Price Informativeness and Business Cycle Misallocation

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Abstract: Recessions are characterized by slow input reallocation and increased measures of misallocation. A usual suspect is declining information quality about new investment opportunities. We study the role of information frictions by measuring how the informativeness of the stock prices changes with business cycles. We first build a stock market model in which both the information content and the noise in prices respond to changes in economic activity, affecting how well those prices reflect firm's performance. Then we incorporate this module in a dynamic model with heterogeneous firms to characterize how stock price informativeness and capital misallocation interact with one another. An increase in liquidity concerns of traders can simultaneously boost information production, decrease stock price informativeness, and increase capital misallocation in the economy.

*** VERY PRELIMINARY AND INCOMPLETE DRAFT ***

1 Introduction

Economic downturns are associated with a decrease in measures of aggregate productivity. Two consistent findings so far are that misallocation of inputs increases during downturns, and capital reallocation decreases.[1] These findings contradict the Schumpeterian growth theory, which suggests decreased demand makes recessions ideal for reallocation. One suspect is the counter-cyclical information quality on investment opportunities. In particular, stock prices reflect the actions of traders who spend time, money, and effort to evaluate firms' future potential and are commonly used to guide investment.[2] Hence, understanding the cyclical behavior of stock price informativeness, i.e., the amount of information revealed by stock prices might shed some light on the capital (re)allocation puzzle.

Stock price informativeness crucially depends on (1) how informed traders are in the first place and (2) how well prices reflect their information. In this paper, we theoretically analyze how asset liquidity interacts with these two channels. We first build a model of information acquisition and trading where the noise in prices is endogenously determined. Second, we incorporate it into a neoclassical growth model where stocks are claims on real assets, and the real sector learns from the trades in the stock markets. Then, we study how the informativeness of these prices interacts with shocks to the economy.

A comprehensive analysis of price informativeness requires relaxing the commonplace assumption of exogenous noise in prices, which followed Grossman and Stiglitz (1980). Exogenous noise prevents prices from perfectly revealing and maintains an equilibrium with incentives to acquire costly information. However, the assumptions made about the noise's exogenous characteristics also dictate how it responds to shocks. We start by building a model with different trading motives across investors. Specifically, we model two types of traders -day and night- interested in different asset properties -liquidity risk and fundamental payoff-. This structure creates endogenous noise in prices:

[1] See Eisfeldt and Rampini (2006).

[2] Baker et al. (2003) shows stock prices are important for the corresponding firm's investment when the firm is dependent on equity with external financing needs. Chen et al. (2006) and Bennett et al. (2019) show that the sensitivity of the firm's investment and CEO turnover on its stock price increases as empirical measures of price informativeness increase. Edmans et al. (2012) shows a decrease in stock prices increases the likelihood that the corresponding firm will be subject to a takeover. Feldman and Schmidt (2003) describes how regulators use stock prices in their decision making.

a high price may indicate a low liquidity risk or a high fundamental payoff.

The stock trading model offers several insights by itself. First, when a larger share of traders are worried about the liquidity risk, the price informativeness on the fundamental payoff goes down. Second, traders may acquire more information about the firm's performance even when nothing changes about its profitability outlook. In particular, information acquisition intensifies when (1) more traders care about its stock's liquidity, (2) the quality of public information about its liquidity decreases, and (3) the quality of costly information about its liquidity increases. Third, in an otherwise symmetrical setting, the total resources spent on information acquisition are the largest when there is an equal number of day and night traders.

Next, we incorporate this stock market module inside a neoclassical growth model with heterogeneous firms. In the absence of information frictions, the investors would allocate capital across firms based on their true idiosyncratic productivity. In our model, the allocation is based on the investors' best guess, given the stock prices. While the stock traders acquire costly information and trade based on their information, the economy's real side uses the prevailing stock prices as signals of true productivity. Hence, the stock markets affect the resource allocation in the real sector through prevailing prices. On the other hand, real shocks affect the stock markets by changing the profitability of firms. This structure allows the amplification of small shocks through feedback loops between the real and financial sectors.

