OR 505 - HW2

1.1

a, Min: $4x_1 + \sqrt{2}x_2 - 0.35x_3$

Subject to

$$-0.001x_1 + 200x_2 \geq 7\sqrt{261} \quad -(1)$$
$$7.07x_2 - 2.62x_3 \leq -4 \quad -(2)$$
$$x_1, x_3 \geq 0$$

Converting to standard form.

i Eliminating inequalities by introducing surplus variable in equation (1) & slack variable in equation (2).

$$\Rightarrow \quad -0.001x_1 + 200x_2 - e_1 = 7\sqrt{261}$$
$$7.07x_2 - 2.62x_3 + s_1 = -4.$$

ii Converting unrestricted variables x_2 to non negative variable

let $x_2 = \bar{x}_2 - \hat{x}_2$, where $\bar{x}_2 \geq 0, \hat{x}_2 \geq 0$

$$\Rightarrow \quad -0.001x_1 + 200(\bar{x}_2 - \hat{x}_2) - e_1 = 7\sqrt{261}$$
$$7.07(\bar{x}_2 - \hat{x}_2) - 2.62x_3 + s_1 = -4.$$

$$\Rightarrow \quad -0.001x_1 + 200\bar{x}_2 - 200\hat{x}_2 - e_1 = 7\sqrt{261}$$
$$7.07\bar{x}_2 - 7.07\hat{x}_2 - 2.62x_3 + s_1 = -4$$

standard form

$$\text{Min } z = 4x_1 + \sqrt{2}\bar{x}_2 - \sqrt{2}\hat{x}_2 - 0.35x_3$$

subject to:

$$-0.001x_1 + 200\bar{x}_2 - 200\hat{x}_2 - e_1 = 7\sqrt{261}$$

$$7.07\bar{x}_2 - 7.07\hat{x}_2 - 2.62x_3 + S_1 = -4$$

where $x_1, \bar{x}_2, \hat{x}_2, x_3, e_1, S_1 \geq 0$

b) Maximize $z = -3.1x_1 + 2\sqrt{2}x_2 - x_3$

subject to

$$100x_1 - 20x_2 = 7 \quad -(1)$$

$$-11x_1 - 7\pi x_2 - 2x_3 \leq 400 \quad -(2)$$

where $x_1 \geq 20$, $x_2 \geq 0$, $x_3 \geq -15$

Convert to standard form.

i) eliminate inequality in eq (2)

$$\Rightarrow -11x_1 - 7\pi x_2 - 2x_3 + S_1 = 400$$

ii) converting restricted variables to nonnegative variables.

$x_1 \geq 20$, $x_1 = \bar{x}_1 + 20$ where $\bar{x}_1 \geq 0$

$x_3 \geq -15$, $x_3 = \bar{x}_3 - 15$ where $\bar{x}_3 \geq 0$

the new constraints will be.

$$100(\bar{x}_1+20) - 20x_2 = 7$$
$$\Rightarrow 100\bar{x}_1 + 2000 - 20x_2 = 7$$
$$\Rightarrow 100\bar{x}_1 - 20x_2 = -1993$$

$$-11(\bar{x}_1+20) - 77x_2 - 2(\bar{x}_3-15) + s_1 = 400$$
$$\Rightarrow -11\bar{x}_1 - 220 - 77x_2 - 2\bar{x}_3 + 30 + s_1 = 400$$
$$-11\bar{x}_1 - 77x_2 - 2\bar{x}_3 + s_1 = 590$$

the objective function will be.

$$\max z = -3.1(\bar{x}_1+20) + 2\sqrt{2}\,x_2 - (\bar{x}_3-15)$$
$$= -3.1\bar{x}_1 - 62 + 2\sqrt{2}x_2 - \bar{x}_3 + 15$$
$$= -3.1\bar{x}_1 + 2\sqrt{2}x_2 - \bar{x}_3 - 47.$$

Converting max to min.

