

# pencal: Dynamic Prediction with Numerous Longitudinal Covariates

Mathematical Institute Leiden University

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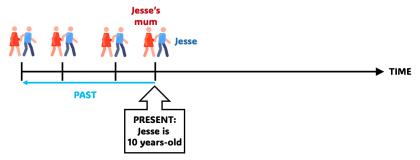
Vignette: bit.ly/pencal-UseR

in: in/signorelli

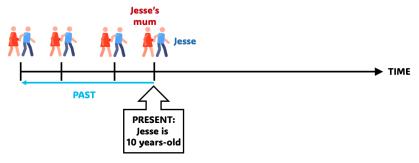
### Dynamic prediction and the R package pencal





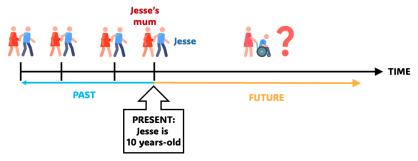






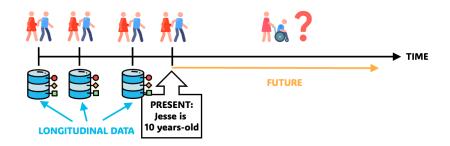
- Jesse suffers from Duchenne muscular dystrophy (DMD)
- Usually: loss of ambulation during adolescence





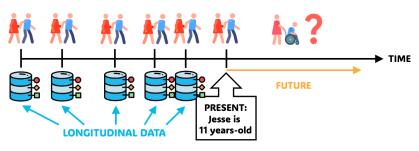
Mum: what's the probability that in 3 year's time, Jesse will need a wheelchair?







One year later:



### Dynamic prediction goals



#### Goals of dynamic prediction:

- 1. predict future survival  $S(t|\ell_1) = P(T > t|T > \ell_1), \ t > \ell_1$ , using repeated measurements over  $[0, \ell_1]$
- 2. update predictions when newer information becomes available, i.e. update  $S(t|\ell_2)$  given repeated measurements over  $[0,\ell_2],$   $t>\ell_2>\ell_1$

### Motivation



- Modern biomedical datasets can feature tens / hundreds / thousands of longitudinal predictors
- Traditional methods for dynamic prediction (joint modelling / landmarking) struggle with such datasets
- Problem: how to do dynamic prediction with "many" longitudinal predictors?

## The method: Penalized Regression Calibration (PRC)



 Statistical method: Penalized Regression Calibration (PRC, Signorelli et al. (2021))



First published: 31 August 2021 | https://doi.org/10.1002/sim.9178

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### The R package: pencal



▶ PRC implemented in the R package pencal



### The R package: pencal



#### Available on CRAN:

pencal: Penalized Regression Calibration (PRC) for the Dynamic Prediction of Survival

Computes penalized regression calibration (PRC), a statistical method for the dynamic prediction of survival when many longitudinal predictors are available. PRC is described in Signorelli (2024) <a href="doi:10.48550/arXiv.2309.15600">doi:10.48550/arXiv.2309.15600</a> and in Signorelli et al. (2021) <a href="doi:10.1002/sim.9178">doi:10.1002/sim.9178</a>>.

Version: 2.2.2 Depends:  $R (\ge 4.1.0)$ 

Imports:

doParallel, dplyr, foreach, glmnet, lcmm, magic, MASS, Matrix, methods, nlme, purrr, riskRegression,

stats, <u>survcomp</u>, <u>survival</u>, <u>survivalROC</u>
Suggests: <u>knitr</u>, <u>ptmixed</u>, <u>rmarkdown</u>, <u>survminer</u>

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License: <u>GPL (≥ 3)</u>

URL: <a href="https://mirkosignorelli.github.io/r">https://mirkosignorelli.github.io/r</a>

NeedsCompilation: no

Citation: pencal citation info
Materials: NEWS
CRAN checks: pencal results

Documentation:

Reference manual: pencal.pdf

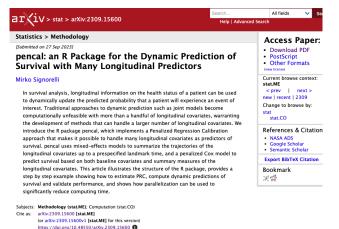
Vignettes: pencal: an R Package for the Dynamic Prediction of Survival with Many Longitudinal Predictors

Vignette: bit.ly/pencal-UseR in: in/signorelli

### Vignette



► Methodology, R package and example code illustrated in Signorelli (2024), available at arXiv:2309.15600 (and on CRAN):



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### Input data

### Example (toy) dataset



- Data from the PBC2 clinical trial (1974-1984)
  - n = 312
  - Outcome (in this example): time to death

```
library(pencal)
data(pbc2data)
sdata = pbc2data*baselineInfo
ldata = pbc2data*longitudinalInfo
```

NB: pencal designed to deal with larger p (and n), but I cannot share more recent datasets with larger p  $\odot$ 

### Input data



- ho  $\ell$  = landmark time ("PRESENT")
  - $i \in \{1, ..., n_{\ell}\}$  subjects still at risk at time  $t = \ell$
- ▶ Observation window ("PAST")  $t \in [0, \ell]$ :
  - $\blacktriangleright$  k baseline covariates  $(x_{1i},...,x_{ki})$  measured at  $t_i=0$  (study entry)
  - $\triangleright$  p longitudinal covariates  $(y_{1ii},...,y_{pii})$  measured at  $t_{i1},...,t_{im_i} \in [0,\ell]$
- ▶ Prediction window ("FUTURE")  $t \in (\ell, +\infty)$ :
  - $T_i = \min(T_i^*, C_i)$  observed survival time<sup>1</sup>
  - $\delta_i = I(T_i = T_i^*)$  event indicator



 $<sup>{}^{1}</sup>T_{i}^{*}$  true survival time,  $C_{i}$  censoring time Vignette: bit.ly/pencal-UseR

### Data preparation steps



1. Choose the landmark. Here:  $\ell=2$ 

```
lmark = 2
```

2. Select the correct subjects and corresponding data

```
# remove subjects with event / censoring before landmark:
sdata = subset(sdata, time > lmark)
ldata = subset(ldata, id %in% sdata$id)
# remove repeated measurements taken after landmark:
ldata = subset(ldata, fuptime <= lmark)</pre>
```

3. Further preprocessing (if desirable / needed)

```
# let's log-transform some highly-skewed predictors:
ldata$logSerBil = log(ldata$serBilir)
ldata$logSerChol = log(ldata$serChol)
ldata$logAlk = log(ldata$alkaline)
ldata$logSGOT = log(ldata$SGOT)
ldata$logProthr = log(ldata$prothrombin)
```

Vignette: bit.ly/pencal-UseR

### Inputs



1. A dataset (ldata) with the longitudinal covariates measured up to the landmark time:

```
##
           age fuptime logSerBil logSerChol albumin logAlk
      id
## 3
       2 56.45
                  0.00
                             0.10
                                         5.71
                                                 4.14
                                                         8.91
## 4
       2 56.95
                  0.50
                            -0.22
                                           NA
                                                 3.60
                                                        7.65
       2 57.45
                             0.00
                                                 3.55
## 5
                1.00
                                           NA
                                                        7.44
## 16
       4 54.74
                 0.00
                             0.59
                                         5.50
                                                 2.54
                                                        8.72
## 17
       4 55.26
                 0.51
                             0.47
                                           NA
                                                 2.88
                                                        7.07
## 18
       4 55.76
                  1.02
                             0.53
                                           NA
                                                 2.80
                                                        7.05
## 19
       4 56.74
                  2.00
                             1.16
                                           NA
                                                 2.92
                                                         7.07
##
      logSGOT platelets logProthr
## 3
         4.73
                     221
                              2.36
## 4
         4.94
                     188
                              2.40
## 5
         4.97
                     161
                              2.45
## 16
         4.10
                     183
                              2.33
## 17
         5.13
                     240
                              2.94
## 18
         5.11
                     251
                              2.45
## 19
         5.12
                     220
                              2.38
```