The main mechanism of the model works as follows. A shock that increases liquidity traders' share in the economy masks the information about fundamentals in prices. This mechanism raises two real distortions: (1) increased spending on acquiring costly information by traders and (2) worse allocation of capital across firms by the investors. The first effect allocates resources away from productive capital and towards information producing, while the second causes a misallocation of capital across firms and lead to lower aggregate investment. Hence a financial shock that increases the liquidity needs increases misallocation, decreases aggregate investment, and increases the resources spent on information production, consistent with the patterns observed during recessions.[3]

The model also allows separately accounting for the efficiency losses through different

[3]See Jiang et al. (2015) and Loh and Stulz (2018) for evidence on counter-cyclical information production by investors.

channels. We look at the efficiency loss due to capital allocated to information production and the loss due to misallocation. We find the efficiency loss due to information production disappears when the share of liquidity traders is either close to 0 or 1.

Our paper lies at the intersection of the literature on price informativeness and the literature on input misallocation. The vast majority of the theoretical literature on price informativeness assumes an exogenous source of noise to prevent prices from being perfectly informative, following the impossibility theorem of Grossman and Stiglitz (1980). We endogenize the information/noise ratio by assuming two dimensions of information condensed in a single price. The closest papers to ours here are Stein (1987) and Vives (2014). The former uses heterogeneity in market access while the latter uses heterogeneity in preferences to generate imperfectly informative prices without exogenous noise. Both papers are theoretical and restrict attention to implications for the stock markets. We extend their framework and allow the heterogeneity itself to change over time. The empirical literature focuses on measuring price informativeness. Dávila and Parlatore (2018) are the first to map empirical regression estimates to parameters in a Grossman and Stiglitz (1980) structure to infer price informativeness. They use time-series regressions to measure price informativeness for each stock, which requires them to make assumptions on how model parameters change over time to keep the cross-sectional variation flexible. We, on the other hand, use cross-sectional regressions to measure price informativeness over time. Thus, we make assumptions on the extent of heterogeneity across stocks to allow parameters to change flexibly over time. Bai et al. (2016), similar to us, analyzes the long-run trend in price informativeness using cross-sectional regressions. However, they are interested in the ability of prices to predict future stock performance, which is determined jointly by the availability of information on future prices and the ability of stock markets to communicate such information. Our model allows disentangling the two components.

One strand of the literature on input misallocation analyzes the patterns of input misallocation across firms in recessions. Foster et al. (2016) find the extent of reallocation across the U.S. firms has declined during the great recession. Kehrig (2015) finds dispersion of productivity distribution in the U.S. is larger in recessions than booms.[4] Furthermore, Eisfeldt and Rampini (2006) shows the amount of capital reallocation is procyclical

[4]The increased misallocation is not specific to the U.S. and has been documented for other countries in economic crises as well. See Oberfield (2013) for Chile, Sandleris and Wright (2014) for Argentine, Dias et al. (2016) for Portugal and Di Nola (2016) for the U.S.

and large countercyclical reallocation costs are required to justify it. Tighter financial constraints, counter-cyclical adverse selection in the market for used-capital, managers' incentives to hide reallocation needs from owners during recessions have been proposed as potential mechanisms for the counter-cyclical misallocation.[5] On the other hand, others take increased uncertainty/misallocation as a primitive shock and analyze its effects to understand business cycles.[6] We contribute to this literature by first proposing a novel mechanism that creates misallocation and analyzing how the misallocation caused by the drop in productivity can feedback to the price informativeness and amplify the initial shock.

There is a recent, but growing, literature at the intersection of these two strands. Benhabib et al. (2019) does a theoretical analysis similar to ours with two-way learning between the real and the financial sectors. Their model exhibits complementarity in information acquisition. Thus, a shock that reduces incentives to acquire information in one sector induces the other to reduce information acquisitions, creating equilibrium switches that amplify the initial shock. David et al. (2016) and David and Venkateswaran (2019) are the closest papers to our study. The former focuses on the role of informational frictions in resource allocation and measures how much each source of information contributes to productivity gaps. The latter has a larger scope and incorporates many potential frictions that can distort resource allocation on top of informational frictions. Both analyses provide static measures; thus, they are silent about cyclicality. While our framework restricts attention to stock markets as the main source of information, we introduce endogenous noise, time-varying model parameters, and a two-way interaction between real and financial sectors.

The remainder of the paper is structured as follows. Section 2 introduces the stock market model with endogenous noise, and Section 3 incorporates it into an RBC model. Section 4 concludes.

