A Network-Based Productivity Growth Model with Observer-Dependent Dynamics

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Motivation: Traditional macroeconomic growth models, such as the Solow-Swan framework, assume uniform productivity growth across agents, failing to capture the heterogeneous and subjective nature of technological progress. This model introduces a network-based approach where productivity evolves differently for each node (e.g., firms, regions) based on its network position and observer perspective. Motivated by the observation that insiders (e.g., Silicon Valley firms) experience smooth, exponential productivity growth while outsiders (e.g., technology adopters) perceive discrete jumps, the model aims to explain persistent inequality and differential technology diffusion in a complex, interconnected economy.

Model Setup: Consider a dynamic network of $N(t)$ nodes connected by a graph $G(t) = (V, E(t))$, with edge weights $w_{ij}(t)$. Each node i produces output via a Cobb-Douglas function:

$$Y_i(t) = A_i(t) \cdot K_i(t)^\alpha \cdot L_i(t)^{1-\alpha}, \tag{1}$$

where $A_i(t)$ is productivity, $K_i(t)$ is capital, $L_i(t)$ is labor, and $\alpha \in (0,1)$.

Productivity Dynamics: Productivity evolves heterogeneously, reflecting observer-dependent perspectives:

$$\frac{dA_i(t)}{dt} = \begin{cases} \beta_i \sum_{j \neq i} w_{ij}(t) \left(A_j(t) - A_i(t)\right) + \psi_i(t)\gamma_i^{\max} + \sigma_i A_i(t) dW_i(t), & \text{if insider or intermediate} \\ 0, & \text{if outsider, unless } \sum_j w_{ij}(t) A_j(t) \end{cases} \tag{2}$$

where:

- β_i: Diffusion rate ($\beta_{\text{insider}} > \beta_{\text{intermediate}} > \beta_{\text{outsider}}$).
- $w_{ij}(t)$: Edge weight from node j to i.
- $\psi_i(t) \in [0,1]$: Institutional effectiveness.
- $\gamma_i^{\max}$: Intrinsic growth potential.
- $\sigma_i A_i(t) dW_i(t)$: Stochastic shock (Brownian motion).
- Outsiders jump: $A_i(t) \leftarrow A_i(t) + \Delta A_i$ when threshold met.

Network Dynamics: Edge weights evolve based on productivity gaps:

$$\frac{dw_{ij}(t)}{dt} = \eta \cdot |A_j(t) - A_i(t)| \cdot w_{ij}(t) - \delta \cdot w_{ij}(t), \tag{3}$$

where $\eta > 0$ governs link strengthening and $\delta > 0$ ensures decay.

Node Birth and Death: Nodes may exit (e.g., if $A_i(t) < A_{\min}$) or enter (e.g., if $\sum_j w_{ij}(t) A_j(t) > A_{\text{spawn}}$), mimicking firm entry and exit.

Usefulness: The model generates testable predictions about inequality and technology diffusion, capturing why central nodes (insiders) experience smooth growth while peripheral nodes (outsiders) see discrete jumps. It extends neoclassical growth by incorporating network effects and subjective productivity dynamics, aligning with real-world TFP dispersion. For the author, a student aiming for quantitative finance, the model serves as a practical tool to develop skills in linear algebra (e.g., adjacency matrix analysis), stochastic differential equations (e.g., simulating shocks), and Python programming (e.g., network simulations), directly applicable to modeling financial markets and portfolio optimization.