Products and Politics: Comparative Advertising and Competitive Positioning

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Abstract

Comparative advertising promotes a product through a comparison with competitors' products, often highlighting the weaknesses of the latter. We study comparative advertising with a focus on how it impacts product positioning for profit maximizing firms. We find that factors such as the negative spillover of comparative advertising and heterogeneity in consumer tastes are important determinants of how firms position themselves in the market and whether they engage in comparative advertising. In certain settings, the threat of comparative advertising can result in lower positional differentiation along with positive advertising. We derive welfare implications of comparative advertising; for instance, allowing comparative advertising, as the FTC does, may lead to lower innovation by firms and lower consumer welfare, without comparative advertising being actually used in equilibrium. We also study the context of political competition, where a candidate's objective is winning by plurality. We find that, due to this difference in objective (compared to profit-maximizing firms), the equilibrium outcome supports high positional differentiation along with comparative advertising. This can help to explain the often observed polarization in political campaigns.

Key Words: Comparative Advertising, Entry, Negative Advertising, Product Innovation, Political Campaigns

JEL codes: M37, D43

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1 Introduction

Consider a firm entering a new market. One key strategic decision that the entrant has to make is how to position itself in the market. A major consideration in this decision is the positioning of the incumbents (Cooper and Kleinschmidt, 1987; Montoya-Weiss and Calantone, 1994). Entrants can carve out a position for themselves by either differentiating their product designs or choosing a brand image that differentiates the company from the competitors (Fuchs and Diamantopoulos, 2010; Maarit Jalkala and Keränen, 2014). Advertising, as a medium of communication, plays an important role for firms to achieve such differentiation (Meenaghan, 1995; Alden et al., 1999). To make positional differences salient, new entrants may use comparative advertising. The mention of a competitor, or a comparison against its product, is known to help firms to emphasize the dimensions of differentiation to consumers (Grewal et al., 1997; Jewell and Saenger, 2014).

As an entrant positions itself in the market, the incumbents may feel a need to respond by re-positioning themselves. In response to the entrant, they may modify the design of their product (Carpenter, 1989; Ellickson et al., 2012; Seamans and Zhu, 2017); intensify advertising to remind consumers of their product, or tap into comparative advertising to showcase the superiority of their product (Hauser and Shugan, 1983; Hauser and Gaskin, 1984; Kumar and Sudharshan, 1988). As modifications to product designs take longer, many firms focus on advertising as the first response strategy (Cubbin and Domberger, 1988; Thomas, 1999).

Examples of incumbents utilizing comparative advertising after a new entry are plenty. American Express faced abundant negative advertising from Visa and Mastercard during the introduction of its new card Optima, where the ads attacked its narrow merchant coverage (Stevenson, 1988). The aggressive comparative advertising was so effective that American Express ended up downplaying its product introduction to avoid further advertising war (Robinson, 1988). After the deregulation of the Australian telephone industry, incumbent Telstra responded to the entry of Optus by running ads emphasizing the affordable buying plans for Telstra and how Telstra was a homegrown company compared to Optus which was a foreign brand (Roberts, 2005). Elite, an Israeli coffee producer, defended its position against the newcomer Nestle via comparative advertising. While Nestle emphasized its global success while entering the Israeli market, Elite highlighted

that its products are closer to the local taste (Greenblatt, 1996).

Similar to incumbents, entrants also utilize comparative advertising as a strategic tool accompanying their entry. The entry of Merck to angiotensin-converting enzyme inhibitor market with Vasotec was accompanied with fierce competitive advertising against Bristol-Myers Squibb's Capoten. Merck ads emphasized that results from scientific research studies could not conclusively determine that side-effects of Capoten do not exist, while BMS defended itself against these claims, emphasizing that studies could not confirm side-effects. In the pain reliever market, McNeil's Tylenol faced two entrants, Datril, and Anacin, whose comparative ads claimed inflammatory side effects from Tylenol (Knight, 1978; Robinson, 1988). Anticipating a "re-positioning response" by the incumbent, a rational entrant should choose its entry position taking into consideration a potential advertising war down the road. Robinson (1988) points out that "... if aggressive and damaging reactions are expected, the entrant can be frightened off or choose to enter on a less ambitious scale" (p. 368).

In this study, we model how a firm's market positioning decision is determined by the threat of later advertising competition. We build a theoretical model of profit-maximizing competing firms making positioning choices, studying the case of an entrant in a new market taking into consideration the position of the incumbent and the risk of incumbent's comparative advertising response post-entry. Specifically, we model the advertising response of the competitor as a choice over comparative advertising content — firms can choose to advertise only the positive aspects of their products ("positive advertising"), or point out the negatives of the competitor's product ("negative advertising"). Managers and advertisers have long been complaining how comparative advertising wars harm all affected parties and decrease industry demand altogether.[1] We model these concerns

[1]In his comprehensive analysis, Beard (2010) documents how industry leaders would regret that escalation of the advertising war would damage both combatants: "Writing about George Washington Hill's war on the sweets industry, the president of the New York Coffee and Sugar Exchange Inc., observed, "Has Mr. Hill forgotten that it was only a short time ago when some of our states, on health grounds, were legislating against cigarettes and that the term 'coffin nails' was applied to them? Would it not be well for the American Tobacco Company to 'Let sleeping dogs lie?'" (Lowry 1929, 790). A Printers' Ink author many years after the Baking Powder War warned that this kind of damage could last for years: "Lots of people still alive and well can vividly recall the days of some years ago when they were repeatedly warned to beware of 'benzoate of soda.' . . . Eventually, the various manufacturers and advertisers of foods discovered the alarming effect such copy was having on their business, and they recovered their reason by stopping all such publicity" (Hanley 1927, 10)." He substantiates the argument adding that "A Printers' Ink author stated the theme even more directly: "But when one party says one thing and the other states what seems to be a directly conflicting fact, the likely conclusion of the consumer is that one of the two is a liar—probably

as a trade-off. If the compared products are not sufficiently differentiated, pointing out to the shortcomings of the competitor's product may negatively impact the demand of the advertiser's own product as well (Wright, 1973; Swinyard, 1981; Jain and Posavac, 2004). Then, an advertiser would engage in negative advertising only when its product is sufficiently differentiated from its competitor's. We analyze how this trade-off shapes the positioning and advertising strategies of firms.

Our first main result is that negative advertising is observed in markets where brands are sufficiently differentiated from each other in positioning or in markets where products are more dissimilar. However, when both firms run negative advertising, the overall consumer base shrinks and the industry surplus becomes lower. Therefore, if firms could coordinate on their effort after they choose their position, they would avoid negative advertising. Our second main result is, at the product design stage, firms would have incentives to produce similar products to commit to avoiding a negative advertising war later. If the benefit of avoiding a negative advertising war is larger than the cost of increased competition, firms choose designs that show higher similarity in equilibrium.

In an extension, we discuss a market where negative advertising is very common: political competition. We modify the model to consider an electoral competition and show that political competitors are more likely to engage in negative advertising than firms. This is because the objective of a politician is to win by plurality (i.e., each voter is allowed to vote for only one candidate, and the winner of the election is whichever candidate receives the largest number of votes), rather than maximizing the total votes. This slight modification to the objective function implies that a decrease in the overall voter base is not inherently bad for two competing politicians, thus the damage from running negative advertising is smaller for the candidates. We indeed observe that, even in electoral races where competitors have very similar policy agendas, candidates still use negative adverting.

both" (Erbes 1934, 46)." and "In a speech to the Advertising Club of Greater Boston, David C. Stewart, president of agency Kenyon & Eckhardt, summarized this belief: "There are certain industries and certain product areas today in which the battle of competitive advertising claims has reached the harsh crescendo of jungle warfare . . . public confidence [once] shaken . . . [usually exerts] a stern reaction against the industries themselves" (as cited in Overly competitive ads invite action by U.S. 1965, 68)."

Scholars have focused on the costs and benefits of comparative advertising in both theoretical and empirical studies. One branch of literature focuses on the informative aspect of comparative advertising. Aluf and Shy (2001) and Anderson and Renault (2009) model comparative advertising as a tool for the revelation of horizontal match characteristics of products, therefore it reduces competition between firms and can potentially increase profits while reducing consumer welfare. However, in both, it is assumed that the overall size of the consumer base is fixed, which kills most of the potential losses from negative advertising. Barigozzi et al. (2009) model how informativeness of comparative advertising can be welfare improving but they also take the total demand for the product as given. Singh and Iyer (2020) present a consumer persuasion-based theory of the incentives that competing firms have to release their own or their rivals' information.

Modeling comparative advertising as a vehicle to communicate product characteristics sounds intuitive. However, the empirical evidence is not clear at all. Multiple studies show there is no consistent difference between comparative and non-comparative advertisement in changing consumer awareness or recall.[2] On the other hand, the effect of comparative ads on sales is found to be much larger than non-comparative ads.[3] The differential effect rather seems to be through reducing demand for the competitor, which motivates our modeling approach.

