Quantum Communications

Contents

1. Introduction

1.1. Purpose of the project

Quantum communication is an essential area of study for enabling secure and efficient information transfer using the principles of quantum mechanics. The purpose of this project is to understand the foundations of quantum communication, especially in addressing the challenges that appear in the transmission of quantum information over long distances.

Long-distance quantum communication faces significant obstacles such as signal loss and degradation of quantum state fidelity due to noise and environmental interference. Quantum repeaters play a critical role in addressing these obstacles by dividing the communication channel into smaller segments and applying techniques such as entanglement swapping and error correction to maintain the quality of quantum states.

This project aims to simulate quantum communication channels using the Qiskit framework in Python. Through this simulation, it will be possible to evaluate the impact of quantum repeaters on improving performance, increasing transmission success rates, and reducing errors. The work will involve building a basic quantum communication model, integrating quantum repeaters, and analyzing different performance metrics such as fidelity.

2. Theoretical Overview

2.1. Principles of Quantum Communication

Quantum communication leverages the principles of quantum mechanics to transmit information in ways that are different from classical communication. Quantum communication relies on the use of qubits, which can exist in superpositions of states, and entanglement, a phenomenon where qubits become correlated in such a way that the state of one qubit is directly related to the state of another, even when separated by large distances. These properties are used in protocols like quantum key distribution (QKD), which allows two parties to generate a secure cryptographic key with information-theoretic security, meaning that any attempt of reading the transmited information can be detected.

Quantum communication also uses quantum teleportation, a process where the state of a qubit is transferred from one location to another without physically moving the qubit itself. This is achieved through the use of entangled pairs and also classical communication. These principles create the foundation of quantum communications.

2.2. Challenges in Long-Distance Quantum Communication

Despite the potential of quantum communication, transmitting quantum information over long distances faces significant challenges. One of the biggest issues is quantum decoherence, where qubits lose their quantum properties due to interactions with their environment. This is particularly problematic in optical fibers, where depolarization errors increase exponentially with distance. For example, in optical fibers, the probability of a photon being lost or its state being altered augments exponentially with the length of the fiber, making direct transmission over kilometers really problematic.

Another challenge is the no-cloning theorem, which states that it is impossible to create an exact copy of an unknown quantum state, due to the fact that we would need to measure the state and any measurement would 'disturb' the state, making it impossible to recover the original information without error. This theorem prevents the use of classical amplification techniques to boost quantum signals. As a result, traditional methods of signal amplification, which are commonly used in classical communication, cannot be used in quantum communication.

These challenges show the need for new solutions to extend the range of quantum communication, particularly for applications like quantum key distribution and also distributed quantum computing.

2.3. The Role of Quantum Repeaters in Quantum Networks

To overcome the limitations of long-distance quantum communication, quantum repeaters have become a key component of quantum networks. Quantum repeaters are devices that segment a long communication channel into shorter segments, so that each segment can be managed independently to reduce the impact of decoherence and loss. The primary function of a quantum repeater is to generate and distribute entangled pairs of qubits between different nodes, which can then be used for quantum communication protocols.

Quantum repeaters function by creating entangled pairs between neighboring nodes and then using techniques such as entanglement swapping to extend the entanglement over longer distances. This process allows for the creation of entangled pairs between distant nodes, even if there is presence of noise and loss. By dividing the communication channel into smaller segments, quantum repeaters reduce the exponential decay of quantum signals, achieving reliable communication over much greater distances.

In addition to extending the range of quantum communication, quantum repeaters also play an important role in error correction. Quantum error correction codes can be implemented at each repeater node to detect and correct errors that occur during transmission, improving the fidelity of the communication. This combination of entanglement distribution and error correction makes quantum repeaters so great for the development of large quantum networks.

2.3..1 Entanglement Swapping

Entanglement swapping is a process that allows the extension of entanglement over longer distances. In this process, two entangled pairs are created between neighbor nodes. For example, if node A is entangled with node B, and node B is entangled with node C, a Bell state measurement can be made at node B to entangle node A with node C, even though they were not directly connected. This process ”swaps” the entanglement, allowing the creation of entangled pairs between nodes that are not directly linked.