[5] See Ordonez (2013), Khan and Thomas (2013), Fajgelbaum et al. (2017) and Straub and Ulbricht (2017) for tighter financial constraints, Fuchs et al. (2016) for adverse selection, and Eisfeldt and Rampini (2008) for managerial incentives.

[6] See Christiano et al. (2014), Arellano et al. (2016), and Bloom et al. (2018).

2 Static Model

In this section, we present a simple static environment with a single risky asset. The setting is designed to be symmetric to flesh out the main mechanics of our mechanism. The price functions as a signal for two properties, which are valued differently by different traders. The trading behavior of one type of trader masks the information for the other type.

2.1 Environment

Preferences There is a measure one of traders with CARA period utility functions. That is, utility from consuming an amount W is given by $V(W) = -e^{-aW}$. A $\gamma \in (0, 1)$ fraction of traders live on a sunny island (sunny traders) and $1 - \gamma$ fraction live on a cloudy island (cloudy traders).

Technology There is a safe asset (money) with return R regardless of where it is consumed. There is also a risky asset (orchid) which gives a random return u_1 when planted in the sunny island and u_2 when planted in the cloudy island. Returns of this risky asset consist of two parts:

$$u_1 = \theta_1 + \epsilon_1$$

$$u_2 = \theta_2 + \epsilon_2$$

where θ_k can be privately observed at a cost $c_k(.)$ while ϵ_k are unobservable. Both θ_k and ϵ_k are random variables. The cost of acquiring information $c_k(.)$ is assumed to be an increasing function of the number of informed traders[7].

Endowments Trader j is assumed to be endowed with $\bar{M}_j$ of safe asset and $\bar{X}_j$ of the risky asset. The total supply of risky asset in the economy is assumed to be 0 and it is common knowledge. We denote the price of the risky asset with P where the price of safe asset is normalized to 1. Trader j's starting wealth becomes $\bar{M}_j + P\bar{X}_j$.

Distributional Assumptions We assume $\theta_k \sim \mathcal{N}(\bar{\theta_k}, \sigma^2_{\theta_k})$ and $\epsilon_k \sim \mathcal{N}(0, \sigma^2_{\epsilon_k})$. We also

[7]This rules out any complementarity in information acquisition and prevents multiple equilibria. This assumption can be derived from a model where the cost of acquiring information is heterogeneous across traders and those with the smallest cost acquire the information first.

assume $\theta_1, \theta_2, \epsilon_1, \epsilon_2$ are jointly independent.

2.2 Portfolio Choice Problem

Both the sunny and cloudy traders first decide whether to pay the cost $c_k(\lambda_k)$ to get information and then decide on a portfolio of assets. After the informational decision, the portfolio choice problem of trader j from island k becomes

$$\begin{aligned} &\max_{M_j, X_j} E\Big[-exp\Big[-a[RM_j + u_k X_j]\Big]\Big] \\ &s.t. \;\; M_j + PX_j = \bar{M}_j + P\bar{X}_j \end{aligned} \tag{1}$$

where M_j and X_j are safe and risky asset demands of trader j. Let's define $W_{0j} \equiv \bar{M}_j + P\bar{X}_j$. Since u_k is normally distributed, the end-of-period wealth of trader j is also normally distributed. Therefore, we can re-write the problem as

$$\max_{M_j, X_j} -exp\Big[-a\Big[RW_{0j} + X_j\left[E[u_k] - RP\right] - \frac{a}{2}X_j^2 Var[u_k]\Big]\Big] \tag{2}$$

which yields

$$X_j^* = \frac{E[u_k] - RP}{aVar[u_k]}$$

Informed traders know the relevant θ. Therefore, they form their expectations on u_k based on θ and do not need to use the market price P. Uninformed traders look at the market price to form their expectations. For these traders, the price is not perfectly informative. The reason is, agents only care about the asset's payoff on the island they plant it but the market price is a function of payoffs in both periods. Therefore, a high price might be the outcome of a high payoff on island 1 as well as in island 2.

