A Network-Based Productivity Growth Model with Dynamic Networks and Shocks

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Abstract

This paper proposes a network-based model of productivity growth, extending the neoclassical framework to incorporate heterogeneous productivity dynamics, dynamic network interactions, and stochastic shocks. Nodes (e.g., countries, firms, or regions) produce output via a Cobb-Douglas function, with productivity evolving through network spillovers, intrinsic growth modulated by institutional effectiveness, and random shocks. Edge weights evolve dynamically based on productivity differences, capturing adaptive economic relationships. The model predicts that dense networks may accelerate convergence, but institutional heterogeneity and network dynamics can sustain persistent productivity gaps, amplifying inequality under shocks.

1 Introduction

Traditional growth models, such as the Solow-Swan framework, often assume homogeneous technological progress or uniform diffusion. This paper introduces a network-based model where productivity evolves heterogeneously across nodes connected by a dynamic graph. By incorporating spillovers, institutional factors, evolving connections, and shocks, the model captures real-world complexities like persistent inequality and differential responses to disruptions, generalizing neoclassical growth to networked economies.

2 Model Setup

Consider a network of N nodes (e.g., countries, firms, or regions) connected by a directed graph $G(t) = (V, E(t))$, where $V = \{1, \dots, N\}$ and $E(t)$ denotes time-varying edges with weights $w_{ij}(t)$.

2.1 Production

Each node $i \in \{1, \dots, N\}$ has capital $K_i(t)$, labor $L_i(t)$, and productivity $A_i(t)$. Output is given by a Cobb-Douglas production function:

$$Y_i(t) = A_i(t) \cdot K_i(t)^\alpha \cdot L_i(t)^{1-\alpha}, \tag{1}$$

where $\alpha \in (0, 1)$ is the capital share.

2.2 Productivity Dynamics

Productivity $A_i(t)$ evolves according to:

$$\frac{dA_i(t)}{dt} = \beta \sum_{j \neq i} w_{ij}(t) \left(A_j(t) - A_i(t) \right) + \psi_i(t) \cdot \gamma_i^{\max} + \epsilon_i(t), \tag{2}$$

where:

- $\beta > 0$: Diffusion rate, governing the speed of productivity spillovers.
- $w_{ij}(t) \geq 0$: Weight of the edge from node j to i, reflecting the strength of economic or technological connections (e.g., trade, knowledge flows).
- $\psi_i(t) \in [0, 1]$: Institutional effectiveness (e.g., governance, trust), varying over time and nodes.
- $\gamma_i^{\max} \geq 0$: Intrinsic growth potential of node i.
- $\epsilon_i(t)$: Stochastic shock term, capturing disruptions (e.g., technological breakthroughs or crises). Typically, $\epsilon_i(t) \sim N(0, \sigma^2)$, or one-off shocks like $\epsilon_i(t) = \sigma$ at specific times.

The first term captures network spillovers, where node i adopts productivity from more advanced neighbors. The second term reflects internal innovation, modulated by institutional quality. The shock term introduces volatility.

2.3 Dynamic Network

The graph $G(t)$ evolves as edge weights $w_{ij}(t)$ adapt based on productivity differences:

$$\frac{dw_{ij}(t)}{dt} = \eta \cdot |A_j(t) - A_i(t)| \cdot w_{ij}(t) - \delta \cdot w_{ij}(t), \tag{3}$$

where:

- $\eta > 0$: Rate at which productivity gaps strengthen economic ties.
- $\delta > 0$: Decay rate, ensuring weights remain bounded.

This formulation implies that larger productivity differences incentivize stronger connections (e.g., increased trade or tech transfer), but links decay without reinforcement.

2.4 Shocks

Shocks are modeled through $\epsilon_i(t)$ or discrete interventions:

- **Productivity shocks**: Sudden changes, e.g., $A_i(t) \leftarrow A_i(t) + \sigma$, representing innovations.
- **Institutional shocks**: Adjustments to $\psi_i(t)$, e.g., $\psi_i(t) \leftarrow 0.1$ for a governance crisis or 0.9 for reforms.
- **Network shocks**: Alterations to $w_{ij}(t)$, e.g., setting $w_{ij}(t) = 0$ for severed links (trade wars) or adding new edges (globalization).