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1) i) $\begin{pmatrix} a^2 & ab \\ ab & b^2 \end{pmatrix}$

Determinant $= a^2b^2 - (ab)ab$

$= \boxed{0}$

ii) $\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix} = \cos^2\alpha - (-\sin^2\alpha)$

$= \boxed{1}$

iii) $\begin{vmatrix} 1 & x & x \\ x & 2 & x \\ x & x & 3 \end{vmatrix} = 1(6-x^2) - x(3x - x^2) + x(x^2 - 2x)$

$= 6 - x^2 - 3x^2 + x^3 + x^3 - 2x^2$

$= \boxed{2x^3 - 6x^2 + 6} \rightarrow$ Ans

iv) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = 1(bc^2 - cb^2) - a(c^2 - b^2) + a^2(c-b)$

$= \boxed{bc^2 - cb^2 - ac^2 + ab^2 + a^2c - a^2b}$

v) $\begin{vmatrix} 1 & 0 & 2 & a \\ 2 & 0 & b & 0 \\ 3 & c & 4 & 5 \\ d & 0 & 0 & 0 \end{vmatrix} = 1\begin{vmatrix} 0 & b & 0 \\ c & 4 & 5 \\ 0 & 0 & 0 \end{vmatrix} - 2\begin{vmatrix} 0 & 2 & a \\ c & 4 & 5 \\ 0 & 0 & 0 \end{vmatrix}$

$+ 3\begin{vmatrix} 0 & 2 & a \\ 0 & b & 0 \\ 0 & 0 & 0 \end{vmatrix} - d\begin{vmatrix} 0 & 2 & a \\ 0 & b & 0 \\ c & 4 & 5 \end{vmatrix}$

$= -0 - 2\times 0 + 3(0) - d(c(-ab))$

$= \boxed{abcd} \rightarrow$ Ans

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vi) $\begin{vmatrix} a & 1 & 0 & 0 \\ -1 & b & 1 & 0 \\ 0 & -1 & c & 1 \\ 0 & 0 & -1 & d \end{vmatrix} = a\begin{vmatrix} b & 1 & 0 \\ -1 & c & 1 \\ 0 & -1 & d \end{vmatrix} + 1\begin{vmatrix} 1 & 0 & 0 \\ -1 & c & 1 \\ 0 & -1 & d \end{vmatrix}$

$= a\left[b(cd+1) + 1(d)\right]$

$+1\left[(cd+1)) + 1(0)\right]$

$= \boxed{abcd + a + ad + cd + 1}$

2) i) $2x_1 - \frac{1}{2}x_2 - \frac{1}{2}x_3 = 0$

$-\frac{1}{2}x_1 + 2x_2 - \frac{1}{2}x_4 = 3$

$-\frac{1}{2}x_1 + 2x_3 - \frac{1}{2}x_4 = 3$

$-\frac{1}{2}x_2 - \frac{1}{2}x_3 + 2x_4 = 0$

$\begin{bmatrix} 2 & -\frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & 2 & 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & 2 & -\frac{1}{2} \\ 0 & -\frac{1}{2} & -\frac{1}{2} & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 3 \\ 0 \end{bmatrix}$

$R_1 \to R_1/2$ ; $R_2 \to R_2/2$ ; $R_3 \to R_3/2$ ; $R_4 \to R_4/2$

$\begin{bmatrix} 1 & -\frac{1}{4} & -\frac{1}{4} & 0 \\ -\frac{1}{4} & 1 & 0 & -\frac{1}{4} \\ -\frac{1}{4} & 0 & 1 & -\frac{1}{4} \\ 0 & -\frac{1}{4} & -\frac{1}{4} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 3/2 \\ 3/2 \\ 0 \end{bmatrix}$

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$R_3 \to R_3 - R_2$ ; $R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & -\frac{1}{4} & -\frac{1}{4} & 0 \\ -\frac{1}{4} & 1 & 0 & -\frac{1}{4} \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{3}{2} \\ 0 \\ 0 \end{bmatrix}.$$