Demand for the risky asset for an informed trader j that lives in island k becomes

$$X_j^* = \frac{\theta_k - RP}{a\sigma_{\epsilon_k}^2}$$

Demand for the risky asset for an uninformed trader that lives in island k becomes

$$X_j^* = \frac{E[u_k|P] - RP}{aVar[u_k|P]}$$

2.3 Solution

Let's define λ_1 and λ_2 as the fraction of traders that pay the cost to be informed on island 1 and 2 respectively. We denote $\lambda \equiv \{\lambda_1, \lambda_2\}$. Market clearing for the risky asset requires

$$\begin{aligned}\gamma\left[\lambda_1\left[\frac{\theta_1 - RP}{a\sigma^2_{\epsilon_1}}\right]+(1-\lambda_1)\left[\frac{E[u_1|P]-RP}{aVar[u_1|P]}\right]\right]& \\ +(1-\gamma)\left[\lambda_2\left[\frac{\theta_2 - RP}{a\sigma^2_{\epsilon_2}}\right]+(1-\lambda_2)\left[\frac{E[u_2|P]-RP}{aVar[u_2|P]}\right]\right] &= 0\end{aligned} \tag{3}$$

Lemma 1. *Given the distributional assumptions, there exists a market price for a given λ with the form*

$$P_\lambda(\theta) = \alpha_{0\lambda} + \alpha_{0\lambda}\left(\theta_2 + \frac{\gamma\lambda_1\sigma^2_{\epsilon_2}}{(1-\gamma)\lambda_2\sigma^2_{\epsilon_1}}\theta_1\right) \tag{4}$$

where $\alpha_{0\lambda}$ and $\alpha_{0\lambda}$ are real numbers that possibly depend on λ but not on θ_1 or θ_2.

Proof. Appendix 4.1. □

Corollary 1. *For a given λ, price becomes more informative about θ_1 when*
(i) a larger fraction of traders are from island 1
(ii) a larger fraction of island 1 traders are informed compared to island 2 traders
(iii) the payoff at island 1 is less volatile conditional on information on θ compared to payoff at island 2.

Proof. Immediately follows from Equation 4. □

Let's denote the end-of-period wealth for trader j from island k for a given λ as $W_j^{\lambda k}$. That is

$$W_{Ij}^{\lambda k} = R(W_{0j} - c^k(\lambda_k)) + [u^k - RP_\lambda(\theta)]X_I^j(P_\lambda(\theta), \theta) \tag{5}$$

$$W^{\lambda k}_{Uj} = RW_{0j} + [u^k - RP_\lambda(\theta)]X^j_U(P_\lambda(\theta), P^*_\lambda) \tag{6}$$

where subscripts I, U refer to being informed and uninformed and P^*_λ is the realized market price. Trader j would be willing to pay to be informed if and only if $E[V(W^{\lambda k}_{Ij})|P_\lambda] \geq E[V(W^{\lambda k}_{Uj})|P_\lambda]$.

Lemma 2. *Under the distributional assumptions,*

$$\frac{E[V(W^{\lambda k}_{Ij})|P_\lambda]}{E[V(W^{\lambda k}_{Uj})|P_\lambda]} = e^{ac_k(\lambda_k)}\sqrt{\frac{Var[u_k|\theta_k]}{Var[u_k|P_\lambda]}} \equiv \psi^k(\lambda) \tag{7}$$

for $k \in \{1, 2\}$ *and* $j \in [0, 1]$.

Proof. Appendix 4.1. □

Corollary 2. $\psi^k(\lambda)$ *is monotone in each* λ_k*. Therefore,*
(i) If $\psi^k(\lambda) > 1 \ \forall \lambda_k \in [0, 1]$*, all island* k *traders become informed, i.e.* $\lambda^*_k = 1$.
(ii) If $\psi^k(\lambda) < 1 \ \forall \lambda_k \in [0, 1]$*, no island* k *traders become informed, i.e.* $\lambda^*_k = 0$.
(iii) Otherwise, λ^*_k *as a function of* λ_{-k} *is given by* $\psi^k(\lambda) = 1$.

Definition $P(\theta)^*, \lambda^*_1, \lambda^*_2$ constitutes a stochastic Rational Expectations Equilibrium (REE) such that
(i) λ^*_1, λ^*_2 are given by Corollary 2, given $P(\theta)^*$.
(ii) $P(\theta)^*$ satisfies the market clearing condition given in Equation 3, given λ^*_1, λ^*_2.

Proposition 1. *Let* $c(\lambda_k)$ *be strictly increasing and strictly concave. There exists a unique linear REE where price has the form given in Lemma 4.*

Proof. There exists a $P(\theta)^*$ for any $\lambda^*_1, \lambda^*_2 \in [0, 1]$ since market clearing condition is unbounded below and above for $P(\theta) \in R^+$. Therefore, showing $\lambda^*_1, \lambda^*_2 \in [0, 1]$ exists that satisfies Corollary 2 is sufficient. Given the structure of Corollary 2, $\lambda^*_1, \lambda^*_2 \in [0, 1]$ has to exist. Thus an equilibrium, which is not guaranteed to be interior, exists.

