\ x_3,\ e_1 \geq 0$

(d) linear problem - yes

please solve it!

original problem - no except if $\bar{x}_1$ is a square of an integer

(Won't take pts off this time)

1.4

Minimize $|x_1| + 2|x_2| - |x_3|$

s.t. $x_1 + x_2 - x_3 \le 10$

$x_1 - 3x_2 + 2x_3 = 12$

(a) no, the absolute value function is non-linear

(b) yes, set new non-negative decision variables v_1, v_2, v_3 and u_1, u_2, u_3 such that $|x_1| = u_1 + v_1$ and $x_1 = u_1 - v_1$

$|x_2| = u_2 + v_2$ and $x_2 = u_2 - v_2$

$|x_3| = u_3 + v_3$ and $x_3 = u_3 - v_3$

then convert existing equations

please write it down -1

(c) Minimize $|x_1 - 5| + |x_2 + 4|$

s.t. $x_1 + x_2 \le 10$

$x_1 - 3x_2 \ge 2$

substitutions: $y_1 + 5 = x_1$, $y_2 - 4 = x_2$

$|y_1| = \bar{y}_1 + \hat{y}_1$ $\quad |y_2| = \bar{y}_2 + \hat{y}_2$

$y_1 = \bar{y}_1 - \hat{y}_1$ $\quad y_2 = \bar{y}_2 - \hat{y}_2$

Minimize $\bar{y}_1 + \hat{y}_1 + \bar{y}_2 + \hat{y}_2$

s.t. $\bar{y}_1 - \hat{y}_1 + \bar{y}_2 - \hat{y}_2 + s_1 = 9$

$-\bar{y}_1 + \hat{y}_1 + 3\bar{y}_2 - 3\hat{y}_2 + s_2 = 15$

$\bar{y}_1, \hat{y}_1, \bar{y}_2, \hat{y}_2, s_1, s_2 \ge 0$

1.5

$$\text{Maximize } 15x_1 + 25x_2$$
$$\text{s.t. } 3x_1 + 4x_2 \le 100$$
$$2x_1 + 3x_2 \le 70$$
$$x_1 + 2x_2 \le 30$$
$$x_1 \ge 0,\ x_2 \ge 3$$

x_1 = # chips 1 produced
x_2 = # chips 2 produced

1.6

$$\text{Minimize } 5x_{A1} + 5x_{A2} + 7x_{A3} + 4x_{A4} + 8x_{A5}$$
$$+ 6x_{B1} + 5x_{B2} + 8x_{B3} + 3x_{B4} + 7x_{B5}$$
$$+ 6x_{C1} + 8x_{C2} + 9x_{C3} + 5x_{C4} + 10x_{C5}$$
$$+ 7x_{D1} + 6x_{D2} + 6x_{D3} + 3x_{D4} + 6x_{D5}$$
$$+ 6x_{E1} + 7x_{E2} + 10x_{E3} + 6x_{E4} + 11x_{E5}$$

$$\text{s.t. } x_{A1} + x_{A2} + x_{A3} + x_{A4} + x_{A5} = 1$$
$$x_{B1} + x_{B2} + x_{B3} + x_{B4} + x_{B5} = 1$$
$$x_{C1} + x_{C2} + x_{C3} + x_{C4} + x_{C5} = 1$$
$$x_{D1} + x_{D2} + x_{D3} + x_{D4} + x_{D5} = 1$$
$$x_{E1} + x_{E2} + x_{E3} + x_{E4} + x_{E5} = 1$$
$$x_{A1} + x_{B1} + x_{C1} + x_{D1} + x_{E1} = 1$$
$$x_{A2} + x_{B2} + x_{C2} + x_{D2} + x_{E2} = 1$$
$$x_{A3} + x_{B3} + x_{C3} + x_{D3} + x_{E3} = 1$$
$$x_{A4} + x_{B4} + x_{C4} + x_{D4} + x_{E4} = 1$$
$$x_{A5} + x_{B5} + x_{C5} + x_{D5} + x_{E5} = 1$$

for $i = A{:}E$, $j = 1{:}5$ $\quad x_{ij} \ge 0$, x_{ij} integer

1.7 for x_{ij},

$i=1 \Rightarrow$ Hong Kong, $i=2 \Rightarrow$ Taiwan

$j=1 \Rightarrow$ China, $2 \Rightarrow$ India, $3 \Rightarrow$ Philippino, $4 \Rightarrow$ US, $5 \Rightarrow$ France

(a) Minimize $50x_{11} + 90x_{12} + 70x_{13} + 150x_{14} + 180x_{15}$
$+ 60x_{21} + 95x_{22} + 50x_{23} + 130x_{24} + 200x_{25}$

s.t.

$$x_{11} + x_{21} = 60$$
$$x_{12} + x_{22} = 45$$
$$x_{13} + x_{23} = 30$$
$$x_{14} + x_{24} = 80$$
$$x_{15} + x_{25} = 55$$
$$x_{11} + x_{12} + x_{13} - x_{14} - x_{15} = 0$$
$$x_{21} + x_{22} + x_{23} - x_{24} - x_{25} = 0$$
$$\text{all } x_{ij} \geq 0$$

(b) add constraints: $x_{11} + x_{12} + x_{13} \leq 60$

(c) add constraints: $x_{11} + x_{12} + x_{13} = 60$

$x_{21} + x_{22} + x_{23} = 50$

also

modify constraints

$$x_{11} + x_{21} \leq 60$$
$$x_{12} + x_{22} \leq 45$$
$$x_{13} + x_{23} \leq 30$$
$$x_{14} + x_{24} \leq 80$$
$$x_{15} + x_{25} \leq 55$$