~~$R_3 \to R_3 -$~~

$x_1 - \frac{1}{4}x_2 - \frac{1}{4}x_3 = 0$

$-\frac{1}{4}x_1 + x_2 - \frac{1}{4}x_4 = \frac{3}{2}$

$-x_2 + x_3 = 0 \Rightarrow x_2 = x_3$

$-x_1 + x_4 = 0$

$x_1 = x_4$

$\Rightarrow x_1 - \frac{1}{4}x_2 - \frac{1}{4}x_2 = 0$

$\Rightarrow -\frac{1}{4}x_1 + x_2 - \frac{1}{4}x_1 = \frac{3}{2}$

$\therefore x_1 = \frac{1}{2}x_2$

$x_2 = \frac{3}{2} + \frac{1}{2}x_1$

$= \frac{3}{\cancel{4}\,2} + \frac{1}{4}x_2$

$\frac{\cancel{3}}{\cancel{4}\,2}x_2 = \frac{\cancel{3}}{\cancel{2}}$

$x_2 = 2$ $\therefore$ Ans $\to [1, 2, 2, 1]$

$x_1 = 1$

$x_4 = 1$

$x_3 = 2$

2) ii) $2x_1 + 3x_2 + 5x_3 + x_4 = 3$

$3x_1 + 4x_2 + 2x_3 + 3x_4 = -2$

$x_1 + 2x_2 + 8x_3 - x_4 = 8$

$7x_1 + 9x_2 + x_3 + 8x_4 = 0$

$$\left[\begin{array}{cccc|c} 2 & 3 & 5 & 1 & 3 \\ 3 & 4 & 2 & 3 & -2 \\ 1 & 2 & 8 & -1 & 8 \\ 7 & 9 & 1 & 8 & 0 \end{array}\right]$$

$R_2 \to R_2 - R_1$

$$\left[\begin{array}{cccc|c} 2 & 3 & 5 & +1 & 3 \\ 1 & 1 & -3 & 2 & -5 \\ 1 & 2 & 8 & -1 & 8 \\ 7 & 9 & 1 & 8 & 0 \end{array}\right]$$

$R_3 \to R_3 - R_1$

$$\left[\begin{array}{cccc|c} 2 & 3 & 5 & 1 & 3 \\ +1 & +1 & -3 & +2 & -5 \\ -1 & -1 & 3 & -2 & 5 \\ 7 & 9 & 1 & 8 & 0 \end{array}\right]$$

$R_3 \to R_3 + R_2$

$$\left[\begin{array}{cccc|c} 2 & 3 & 5 & 1 & 3 \\ 1 & 1 & -3 & 2 & -5 \\ 0 & 0 & 0 & 0 & 0 \\ 7 & 9 & 1 & 8 & 0 \end{array}\right]$$

$\therefore$ These equations have $\infty$ solutions

(6)

4) $x_1 + x_2 + x_3 = 1$

$2x_2 + 2x_3 = 1$

$x_1 - x_2 = 2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 1 & -1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & +1 \\ 1 & -2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 3 \\ -1 \end{bmatrix} \times \frac{1}{2}$$

$\therefore x_1 = 3/2 ; x_2 = 3/2 ; x_3 = -1/2$

5) i) False

let $A = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -2 \\ -3 & 6 \end{bmatrix}$

A & B are both not invertible

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But $A+B$

$$= \begin{bmatrix} 3 & 2 \\ -2 & 8 \end{bmatrix} \rightarrow \text{which is invertible}$$

5 ii) True

$(AB)(AB)^{-1} = I$

$\therefore ABB^{-1}A^{-1} = I$

For this condition to satisfy $\rightarrow BB^{-1} = I$

$\therefore AIA^{-1} = I$

For this condition to satisfy $\rightarrow AA^{-1} = I$

$\therefore AA^{-1} = I$

$$\left.\begin{array}{l} \therefore BB^{-1} = I \\ \& \\ AA^{-1} = I \end{array}\right\} \Rightarrow \text{A \& B are both invertible}$$

5iii) False

$(AB)(AB^{-1}) \neq I$

Let $B = \begin{bmatrix} 3 & 2 \\ -2 & 8 \end{bmatrix}$

$A = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$

$\Rightarrow B$ is inversible; $A \rightarrow$ not inversible.

$\therefore (AB)(AB)^{-1}$

$= ABB^{-1}A^{-1} = AIA^{-1}$

$= AA^{-1} \neq I$

$\therefore AB$ is not invertible if either A or B is ~~invertt~~ not invertible.

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5) iv) True

$AA^{-1} = I$

$kA = B$

$BB^{-1} = (kA)(kA)^{-1}$

$= kA(\frac{1}{k}A^{-1})$

$= AA^{-1} = I$

7) $\alpha = (1, 2, 1, 1)$; $\alpha_1 = (1, 1, 1, 1)$; $\alpha_2 = (1, 1, -1, -1)$
$\alpha_3 = (1, -1, 1, -1)$, $\alpha_4 = (1, -1, -1, 1)$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

where $a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 + a_4\alpha_4 = \alpha$
$a_1, a_2, a_3, a_4 \rightarrow$ constants.