Entanglement swapping is a fundamental technique in quantum repeaters, as it enables the creation of long-distance entanglement by connecting shorter segments. This process can be repeated as many times as needed by the network.

2.3..2 Error Correction

Error correction is another important technique used in quantum repeaters. Due to the nature of quantum states, errors can easily occur during transmission, leading to decoherence and loss of information. Quantum error correction codes are designed to detect and correct these errors, ensuring that the transmitted quantum states remain intact.

One common approach involves encoding quantum information in multiple physical qubits to create logical qubits that are more stronger against errors. When errors occur during transmission, they can be detected through measurements and corrected through specific quantum operations. For example, phase-flip errors can be addressed using X-basis measurements followed by Z-gate corrections, while bit-flip errors can be handled through Z-basis measurements paired with X-gate corrections.

By combining entanglement swapping and error correction, quantum repeaters provide a powerful solution to the issues of long-distance quantum communication..

3. Implementation Process

3.1. Simulating a Basic Quantum Communication Channel

3.1..1 Description of the Model

The simulation of a basic quantum communication channel is implemented using Qiskit, a quantum computing framework. The model consists of the following key components:

- Quantum Circuit: A quantum circuit is created to simulate the transmission of quantum information over a distance. The circuit includes qubits representing the sender, receiver, and any intermediate repeater nodes.

- Noise Model: To simulate real-world conditions, a noise model is introduced. This model includes depolarizing errors and thermal relaxation errors, which mimic the effects of decoherence and imperfect operations in quantum systems.

- Bell Pair Generation: The simulation starts by generating Bell pairs (maximally entangled qubit pairs) between neighboring nodes. These pairs are the foundation for quantum communication protocols like entanglement swapping.

- Measurement and Fidelity Calculation: After simulating the transmission, the final state of the qubits is measured, and the fidelity of the Bell pairs is calculated.

3.1..2 Impact of Noise and Losses Over Long Distances

In the simulation, the impact of noise and losses over long distances is calculated using the following parameters:

- Attenuation Coefficient: This parameter represents the loss of quantum information due to the physical medium (optical fiber in out case). The attenuation coefficient is set to 0.2 dB/km, which is a realistic value for optical fibers.

- Error Probability: The error probability increases with distance, as the likelihood of decoherence and depolarization grows. The error probability is calculated as:

$$\text{error\_prob} = 1 - \exp\left(-\frac{\text{segment\_length} \times \text{attenuation\_coefficient}}{10}\right)$$

  This formula ensures that errors become more frequent as the distance between nodes increases.

- Thermal Relaxation Errors: These errors model the loss of coherence over time, with a coherence time of 0.1 ms. This simulates the gradual decay of quantum states due to interactions with the environment.

- The simulation shows that as the distance increases, the fidelity of the transmitted quantum states decreases significantly. This highlights the challenges of long-distance quantum communication and the need for techniques like quantum repeaters to mitigate these effects.

3.2. Integration of Quantum Repeaters

Quantum repeaters are integrated into the simulation to address the challenges of long-distance quantum communication. The steps in the integration are:

- Segmenting the Channel: The total communication distance is divided into smaller segments, each managed by a quantum repeater. The number of repeaters is specified by the user, and the segment length is calculated as:

$$\text{segment\_length} = \frac{\text{distance}}{\text{num\_repeaters} + 1}$$

- Entanglement Swapping: At each repeater node, entanglement swapping is performed to extend entanglement over longer distances. This involves creating Bell pairs between neighboring nodes, then, performing a Bell state measurement (BSM) on intermediary qubits and finally, applying conditional Pauli-X and Pauli-Z gates to correct the final entangled state based on the measurement outcomes.

- Error Correction: The conditional operations in the entanglement swapping process act as a simple form of error correction. By applying Pauli-X and Pauli-Z gates based on measurement results, the simulation ensures that the final entangled state is preserved despite errors introduced during transmission.

The integration of quantum repeaters improves the fidelity of the transmitted quantum states. This showcases the effectiveness of quantum repeaters in overcoming the limitations of direct quantum communication.