Vignette: bit.ly/pencal-UseR

### Inputs



### 2. A dataset (sdata) with the survival outcome, and baseline covariates:

```
time event baselineAge
##
     id
                                      sex treatment
## 3
      2 14.152338
                          56.44782 female D-penicil
         2.770781
                          70.07447
                                     male D-penicil
                          54.74209 female D-penicil
## 16
         5.270507
## 23 5 4.120578
                          38.10645 female
                                           placebo
## 29
        6.853028
                     1 66.26054 female
                                           placebo
## 35
         6.847552
                          55.53609 female
                                           placebo
```

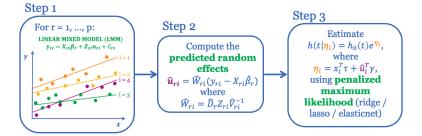
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#### Model estimation

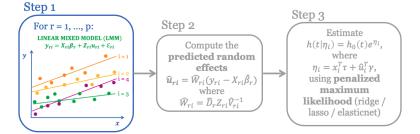
### PRC: modelling steps





### Step 1: model the longitudinal data





## Step 1: model the longitudinal data



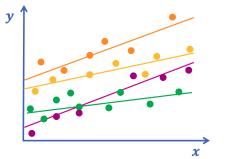
▶ Step 1: fit to each longitudinal  $Y_r$  a LMM:  $y_{ri} = X_{ri}\beta_r + Z_{ri}u_{ri} + \varepsilon_{ri}$ 

### Step 1: model the longitudinal data



Example:

$$y_{rij} = \beta_{r0} + u_{r0i} + (\beta_{r1} + u_{r1i})a_{ij} + \varepsilon_{rij},$$
 where  $u_{ri} = (u_{r0i}, u_{r1i}) \sim N(0, D_r)$  and  $\varepsilon_{ri} \sim N(0, \sigma_r^2 I_{m_i})$ 



### Step 1: fit lmms



ranefs argument follows the nlme package notation for the random effects formula (random intercept: ~ 1 | id, random slope: ~ x | id, ...)

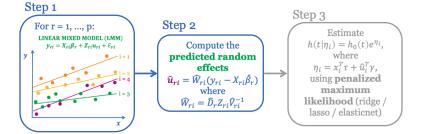
### Extracting output from the fitted LMMs



```
summary(step1, yname = 'logSerBil', what = 'betas') |> round(6)
## (Intercept) age
     0.518320 -0.001045
##
summary(step1, yname = 'logSerBil', what = 'tTable') |> round(4)
##
               Value Std.Error DF t-value p-value
## (Intercept) 0.5183 0.2788 566 1.8590 0.0636
## age -0.0010 0.0055 566 -0.1884 0.8506
summary(step1, yname = 'logSerBil', what = 'variances')
## id = pdLogChol(age)
             Variance StdDev Corr
##
## (Intercept) 7.332118e-01 0.856277849 (Intr)
## age 4.731627e-05 0.006878682 0.103
## Residual 1.437622e-01 0.379159888
```

### Step 2: summarize the longitudinal predictors





### Step 2: summarize the longitudinal predictors



- Derive subject-specific summaries of the longitudinal trajectories from the mixed-effects models
- ▶ We use the predicted random effects from the LMM:

$$\hat{u}_{ri} = E(u_{ri}|Y_{ri} = y_{ri}) = \hat{D}_r Z_i^T \hat{V}_{ri}^{-1} (y_{ri} - X_i \hat{\beta}_r),$$

where  $V_{ri} = Z_i D_r Z_i^T + \sigma_r^2 I_{m_i}$  is the marginal covariance matrix of subject i