Our paper contributes to three different strands of the literature. The first strand analyzes the effect of comparative advertising on industry dynamics and consumer welfare. Anderson et al. (2016) study an environment similar to ours, where the size of the consumer base in the market can change. Similar to our study, the authors allow the comparative ad to decrease utility from the competitor's product. The authors develop a theoretical model where brands simultaneously choose prices, self-promotion strategy, and comparative advertising expenditure but the product design is taken as given. The model is then estimated by advertising data on the OTC industry where comparative advertising is prevalent. They conclude that comparative advertising is more effective than generic advertising but mainly due to stolen customers and not a larger market. Second, the benefit of comparative advertising to the advertiser is smaller than the damage on the competitor, i.e., the overall demand shrinks. Third, the other firms in the sector benefit from comparative

[2]See Prasad (1976), Jain and Hackleman (1978), Shimp and Dyer (1978), and Pechmann and Stewart (1990).
[3]See Demirdjian (1983).

advertising more than the advertiser, i.e., there is evidence for sizable negative backlash. In a counter-factual, they show that the industry would be better off when comparative advertising is banned. We endogenize product design choices and show that comparative advertising can be welfare-reducing for consumers as well as for the industry. Johnson and Myatt (2006) analyze how product design changes with the dispersion of consumer preferences without distinguishing positive and negative advertising. They find firms do better when there is minimum or maximum dispersion by producing general products in the former and niche products in the latter.

The second strand analyzes the differences in the prevalence of comparative advertising on industry and political competition. The core mechanism that discourages firms from negative advertising in our case is the shrinking consumer base due to the negative advertising war. The literature on political competition does not agree on the effect of negative advertising on voter turnout.[4] One puzzle is why politicians utilize negative advertising despite the ambiguous effect of the negative tone of advertising on their voter base. We contribute to this literature by demonstrating how, even in an environment in which negative advertising demobilizes one's own voters, candidates may use more negative advertising relative to firms.

Finally, our paper is also related to the literature on product design and the positioning of products. Gavish et al. (1983), Moorthy (1988) and Horsky and Nelson (1992) are among a large body of papers that theoretically study pricing and positioning decisions together in the spirit of Hotelling (1929). Thomadsen (2007) analyzes how positioning depends on the degree of asymmetry between the firms while Kuksov (2004) studies how the existence of search costs interacts with the positioning decision. These papers study how the positioning decision interacts with pricing decision while ignoring the advertising decisions. Erickson (1985) studies advertising competition in a dynamic setting while abstracting from both the positioning and pricing decisions. Bass et al. (2005) shows in a dynamic setting, firms have the incentive to do positive advertising to enlarge the market in the short run and negative advertising to steal consumers from competitors in the long run. The closest study to ours is by Chen et al. (2009) who build a model with advertising and pricing decisions. They show that increasing cost of advertising can help the firms by preventing a

[4]See Lau et al. (2007) and Arceneaux and Nickerson (2010) for no effect, Niven (2006) and Barton et al. (2016) for possibly increased voter turnout and Ansolabehere et al. (1994) for decreased voter turnout.

prisoner's dilemma with a pricing war that results from an advertising war. We instead model the positioning decision and show how the availability of negative advertising leads to inefficiently low product differentiation.

The rest of the paper is organized as follows. In Section 2, we present the general model in which competing entities have to make positioning and advertising decisions. In Section 3, we analyze the model for the case of firms selling products that have the objective of maximizing profits and obtain our key insights on positioning and advertising. In Section 4, we analyze the model for the case of political parties that have the objective of winning an election by plurality, and obtain certain results on positioning and advertising that provide interesting contrasts with the case of firms maximizing profits. In Section 5, we conclude. All proofs are provided in the appendix to the paper.

2 Model

In this section, we develop our model. We will use this model for both the case of firms selling products and for the case of political parties aiming to win consumer votes. For expositional simplicity, we will develop the model in the context of firms selling products.

Market Structure and Product Positioning

Consider a market with two ex-ante identical firms producing substitute goods, where each chooses its product design and its advertising strategy. The choice of product design is represented by a choice between two locations on a design spectrum as shown in Figure 1. Specifically, there are two locations, L and R, that are a distance d apart. Firms' locations at the same versus different points can be interpreted as low versus high innovation in the market. Location L has a mass 1 of consumers, while location R has a mass $m \gtreqless 1$ of consumers. This specification implies horizontal differentiation between products. Therefore, the parameter d acts as a measure of the heterogeneity in consumer taste. We assume that each unit of a product sold gives a firm a marginal profit of 1.

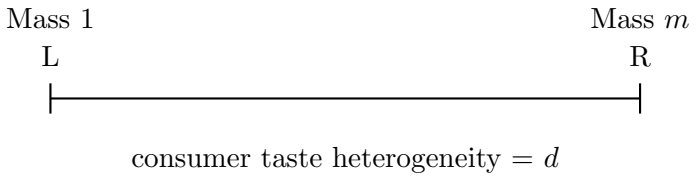

Figure 1: Positions of Consumers and Firms

Consumer Utility

Consumers are heterogeneous in product design preference denoted with x_j and the reservation value for the product denoted with R_j. As mentioned earlier, mass 1 of consumers are located on the left end of the spectrum and mass m are located on the right end. Consumer j has a reservation value $R_j \sim F[\underline{R}, \overline{R}]$ and has an ideal product located at $x_j \in \{L, R\}$. Consumers prefer products that are closer to their ideal product and incur a disutility from any deviation from this point, proportional to the distance between x_i and x_j, which is assumed to be equal to d for $i \neq j$ and equal to 0 for $i = j$.

We model ads as complements to the product so that they change the utility gained from consuming the product as in Carpenter (1989) and Becker and Murphy (1993). Let A_i represent the composite effect of advertising on the product of Firm i. Let the utility consumer j derives from purchasing Firm i's product be:

$$U_{ij} = R_j - |x_i - x_j| + A_i \tag{1}$$

Comparing utility gains, each consumer either buys the product of Firm i, the product of Firm k, or does not buy any product and gets 0 utility. Therefore, the total demand can shrink or enlarge depending on firms' decisions.

Advertising Decisions

The advertising action set of each firm consists of running a positive or a negative ad (we use "negative" and "comparative" interchangeably throughout the paper). We assume that advertising influences consumer decision making in two distinct ways. First, ads are assumed to have a *direct*

effect on the demand for the advertised good. Positive ads about a product increase the utility consumers derive from that product, while negative ads about the competitor decrease the utility of consumers from the competitor's product. Second, negative ads can have a *spillover effect* on advertiser's own product. Specifically, if a firm chooses to air negative ads, the utility derived from the advertiser's own product is negatively affected, more so when the competitor's and own products are similar to each other. A_i denotes the sum of these three effects for product i.

In our benchmark model, we will define b to be the direct effect of negative ads on competitor's product and normalize the direct effect of positive ads on own product to 0, without loss of generality. When $b > 0$, the direct effect of negative ads is stronger than that of positive ads, in the absence of spillovers.[5]

We represent the spillover effect of negative advertising with β. The negative spillover on the advertiser's own product happens because advertisements act as a call for the evaluation of products for consumers. As consumers evaluate the two products on the mentioned attributes, an advertiser with a high degree of similarity to the product in negative advertising may also see a decline in demand. For simplicity, we will analyze the negative spillover impact at two levels. When products share a high degree of similarity (i.e., when they co-locate), the magnitude of negative spillover is β, and if they choose to vary their locations, then we normalize the magnitude to zero.

Table 1 summarizes how the comparative advertising tone (A_i, A_k) of firms i and k influences consumer's utility, conditional on their location choices.

We make the assumption that advertising alone cannot reverse consumer preferences. So firms cannot attract all consumers whose preferences are not identical with the product location, even with the highest combined advertising effect. Moreover, even with a strong positive advertising strategy, some consumers whose preferences do not coincide with the firm location will choose to buy from its competitor or not to buy at all. Similarly, even when both firms utilize strong negative advertising, some consumers with both design preferences will still choose to buy the product. Formally, this implies the following assumption.[6]

[5]The direct negative impact of comparative ads has been documented by Grewal et al. (1997) and Liaukonyte (2015).

[6]This assumption saves us from dealing with corner cases.

		Firm k	
		Positive	Negative
Firm i	Positive	$0, 0$	$-b, -\beta$
	Negative	$-\beta, -b$	$-b-\beta, -b-\beta$

(a) Under Co-location

		Firm k	
		Positive	Negative
Firm i	Positive	$0, 0$	$-b, 0$
	Negative	$0, -b$	$-b, -b$

(b) Under Differentiation

Table 1: Composite Effects of Positive and Negative Advertising for Firms i and k

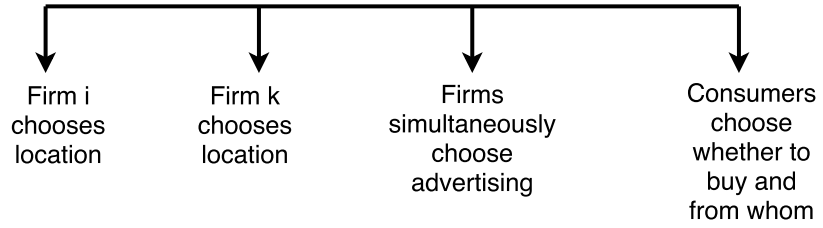

Figure 2: Timeline of the Game

Assumption 1. *$\underline{R} < d$, where $\underline{R}$ is the lower bound of the support of consumer reservation value distribution and $\overline{R} > d + b + \beta$, where $\overline{R}$ is the upper bound of the support of consumer reservation value distribution. These two requirements imply $F(d + b + \beta) < 1$ and $F(d) > 0$.*

Timing

The timing of decisions is given in Figure 2. First Firm i, which we call the incumbent, chooses to position itself on L or R. Next, Firm j, which we call the entrant, chooses to co-locate or differentiate itself from the incumbent. Following this, the firms simultaneously choose their advertising strategies. Finally, consumers make their product purchase decisions.