$$\max(z) = -\min(z)$$
$$\Rightarrow -(-3.1\bar{x}_1 + 2\sqrt{2}x_2 - \bar{x}_3 - 47)$$
$$\Rightarrow 3.1\bar{x}_1 - 2\sqrt{2}x_2 + \bar{x}_3 + 47.$$

standard form is

$$\min z = 3.1\bar{x}_1 - 2\sqrt{2}x_2 + \bar{x}_3 + 47$$

subject to

$$100\bar{x}_1 - 20x_2 = -1993$$
$$-11\bar{x}_1 - 77x_2 - 2\bar{x}_3 + s_1 = 590$$

where $\bar{x}_1, x_2, \bar{x}_3, s_1 \geq 0$

c, max $z = x_1 + 3x_2 - 2x_3$

subject to:

$-2 \leq 3x_1 - 5x_2 \leq 15$ — ①

$11 \leq -5x_1 + 20x_2 \leq 40$ — ②

$x_2 \geq 0, x_3 \leq 10$

converting to standard form.

lets break constraints ① & ②

$\Rightarrow -2 \leq 3x_1 - 5x_2 \leq 15$

$\Rightarrow 3x_1 - 5x_2 \geq -2$ &

$3x_1 - 5x_2 \leq 15$

adding slack & surplus variables.

$3x_1 - 5x_2 - e_1 = -2$

$3x_1 - 5x_2 + s_1 = 15$

$\Rightarrow 11 \leq -5x_1 + 20x_2 \leq 40$

$\Rightarrow -5x_1 + 20x_2 \geq 11$

& $-5x_1 + 20x_2 \leq 40$

adding slack & surplus variables.

$-5x_1 + 20x_2 - e_2 = 11$

$-5x_1 + 20x_2 + s_2 = 40$

Converting restricted & unrestricted variables to non negative variables.

let $x_1 = \bar{x}_1 - \hat{x}_1$, where $\bar{x}_1, \hat{x}_1 \geq 0$

$x_3 \leq 10 \Rightarrow$ let $x_3 = 10 - \bar{x}_3$, where $\bar{x}_3 \geq 0$

the new constraints would be.

$$3(\bar{x}_1 - \hat{x}_1) - 5x_2 - e_1 = -2$$
$$\Rightarrow 3\bar{x}_1 - 3\hat{x}_1 - 5x_2 - e_1 = -2$$

$$3(\bar{x}_1 - \hat{x}_1) - 5x_2 + s_1 = 15$$
$$\Rightarrow 3\bar{x}_1 - 3\hat{x}_1 - 5x_2 + s_1 = 15$$

$$-5(\bar{x}_1 - \hat{x}_1) + 20x_2 - e_2 = 11$$
$$\Rightarrow -5\bar{x}_1 + 5\hat{x}_1 + 20x_2 - e_2 = 11$$

$$-5(\bar{x}_1 - \hat{x}_1) + 20x_2 + s_2 = 40$$
$$\Rightarrow -5\bar{x}_1 + 5\hat{x}_1 + 20x_2 + s_2 = 40$$

the new objective function would be.

$$\max z = x_1 + 3x_2 - 2(10 - \bar{x}_3)$$
$$= x_1 + 3x_2 - 20 + 2\bar{x}_3$$
$$= x_1 + 3x_2 + 2\bar{x}_3 - 20$$

converting $\max(z)$

$$\max(z) = -\min(z)$$
$$\Rightarrow -x_1 - 3x_2 - 2\bar{x}_3 + 20$$

standard form is

$$\min \ z = -x_1 - 3x_2 - 2\vec{x}_3 + 20$$

subject to

$$3\bar{x}_1 - 3\hat{x}_1 - 5x_2 - e_1 = -2$$
$$3\bar{x}_1 - 3\hat{x}_1 - 5x_2 + s_1 = +5$$
$$-5\bar{x}_1 + 5\hat{x}_1 + 20x_2 - e_2 = 11$$
$$-5\bar{x}_1 + 5\hat{x}_1 + 20x_2 + s_2 = 40$$