There exists a unique $P(\theta)^*$ for any $\lambda^*_1, \lambda^*_2 \in [0, 1]$ since market clearing condition is strictly monotonic in price. By Lemma 3, λ_k is unique for a given λ_{-k} since $\psi^k(\lambda)$ is strictly monotonic in λ_k. Therefore, there cannot exist multiple equilibria that are not interior. For interior equilibria, $\lambda^*_k(\lambda_{-k})$ is a proper function and is strictly increasing and concave in λ_{-k} for $k = 1, 2$. Therefore, $\lambda_1^*(\lambda_2)$ and $\lambda_2^*(\lambda_1)$ can only have a single crossing inside $[0, 1] \times [0, 1]$. Thus the equilibrium is unique. $\square$

Corollary 3. *Let $\lim_{\lambda_k \to 0} c(\lambda_k) = 0$ and $\lim_{\lambda_k \to 1} c(\lambda_k) = \bar{C}$ where $\bar{C}$ is large enough. Then the unique linear REE is interior.*

2.4 Equilibrium Characterization

Here, we focus on interior equilibria and follow Goldstein et al. (2014) to define price informativeness as the reduction in payoff variance faced after observing the price. However, we normalize it with the reduction in payoff variance after acquiring the costly signal.

Definition Price informativeness measure for island 1 traders is defined as

$$PI_1 = \frac{1}{1 + \frac{\sigma_{\theta_2}^2}{\sigma_{\theta_1}^2} \left(\frac{(1-\gamma)\lambda_2 \sigma_{\epsilon_1}^2}{\gamma \lambda_1 \sigma_{\epsilon_2}^2} \right)^2} \tag{8}$$

and for island 2 traders is defined as

$$PI_2 = \frac{1}{1 + \frac{\sigma_{\theta_1}^2}{\sigma_{\theta_2}^2} \left(\frac{\gamma \lambda_1 \sigma_{\epsilon_2}^2}{(1-\gamma)\lambda_2 \sigma_{\epsilon_1}^2} \right)^2} \tag{9}$$

An immediate implication is whatever increases the informativeness of the price for the island 1 traders decreases it for the island 2 traders and vice versa.

Using Lemma 3, in interior equilibria, we can write price informativeness for island i trader (to be denoted as PI_i) as

$$PI_i = \sigma_{\theta_i}^2 + \sigma_{\epsilon_i}^2 (1 - e^{2ac_i(\lambda_i)}) \tag{10}$$

In Grossman and Stiglitz (1980), equilibrium objects do not appear on the RHS, thus comparative statics become trivial. In our case, combining Definition 2.4 and (10) we have two equations to solve for two equilibrium objects λ_1 and λ_2. We further assume a cost function form $c(\lambda_k) = C_k \mathcal{C}(\lambda_k)$ where C_k is a parameter.

Proposition 2. *In interior equilibria, fraction of informed island* 2 *traders* λ_2 *increases as*
(1) a, C_1, C_2 *and* $\sigma^2_{\epsilon_1}$ *decreases*
(2) $\sigma^2_{\theta_1}$ *and* γ *increases*
(3) $\sigma^2_{\epsilon_2}$ *decreases and* $\sigma^2_{\theta_2}$ *increases.*
Symmetric results hold for island 1 *traders.*

Proof. Appendix 4.1. □

In Proposition 3, the comparative statics w.r.t. the parameters of the island 2 payoffs are similar to Grossman and Stiglitz (1980). A larger fraction of island 2 traders acquire information when they are less risk averse, information acquisition is cheaper, pre-acquisition uncertainty is higher, and post-acquisition uncertainty is lower.

What is new to our setting is the comparative statics w.r.t. to the payoffs in the other island. In particular, Corollary 1 shows the informed trades by one island masks the information about the other island in prices. Proposition 3 adds to it by describing how traders respond by changing their information acquisition. In particular, a larger fraction island 2 traders acquire information when for island 1 traders information acquisition is cheaper, pre-acquisition uncertainty is higher, and post-acquisition uncertainty is lower.