Discussion of Modeling Choices

Before we proceed to the analysis, we discuss some deliberate simplifications that we have chosen to make in the model to maintain tractability and clarity of exposition.

First, we note that while we have made a modeling choice to have two-segment consumer heterogeneity, choosing a *continuous distribution of consumers* (such as assuming that consumers are distributed uniformly on a line segment between L and R) would lead to qualitatively the same insights as from our model. However, assuming a continuous distribution of consumers makes the model derivations analytically cumbersome and sometimes intractable. Therefore, we maintain the discrete consumer distribution model. Previous work has used such a discrete consumer distribution when product positioning is a decision, e.g., Kuksov (2004) uses a model essentially identical to ours.

Second, in this model we have assumed firms to be are price takers. While price is a natural decision variable when firms are selling products, it is not a natural decision variable when political parties or candidates are vying for the votes of voters. Since we want our core model to be applicable for the cases of product competition as well as political competition, we do not include price in it. This simplification also makes it easier to comprehend the advertising-related forces at play and how they interact with positioning, and deliver the key insights. However, for the product competition case, we do include *price* as a decision variable in an extension of the model in Section 3.1.2. The main tension is that price competition is higher with co-location than with differentiation; we build this tension into the model in the extension with price. An important aspect here is that on co-location the price should not be equal to marginal cost, as in this case the firms will make no profit on co-location; as Kuksov (2004) shows, realistic and relatively straightfoward mechanisms can be included in the model that ensure this.[7]

Third, we have proposed a simple reduced-form utility function in (1). In Appendix A, we show how this simple structure can be derived from two *microfounded models*, one based on informative advertising and the other based on persuasive advertising.

[7]We can address the first two critiques here by employing a standard Hotelling model with consumers distributed uniformly on a line segment, continuous prices, any allowed locations for firm positioning and positive and negative advertising. This model, which we solve numerically, provides results that are qualitatively similar to the results that we report here.

3 Product Positioning and Advertising

In this section, we consider the game in which two firms are profit maximizers and have to make their decisions on product positioning and advertising strategies.

We start by considering how an entrant would choose its advertising and product design strategies when in competition with an incumbent. Let i be the incumbent firm and k be the entrant. Without loss of generality, we will assume that the incumbent i is located at the left end of the spectrum of consumer tastes (as provided in Figure 1). We will further assume that $m = 1$, i.e. locations are ex-ante identical. We will relax this assumption later in the paper.

The entrant k will first determine its location on entering the market, and then both firms will choose their advertising strategies simultaneously. We determine the pure strategy subgame perfect equilibria in this setting. A pure strategy SPE in this setting is a set of strategies ($x^k \in \{L, R\}, A^k(x_i, x_k) \in \{+, -\}$) and ($A^i(x_i, x_k) \in \{+, -\}$) such that

1. $A^k(x_i, x_k) \in \{+, -\}$ and $A^i(x_i, x_k) \in \{+, -\}$ constitute a Nash equilibrium of the advertising subgame given x_i and x_k.

2. $x^k \in \{L, R\}$ maximizes firm k's profit given x_i, $A^k(x_i, x_k)$ and $A^i(x_i, x_k)$.

where $x^k(.)$ and $A^k(.)$ denote firm k's location and advertising policy functions and A^i denotes firm i's advertising policy function.[8]

No Negative Advertising

We start by considering an environment where negative advertising is regulated and is not a feasible action for firms. Thus, firms can only run positive advertising. In the absence of a negative advertising option, if the entrant chooses to co-locate with the incumbent, it would expect to share the same market equally. More specifically, they equally share a total demand that composes of all the consumers at the left and some of the consumers at the right mass point. If, on the other hand, the entrant chooses to locate at the right point, all the consumers would buy and each firm would profit more. Therefore, in equilibrium, the entrant would choose to differentiate its position from the incumbent.

[8] We use superscripts do denote policy functions and subscripts to denote realized decisions.

		Firm k	
		Positive	*Negative*
Firm i	*Positive*	$1-\frac{F(d)}{2}, 1-\frac{F(d)}{2}$	$0, 2-F(\beta)-F(\beta+d)$
	Negative	$2-F(\beta)-F(\beta+d), 0$	$1-\frac{F(b+\beta)+F(b+\beta+d)}{2}, 1-\frac{F(b+\beta)+F(b+\beta+d)}{2}$

Table 2: Payoffs from Advertisement Subgame with Co-location

Proposition 1. (Positioning in the Absence of Negative Advertising) *If negative advertising is not allowed, the entrant would choose to differentiate its position from the position of the incumbent.*

Negative Advertising

If negative advertising is possible, then we have the following analysis. Suppose the entrant firm k co-locates with the incumbent firm i at L. Then there are four cases for advertising since firm i and firm k can do positive or negative advertising. The effects of advertising, A_i and A_k are as per Table 1(a).

Now suppose both firms do positive advertising. Then a consumer j located at L has utility $U_{ij} = R_j - 0 - 0$ from firm i and utility $U_{kj} = R_j - 0 - 0$ from firm k. Therefore, all consumers at L will buy the product and these consumers will be split equally between the two firms, i.e., each firm will get mass $\frac{1}{2}$ of consumers from L. On the other hand, a consumer j located at R has utility $U_{ij} = R_j - d - 0$ from firm i and utility $U_{kj} = R_j - d - 0$ from firm k. Therefore, a fraction $1 - F(d)$ of consumers at R will buy the product and these consumers will be split equally between the two firms, i.e., each firm will get mass $\frac{1}{2}(1 - F(d))$ of consumers from R. Overall, each firm will get a mass $1 - \frac{F(d)}{2}$ of consumers, with each consumer providing a profit of 1. This is shown in the relevant entry, i.e., (*Positive*, *Positive*) in Table 2.

Next, suppose that firm i does positive advertising and firm k does negative advertising. Then a consumer j located at L has utility $U_{ij} = R_j - 0 - b$ from firm i and utility $U_{kj} = R_j - 0 - \beta$ from firm k. Since the latter is greater than the former, all consumers who buy the product at L will buy from firm k and the mass of these consumers is $1 - F(\beta)$. On the other hand, a consumer j

located at R has utility $U_{ij} = R_j - d - b$ from firm i and utility $U_{kj} = R_j - d - \beta$ from firm k. Again, since the latter is greater than the former, all consumers who buy the product at R will buy from firm k and the mass of these consumers is $1 - F(d + \beta)$. Overall, in this case, firm i will get no consumers and firm k will get a mass $2 - F(d + \beta) - F(\beta)$ of consumers, with each consumer providing a profit of 1. This is shown in the relevant entry, i.e., (*Positive*, *Negative*) in Table 2. The entry for (*Negative*, *Positive*) is derived similarly.

Now suppose both firms do negative advertising. Then a consumer j located at L has utility $U_{ij} = R_j - b - \beta$ from firm i and utility $U_{kj} = R_j - b - \beta$ from firm k. Therefore, a fraction $1 - F(b + \beta)$ consumers at L will buy the product and these consumers will be split equally between the two firms, i.e., each firm will get mass $\frac{1}{2}(1 - F(b + \beta))$ of consumers from L. On the other hand, a consumer j located at R has utility $U_{ij} = R_j - d - b - \beta$ from firm i and utility $U_{kj} = R_j - d - b - \beta$ from firm k. Therefore, a fraction $1 - F(d + b + \beta)$ of consumers at R will buy the product and these consumers will be split equally between the two firms, i.e., each firm will get mass $\frac{1}{2}(1 - F(d + b + \beta))$ of consumers from R. Overall, each firm will get a mass $1 - \frac{F(b+\beta)+F(d+b+\beta)}{2}$ of consumers, with each consumer providing a profit of 1. This is shown in the relevant entry, i.e., (*Negative*, *Negative*) in Table 2.

Now suppose both firm do positive advertising. Then a consumer j located at L has utility $U_{ij} = R_j - 0 - 0$ from firm i and utility $U_{kj} = R_j - 0 - 0$ from firm k. Therefore, all consumers at L will buy the product and these consumers will be split between the two firms, i.e., each firm will get mass $\frac{1}{2}$ of consumers from L. On the other hand, a consumer j located at R has utility $U_{ij} = R_j - d - 0$ from firm i and utility $U_{kj} = R_j - d - 0$ from firm k. Therefore, a fraction $1 - F(d)$ of consumers at R will buy the product and these consumers will be split between the two firms, i.e., each firm will get mass $\frac{1}{2}(1 - F(d))$ of consumers from R. Overall, each firm will get a mass $1 - \frac{F(d)}{2}$ of consumers, with each consumer providing a profit of 1. This is shown in the relevant entry, i.e., (*Positive*, *Positive*) in Table 2.

