Jonathan Lovin
September 5, 2017
OR 505
Dr. Fang

Homework 2

1.1 Convert the following linear programming problems into standard form.

a) Min $4x_1 + \sqrt{2}\, x_2 - 0.35x_3$

s.t. $-0.001x_1 + 200x_2 \geq 7\sqrt{261}$

$7.07x_2 - 2.62x_3 \leq -4$

$x_1, x_3 \geq 0$

x_2 unrestricted, so replace with $x_2^+ - x_2^-$ and add constraints $x_2^+, x_2^- \geq 0$

C_1 - subtract a surplus variable e_1, $e_1 \geq 0$

C_2 - add a slack variable s_2, $s_2 \geq 0$

$$\boxed{\begin{array}{l} \text{Min } 4x_1 + \sqrt{2}(x_2^+ - x_2^-) - 0.35x_3 \\ \text{s.t. } -0.001x_1 + 200(x_2^+ - x_2^-) - e_1 = 7\sqrt{261} \\ \quad 7.07(x_2^+ - x_2^-) - 2.62x_3 + s_2 = -4 \\ \quad x_1, x_2^+, x_2^-, x_3, e_1, s_2 \geq 0 \end{array}}$$

b) Max $-3.1x_1 + 2\sqrt{2}\, x_2 - x_3$

s.t. $100x_1 - 20x_2 = 7$

$-11x_1 - 7\pi x_2 - 2x_3 \leq 400$

$x_1 \geq 20,\ x_2 \geq 0,\ x_3 \geq -15$

Objective is a max function so multiply by -1 to convert to a min function.

$C_2 \rightarrow$ add a slack variable s_2, $s_2 \geq 0$

$x_1 \geq 20 \rightarrow$ replace x_1 with $\bar{x}_1 + 20$ and replace $x_1 \geq 20$ with $\bar{x}_1 \geq 0$

$x_3 \geq -15 \rightarrow$ replace x_3 with $\bar{x}_3 - 15$ and replace $x_3 \geq -15$ with $\bar{x}_3 \geq 0$

$$\boxed{\begin{array}{l} \text{Min } 3.1(\bar{x}_1 + 20) - 2\sqrt{2}\, x_2 + (\bar{x}_3 - 15) \\ \text{s.t. } 100(\bar{x}_1 + 20) - 20x_2 = 7 \\ \quad -11(\bar{x}_1 + 20) - 7\pi x_2 - 2(\bar{x}_3 - 15) + s_2 = 400 \\ \quad \bar{x}_1, x_2, \bar{x}_3, s_1 \geq 0 \end{array}}$$

c) Max $x_1 + 3x_2 - 2x_3$

s.t. $-2 \le 3x_1 - 5x_2 \le 15$

$11 \le -5x_1 + 20x_2 \le 40$

$x_2 \ge 0,\ x_3 \le 10$

Simplify Constraints

Max $x_1 + 3x_2 - 2x_3$ → multiply by -1 to convert to a min function

s.t. $3x_1 - 5x_2 \ge -2$ → subtract a surplus variable e_1, $e_1 \ge 0$

$3x_1 - 5x_2 \le 15$ → add a slack variable s_2, $s_2 \ge 0$

$-5x_1 + 20x_2 \ge 11$ → subtract a surplus variable e_3, $e_3 \ge 0$

$-5x_1 + 20x_2 \le 40$ → add a slack variable s_4, $s_4 \ge 0$

$x_2 \ge 0,\ x_3 \le 10$ → x_1 unrestricted, so replace with $x_1^+ - x_1^-$ and add $x_1^+, x_1^- \ge 0$

→ $x_3 \le 10$ → replace x_3 with $10 - \bar{x}_3$ and $x_3 \le 10$ with $\bar{x}_3 \ge 0$

$$\boxed{\begin{array}{ll}
\text{Min} & -(x_1^+ - x_1^-) - 3x_2 + 2(10 - \bar{x}_3) \\
\text{s.t.} & 3(x_1^+ - x_1^-) - 5x_2 \qquad - e_1 = -2 \\
& 3(x_1^+ - x_1^-) - 5x_2 \qquad + s_2 = 15 \\
& -5(x_1^+ - x_1^-) + 20x_2 \qquad - e_3 = 11 \\
& -5(x_1^+ - x_1^-) + 20x_2 \qquad + s_4 = 40 \\
& x_1^+, x_2^+, x_2, \bar{x}_3, e_1, s_2, e_3, s_4 \ge 0
\end{array}}$$

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1.2. Consider a linear programming problem:

$$\text{Min } 2x_1+6x_2+8x_3$$
$$\text{s.t. } x_1+2x_2+x_3=5$$
$$4x_1+6x_2+2x_3=12$$
$$x_2, x_3 \geq 0$$

a) Convert this problem to standard form.

