A Network-Based Model of Productivity Growth: From Local Diffusion to Fractal Structure

Gorka Bravo Díaz

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Abstract

This paper proposes a network-based model of productivity growth that generalizes and challenges the assumptions of traditional macroeconomic growth frameworks. We allow productivity to evolve heterogeneously across a graph of agents, with spillovers occurring through structured diffusion and innovation driven by node-specific factors. We further extend the model by introducing a fractal, multi-scale interpretation in which nodes themselves may represent recursively defined sub-networks. This approach captures both bottom-up emergence and top-down influence in a single formalism.

1 Model Framework

1.1 1. Network Setup

We define a network of N nodes (agents, firms, regions, or countries) with graph structure:

- $G = (V, E)$ where $V = \{1, \ldots, N\}$
- Each node i has:
  - Productivity $A_i(t)$
  - Capital $K_i(t)$
  - Labor $L_i(t)$

Output follows a Cobb-Douglas form:

$$Y_i(t) = A_i(t)K_i(t)^{\alpha}L_i(t)^{1-\alpha}$$

1.2 2. Productivity Dynamics

Productivity at each node evolves via both local diffusion and endogenous innovation:

$$\frac{dA_i(t)}{dt} = \beta \sum_{j \in \mathcal{N}_i} w_{ij}(A_j(t) - A_i(t)) + \psi_i(t) \cdot \gamma_i + \epsilon_i(t)$$

Where:

- β is the diffusion rate
- w_{ij} is the weight on the edge between i and j
- γ_i is the innovation/absorption potential
- $\psi_i(t) \in [0, 1]$ is the institutional effectiveness of node i
- $\epsilon_i(t)$ is a noise or shock term

1.3 3. Dynamic Network Topology

Edge weights can evolve over time based on productivity differentials:

$$\frac{dw_{ij}(t)}{dt} = \eta|A_j(t) - A_i(t)|w_{ij}(t) - \delta w_{ij}(t)$$

Where η controls sensitivity to productivity gaps, and δ is the decay rate.

1.4 4. Shock Framework

We incorporate three types of shocks:

- **Productivity shocks** to A_i directly (e.g., tech breakthroughs)
- **Institutional shocks** to $\psi_i(t)$ (e.g., governance failures)
- **Network shocks** to w_{ij} (e.g., sanctions, decoupling)

2 Fractal Extension: Multi-Scale Generalization

2.1 1. Recursive Graphs by Level

Let l denote the level of scale (e.g., $l = 0$ for firms, $l = 1$ for regions, $l = 2$ for countries). Each level has its own graph:

$$G^{[l]} = (V^{[l]}, E^{[l]})$$

Where nodes at level $l + 1$ are composed of subgraphs at level l.

2.2 2. Layer-Specific Dynamics

At any level l, productivity evolves via:

$$\frac{dA_i^{[l]}(t)}{dt} = \beta^{[l]} \sum_{j \in \mathcal{N}_i^{[l]}} w_{ij}^{[l]}(A_j^{[l]} - A_i^{[l]}) + \psi_i^{[l]}(t) \cdot \gamma_i^{[l]} + \xi_i^{[l]}(t)$$

2.3 3. Cross-Level Coupling

Higher-level innovation depends on lower-level aggregation:

$$\gamma_i^{[l+1]} = f\left(\{A_j^{[l]} \mid j \in V_i^{[l]}\}\right)$$

Lower-level institutions may depend on macro context:

$$\psi_j^{[l]}(t) = g\left(A_i^{[l+1]}, \text{policy}, \text{external conditions}\right)$$

This allows top-down influence and bottom-up emergence to coexist.

3 Conclusion

This model allows us to capture localized growth, diffusion, and divergence in a way that traditional macroeconomic models cannot. The fractal extension further permits a unified framework across scales, making it suitable for simulation, policy experimentation, and inequality tracking across nested economic structures.