Table 2 shows the outcomes for co-location. (*Negative*, *Negative*) is trivially an equilibrium given Assumption 1. For (*Positive*, *Positive*) to be an equilibrium too, we need $1 - \frac{F(d)}{2} \geq 2 - F(\beta) - F(\beta + d)$ which can be simplified into the above expression.

The Nash equilibria of the advertising subgame conditional on entrant's location choice, is given in the following lemma. Throughout the rest of the analysis, we will ignore boundary cases (i.e., $b \neq \beta$ and $b \neq d$).

Lemma 1. (Negative Advertising under Minimal Differentiation) *If the entrant co-locates with the incumbent, a unique equilibrium where both advertisers follow negative advertising arises if* $b > \beta$ *and* $F(\beta) + F(\beta + d) < 1 + \frac{F(d)}{2}$. *All other parameter sets under co-location of the firms involve an equilibrium where both firms utilize positive advertising.*

Lemma 1 states that the first major determinant of whether firms will find themselves in a negative advertising war is the degree of negative spillover between the firms. In particular, both conditions can be interpreted as the spillover effect β begin small enough. Therefore, if the size of the negative spillover (β) is low, in equilibrium, firms find it more profitable to utilize negative advertising. However, for large β, the threat of a negative spillover makes negative advertising not preferable, unless the competitor goes negative.

Next, based on a similar derivation, we state the following lemma.

Lemma 2. (Negative Advertising under Maximal Differentiation) *If the entrant chooses to locate differently than the incumbent, both firms utilize negative advertising if its direct effect is strong enough, i.e.,* $b > d$. *For* $b < d$, *(positive, positive) is the only equilibrium that survives simple selection criteria.*

Lemma 2 shows the second factor which influences firms' choice of negative advertising: heterogeneity in the market. Specifically, the degree of customer taste heterogeneity determines how costly it is for the customer to switch from one product to another. When d is high, the firm which provides the worse-fit for a customer yields even less utility. When the differentiation between the products is small, a firm can persuade the competitor's customers to switch by running negative advertising. But for a high degree of differentiation, it will not be sufficient to attract these customers. So even with a very small cost of negative advertising, firms would avoid it. As a result, when $b < d$ holds, we follow an equilibrium selection to favor the positive advertising equilibrium (positive, positive).

	Conditions for Advertising Equilibrium	
Entrant's Position Relative to Incumbent	(Positive, Positive)	(Negative, Negative)
Colocation	$\beta > b > d$ or $b > d, \beta$ and $F(\beta) + F(\beta + d) \geq 1 + \frac{F(d)}{2}$	
Differentiation	$b < d$	$b > d$, β and $F(\beta) + F(\beta + d) < 1 + \frac{F(d)}{2}$ or $b > d, \beta$ and $F(\beta) + F(\beta + d) \geq 1 + \frac{F(d)}{2}$

Table 3: Firm Positions and Conditions for Advertising Equilibrium

We combine the findings from the two lemmas to characterize the equilibrium outcome.

Proposition 2. (Positioning and Advertising Strategy)

(i) If customer taste heterogeneity is large enough, in equilibrium, entrant differentiates its product and both firms run positive advertising.

Otherwise, equilibrium depends on the size of the spillover (β).

(ii) For large values of the spillover effect, β, entrant co-locates with incumbent and both firms run positive advertising.

(iii) For small values of the spillover effect, β, entrant chooses to differentiate its product and both firms run negative advertising.

(iv) For intermediate values, there are two possible outcomes. In the first, the entrant differentiates its position and both firms run negative advertising. In the second, the entrant does not differentiate and both firms run positive advertising. Profits are higher for both firms under the second outcome.

Table 3 summarizes the conditions under which each of the possible equilibria can arise.

To see the intuition of Proposition 2, recall that if negative advertising was regulated, the unique market outcome in firm position would be to differentiate products while running positive advertisement. The first part of Proposition 2 suggests that when $b < d$, even if negative advertising is allowed, firms would not utilize it. This is because consumer tastes are so heterogeneous that none would switch to the competitor firms' product under negative advertising.

In the second part of the proposition, $b > d$, so negative advertising allows firms to steal each others' consumers when products are differentiated. However, under colocation, the negative spillovers from negative advertising are prohibitively high ($\beta > b$) to prevent any negative advertising. Thus, in order to reduce the attraction of negative advertising to the competitor, the entrant

chooses to enter into the market offering an identical product to the one that is offered by the incumbent. This positioning strategy prevents a negative advertising war because the incumbent would observe a high negative impact on its own product should he choose to utilize a negative advertising campaign.

In the third part of the proposition, $b > d$ therefore, some consumers can switch to the competitor, but at the same time the size of the negative spillover is too low. As a result, colocating does not rule out negative advertising for the firms. Therefore firms utilize negative advertising while offering differentiated products to enjoy a broader consumer base. This equilibrium outcome is one that is in line with the goals of a policy maker such as the Federal Trade Commission (FTC), since it supports innovation and product differentiation in the marketplace.

In the fourth part, we observe that there are two possible equilibria. In the first of these, the location decision of the firms are not affected but the benefits of negative advertising can materialize as in the second part. In the other one, however, firms reduce product differentiation in the economy, without the benefits of negative advertising as in the third part. Moreover, the size of the pie from the latter is larger than the one in the former. That is, any coordination among firms at the location choice stage would result in the latter equilibrium outcome.

Corollary 1. (Consumer, Firm and Total Surplus) *For all parameter sets, both consumer and firm surplus are highest when firms locate apart and run positive advertising.*

Because the price is fixed, firm surplus is increasing in the number of consumers. The total number of consumers is maximized when firms differentiate their products and run positive ads. Both the number of consumers and the utility a given consumer derives from consuming are the highest when firms differentiate and run positive ads. Therefore, consumer surplus is maximized as well.

In summary, depending on the parameters, some sectors would be better off by prohibiting negative advertising. The possibility of a negative advertising war might scare firms from bringing innovative products to the market in the first place.[9]

[9]In our model, having two firms that co-locate and run positive ads and having a single firm would be identical. Therefore, another way to interpret our results is in terms of the decision to enter. If product designs were not choices, but were randomly drawn by entrants, the possibility of a negative advertising war could deter entry as well.

Next, we analyze how the parameters of the model influence the key findings from the paper. We can think of an industry sector as the combination of three numbers: $\{d, b, \beta\}$, i.e., how much dispersion there is in terms of consumer tastes, how effective negative advertising is and the degree of spillover that negative advertising creates, respectively. Then, comparative statics on parameters can be translated into different policy recommendations for different industry sectors. We state the following propositions.

Proposition 3. (Severity of Spillover) *When the consumer taste heterogeneity is large enough, i.e., $d > b$, the firms differentiate in spite of the possibility of negative advertising.*

Proposition 4. (Degree of Heterogeneity in Consumer Tastes) *When spillovers from negative advertising are large but consumer taste heterogeneity is small, the firmd will co-locate and use positive advertising.*

When spillover is more severe, it becomes a better deterrent for negative advertising in case firms produce similar products. Consequently, firms are more inclined to believe that producing similar products will ensure a positive advertising equilibrium. Therefore, avoiding innovative products becomes a safer choice. To give an example, negative advertising tends to create bigger spillovers in sectors where people are more risk averse, such as pharmaceuticals. Therefore, pharmaceutical companies would be more inclined to avoid differentiating on their products in ways that could allow competitors to use aggressive negative advertising.

The parameter d gives a simple measure of how dispersed the consumers are in their preferences. When d is small, everyone has similar tastes and when d is large, there are two clusters of consumers with very dissimilar tastes. In some sectors, consumers have strict preferences over what type of a product they demand (such as computer operating systems) while in others consumers readily switch between different characteristics (such as computer screens). Then, we might want to ask how does the effect of comparative advertising on firm behavior changes for sectors with varying consumer taste heterogeneity.

When consumer tastes heterogeneity is low, negative advertising becomes more attractive for firms located apart, as it allows stealing most of the market at the other point. As a result, negative

advertising wars regarding dissimilar products are more likely. To avoid this situation, firms would prefer to co-locate more strongly. Johnson and Myatt (2006) also concluded that smaller dispersion would lead to more generic product design, because firms want to be able to market to a larger audience. Our analysis adds one more mechanism that brings this result: the incentive to avoid a negative ad war.

3.1 Extensions to Product Positioning and Advertising Analysis

In this section, we consider two variations to the framework that we used for the product positioning and advertising analysis — simultaneous entry of firms and endogenous pricing.

3.1.1 Simultaneous Entry of Competing Firms

In the main model, we demonstrated that the desire to avoid a negative advertising war can cause an entrant to choose a product positioning that is similar to the existing offering in the market. Next, we consider the case where two competing firms enter a new market simultaneously and choose their positions, expecting that the degree of overlap between their product offerings can influence their competitor's choice of negative advertising in the second stage.