where

$$\bar{x}_1, \hat{x}_1, x_2, \bar{x}_3, e_1, e_2, s_1, s_2 \geq 0$$

1.2

$$\min \ z = 2x_1 + 6x_2 + 8x_3$$

subject to

$$x_1 + 2x_2 + x_3 = 5 \quad (1)$$
$$4x_1 + 6x_2 + 2x_3 = 12 \quad (2)$$
$$x_2 \geq 0, \ x_3 \geq 0$$

a) convert to standard form.

let $x_1 = \bar{x}_1 - \hat{x}_1$, where $\bar{x}_1, \hat{x}_1 \geq 0$

$$\min \ z = 2(\bar{x}_1 - \hat{x}_1) + 6x_2 + 8x_3$$
$$= 2\bar{x}_1 - 2\hat{x}_1 + 6x_2 + 8x_3$$

new constraints

$$\bar{x}_1 - \hat{x}_1 + 2x_2 + x_3 = 5.$$

$$4(\bar{x}_1 - \hat{x}_1) + 6x_2 + 2x_3 = 12.$$

$$\Rightarrow 4\bar{x}_1 - 4\hat{x}_1 + 6x_2 + 2x_3 = 12.$$

$\therefore$ standard form is

$$\min \ z = 2\bar{x}_1 - 2\hat{x}_1 + 6x_2 + 8x_3$$

subject to

$$\bar{x}_1 - \hat{x}_1 + 2x_2 + x_3 = 5$$
$$4\bar{x}_1 - 4\hat{x}_1 + 6x_2 + 2x_3 = 12$$

where $\bar{x}_1, \hat{x}_1, x_2, x_3 \geq 0$

b.

~~sol~~

from ①

let $x_1 = 5 - 2x_2 - x_3$

substituting x_1 in ② & objective function

Eq ②

$$\Rightarrow 4(5 - 2x_2 - x_3) + 6x_2 + 2x_3 = 12$$

$$= 20 - 8x_2 - 4x_3 + 6x_2 + 2x_3 = 12$$

$$-2x_2 - 2x_3 = -8$$

$$\Rightarrow 2x_2 + 2x_3 = 8$$

$$x_2 + x_3 = 4$$

z.

$$\Rightarrow 2(5 - 2x_2 - x_3) + 6x_2 + 8x_3$$

$$= 10 - 4x_2 - 2x_3 + 6x_2 + 8x_3$$

min. z. $= 2x_2 + 6x_3 + 10$.

~~where~~ ~~x_2, x_3~~

The new linear programming would be.

min $z = 2x_2 + 6x_3 + 10$

subject to

$$x_2 + x_3 = 4$$

where $x_2, x_3 \geq 0$

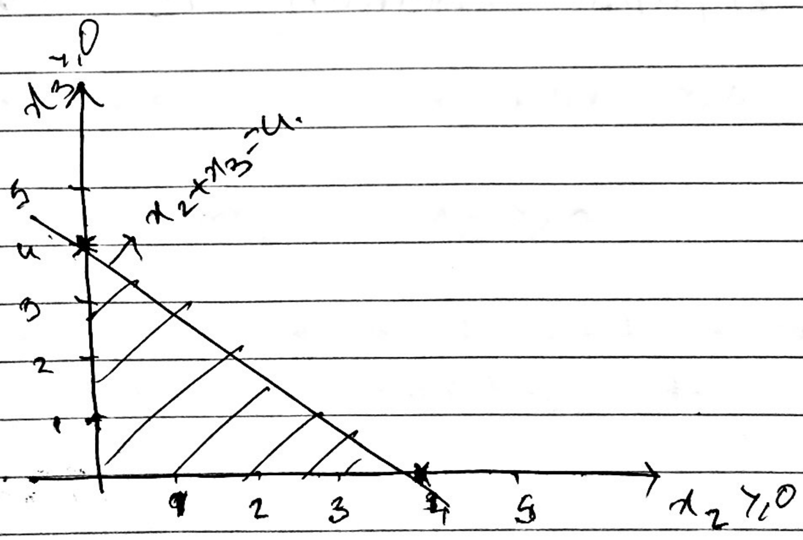
c) the converted above linear program form is in standard form.

d) Using geometrical method.