3.3. Step-by-Step Implementation in Qiskit

The implementation of the quantum communication channel with repeaters is implemented in Qiskit as follows:

Setup

- Install the required libraries (`qiskit`, `qiskit-aer`, `numpy`, `matplotlib`, `scipy`).
- Define the `QuantumCommunicationChannel` class

Noise Model Creation

The noise model in the simulation is designed to resemble realworld imperfections that affect quantum systems. It includes two main types of errors: depolarizing errors and thermal relaxation errors. These errors are added to the quantum gates to simulate the impact of noise on the quantum communication channel.

Depolarizing Errors

Depolarizing errors are a type of quantum error that randomize the state of a qubit. This type of error is particularly relevant in quantum communication, as it dictates the effect of noise on qubits during transmission or gate operations.

Single-Qubit Depolarizing Errors: These errors affect individual qubits and are applied to single-qubit gates like the identity gate (`id`), Pauli-X gate (`x`), and Hadamard gate (`h`).

The error probability for single-qubit depolarizing errors is calculated as:

$$\text{error\_prob} = 1 - \exp\left(-\frac{\text{segment\_length} \times \text{attenuation\_coefficient}}{10}\right)$$

This formula ensures that the error probability increases with the segment length.

The depolarizing error is implemented using `depolarizing_error(error_prob, 1)`, where 1 indicates that it is a single-qubit error.

Two-Qubit Depolarizing Errors: These errors affect pairs of qubits and are applied to two-qubit gates like the CNOT gate (`cx`).

The error probability for two-qubit depolarizing errors is set to be twice that of single-qubit errors, up to a maximum of 100%:

$$\text{two\_qubit\_error\_prob} = \min\left(2 \times \text{error\_prob}, 1.0\right)$$

This reflects the fact that two-qubit gates have generally more probability to errors than single-qubit gates.

The depolarizing error is implemented using `depolarizing_error(two_qubit_error_prob, 2)`, where 2 indicates that it is a two-qubit error.

Thermal Relaxation Errors

Thermal relaxation errors model the loss of coherence in qubits over time due to interactions with the environment. These errors are particularly important for simulating the decoherence of quantum states.

T1 and T2 Times:

- **T1 (longitudinal relaxation time):** This is the time it takes for a qubit to decay from the excited state ($|1\rangle$) to the ground state ($|0\rangle$). It represents energy dissipation.
- **T2 (transverse relaxation time):** This is the time it takes for the qubit's phase coherence to decay. It represents the loss of quantum information due to dephasing.

In the simulation, T1 is set to 0.1 ms, and T2 is set to half of T1 ($T2 = \frac{T1}{2}$), which is a common assumption in quantum systems.

Thermal Relaxation Error Model: The thermal relaxation error is implemented using `thermal_relaxation_error(t1, t2, gate_time)`, where:

- `t1` and `t2` are the coherence times.
- `gate_time` is the duration of the gate operation, which is set to 0 in this case because they are practically instantaneous.

This error is added to single-qubit gates (`id`, `x`, `h`) to simulate the gradual decay of quantum states over time.

Bell Pair Generation

- Define a method `create_bell_pair()` to generate a Bell pair using a Hadamard gate (`h`) and a CNOT gate (`cx`).

Entanglement Swapping

Entanglement swapping is a technique in quantum communication that allows the extension of entanglement over longer distances by connecting shorter entangled segments. The `perform_entanglement_swapping()` method implements this process, which involves the following steps:

Steps in Entanglement Swapping

Apply a CNOT Gate and a Hadamard Gate:

- A CNOT gate is applied to the intermediary qubits (`q2` and `q3`). This gate entangles the two qubits, creating a correlation between them.
- A Hadamard gate is then applied to `q2`. This gate puts `q2` into a superposition state, which is necessary for the measurement.

Measure the Intermediary Qubits:

- The intermediary qubits (`q2` and `q3`) are measured, and the results are stored in classical registers (`cr_offset` and `cr_offset + 1`).
- These measurements determine the state of the final entangled pair (`q1` and `q4`).