• x_1 unrestricted, replace with $x_1^+ - x_1^-$ and add $x_1^+, x_1^- \geq 0$

$$\text{Min } 2(x_1^+ - x_1^-)+6x_2+8x_3$$
$$\text{s.t. } (x_1^+ - x_1^-)+2x_2+x_3=5$$
$$4(x_1^+ - x_1^-)+6x_2+2x_3=12$$
$$x_1^+, x_1^-, x_2, x_3 \geq 0$$

b) Can you find an equivalent linear programming problem with only 2 variables?

$x_1+2x_2+x_3=5$ $\quad$ $4x_1+6x_2+2x_3=12$

$x_1=5-2x_2-x_3$ $\quad$ $x_1=3-1.5x_2-0.5x_3 \Rightarrow 6-3x_2-x_3=2x_1$

$$\text{Min } 2(5-2x_2-x_3)+6x_2+8x_3$$
$$3-1.5x_2-0.5x_3+2x_2+x_3=5$$
$$20-8x-4x_3+6x_2+2x_3=12$$

$\Rightarrow$

$$\text{Min } 10+2x_2+6x_3$$
$$\text{s.t. } 0.5x_2+0.5x_3=2$$
$$-2x_2-2x_3=-8$$
$$x_2, x_3 \geq 0$$

They are the same

c) Convert to standard form.

(Already in standard form)

$$\text{Min } 10+2x_2+6x_3$$
$$\text{s.t. } x_2+x_3=4$$
$$-2x_2-2x_3=-8$$
$$x_2, x_3 \geq 0$$

d) Solve the problem.

$x_2+x_3=4$ $\quad$ $-2(4-x_3)-2x_3=8$

$x_2=4-x_3$ $\quad$ $-8-2x_3-2x_3=8$

$x_2=4-0$ $\quad$ $-4x_3=0$

$x_2=4$ $\quad$ $x_3=0$

$x_1=5-2(4)-0=-3$

Objective function value of new problem
$=10+2(4)+6(0)=18$

Objective function value of original problem
$=2(-3)+6(4)+8(0)=18$

1.3 Consider the following problem:

$$\begin{aligned} \text{Min} \quad & x_1^2 + x_2 + 4x_3 \\ \text{s.t.} \quad & x_1^2 - x_2 = 0 \\ & 2x_2 + 4x_3 \geq 4 \\ & x_1 \geq 0,\ x_2 \geq 2,\ x_3 \geq 0 \end{aligned}$$

a) No, x_1^2 is not a linear function of x_1.

b) $x_1^2 - x_2 = 0 \Rightarrow x_1^2 = x_2 \rightarrow$

$$\begin{aligned} \text{Min} \quad & x_2 + x_2 + 4x_3 \\ \text{s.t.} \quad & \cancel{x_2 - x_2} = 0 \\ & 2x_2 + 4x_3 \geq 4 \\ & x_2 \geq 2 \quad x_3 \geq 0 \end{aligned}$$

$$\begin{aligned} \text{Min} \quad & 2x_2 + 4x_3 \\ \text{s.t.} \quad & 2x_2 + 4x_3 \geq 4 \\ & x_2 \geq 2,\ x_3 \geq 0 \end{aligned}$$

c) Convert to standard form.