$x_2 + x_3 = 4$.

let $x_2 = 0 \Rightarrow x_3 = 4$.
let $x_3 = 0 \Rightarrow x_2 = 4$.

$\therefore$



checking for the extreme points in the feasible region ~~with the~~ in the objective function

the extreme points being

$(0,0)$, $(4,0)$, $(0,4)$

$$z = 2x_2 + 6x_3 + 10$$

$(0,0)$

$$z = 10$$

$(4,0)$

$$z = 8 + 10 = 18$$

$(0,4)$

$$z = 6(4) + 10$$
$$= 34.$$

the optimal solution is. $x_2 = 0$, $x_3 = 4$.

1.3, min $z = x_1^2 + x_2 + 4x_3$

subject to $x_1^2 - x_2 = 0$ — ①

$2x_2 + 4x_3 \geq 4$ — ②

$x_1 \geq 0, \; x_2 \geq 2, \; x_3 \geq 0$

a, No, this is not a linear program in this form.

b, Yes, we can convert into a linear program.

let $x_1^2 = x_2$ from constraint ①

$$z = x_2 + x_2 + 4x_3$$
$$= 2x_2 + 4x_3.$$

the equivalent linear program is

min $z = 2x_2 + 4x_3$

subject to $2x_2 + 4x_3 \geq 4.$

$x_2 \geq 2, x_3 \geq 0.$

c, Yes, to convert to standard form, we need to convert restricted variables to non negative variables.

$x_2 \geq 2 \Rightarrow$ let $x_2 = \bar{x}_2 + 2$.