Apply Conditional Pauli-X and Pauli-Z Gates:

- Based on the measurement results, conditional Pauli-X and Pauli-Z gates are applied to the final qubits (`q1` and `q4`) to correct the state and ensure that the final entanglement is conserved.

Conditional Corrections

The conditional corrections are applied based on the measurement outcomes of `q2` and `q3`. The logic behind these corrections is the following.

First Conditional (Pauli-X Gates):

- The first condition checks if the measurement result of `q2` (stored in `cr_offset`) is `True` (the qubit was measured as 1).
- If this condition is met, Pauli-X gates are applied to `q1` and `q4`.
- **Why Pauli-X?** The Pauli-X gate flips the state of the qubit ($|0\rangle \to |1\rangle$ and $|1\rangle \to |0\rangle$). This correction is necessary because the measurement of `q2` as 1 indicates that the state of `q1` and `q4` needs to be flipped to maintain the correct entanglement.

Second Conditional (Pauli-Z Gates):

- The second condition checks if the measurement result of `q3` (stored in `cr_offset + 1`) is `True` (the qubit was measured as 1).
- If this condition is met, Pauli-Z gates are applied to `q1` and `q4`.
- **Why Pauli-Z?** The Pauli-Z gate introduces a phase flip ($|1\rangle \to -|1\rangle$). This correction is necessary because the measurement of `q3` as 1 indicates that the phase of the state of `q1` and `q4` needs to be corrected to maintain the correct entanglement.

When Both Conditions Are Met:

- If both conditions are met (both `q2` and `q3` are measured as 1), both Pauli-X and Pauli-Z gates are applied to `q1` and `q4`.
- **Why Both?** In this case, both a bit-flip and a phase-flip correction are needed. The Pauli-X gate corrects the bit-flip, and the Pauli-Z gate corrects the phase-flip, ensuring that the final state is properly entangled.

Why These Corrections Are Necessary

The corrections are necessary because the measurement of the intermediary qubits (`q2` and `q3`) affects the state of the final qubits (`q1` and `q4`). Without these corrections, the final entangled state would not be preserved, and the entanglement swapping process would fail. The conditional Pauli-X and Pauli-Z gates ensure that the final state is corrected based on the measurement outcomes, maintaining the fidelity of the entanglement.

By applying these conditional corrections, the `perform_entanglement_swapping()` method ensures that the final entangled state between `q1` and `q4` is preserved, regardless of the measurement outcomes of the intermediary qubits. This makes entanglement swapping a powerful technique for extending entanglement over long distances in quantum communication networks.

Simulation Execution

- Define the `simulate_communication()` method to simulate the quantum communication process:
  - Create a quantum circuit with the required number of qubits and classical registers.
  - Generate Bell pairs between neighboring nodes.
  - Perform entanglement swapping at each repeater node.
  - Measure the final qubits and calculate the fidelity of the Bell pairs.

Performance Analysis

- Define the `analyze_performance()` function to evaluate the performance of the quantum communication channel for different distances and repeater configurations.
- Run the simulation for various distances the different distances and repeater counts.
- Collect and analyze the results, including the fidelity of the transmitted quantum states.

4. Performance Analysis

System Performance Without Quantum Repeaters

When quantum repeaters are not used, the performance of the quantum communication channel degrades significantly as the distance increases. This is due to the exponential increase in error probability caused by decoherence and depolarization errors. The results for different distances without repeaters are as follows:

- **1 km:**
  - Error Probability: $\sim 1.98\%$
  - Fidelity: 0.0020
- **5 km:**
  - Error Probability: $\sim 9.52\%$
  - Fidelity: 0.0020
- **10 km:**
  - Error Probability: $\sim 18.13\%$
  - Fidelity: 0.0020
- **50 km:**
  - Error Probability: $\sim 63.21\%$
  - Fidelity: 0.0020

Observations:

- The fidelity remains extremely low (0.0020) across all distances, indicating that the quantum communication channel is practically unusable without repeaters.
- The error probability increases significantly with distance, reaching over 60% at 50 km. This highlights the fundamental challenge of long-distance quantum communication without error mitigation techniques.