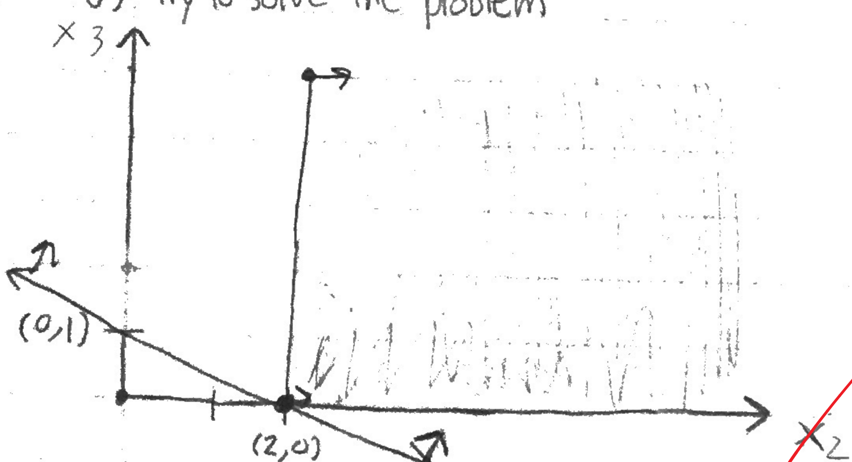
- $2x_2 + 4x_3 \geq 4$: subtract a surplus variable e_1
- $x_2 \geq 2$: replace x_2 with $\bar{x}_2 + 2$ and $x_2 \geq 2$ with $\bar{x}_2 \geq 0$

$$\begin{aligned} \text{Min} \quad & 2(\bar{x}_2 + 2) + 4x_3 \\ \text{s.t.} \quad & 2(\bar{x}_2 + 2) + 4x_3 - e_1 = 4 \\ & e_1, \bar{x}_2, x_3 \geq 0 \end{aligned}$$

$\rightarrow$

$$\begin{aligned} \text{Min} \quad & 4 + 2\bar{x}_2 + 4x_3 \\ \text{s.t.} \quad & 2\bar{x}_2 + 4x_3 - e_1 = 0 \\ & \bar{x}_2, x_3, e_1 \geq 0 \end{aligned}$$

d) Try to solve the problem



$$\boxed{\begin{aligned} & x_2 = 2 \\ & x_3 = 0 \\ & z = 4 = 2(2) + 4(0) \end{aligned}}$$

$(2,0)$: $2(2) + 4(0) = 4$

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1.4 Consider the following optimization problem:

Min $|x_1| + 2|x_2| - |x_3|$

s.t. $x_1 + x_2 - x_3 \le 10$

$x_1 - 3x_2 + 2x_3 = 12$

a) No, the absolute value function of unrestricted variables prevents this from being a linear programming problem.

b) Can you convert it into a linear program in standard form?

$|x_1|: \bar{x}_1 + \hat{x}_1 = |x_1|;\ \bar{x}_1 - \hat{x}_1 = x_1;\ \bar{x}_1, \hat{x}_1 \ge 0$

$|x_2|: \bar{x}_2 + \hat{x}_2 = |x_2|;\ \bar{x}_2 - \hat{x}_2 = x_2;\ \bar{x}_2, \hat{x}_2 \ge 0$

$|x_3|: \bar{x}_3 + \hat{x}_3 = |x_3|;\ \bar{x}_3 - \hat{x}_3 = x_3;\ \bar{x}_2, \hat{x}_2 \ge 0$

$x_1 + x_2 - x_3 \le 10$: add a slack variable s_1 and constraint $s_1 \ge 0$

$$\boxed{\begin{array}{ll} \text{Min} & (\bar{x}_1 + \hat{x}_1) + 2(\bar{x}_2 + \hat{x}_2) - (\bar{x}_3 + \hat{x}_3) \\ \text{s.t.} & (\bar{x}_1 - \hat{x}_1) + (\bar{x}_2 - \hat{x}_2) - (\bar{x}_3 - \hat{x}_2) + s_1 = 10 \\ & (\bar{x}_1 - \hat{x}_1) - 3(\bar{x}_2 - \hat{x}_2) + 2(\bar{x}_3 - \hat{x}_3) = 12 \\ & \bar{x}_1, \hat{x}_1, \bar{x}_2, \hat{x}_2, \bar{x}_3, \hat{x}_3, s_1 \ge 0 \end{array}}$$

c) Convert the following problem into a linear program in standard form:

Min $|x_1 - 5| + |x_2 + 4|$

s.t. $x_1 + x_2 \le 10$

$x_1 - 3x_2 \ge 2$

- $|x_1 - 5|: |x_1 - 5| = (\bar{x}_1 - 5) + (\hat{x}_1 - 5)$, $\bar{x}_1 - 5 \ge 0 \Rightarrow \bar{x}_1 \ge 5$, $\hat{x}_1 - 5 \ge 0 \Rightarrow \hat{x}_1 \ge 5$
- $|x_2 + 4|: |x_2 + 4| = (\bar{x}_2 + 4) + (\hat{x}_2 + 4)$, $\bar{x}_2 + 4 \ge 0 \Rightarrow \bar{x}_2 \ge -4$, $\hat{x}_2 + 4 \ge 0 \Rightarrow \hat{x}_2 \ge -4$
- $x_1: x_1 = \bar{x}_1 - \hat{x}_1$
- $x_2: x_2 = \bar{x}_2 - \hat{x}_1$