the new objective function would be

$$\min z = 2(\bar{x}_2 + 2) + 4x_3$$

$$= 2\bar{x}_2 + 4x_3 + 4.$$

the new constraints would be

$$2(\bar{x}_2 + 2) + 4x_3 \geq 4.$$

$$\Rightarrow 2\bar{x}_2 + 4x_3 \geq 0$$

where

$$\bar{x}_2, x_3 \geq 0$$

standard form would be

$$\min z = 2\bar{x}_2 + 4x_3 + 4.$$

subject to: $2\bar{x}_2 + 4x_3 - e_1 = 0$

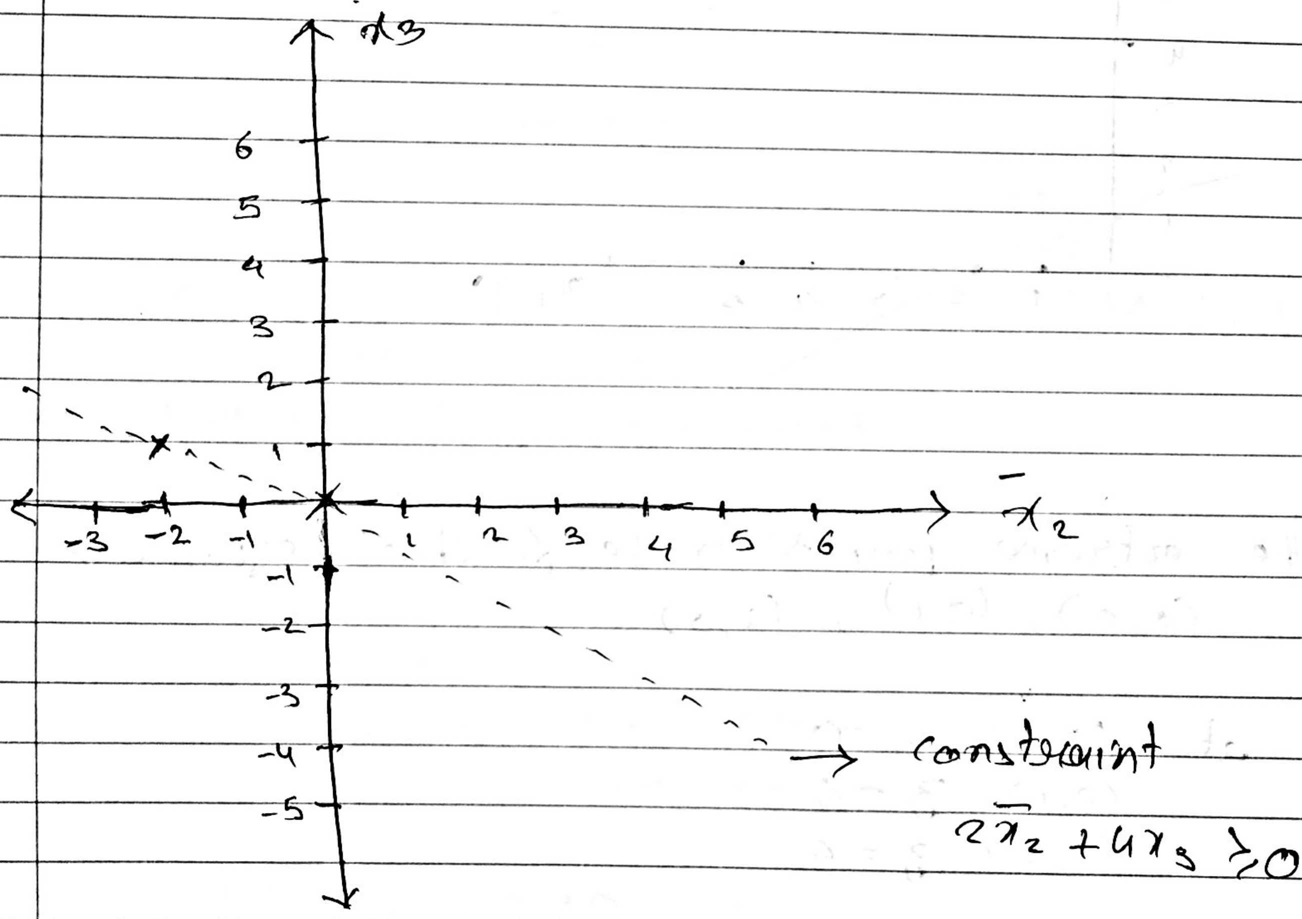
where $\bar{x}_2, x_3, e_1 \geq 0$

2) Solving using geometrical method.

$$2\bar{x}_2 + 4x_3 - e_1 = 0$$

let $\bar{x}_2 = 0 \Rightarrow x_3 = 0$

let $x_3 = 1 \Rightarrow \bar{x}_2 = -2$



there is only one point in the feasible region

$\therefore$ the solution is $\bar{x}_2 = 0$, $x_3 = 0$

1-5 let x_1 be no. of chip-1 chips.
let x_2 be no. of chip-2 chips.

$$\max z = 15x_1 + 25x_2$$

① - $3x_1 + 4x_2 \leq 100$ - skilled labor constra

② - $2x_1 + 3x_2 \leq 70$. - unskilled labor constraint

③ - $x_1 + 2x_2 \leq 30$. - raw material constraint

$x_2 \geq 3$

$x_1 \geq 0$

1.6:

	1	2	3	4	5
A	5	5	7	4	3
B	6	5	8	3	7
C	6	8	9	5	10
D	7	6	6	3	6
E	6	7	10	6	11

standard wage

= \$60 /person/day.