Again, we will focus on pure strategy SPE, and we will demonstrate it as the set of strategies $(x^k \in \{L, R\}, A^k(x_i, x_k) \in \{+, -\})$ and $(x^i \in \{L, R\}, A^i(x_i, x_k) \in \{+, -\})$ such that:

1. $A^k(x_i, x_k) \in \{+, -\}$ and $A^i(x_i, x_k) \in \{+, -\}$ constitute a Nash equilibrium of the advertising subgame given x_i and x_k.

2. $x^k \in \{L, R\}$ and $x^i \in \{L, R\}$ constitute a Nash equilibrium given $A^k(x_i, x_k) \in \{+, -\}$ and $A^i(x_i, x_k) \in \{+, -\}$.

When two firms simultaneously enter the market, the second stage of the game where firms determine their advertising strategy is identical to the one in the benchmark model. Therefore Lemmas 1 and 2 apply in the backward induction solution. What is different is the first-stage decisions about positioning. Yet, it turns out that the equilibria of the simultaneous entry game are identical, with the slight modification that firms can choose to locate both on the right and left end of the line of consumer heterogeneity:

Proposition 5. (Positioning under Simultaneous Entry)

(i) For each pure strategy equilibrium of the colocation of the entrant-incumbent game, there are two equivalent pure strategy equilibria in the simultaneous entry game where both firms choose to locate on either the right or the left end of the consumer heterogeneity spectrum.

(ii) For each pure strategy equilibrium for location differentiation of the entrant-incumbent game, there are two equivalent pure strategy equilibria in the simultaneous entry game where one firm locates on the right and one firm on the left end of the consumer heterogeneity spectrum.

Proposition 5 demonstrates that the key insights of the benchmark model are not sensitive to the simultaneous or sequential entry of the firms. The equilibrium outcomes map under a simultaneous game to that of the sequential game.

3.1.2 Pricing

The basic model of competing firms derives some key insights under the assumption that firms are price takers. However, modeling prices in our setting is important for two reasons. First, introducing dissimilar products should help alleviate price competition between firms. Then, pricing might act as a counter-balancing force to the innovation discouraging the effect of a negative advertising war. Second, the FTC states that negative advertising helps consumers by encouraging innovation and decreasing prices. We have already shown that, in some sectors, innovation decreases with likelihood of negative advertising. Here, we will show that the effect on prices is ambiguous.

Due to the discrete nature of our problem and the number of parameters, adding an explicit pricing decision increases the complexity of the model significantly. Here, we will introduce a simple pricing decision and will show analytically that: (1) under a large set of parameters, our results in the previous sections continue to hold, and (2) introduction of negative advertising can even increase the prices in some sectors.

In the modified game, we will assume that, in the second stage of the game, firms simultaneously choose their prices together, along with advertising tone. The main impact of making the pricing decision endogenous is that price competition is higher with co-location than with differentiation, and we build this tension into the model. Our modeling requirement here is that on co-location the

price should not be zero (i.e., equal to marginal cost), as in this case the firms will make no profit on co-location and this will not be a viable option for the entrant. However, as Kuksov (2004) shows, realistic and relatively straightfoward mechanisms can be included in the model to ensure that co-location does not lead to zero prices.[10] To reduce complexity, we do not explicitly include such a mechanism in our model, but directly assume that co-location does not lead to plummeting prices.[11]

We assume that firms can charge a low price normalized to 1 or a slightly higher price $H > 1$. In that case, the utility derived from that product drops by η. For the pricing decision to be meaningful, we need $H(1 - F(\eta)) > 1$, that is, firms prefer to charge a high price in the absence of direct competition.

Assumption 2. $d + \eta > b > \eta > d$

Assumption 3. $2 - F(d) > H(1 - F(\eta))$

Assumption 2 says firms that are located apart (1) can charge their consumers a higher price if they run negative advertising while competitor does not and (2) can steal competitor's consumers if they charge a lower price unless they run positive advertising and competitor runs negative advertising. Assumption 3 says firms that are located apart prefer charging a lower price and stealing competitor's consumers to charging a high price.

Proposition 6. (Impact of Pricing) *Under Assumptions 2 and 3, in the absence of negative advertising, firms would locate apart and charge the low price.*

Now, let $b > \beta$, i.e., negative advertising is not trivially dominated. We already assumed $b > d$. In this case, our original model without pricing decision predicted two possible pure strategy equilibria. In one of them, entrant locates apart and runs negative advertising, in the other, entrant co-locates and they runs positive advertising.

[10] In Kuksov (2004), firms do not know the exact valuations that consumers have of their products but get an imperfect signal of the same, and consumers have to search for prices.

[11] We can also assume a more standard Hotelling model with continuously distributed consumers on a line segment, continuous prices and any allowed locations for firms. This model is too complicated to solve analytically but a numerical solution produces results qualitatively similar to the results that we report here.

Proposition 7. *Under Assumptions 2 and 3, where $b > \beta$, there is a pure strategy equilibrium where entrant locates apart, they both run negative advertising and charge the high price if $H \geq \frac{2-F(d+b)}{1-F(b+\eta)}$.*

Proposition 7 suggests allowing negative advertising can increase the prices in certain sectors. The intuition is as follows: negative advertising brings another dimension where firms can distinguish themselves. Firms who did not charge a high price for the fear of losing the market can be more aggressive if they can create an advantage through negative advertising. Our main result in the previous section suggested FTC's policy might have unintended consequences in sectors where negative advertising is not utilized. Proposition 7 suggests unintended consequences might also arise in sectors where firms use negative advertising.

4 Political Positioning and Advertising

Our results so far suggest that firms have incentives to avoid negative advertising competition, and this might even induce them to avoid differentiating. The reader may agree that numerous firms do not engage in comparative claims in their advertising, even though it is legal.

In political competition, however, comparative advertising with a strong negative tone is utilized frequently. Political advertising campaigns are among the most sophisticated advertising campaigns in the US (Petrova et al., 2017). In this section, we extend our competition model to study the competition of two political parties. This section will demonstrate the following key insights: (1) since political parties do not care about a drop in voter turnout if both parties lose voters proportionally, they tend to run negative advertising much more often then firms do,[12] and (2) changes in policy decisions of parties are much harder to anticipate than product decisions of firms.

Similar to the previous section, we study two situations where the presence of negative advertising will affect the political positioning of the parties. First, we will consider a market with an incumbent politician and a challenger running for the office. While the position and policy stance of the incumbent is generally known by the electorate, the entrant can choose her position. Following this, we will consider a simultaneous entry game in a political race.

[12]This result heavily hinges on the assumption that only two parties compete, however empirical studies show that in the U.S., most elections are bipartisan (Garcia-Jimeno and Yildirim, 2017).

We will set the political competition as close to our setting for the commercial firms as possible. Of course, in the context of political races, consumers are replaced by voters and brands are replaced by political parties. Moreover, a party's goal is to maximize the vote differential instead of profit/demand.[13] Put differently, each party is interested in winning an election by plurality, i.e., the candidate with more votes cast in his/her favor wins the election.

In line with our benchmark specification, we will have the utility voter j derives from voting for party i as

$$U_{ij} = R_j - |x_i - x_j| + A_i \tag{2}$$

We assume a voter can decide to cast a vote for party i, party k or may decide not to vote if she has negative utility.

Consider the case where the candidate of party i has already chosen a policy position to the left end of the political spectrum, as in Figure 1. Party k will first choose which point to locate its product and both parties will decide on which ads to run simultaneously. Strategy sets and equilibrium definition are identical to the competition of the firms.

In the absence of negative advertising as a strategy, both candidates run positive advertising. If the entrant chooses to locate on the left point, the votes are divided equally between them. If entrant differentiates in location, it will get all the right-wing voters. For any $m > 1$, it chooses to locate apart and for any $m < 1$, it chooses to co-locate. For $m = 1$, the incumbent is indifferent.

When negative advertising is possible, the mechanics differ significantly from that in the commercial competition. Since parties only care about the vote differentials, they are not bothered by the loss of votes on their side as long as the competing party loses at least the same amount of voters. This makes spillover caused by running negative advertising less costly. This difference in objective functions, by itself, provides a clear explanation for why negative advertising is observed much more frequently in political competition.

We will again assume $m = 1$ and the exact equivalent of Assumption 1 for parties, where only the interpretation changes. If both policies are the same in the political spectrum, then at least

[13] We prefer vote differential instead of winning probability for two reasons. First, it is much easier to deal with mathematically. Second, all the main results are identical with the case of winning probability. This objective assumes that the parties want to increase their winning margin (or decrease losing margin) on top of winning.

some voters with different preferences would choose not to vote at all. Moreover, there are always people willing to vote regardless of policy and advertisement strategies of parties.

Lemma 3. *In equilibrium, political candidates*
(i) always run negative ads if they have opposite policy positions,
(ii) run positive ads if $b < \beta$ and they have identical policy positions,
(iii) run negative advertising if $b > \beta$ and they have identical policy positions.