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

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$$\begin{bmatrix}a_1\\a_2\\a_3\\a_4\end{bmatrix} = \frac{1}{4}\begin{bmatrix}1&1&1&1\\1&1&-1&-1\\1&-1&1&-1\\1&-1&-1&1\end{bmatrix}\begin{bmatrix}1\\2\\1\\1\end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix}5\\1\\-1\\-1\end{bmatrix}$$

$\therefore a_1 = 5/4;\ a_2 = 1/4;\ a_3 = -1/4;\ a_4 = -1/4$

$\therefore \frac{5}{4}\alpha_1 + \frac{1}{4}\alpha_2 - \frac{1}{4}\alpha_3 - \frac{1}{4}\alpha_4 = \alpha$

8) $\alpha_1 = \begin{bmatrix}1\\1\\1\end{bmatrix};\ \alpha_2 = \begin{bmatrix}0\\2\\5\end{bmatrix};\ \alpha_3 = \begin{bmatrix}1\\3\\6\end{bmatrix}$

for linear independency.

$a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 = 0$

where, $a_1$, $a_2$ & $a_3$ are zero (constants)

$$\therefore \begin{bmatrix}1&0&1\\1&2&3\\1&5&6\end{bmatrix}\begin{bmatrix}a_1\\a_2\\a_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

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$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 1 & 2 & 3 & 0 \\ 1 & 5 & 6 & 0 \end{array}\right]$$

$R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 1 & 2 & 3 & 0 \\ 0 & 5 & 5 & 0 \end{array}\right]$$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 5 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2/2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 5 & 5 & 0 \end{array}\right]$$

$\cancel{R_2 \to R_2 - 0R_2}$ $\cancel{R_2 \to R_2 - R_1}$

$a_1 + a_3 = 0$

$a_2 + a_3 = 0$

$5a_2 + 5a_3 = 0$

$\Rightarrow a_1 = a_2 = a_3 = 0$

$\Rightarrow \alpha_1, \alpha_2$ & $\alpha_3$ are linearly idependent.

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10) i) $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 1 & 2 & -1 \end{bmatrix}$

$R_4 \to R_4 + R_2 + R_3$

$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$\therefore$ Rank of matrix $= 3$

ii) $\begin{bmatrix} 3 & 2 & -1 & -3 & -2 \\ 2 & -1 & 3 & 1 & -3 \\ 4 & 5 & -5 & -6 & 1 \end{bmatrix}$

$R_1 \to R_1 - R_2$

$\begin{bmatrix} 1 & 3 & -4 & -4 & 1 \\ 2 & -1 & 3 & 1 & -3 \\ 4 & 5 & -5 & -6 & 1 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$

$\begin{bmatrix} 1 & 3 & -4 & -4 & 1 \\ 0 & -7 & 11 & 9 & -5 \\ 4 & 5 & -5 & -6 & 1 \end{bmatrix}$ $\Rightarrow$ Rank of matrix $= 3$.

33) $A = P\Lambda Q$

$P = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $\Lambda = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $Q = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

$P\Lambda Q = \begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 7 & -12 \\ 4 & -7 \end{bmatrix}$

$A^2 = \begin{bmatrix} 7 & -12 \\ 4 & -7 \end{bmatrix} \begin{bmatrix} 7 & -12 \\ 4 & -7 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$\therefore A^3 = I$

$\therefore A^8 = I$

$A^9 = I$

$A^{2n} = I$

$A^{2n+1} = I$

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a) For $\delta_1+\delta_2$, $\delta_2+\delta_3$ & $\delta_3+\delta_1$ to be linearly independent

$$a_1(\delta_1+\delta_2)+a_2(\delta_2+\delta_3)+a_3(\delta_3+\delta_1)=0$$

$$\Rightarrow a_1=a_2=a_3=0$$

$$\therefore \left[\begin{array}{c|c} \delta_1+\delta_2 & 0 \\ \delta_2+\delta_3 & 0 \\ \delta_3+\delta_1 & 0 \end{array}\right]$$

$R_1 \to R_1 - R_2 + R_3$

$$\begin{bmatrix} 2\delta_1 \\ \delta_2+\delta_3 \\ \delta_3+\delta_1 \end{bmatrix}$$

$R_2 \to R_2 - R_3 + \frac{1}{2}R_1$

$$\begin{bmatrix} 2\delta_1 \\ \delta_2 \\ \delta_3+\delta_1 \end{bmatrix}$$

$R_3 \to R_3 - \frac{1}{2}R_1$

$$\begin{bmatrix} 2\delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix}$$

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$R_1 \rightarrow \frac{1}{2} R_1$

$$\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix}$$

$\therefore$ After transformation it implies that for $(\delta_1+\delta_2), (\delta_2+\delta_3), (\delta_3+\delta_1)$ to be independent. $\delta_1, \delta_2$ & $\delta_3$ has to be linearly independent.