Conclusion: Without quantum repeaters, the system is unable to maintain the fidelity of the transmitted quantum states, even over relatively short distances. This demonstrates the necessity of quantum repeaters for reliable long-distance quantum communication.

System Performance With Quantum Repeaters

The introduction of quantum repeaters significantly improves the performance of the quantum communication channel. By segmenting the communication path and performing entanglement swapping, the system can maintain higher fidelity over longer distances. The results for different distances with different numbers of repeaters are the following:

- **1 km:**
  - **1 Repeater:**
    - Error Probability: $\sim 0.99\%$
    - Fidelity: 0.0078
  - **2 Repeaters:**
    - Error Probability: $\sim 0.66\%$
    - Fidelity: 0.0264
  - **4 Repeaters:**
    - Error Probability: $\sim 0.40\%$
    - Fidelity: 0.2432
  - **5 Repeaters:**
    - Error Probability: $\sim 0.33\%$
    - Fidelity: 0.4004
- **5 km:**
  - **1 Repeater:**
    - Error Probability: $\sim 4.88\%$
    - Fidelity: 0.0078
  - **2 Repeaters:**
    - Error Probability: $\sim 3.28\%$
    - Fidelity: 0.0303
  - **4 Repeaters:**
    - Error Probability: $\sim 1.98\%$
    - Fidelity: 0.2852
  - **5 Repeaters:**
    - Error Probability: $\sim 1.65\%$
    - Fidelity: 0.4199
- **10 km:**
  - **1 Repeater:**
    - Error Probability: $\sim 9.52\%$
    - Fidelity: 0.0078
  - **2 Repeaters:**
    - Error Probability: $\sim 6.45\%$
    - Fidelity: 0.0312
  - **4 Repeaters:**
    - Error Probability: $\sim 3.92\%$
    - Fidelity: 0.2871
  - **5 Repeaters:**

    * Error Probability: ∼ 3.28%
    * Fidelity: 0.3965

- **50 km:**
  - **1 Repeater:**
    * Error Probability: ∼ 39.35%
    * Fidelity: 0.0078
  - **2 Repeaters:**
    * Error Probability: ∼ 28.35%
    * Fidelity: 0.0312
  - **4 Repeaters:**
    * Error Probability: ∼ 18.13%
    * Fidelity: 0.3145
  - **5 Repeaters:**
    * Error Probability: ∼ 15.35%
    * Fidelity: 0.4326

Conclusion: Quantum repeaters significantly enhance the performance of the quantum communication channel, especially over long distances. However, the results also show the need for more optimization, particularly for the cases with a small number of repeaters.

Observations:

- As the number of repeaters increases, the error probability decreases, and the fidelity improves significantly. For example, at 50 km, the fidelity increases from 0.0020 (no repeaters) to 0.4326 (5 repeaters).
- The improvement in fidelity is stronger at longer distances. For instance, at 50 km, adding 5 repeaters increases the fidelity by over 200 times compared to the no-repeater case.
- However, even with repeaters, the fidelity does not reach ideal values (close to 1.0). This indicates that while repeaters mitigate the effects of noise and loss, there is still a lot of option for improvement.

Potential Issues:

- The fidelity values for 1 repeater and 2 repeaters are strangely low (0.0078 and 0.0264 at 1 km). This could indicate that the error correction mechanism in the entanglement swapping process is not effective for small numbers of repeaters.

5. Conclusion

The simulation of a quantum communication channel, both with and without quantum repeaters, gives valuable information about the challenges and solutions for long-distance quantum communication. Without quantum repeaters, the system suffers from exponential decay in fidelity due to noise and losses, making it impractical for distances beyond a few meters. However, the introduction of quantum repeaters significantly improves the performance by segmenting the communication path and mitigating errors through entanglement swapping and conditional corrections.

In summary, quantum repeaters are essential for building scalable and reliable quantum communication networks.

6. References

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A Appendices

The code for the simulation of the quantum communication channel is available on GitHub. You can access the full implementation in the following repository:

`https://github.com/gorkadabo1/quantum-communications/blob/main/`