Different from our benchmark model and Lemma 1, Part (i) says that when there is no negative spillover from advertising, candidates will always go for negative advertising. So, parties end up running negative advertising, even if there is no possibility of stealing voters from each other. This is different from the firms' case, since firms only care about competitor's loss if it turns a profit for themselves. For political parties, however, making the competitor lose voters is preferable even if their own votes do not go up. Part (ii) gives the same intuitive result with firms' case: if negative advertising is inferior to positive advertising due to a strong negative spillover, parties do not do it. Part (iii) says that if spillover is smaller than the harm on the competitor parties use negative advertising when co-locate. Combined with part (i), parties always run negative advertising. In the firms' case, a positive advertising equilibrium was also possible in this situation, but for the same reason as in (i), it disappears for the parties.

Proposition 8. (Policy Positioning) *In political competition, mimimal and maximal differentiation in positioning are both equilibrium outcomes for all parameter values. Thus negative advertising is more common in political competition relative to competition between commercial firms.*

Therefore, in our symmetric setting, we cannot pin down the policy setting behavior of the parties. However, parties run negative advertising whenever firms do and even for other parameters and location decisions where firms would not. Therefore, we can conclude that parties are expected to run more negative advertising than firms do.

The results extend to the situation where parties decide on policies simultaneously, similar to the firms' case.

4.1 Extensions to Policy Positioning and Advertising Analysis

4.1.1 Analysis for $m \neq 1$

In the incumbent/entrant game, the indeterminacy of party location equilibrium results from two locations being ex-ante identical. We can make stronger conclusions and comparisons between the firm and party competition when $m \neq 1$, i.e., one point has more mass than the other.

Proposition 9. *(i) Let $m > 1$. In the political advertising game, the entrant party always chooses to locate apart. In the product advertising game, however, there exists a parameter set where entrant firm still co-locates for m sufficiently close to 1.*
(ii) Let $m < 1$. In the political advertising game, the entrant party always chooses to co-locate. In the product advertising game, however, there exists a parameter set where entrant firm still locates apart for m sufficiently close to 1.

For $m > 1$, when one player is already settled in the left point with mass 1, this corresponds to a case where a new product (policy) would gather more consumers (supporters) than the product (policy) already supplied by the incumbent. Naive thinking could suggest that a newcomer would always locate at the new point to enjoy larger customer (voter) base. This is true for the political competition scenario, where co-location would result in the same number of votes for both parties and locating at the new point would bring positive vote differential. In the firms' problem, however, we had shown that, for some parameter sets, there were strictly positive gains to co-locate. Therefore, there should be an m, sufficiently close to 1, such that a newcomer still prefers to co-locate at the point with a smaller mass. In short, entrant firms are more likely to avoid going for the innovative idea compared to entrant parties when there is a larger market for the innovative idea compared to the existing one.

For $m < 1$, when one player is already settled in the left point with mass 1, this corresponds to a case where a new product (policy) would gather fewer consumers (supporters) than the product (policy) already supplied by the incumbent. The new party here would always co-locate to not let the opponent gain advantage. In the firms' problem, however, the gains from reduced competition could exceed the benefit of being able to reach a larger consumer base and avoiding negative

advertising. Therefore, depending on the parameters, there could exist values of m, such that newcomer still prefers to locate apart and go to the point with a smaller mass. In short, entrant firms are more likely to go for the innovative idea compared to entrant parties when there is a smaller market for the new idea.

4.1.2 Core Supporters

Here, we assume that a mass h of people at each point (L and R) slightly prefer a different firm (party).[14] This has intuitive appeal as some leftist voters could prefer to vote for Democrats even when both Democrats and Republicans push for leftist policies or some consumers could still prefer Coca Cola even when Lipton produces the same coke. This behavior could be explained by the lack of trust from voters' (consumers') side on how successful can the policy (product) be when it is produced by an unexpected party.

Assume that the left point L has a small mass h favoring firm (party) i and the right point R has a small mass of equal size favoring firm (party) k.

For the political parties, this would immediately pin down the equilibrium. A newcomer would locate on its favored point while in the simultaneous location game both parties would go to their respective favored point.

For the firms, however, an entrant might still choose to co-locate to escape a negative advertising war, even though its favored point is the other one, for h small enough. Similarly, an entrant might choose to locate apart even though its preferred point is the one incumbent is sitting on, for h small enough. Here, we will only analyze the former case for brevity, since (1) mathematics and intuition of the latter case is similar and (2) the former case is arguably empirically more relevant.

Proposition 10. *Let a mass h of people at L prefer the incumbent and a mass h of people at R prefer the entrant. In the political advertising game, the entrant party always chooses to locate apart. In the product advertising game, however, there exists a parameter set such that the entrant firm will choose to co-locate for h small enough.*

[14] $U_{iL} = R_L - |x_i - x_L| + A_i + \delta$ and $U_{kL} = R_L - |x_k - x_L| + A_k$ where $\delta > 0$ is very small. Therefore it affects the preferences of the existing customers (voters) between firms (parties) when they are identical but does not change the total consumption (vote count).

Therefore, although the parameter set where an entrant might co-locate is smaller now, it is still non-trivial. The fear of a negative advertising war might push a new entrant into a product choice where the consumer base is already favoring the incumbent when all else is equal.

5 Conclusion and Discussion

Comparative advertising is a common form of informing and persuading consumers about the relative strengths of one's own brand over the other competing brands. Our paper focuses on the competition of two brands that choose their product location strategy in anticipation of a negative advertising choice by their rivals. The advertisers face a tradeoff — while choosing product locations similar to that of the competitor reduces the likelihood of being subjected to a negative advertising attack, it also leads to under-utilizing the full consumer demand. We find that in certain settings, the threat of comparative advertising can result in lower product differentiation even though firms do not actively run comparative advertising campaigns. We extend our model to the case of political competition where the objective of competitors is to win by plurality (and not profit maximization). We show that this change in the objective function can explain the relatively widespread use of negative advertising in electoral races, as competitors care less about an overall reduction in voter turnout.

While comparative advertising was almost shunned upon in various parts of Europe until '90s,[15] in the United States, the FTC's Regulatory Overboard of Advertising has been actively encouraging firms to name their competitors and draw comparisons about pricing and product attributes.[16] The FTC statement regarding comparative advertising argued that it could "... assist (consumers) in making rational purchase decisions through direct comparisons of brands" and "... encourages product improvement and innovation, and can lead to lower prices in the marketplace." For negative advertisements to inform consumer decision making and incentivize firm innovation, firms should be actually using it in practice. Despite the FTC's encouragement, only about one-third of commercial brands ever engage in comparative ads (Grewal et al., 1997). Our results show that, in some

[15] In EU, until 2000, comparative advertising was regarded as an improper business practice and trading on another firm's reputation and goodwill was considered unfair (Romano, 2004).

[16] https://www.ftc.gov/public-statements/1979/08/statement-policy-regarding-comparative-advertising

sectors, firms might be avoiding product designs that are too different from the existing ones to avoid a subsequent negative advertising war. Therefore, in contrast to the FTC's claims, product improvement and innovation is discouraged in these sectors due to the threat of negative advertising. In such cases, welfare can in fact be improved by restricting negative advertising.

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Appendices

A Model Microfoundations

Consider a setting with two firms i and k where each product consists of m features in addition to the product location: $(c_1^i, ..., c_m^i)$ and $(c_1^k, ..., c_m^k)$. Customers are only aware of first $n < m$ features. Evaluating these n features, customers decide on a willingness to pay as a function of product's quality in each of these features: $f(c_1^i, ..., c_m^i)$ and $f(c_1^k, ..., c_m^k)$. In other words, while customers differ on how they value product location, their preferences towards these m features are identical.

A.1 Informative Advertising

Firms can choose to run positive or negative advertising. Choosing positive advertising allows firm i to introduce a feature $c_{t_i}^i$ where $t_i \in \{n+1, m\}$, to the customers' awareness. Choosing negative advertising allows firm i to introduce a new feature $c_{t_k}^k$ of firm k, where $t_k \in \{n+1, m\}$, to the customers' awareness. However, if firm i and k have the same location, this results in $c_{t_k}^i$ being revealed as well. Let t_i^* and t_k^* denote optimal choices of firm i, conditional on running positive and negative advertising respectively.

A special case of this setting, where

(1) $f(c_1^i, ..., c_n^i) = R$

(2) $f(c_1^i, ..., c_n^i, c_{t_i^*}^i) - f(c_1^i, ..., c_n^i) = 0$

(3) $f(c_1^k, ..., c_n^k, c_{t_k^*}^k) - f(c_1^k, ..., c_n^k) = -b$

(4) $f(c_1^i, ..., c_n^i, c_{t_k^*}^i) - f(c_1^i, ..., c_n^i) = -\beta$

(5) All effects are symmetric for firm i and firm k

corresponds to the reduced form setting we have in the main text.