### Step 2: summarize\_lmms



#### step2 = summarize\_lmms(step1)

Handy: summarize\_lmms automatically inherits relevant arguments from fit\_lmms ©

### Step 2: sample output



#### summary(step2)

```
## Number of predicted random effect variables: 14
## Sample size: 278
```

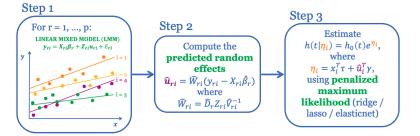
#### round(step2\$ranef.orig[1:4, 1:6], 6)

```
##
    logSerBil_b_int logSerBil_b_age logSerChol_b_int
## 2
         -0.382988
                      -0.001661 -0.071154
## 3
         -0.117107 -0.000584
                                   -0.598453
## 4
         0.168600
                     0.000922 -0.370434
## 5
         0.380035 0.001170
                                 -0.291031
##
    logSerChol_b_age albumin_b_int albumin_b_age
## 2
          0.000660
                      0.179725
                                    3e-06
                                    1e-06
## 3
          0.004916 0.018124
## 4
          0.003468 -0.529776
                                   -7e-06
## 5
          0.002886 -0.148329
                                    8e-06
```

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### Step 3: model the survival outcome





### Step 3: model the survival outcome



Cox model linking survival outcome to baseline covariates and summaries of longitudinal covariates:

$$h(t_i|x_i, \hat{u}_{0i}, \hat{u}_{1i}) = h_0(t_i) \exp(\eta_i),$$

$$\eta_i = \sum_{q=1}^k \theta_q x_{qi} + \sum_{r=1}^p \gamma_r \hat{u}_{r0i} + \sum_{r=1}^p \delta_r \hat{u}_{r1i}$$
(1)

Estimated using penalized maximum likelihood

$$\max_{\xi,\gamma,\delta} \ \ell(\xi,\gamma,\delta) - \lambda p(\xi,\gamma,\delta;\alpha)$$

▶ Penalty functions: ridge ( $\ell^2$ , recommended), lasso ( $\ell^1$ ), elastic net

### Step 3: fit\_prclmm



```
step3 = fit_prclmm(step2, surv.data = sdata,
    baseline.covs = ~ baselineAge + sex + treatment,
    penalty = 'ridge', standardize = T)
```

### Step 3: fitted model



#### summary(step3)

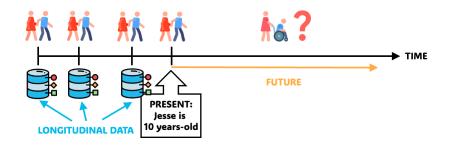
```
## Fitted model: PRC-LMM
## Penalty function used: ridge
## Tuning parameters:
##
       lambda alpha
## 1 0.2126761
## Sample size: 278
## Number of events: 107
## Bootstrap optimism correction: not computed
## Penalized likelihood estimates (rounded to 4 digits):
##
    baselineAge sexfemale treatmentD-penicil logSerBil b int
## 1
         0.0476 - 0.2872
                                      -0.0157
                                                       0.4341
##
    logSerBil b age logSerChol b int logSerChol b age
## 1
            111.3935
                              0.0986
                                              -10.5311
    albumin_b_int albumin_b_age logAlk_b_int logAlk_b_age
##
## 1
          -1.1361
                       23070.92
                                      0.0874 -12.5617
    logSGOT_b_int logSGOT_b_age platelets_b_int
##
## 1
            0.238
                        272,246
                                         -0.0011
##
    platelets_b_age logProthr_b_int logProthr_b_age
## 1
            -0.2046
                             2.8114
                                          -573.3093
```



Prediction

## Back to our goal: predicting $P(T > t | T > \ell)$





## Prediction of survival probabilities



 $lackbox{ Predicted survival probabilities: } \hat{\mathcal{S}}(h|\ell) = \hat{\mathcal{S}}(h) = e^{-\int_0^h \hat{h}_0(s)e^{\hat{\eta}_i}ds}$ 

```
Shat = survpred_prclmm(step1, step2, step3, times = 3:5)
```

