Exercise (1)

We list the key results at each iteration.

Iteration k	$d_y^k = -X_k r^k$	Step length α_k	x^{k+1}	$c^T x^k - c^T x^{k+1}$
0	$(-0.0682, 0.0227, 0.0227)$	14.52	$(0.0025, 0.6650, 0.3325)$	0.0825
1	$(-0.8333, 0.0021, 0.0021) \times 10^{-3}$	1.1880×10^3	$(0.0000, 0.6666, 0.3333)$	8.2500×10^{-4}
2	$(-0.8333, 0.0000, 0.0000) \times 10^{-5}$	1.1880×10^5	$(0.0000, 0.6667, 0.3333)$	8.2500×10^{-6}

Textbook Problem 7.2

(a) At the vertex $x = (0, 0, 0.5, 0.5)$, the reduced costs are $r_1 = -1$ and $r_2 = 0$. So the moving direction is $d_1 = (1, 0, -0.5, -0.5)$.

(b) $x = (0.01, 0.01, 0.49, 0.49)$. Use Karmarkar's algorithm. $d_y^k = (0.0075, -0.0025, -0.0025, -0.0025)$. Since the transformation between x and y is nonlinear, we don't have a moving "direction" in $x-$space.

(c) Use the primal affine scaling algorithm. Moving direction is $(0.9998, -0.0002, -0.4998, -0.4998) \times 10^{-4}$.

(d) Set $\mu = 0.01$. The central force is $(0.0098, 0.098, -0.0098, -0.0098)$.

Moving direction is $(0.0198, 0.0098, -0.0148, -0.0148)$.

(e) Compare the moving directions given by different algorithms. Think about how they are defined. Consider their effects on the performance of the algorithms. Think about the meaning of μ.

Textbook Problem 7.3

(a) Dual problem is

$$\begin{aligned} \max \quad & w_2 + 1 \\ \text{s.t.} \quad & w_2 \leq -1 \\ & w_2 \leq 0 \\ & w_1 + w_2 \leq 0 \\ & -w_1 + w_2 \leq 0 \end{aligned}$$

(b) Obviously.

(c) At $w = (1, -2)$, we find $s = (1, 2, 1, 3)$. $d_w^k = (-0.4848, 0.6061)$. $d_s^k = (-0.6061, -0.6061, -0.1212, -1.0909)$.

(d) This moving direction is pointing to one of the dual optimal solutions.

(e) Set $\mu = 0.01$. Centering force is $(-0.5152, 1.3939)$. The moving direction is $(-47.9697, 59.2121)$.

(f) Whether the direction in (e) is better than in (c) depends on the choice of μ. Please try different μ. Some directions fail to point to an optimal solution.

Textbook Problem 7.6

(a) Based on the representation of $x = y + q$, we may convert the original LP problem to the following LP problem in terms of the new variable y:

$$\begin{aligned} \min \quad & c^T y + c^T q \\ \text{s.t.} \quad & Ay = b - Aq \\ & y \geq 0. \end{aligned}$$

Subtracting the constant $c^T q$ in the objective, it becomes the following standard form LP problem:

$$\begin{aligned} \min \quad & c^T y \\ \text{s.t.} \quad & Ay = b - Aq \\ & y \geq 0. \end{aligned}$$

(b) For the above standard form LP problem, its dual problem is given by

$$\begin{aligned} \min \quad & (b - Aq)^T w \\ \text{s.t.} \quad & A^T w \leq c. \end{aligned}$$

When $q = 0$, the above dual problem becomes

$$\begin{aligned} \min \quad & b^T w \\ \text{s.t.} \quad & A^T w \leq c, \end{aligned}$$

which is actually a regular dual problem for the original LP problem with $q = 0$.