A.2 Persuasive Advertising

Firms can choose to run positive or negative advertising. Choosing positive advertising allows firm i to change customer's evaluation of a feature $c_{t_i}^i$ positively to $c_{t_i}^i + \epsilon^+$ where $t_i \in \{1, n\}$ and $\epsilon^+ > 0$. Choosing negative advertising allows firm i to change customer's evaluation of a feature

$c^k_{t_k}$ negatively to $c^i_{t_i} - \epsilon^-$ where $t_i \in \{1, n\}$ and $\epsilon^- > 0$. However, if firm i and k have the same location, this results in evaluation $c^i_{t_k}$ being negatively affected as well to $c^i_{t_k} - \epsilon^f$ where $\epsilon^f > 0$. Let t^*_i and t^*_k denote optimal choices of firm i, conditional on running positive and negative advertising respectively.

A special case of this setting, where

(1) $f(c^i_1, ..., c^i_m) = R$

(2) $f(c^i_1, ..., , c^i_{t^*_i} + \epsilon^+, ..., c^i_n) - f(c^i_1, ..., c^i_n) = 0$

(3) $f(c^k_1, ..., , c^k_{t^*_k} - \epsilon^-, ..., c^k_n) - f(c^k_1, ..., c^k_n) = -b$

(4) $f(c^i_1, ..., , c^i_{t^*_k} - \epsilon^f, ..., c^i_n) - f(c^i_1, ..., c^i_n) = -\beta$

(5) All effects are symmetric for firm i and firm k

corresponds to reduced form setting we have in the main text.

B Proofs

Proof of Lemmas 1 and 2. We first give a formal restatement of the two lemmas.

(i) If $b < \beta$, (positive,positive) is the unique equilibrium outcome of the subgame after co-location.

(ii) When $b > \beta$ and $F(\beta) + F(\beta + d) \geq 1 + \frac{F(d)}{2}$, there are two equilibrium outcomes for the advertising subgame in co-location: (positive, positive) and (negative, negative), where payoff from the former is strictly preferred to payoff from the latter by both firms. When $F(\beta)+F(\beta+d) < 1+\frac{F(d)}{2}$, (negative, negative) is the unique equilibrium outcome of the subgame after co-location.

(iii) When $b > d$, (negative, negative) is the unique equilibrium outcome of the subgame after locating apart.

(iv) If $b < d$, any two actions constitute an equilibrium after locating apart. However, (positive, positive) is the only surviving equilibrium, if there is a non-zero spillover from negative advertising after locating apart.

Proof is as follows:

(i) Let $b < \beta$. Then spillover effect (β) from negative advertising is larger than the damage

caused on the competing product (b). Therefore, negative advertising is strictly dominated by positive advertising in the advertisement subgame.

(ii) If newcomer chooses to locate on the left point, payoff matrix is as per Table 2 (negative, negative) is trivially an equilibrium given Assumption 1. For (positive,positive) to be an equilibrium too, we need $1 - \frac{F(d)}{2} \geq 2 - F(\beta) - F(\beta + d)$ which can be simplified into the above expression.

(iii) If entrant chooses to locate on the right point, payoff matrix becomes

		Firm k	
		Positive	*Negative*
Firm i	*Positive*	$1, 1$	$0, 2 - F(d)$
	Negative	$2 - F(d), 0$	$1 - F(b), 1 - F(b)$

Table A1: Payoffs from Advertisement Subgame (Locating Apart)

where (negative, negative) is the unique equilibrium by Assumption 1.

iv) Let $b < d$. If entrant chooses to locate on the right point, payoff matrix becomes

		Firm k	
		Positive	*Negative*
Firm i	*Positive*	$1, 1$	$1 - F(b), 1$
	Negative	$1, 1 - F(b)$	$1 - F(b), 1 - F(b)$

Table A2: Payoffs from Advertisement Subgame (Locating Apart)

where all action profiles constitute an equilibrium, as outcomes of either agent do not depend on her own action. Now assume there is an $\epsilon > 0$ spillover from negative advertising, where ϵ is a very small number. The payoff matrix would become

		Firm k	
		Positive	*Negative*
Firm i	*Positive*	$1, 1$	$1 - F(b), 1 - \epsilon$
	Negative	$1 - \epsilon, 1 - F(b)$	$1 - F(b) - \epsilon, 1 - F(b) - \epsilon$

Table A3: Payoffs from Advertisement Subgame (Locating Apart)

Here, (positive, positive) would be the unique equilibrium. □

Proof of Proposition 2. We first give a formal restatement of the proposition.

(i) If $b < d$, entrant chooses to differentiate its product and (Positive, Positive) is the equilibrium advertising outcome.

(ii) If $\beta > b > d$, the entrant collocates with incumbent and both firms run positive advertising.

If $b > d, \beta$ and

(iii) $F(\beta)+F(\beta+d) < 1+\frac{F(d)}{2}$, entrant chooses to differentiate its product and (negative, negative) is equilibrium advertising outcome.

(iv) $F(\beta) + F(\beta + d) \geq 1 + \frac{F(d)}{2}$ there are two pure strategy SPEs. In the first, the entrant differentiates its position and both firms run negative advertising. In the second, the entrant does not differentiate and both firms run positive advertising. Profits are higher for both firms under the second equilibrium.

The proof is as follows: (i) Follows from Lemma 1. The entrant prefers locating apart to co-locating as long as the advertising outcome is not strictly better in co-location, where strictness comes from Assumption 1. Given our equilibrium selection in Lemma 1, (Positive, Positive) surfaces as the unique equilibrium of the SPE.

(ii) The unique advertising equilibrium when entrant locates apart is (negative,negative). Anticipating that, entrant would pick co-locating if potential income from co-locating $(1 - \frac{F(d)}{2})$ is larger than its potential income from locating apart $(1 - F(b))$. Since, the case is defined with $b > d$, we have $1 - \frac{F(d)}{2} > 1 - F(b)$. Thus, entrant would pick co-locating.

(iii) In this scenario, unique advertising equilibrium is (negative,negative) regardless of entrant's

location choice. Therefore, entrant would locate apart as the payoff after locating apart $1 - F(b)$ is larger than payoff after co-locating $1 - (F(b+\beta) + F(b+\beta+d))/2$.

(iv) Here, there are two pure strategy Nash equilibria of the advertising game after co-location. In one, co-location results in negative advertising, thus entrant locates apart and (negative, negative) becomes the equilibrium outcome similar to part (ii). In the other, co-location results in positive advertising. In this case, payoff from co-location $1 - \frac{F(d)}{2}$ is greater than locating apart $1-F(b)$. Therefore, entrant co-locates and both firms run positive advertisements in the equilibrium outcome. □

Proof of Proposition 5. We will provide a proof for only one of the cases, since all follow the same logic. Suppose $b < d$ and $b < \beta$. Here, positive advertising is the unique equilibrium of the subgame. In the location decision, there are two equilibria:

		Firm k	
		Left	*Right*
Firm i	*Left*	$1 - \frac{F(d)}{2}, 1 - \frac{F(d)}{2}$	$1, 1)$
	Right	$1, 1$	$1 - \frac{F(d)}{2}, 1 - \frac{F(d)}{2}$

Table A4: Payoffs from Location Decision

Therefore, there are two equilibria in the game: (i) i locates left, k locates right and they both run positive advertising (ii) i locates right, k locates left and they both run positive advertising. □

Proof of Proposition 3. We define $\beta^*(d)$ s.t. $F(\beta^*(d)) + F(\beta^*(d) + d) = 1 + F(d)/2$.[17]

First part immediately follows from Proposition 2. When $b > d$, there are two possibilities. If $\beta^*(d) > b$, firms locate apart and run negative advertising for small β ($\beta < b$), co-locate and run positive advertising for intermediate values of β ($\beta^*(d) > \beta > b$) and can do either for high values of β ($\beta > \beta^*(d)$). If $\beta^*(d) < b$, firms locate apart and run negative advertising for small β

[17]To be precise, $\beta^*(d)$ need not exist. Depending on the shape of F and value of d, it might be the case that $F(\beta) + F(\beta + d) > 1 + F(d)/2$ or $F(\beta) + F(\beta + d) < 1 + F(d)/2$ for all β. In that case, the result only simplifies and the core message stays the same.

($\beta < \beta^*(d)$), co-locate and run positive advertising for high values of β ($\beta > b$) and can do either for intermediate values of β ($\beta^*(d) < \beta < b$). So, with the possible exception of the transition from intermediate to high values of β when $\beta^*(d) > b$, larger values of β imply a higher probability of observing unintended consequences. □

Proof of Proposition 4. First, let $b < \beta$. When $d > \beta$, availability of comparative advertising has no effect on firm behavior. When $\beta > d > b$, we can have (1) locating apart with (negative, negative) as the unique equilibrium outcome or (2) locating apart with (negative, negative) and co-locating with (positive, positive) as multiple equilibrium outcomes. When $d < b$, co-locating with (positive, positive) is the unique equilibrium outcome.