Min $(\bar{x}_1 - 5) + (\hat{x}_1 - 5) + (\bar{x}_2 + 4) + (\hat{x}_2 + 4)$

s.t. $(\bar{x}_1 - \hat{x}_1) + (\bar{x}_2 - \hat{x}_2) \le 10$ add a slack variable s_1

$(\bar{x}_1 - \hat{x}_1) - 3(\bar{x}_2 - \hat{x}_2) \ge 2$ subtract a surplus variable e_1

$\bar{x}_1, \hat{x}_1 \ge 5 \quad \bar{x}_2, \hat{x}_2 \ge -4$

$\bar{x}_1, \hat{x}_1 \ge 5$: replace $\bar{x}_1$ w/ $\bar{\bar{x}}_1 + 5$ and $\bar{x}_1 \ge 5$ with $\bar{\bar{x}}_1 \ge 0$, replace $\hat{x}_1$ w/ $\hat{\hat{x}}_1 + 5$ and $\hat{x}_1 \ge 5$ w/ $\hat{\hat{x}}_1 \ge 0$

$\bar{x}_2, \hat{x}_2 \ge -4$: replace $\bar{x}_2$ w/ $\bar{\bar{x}}_2 - 4$ and $\bar{x}_2 \ge -4$ with $\bar{\bar{x}}_2 \ge 0$, replace $\hat{x}_2$ w/ $\hat{\hat{x}}_2 - 4$ and $\hat{x}_2 \ge -4$ w/ $\hat{\hat{x}}_2 \ge 0$

$$
\begin{aligned}
\text{Min}\ & (\bar{\bar{x}}_1+5-5)+(\hat{\hat{x}}_1+5-5)+(\bar{\bar{x}}_2-4+4)+(\hat{\hat{x}}_2-4+4) \\
\text{s.t}\ & (\bar{\bar{x}}_1+5)-(\hat{\hat{x}}_1+5) \quad +(\bar{\bar{x}}_2-4)-(\hat{\hat{x}}_2-4) \quad +s_1 = 10 \\
& (\bar{\bar{x}}_1+5)-(\hat{\hat{x}}_1+5) \quad -3[(\bar{\bar{x}}_2-4)-(\hat{\hat{x}}_2-4)] \quad +e_2 = 2 \\
& \bar{\bar{x}}_1, \hat{\hat{x}}_1, \bar{\bar{x}}_2, \hat{\hat{x}}_2, s_1, e_2 \geq 0
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
\text{Min}\ & \bar{\bar{x}}_1+\hat{\hat{x}}_1+\bar{\bar{x}}_2+\hat{\hat{x}}_2 \\
\text{s.t.}\ & \bar{\bar{x}}_1-\hat{\hat{x}}_1+\bar{\bar{x}}_2-\hat{\hat{x}}_2 \quad +s_1 = 10 \\
& \bar{\bar{x}}_1-\hat{\hat{x}}_1-3\bar{\bar{x}}_2+3\hat{\hat{x}}_2+e_2 = 2 \\
& \bar{\bar{x}}_1, \hat{\hat{x}}_1, \bar{\bar{x}}_2, \hat{\hat{x}}_2, s_1, e_2 \geq 0
\end{aligned}
}
$$

1.5 x_1: #chip 1 x_2: #chip 2

x_1 cost \$15 x_2 cost \$25

x_1 { 3hr skilled labor, 2hr unskilled labor, 1 unit raw materral }

x_2 { 4hr skilled labor, 3hr unskilled labor, 2 units raw materral }

Total Resorces

100 hrs skilled labor

70 hrs unskilled labor

30 units raw materral

Requirements

At least 3 x_2 are made, fractional quantities acceptable.

Formulate a linear program.

$$
\begin{aligned}
\max\ & 15x_1 + 25x_2 \\
\text{s.t.}\ & 3x_1 + 4x_2 \leq 100 \\
& 2x_1 + 3x_2 \leq 70 \\
& x_1 + 2x_2 \leq 30 \\
& x_1 \geq 0 \\
& x_2 \geq 3
\end{aligned}
$$

Homework 2

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1.6

	Project # 1	2	3	4	5
1 = A	5	5	7	4	8
2 = B	6	5	8	3	7
3 = C	6	8	9	5	10
4 = D	7	6	6	3	6
5 = E	6	7	10	6	11

1 person per project
1 project per person
\$60 per day

Formulate a integer linear program.