▶ This will compute  $\hat{S}(t|2)$ , t = 3, 4, 5:

```
head(Shat$predicted_survival, 4) |> dfround(3)
```

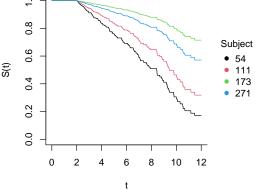
```
## id S(3) S(4) S(5)
## 2 2 0.940 0.887 0.833
## 3 3 0.856 0.739 0.632
## 4 4 0.814 0.671 0.545
## 5 5 0.946 0.898 0.849
```

 Prediction for new subjects? Possible through additional arguments new.longdata and new.basecovs

## Visualizing predictions



```
survplot_prc(step1, step2, step3,
             ids = c(54, 111, 173, 271), tmax = 12)
               8.0
```







Evaluation of predictive performance

## Evaluation of predictive performance



- ► Predictive performance estimated through a "cluster bootstrap optimism correction procedure" (Signorelli et al., 2021)
- Performance measures: C index, time-dependent AUC, Brier score

## Evaluation of predictive performance with pencal



#### 1. Rerun all 3 steps adding n.boots argument to step 1:

▶ Use n.cores argument to automatically parallelize computations

# Evaluation of predictive performance with pencal



2. Call performance\_prc to evaluate the predictive performance:

times indicates the time points at which to evaluate the time-dependent AUC and Brier score

#### Results



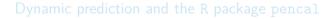
#### cbocp\$tdAUC

```
##
     pred.time tdAUC.naive optimism.correction tdAUC.adjusted
             3
                                         -0.0060
## 1
                    0.9439
                                                         0.9379
## 2
             4
                    0.9351
                                         -0.0152
                                                         0.9199
## 3
                    0.9266
                                         -0.0128
                                                         0.9138
```

### cbocp\$Brier

```
##
     pred.time Brier.naive optimism.correction Brier.adjusted
## 1
             3
                    0.0571
                                         0.0132
                                                         0.0703
## 2
             4
                    0.0699
                                         0.0251
                                                         0.0950
                    0.0844
                                         0.0307
                                                         0.1151
## 3
```

Vignette: bit.ly/pencal-UseR





Wrap-up

## Summary

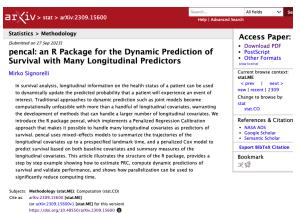


- pencal makes it easy to do dynamic prediction of survival outcomes when you have numerous longitudinal predictors
- pencal automates:
  - 1. model estimation
  - 2. prediction of survival probabilities
  - 3. evaluation of predictive performance
- Easy parallelization: just specify n.cores as argument inside pencal's functions

Vignette: bit.ly/pencal-UseR

## More about pencal

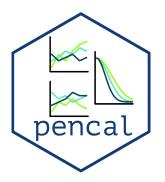
pencal



Accepted for publication in The R Journal

Vignette: bit.ly/pencal-UseR in: in/signorelli





- mirkosignorelli.github.io
- m.signorelli@math.leidenuniv.nl
- in in/signorelli

#### References



Signorelli, M. (2024). pencal: an R Package for the Dynamic Prediction of Survival with Many Longitudinal Predictors. *To Appear in: The R Journal. Preprint: arXiv:2309.15600*.

Signorelli, M., Spitali, P., Al-Khalili Sgyziarto, C., The Mark-MD Consortium, & Tsonaka, R. (2021). Penalized regression calibration: A method for the prediction of survival outcomes using complex longitudinal and high-dimensional data. Statistics in Medicine.

Vignette: bit.ly/pencal-UseR