Now let $b > \beta$. When $d > b$, availability of comparative advertising has no effect on firm behavior. When $d < b$, we can have (1) locating apart with (negative, negative) as the unique equilibrium outcome or (2) locating apart with (negative, negative) and co-locating with (positive, positive) as multiple equilibrium outcomes. □

Proof of Proposition 6. If firms were to locate together, each firm would prefer to undercut the price to get the whole market since $H - 1$ is assumed to be small. Therefore, in the unique equilibrium, they would charge a low price and end up with the payoff 1/2. If they were to locate apart, both would charge low price to steal other firms consumers since $2 - F(d) > H(1 - F(\eta)$. Their equilibrium payoff would be 1. □

Proof of Proposition 7. If firms were to locate together, the payoff matrix from the pricing-advertising subgame would yield three possible equilibria:

(1) ({ Low,Negative },{ Low,Negative })

(2) ({ Low,Positive },{ Low,Positive }) if $F(\beta) + F(\beta + d) \geq 1 + \frac{F(d)}{2}$ and $H \leq \frac{1-F(d)/2}{2-F(\eta)-F(\eta+d)}$.

(3) ({ High,Negative },{ High,Negative }) if $H \geq \frac{4-2F(b)-2F(d+b)}{2-F(\eta+b+\beta)-F(\eta+d+b+\beta)}$.

If firms were to locate apart, the payoff matrix from the pricing-advertising subgame would yield two possible equilibria:

(1') ({ Low,Negative },{ Low,Negative })

(2') ({ High,Negative },{ High,Negative }) if $H \geq \frac{2-F(d+b)}{1-F(\eta+b)}$.

Entrant prefers (2') to (3) and to (1) for small η or large β. □

Proof of Lemma 3. (i) Suppose the parties are located apart. Decrease in the indirect utility from voting for the competitor in negative advertising (b) is larger than the spillovers from negative advertising, which is assumed to be 0 (or a very small number). Since only relative votes matter, parties run negative advertising.

(ii) Let $b < \beta$. Relative decrease $(b - \beta)$ in the indirect utility from voting for the competitor in negative advertising is less than 0. Since only relative votes matter, parties run positive advertising.

(iii) Let $b > \beta$. Relative decrease $(b - \beta)$ in the indirect utility from voting for the competitor in negative advertising is greater than 0. Since only relative votes matter, parties run positive advertising. □

Proof of Proposition 8. Equilibrium of the advertising game is always symmetric (unless parties are indifferent about both advertisement strategies) and the game itself is symmetric. Parties try to maximize voting differentials. Therefore, all equilibria necessarily end with the outcome $(0, 0)$. Thus, parties are indifferent about locating apart or co-locating, as all will results in the outcome $(0, 0)$. □

Proof of Proposition 9. We first give a formal restatement of the proposition.

Let $m > 1$. In political advertising game, entrant party always chooses to locate apart. In the product advertisement game, however, entrant firm will choose to co-locate for $m < \frac{1}{1+F(d)-2F(b)}$ where

(i)$b < \beta, b > d$

(ii) $b > \beta, b > d$ and $F(\beta) + mF(\beta + d) \geq \frac{1+m}{2} + m\frac{F(d)}{2}$.

For the rest of the parameter space, entrant firm will locate apart.

Let $m < 1$. In political advertising game, entrant party always chooses to co-locate. In the product advertisement game, however, entrant firm will choose to locate apart for

(i)$m > \frac{1}{1+F(d)}$ where $b < \beta, b < d$

(ii) $m > \frac{1}{1+F(d)}$ where $b > \beta, b < d$ and $F(\beta) + mF(\beta + d) \geq \frac{1+m}{2} + m\frac{F(d)}{2}$.

(iii) $m > \frac{1-F(b+\beta)}{1+F(b+\beta+d)}$ where $b > \beta, b < d$ and $F(\beta) + mF(\beta + d) < \frac{1+m}{2} + m\frac{F(d)}{2}$.

(iv) $m > \frac{1-F(b+\beta)}{1+F(b+\beta+d)+2F(b)}$ where $b > \beta, b > d$ and $F(\beta) + mF(\beta + d) < \frac{1+m}{2} + m\frac{F(d)}{2}$.

For the rest of the parameter space, entrant firm will co-locate.

The proofs are as follows:

When firms locate apart, payoff matrix for the advertisement game becomes

		Firm k: *Positive*	Firm k: *Negative*
Firm i	*Positive*	$1, m$	$0, m + 1 - F(d)$
	Negative	$m + 1 - F(d), 0$	$1 - F(b), m - mF(b)$

Table A5: Payoffs from Advertisement Subgame (Locating Apart)

for $b > d$ and

		Firm k: *Positive*	Firm k: *Negative*
Firm i	*Positive*	$1, m$	$1 - F(b), m$
	Negative	$m + 1 - F(d), 0$	$1 - F(b), m - mF(b)$

Table A6: Payoffs from Advertisement Subgame (Locating Apart)

for $b < d$. Unique equilibrium outcome from the former is (Negative, Negative). In the latter, everything is a possible equilibrium outcome. We can again introduce an $\epsilon > 0$ spillover from negative advertising after locating apart and get (Positive, Positive) as the unique equilibrium outcome. When firms co-locate, payoff matrix for the advertisement game becomes

		Firm k: *Positive*	Firm k: *Negative*
Firm i	*Positive*	$\frac{1+m-mF(d)}{2}, \frac{1+m-mF(d)}{2}$	$0, 1 - F(\beta) + m - mF(\beta + b)$
	Negative	$1 - F(\beta) + m - mF(\beta + b), 0$	$\frac{1+m-F(b+\beta)-mF(\beta+b+d)}{2}, \frac{1+m-F(b+\beta)-mF(\beta+b+d)}{2}$

Table A7: Payoffs from Advertisement Subgame (Co-location)

for $b > \beta$ and

		Firm k: *Positive*	Firm k: *Negative*
Firm i	*Positive*	$\frac{1+m-mF(d)}{2}, \frac{1+m-mF(d)}{2}$	$1-F(b)+m-mF(b+d), 0$
	Negative	$0, 1-F(b)+m-mF(b+d)$	$\frac{1+m-F(b+\beta)-mF(\beta+b+d)}{2}$, $\frac{1+m-F(b+\beta)-mF(\beta+b+d)}{2}$

Table A8: Payoffs from Advertisement Subgame (Co-location)

for $b < \beta$. Unique equilibrium outcome for the former is (Negative, Negative) for $F(\beta)+mF(\beta+d) < \frac{1+m}{2} + m\frac{F(d)}{2}$. Otherwise, both (Positive, Positive) and (Negative, Negative) are possible equilibrium outcomes. In the latter, (Positive, Positive) is the unique equilibrium outcome.

Given the equilibria of the advertising subgame, we can characterize the entrant's location decision based on m. For $b < \beta$ and $b < d$, entrant (firm k) compares the payoff from upper left cell of the second payoff matrix to the upper-left cell of the fourth payoff matrix. Therefore, locates apart if $m > \frac{1}{1+F(d)}$.

Similarly, for $b < \beta$ and $b > d$, entrant locates apart if $m > \frac{1}{1+F(d)-2F(b)}$ when $1+F(d)-2F(b) > 0$.

For $b > \beta$ and $b < d$, entrant locates apart if $m > \frac{1}{1+F(d)}$. If $m < \frac{1}{1+F(d)}$ and $m > \frac{1-F(b+\beta)}{1+F(b+\beta+d)}$ then entrant locates apart when $F(\beta) + mF(\beta+d) < \frac{1+m}{2} + m\frac{F(d)}{2}$.

For $b > \beta$ and $b > d$, entrant locates apart if $m > \frac{1}{1+F(d)-2F(b)}$ when $1 + F(d) - 2F(b) > 0$. If $m < \frac{1}{1+F(d)-2F(b)}$ and $m > \frac{1-F(b+\beta)}{1+F(b+\beta+d)+2F(b)}$ then entrant locates apart when $F(\beta)+mF(\beta+d) < \frac{1+m}{2} + m\frac{F(d)}{2}$.

For the rest of the parameter space, entrant co-locates. Taking into account whether the bounds on m are greater than or less than 1, the propositions follow. □

Proof of Proposition 10. We first give a formal restatement of the proposition.

Let mass h of people in left point prefer the incumbent and mass h of people in right point prefer the entrant. In political advertising game, entrant party always chooses to locate apart. In the product advertisement game, however, entrant will choose to collocate for

(i)$b < \beta, b > d$

(ii) $b > \beta, b > d$ and $F(\beta) + mF(\beta + d) \geq 1 + (1 + h)\frac{F(d)}{2}$.

For the rest of the parameter space, entrant firm will locate apart.

The proof is as follows:

For the parameter set entrant locates apart in the original case (Proposition 2), it will locate apart here as well, since now there is more incentive to locate apart. Therefore, we only need to check two cases.

For $b < \beta, b > d$, entrant will locate apart if $1 - F(b) \geq \frac{1-h+(1+h)(1-F(d))}{2}$ which requires $h \geq \frac{2F(b)}{F(d)} - 1 > 1$. Therefore, entrant will co-locate.

For $b > \beta, b > d$ and $F(\beta) + mF(\beta + d) \geq 1 + (1 + h)\frac{F(d)}{2}$, there are two advertising equilibria after co-locating. Entrant prefers the better outcome of co-locating (where positive advertising is played by both firms) to locating apart if $1 - F(b) < \frac{1-h+(1+h)(1-F(d))}{2}$, which is always satisfied. Therefore, in one of the equilibria in this parameter set, entrant chooses to co-locate. □