$X_{ij} = \begin{cases} 1, & \text{if job } i \text{ assigned to worker } j \\ 0, & \text{otherwise} \end{cases}$ $\quad i = 1,2,3,4,5$, $j = 1,2,3,4,5$

min $60 \sum_{i=1}^{5} \sum_{j=1}^{5} X_{ij}$

s.t. $\sum_{i=1}^{5} X_{ij} = 1$, for $j = 1, \ldots, 5$

$\sum_{j=1}^{5} X_{ij} = 5$, for $i = 1, \ldots, 5$

$X_{ij} = 0$ or 1 for $i = 1, \ldots, 5$ and $j = 1, \ldots, 5$

why 5?

−1

1.7

i \ j	China (1)	India (2)	Philippines (3)	USA (4)	France (5)
(1) Hong Kong	50/ton	90/ton	70/ton	150/ton	180/ton
(2) Taiwan	60/ton	95/ton	50/ton	130/ton	200/ton

60 tons from China
45 tons from India
30 tons from Phillipines

80 tons to U.S.
55 tons to France

X_{ij} = # tons moved between distribution center i and supplier/market j

C_{ij} = cost/ton moved between distribution center i and supplier/market j

a) min $\sum_{i=1}^{2} \sum_{j=1}^{5} C_{ij} X_{ij}$ $\quad i = 1,2$, $j = 1,2,3,4,5$

s.t. $\sum_{i=1}^{2} X_{i1} = 60$

$\sum_{i=1}^{2} X_{i2} = 45$

$\sum_{i=1}^{2} X_{i3} = 30$

$\sum_{i=1}^{2} X_{i4} = 80$

$\sum_{i=1}^{2} X_{i5} = 55$

$X_{ij} \geq 0$ for $i = 1,2$, $j = 1,2,3,4,5$

b) Add the constraint

$$\sum_{j=1}^{3} X_{1j} \leq 60$$

c) The problem becomes unbalance. We will have more incoming goods than can be processed and will not be able to provide enough processed goods to meet demand.

d) Problem 1: Supply to Distribution

X_{ij} = # tons from supplier i to center j C_{ij} = cost/ton from i to j

Supply = 135 Demand = 110 , introduce a dummy center j

	j=1 Hong Kong	j=2 Taiwan	j=3 Dummy	Supply
i=1 China	50/ton	60/ton	-	60
i=2 India	90/ton	95/ton	-	45
i=3 Philippines	70/ton	50/ton	-	30
Demand	60	50	25	

$$\min \sum_{i=1}^{3} \sum_{j=1}^{3} C_{ij} X_{ij} \qquad i=1,2,3$$

s.t.

$$\sum_{j=1}^{3} X_{1j} = 60 \qquad j=1,2,3$$

$$\sum_{j=1}^{3} X_{2j} = 45$$

$$\sum_{j=1}^{3} X_{3j} = 30$$

$$\sum_{i=1}^{3} X_{i1} = 60$$

$$\sum_{i=1}^{3} X_{i2} = 50$$

$$\sum_{i=1}^{3} X_{i3} = 25$$

$X_{ij} \geq 0$ for all i and j

Not feasible

Problem 2: Distribution to Demand

Demand > Supply (135 > 110)

		j=1 U.S.	j=2 France	Supply
i=1	Hong Kong	150/ton	180/ton	60
i=2	Taiwan	130/ton	200/ton	50
i=3	Dummy	–	–	25
	Demand	80	55	

X_{ij} = # tons from center i to buyer j

C_{ij} = cost/ton from i to j

$$\min \sum_{i=1}^{3}\sum_{j=1}^{2} C_{ij}X_{ij}$$

s.t.

$$\sum_{i=1}^{3} X_{i1} = 80$$

$$\sum_{i=1}^{3} X_{i2} = 55$$

$$\sum_{j=1}^{2} X_{1j} = 60$$

$$\sum_{j=1}^{2} X_{2j} = 50$$

$$\sum_{j=1}^{2} X_{3j} = 25$$

$X_{ij} \geq 0$ for each i and j

Not feasible

please check the solution

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