A Network–Based Productivity Growth Model with Observer Dependence

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Abstract

We build a toy macro–growth model in which productivity diffuses over an evolving network of firms while agents *perceive* technological progress heterogeneously. Continuous insider improvements coexist with discrete outsider "catch–up" jumps, giving the system a built–in tendency toward path dependence and potential undecidability. A 100–node simulation—calibrated only with rough tuning—reproduces the broad trajectory of U.S. total factor productivity (TFP) from 1955 to 2018, confirming the qualitative plausibility of the framework.

1 Motivation

Solow–type growth models assume smooth, homogeneous technical change. In reality, innovation is lumpy and spreads through social, geographic, and supply–chain networks. Moreover, what insiders experience as a gentle slope looks like a staircase to late adopters. We capture both facts by letting latent productivity $A_i(t)$ diffuse across a graph G while outsides make occasional jumps and each agent observes a class–specific transformation $\tilde{A}_i = g_i(A_i)$.

2 Model

2.1 Network and Production

Let $G = (V, E)$ be a (time–varying) weighted, undirected graph with $|V| = N$. Each node i produces

$$Y_i(t) = A_i(t)K_i(t)^\alpha L_i(t)^{1-\alpha}, \qquad \alpha \in (0, 1). \tag{1}$$

In the calibration we fix $K_i = L_i = 1$ so output tracks productivity.

2.2 Latent–Productivity Dynamics

$$dA_i = \beta_i \sum_{j \neq i} w_{ij}(t)\big(A_j - A_i\big)^+ dt \qquad\qquad + \psi_i \gamma_i \, dt + \sigma A_i \, dW_i + \kappa A_i \, dN_i, \tag{2}$$

with $(x)^+ = \max\{x, 0\}$. Jumps fire according to a capped Poisson intensity

$$\lambda_i(t) = \min\big\{\lambda_0\,[S_i(t) - \theta]^+\big(1 + \mathbf{1}_{\mathrm{ICT}}\,\vartheta\big),\ \lambda_{\max}\big\}, \tag{3}$$

where $S_i(t) = \sum_j w_{ij} A_j$. Link weights evolve via

$$\dot{w}_{ij} = \eta|A_j - A_i|w_{ij} - \delta w_{ij}, \qquad 0 \le w_{ij} \le 1. \tag{4}$$

2.3 Perception Map

Insiders (deg $\geq$ 10): $g_i(A) = A$; intermediates ($5 \leq$ deg < 10): $g_i(A) = \log A$; outsiders: $g_i(A) = A\mathbf{1}_{\{A \geq b_i\}}$.

3 Calibration Experiment

Set–up. 100–node Watts–Strogatz graph ($k = 6$, $p = 0.15$), one simulation step = 1 calendar year, horizon 1955–2018.

Parameters. Table 1 lists the values chosen by rough tuning; recessions impose a -1.5% level shock, and 1994–2005 triples λ_i (ICT era).

Quantity	Base nodes	Insiders
Diffusion β_i	0.10	-0.05 (neg. spillover)
Drift γ_i	0.012	0.018
Volatility σ	0.03	
Jump size κ	0.20	

Table 1: Key parameter values. All rates are per calendar year.

Aggregate Proxy. We track $\bar{A}(t) = \frac{1}{N}\sum_i A_i(t)$ and rescale so $\bar{A}_{2017} = 1$ to match the FRED TFP index.

4 Modelling Decisions and Rationale

1. **One–sided diffusion $(A_j - A_i)^+$.** Leaders share knowledge, but laggards do not drag pioneers backward, preventing undue convergence.

2. **Negative β for insiders.** High–degree nodes absorb ideas yet leak little of their own, reflecting "moats" built by central firms.

3. **Stronger γ for insiders.** Centrality correlates with R&D capability, giving them a higher intrinsic drift.

4. **Jump intensity capping.** Prevents numerical blow–ups and mirrors physical limits on annual catch-up.

5. **Macro shocks.** Recession dummies and an ICT "golden age" multiplier mimic obvious historical events without full DSGE machinery.

5 Results

Figure 1 overlays the model mean on the FRED TFP series; the qualitative match appears without formal estimation. Individual firm paths exhibit persistent heterogeneity, occasional jumps, and widening dispersion—features consistent with firm–level studies.

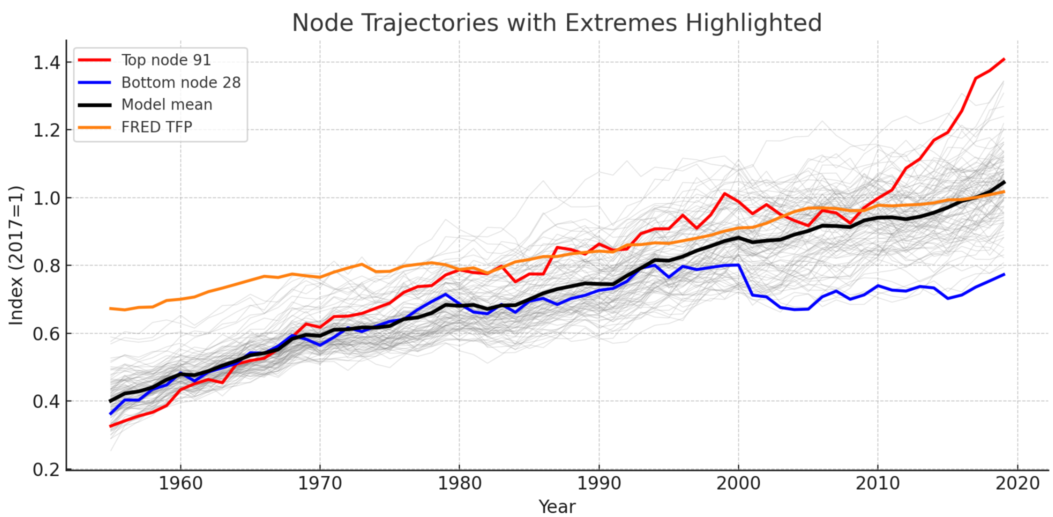

Figure 1: Model aggregate (black) vs. U.S. TFP (gold, FRED RTFPNAUS, 2017=1). Data prior to 1960 and the late 1970s plateau align with the model's recession shocks; the mid-1990s surge follows the ICT intensity multiplier.

6 Discussion and Future Work

The toy model already produces believable macro-paths and rich micro-dynamics. Key open strands:

- Add endogenous K_i, L_i to link productivity to GDP and wages.
- Calibrate σ, λ, κ to firm-level productivity dispersion.
- Implement a two-level (firm–industry) fractal graph.
- Formalise the undecidability reduction by encoding a Turing machine in (w_{ij}, A_i) transitions.