To formulate

let x_{ij} = assignment of person 'i' to project 'j'

objective function would be

$$\min \; z = 60(5x_{A1} + 5x_{A2} + 7x_{A3} + 4x_{A4} + 8x_{A5} + 6x_{B1} + 5x_{B2} + 8x_{B3} + 3x_{B4} + 7x_{B5} + 6x_{C1} + 8x_{C2} + 9x_{C3} + 5x_{C4} + 10x_{C5} + 7x_{D1} + 6x_{D2} + 6x_{D3} + 3x_{D4} + 6x_{D5} + 6x_{E1} + 7x_{E2} + 10x_{E3} + 6x_{E4} + 11x_{E5})$$

constraints would be.

$$x_{A1} + x_{B1} + x_{C1} + x_{D1} + x_{E1} = 1$$
$$x_{A2} + x_{B2} + x_{C2} + x_{D2} + x_{E2} = 1$$
$$x_{A3} + x_{B3} + x_{C3} + x_{D3} + x_{E3} = 1$$
$$x_{A4} + x_{B4} + x_{C4} + x_{D4} + x_{E4} = 1$$
$$x_{A5} + x_{B5} + x_{C5} + x_{D5} + x_{E5} = 1$$

where $x_{ij} = \begin{cases} 0 & \text{when project j not assigned to person i} \\ 1 & \text{when project j assigned to person i} \end{cases}$

1.7,

	China	India	Philippines	USA	France
Hongkong	50	90	70	150	180
Taiwan	60	95	50	130	200

x_{11} = tons shipped from China to Hongkong
x_{21} = " " " India to " "
x_{31} = " " " Philippines to " "
x_{41} = " " " Hongkong to USA
x_{51} = " " " " to France
x_{12} = " " " China to Taiwan
x_{22} = " " " India to Taiwan
x_{32} = " " " Philippines to "
x_{42} = " " " Taiwan to USA
x_{52} = " " " Taiwan to France

a, Min shipping cost

$$\min z = 50x_{11} + 60x_{12} + 90x_{21} + 95x_{22} + 70x_{31} + 50x_{32} + 150x_{41} + 130x_{42} + 180x_{51} + 200x_{52}$$

S.t.

$$x_{11} + x_{12} = 60$$
$$x_{21} + x_{22} = 45$$
$$x_{31} + x_{32} = 30$$
$$x_{41} + x_{42} = 80$$
$$x_{51} + x_{52} = 55$$

where $x_{11}, x_{12}, x_{21}, x_{22}, x_{31}, x_{32}, x_{41}, x_{42}, x_{51}, x_{52} \geq 0$

b) only one additional constraint should be added.

to supply side as $x_{11} + x_{21} + x_{31} \leq 60$

or to demand side as $x_{41} + x_{51} \leq 60$.

c) Similarly, 2 constraints will be added to LP.

i) $x_{11} + x_{21} + x_{31} \leq 60$ or $x_{41} + x_{51} \leq 60$

ii) $x_{12} + x_{22} + x_{32} \leq 50$ or $x_{42} + x_{52} \leq 50$

d) 2 transportation problems can be formulated

i) $\min\ z = 50x_{11} + 60x_{12} + 90x_{21} + 95x_{32} + 70x_{31} + 50x_{32}$

s.t
$$x_{11} + x_{12} = 60$$
$$x_{21} + x_{22} = 45$$
$$x_{31} + x_{32} = 30$$
$$x_{11} + x_{21} + x_{31} \leq 60$$
$$x_{12} + x_{22} + x_{32} \leq 60$$

and $x_{11}, x_{12}, x_{21}, x_{22}, x_{31}, x_{32} \geq 0$

ii) $\min\ z = 150x_{41} + 130x_{42} + 180x_{51} + 200x_{52}$

s.t
$$x_{41} + x_{42} = 80$$
$$x_{51} + x_{52} = 55$$
$$x_{41} + x_{51} \leq 60$$
$$x_{42} + x_{52} \leq 60$$

where $x_{41}, x_{42}, x_{51}, x_{52} \geq 0$