2.5 Comparative Statics

The model does not give closed-form solutions for all equilibrium objects, therefore, we rely on numerical solutions for comparative statics. To isolate the role of price informativeness, we focus on a fully symmetric structure, where beliefs and realizations of each island are identical. The benchmark values for parameters are given in Table 2.5. We analyze a case where both island 1 and island 2 payoffs turn out to be lower than expected. The parameter values are summarized in Table 2.5.

Figure 2.5 presents comparative statics with respect to fraction of island 1 traders: γ. Here, we interpret the comparative statics as a response to a liquidity shock[8]. The equilibrium response crucially depends on the initial and final values of γ. Panel 1 shows

[8] This can be justified in an infinitely played market game where θ realizations are i.i.d. over time, so static equilibria are played in each game. Therefore, transition dynamics can be ignored.

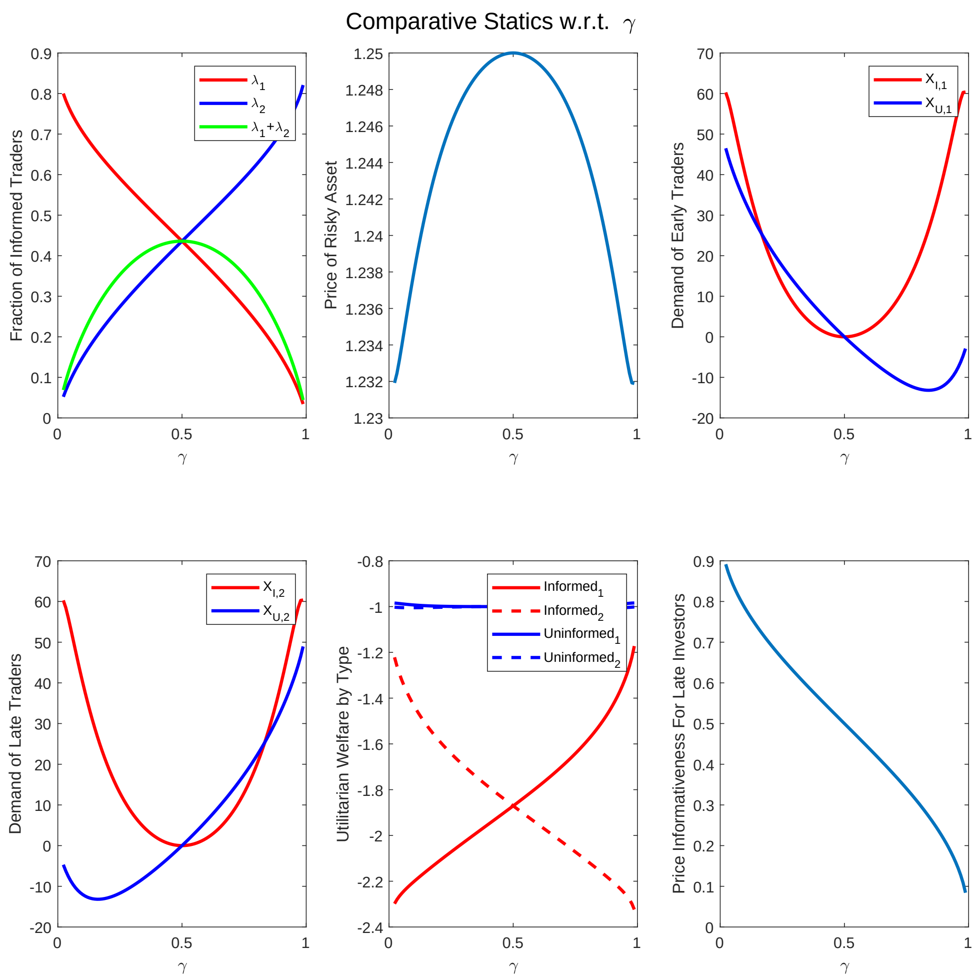

Figure 1: Comparative statics for the benchmark model with respect to γ

Table 1: Comparative Statics Benchmarks

Parameter		Value
Technology and Preferences		
a	Risk Aversion Parameter	0.03
γ	Fraction of Island 1 Traders	0.5
R	Risk Free Return	1
C_1	Cost of Acquiring Information for Island 1 Traders	0.5
C_2	Cost of Acquiring Information for Island 2 Traders	0.5
Beliefs		
$\bar{\theta}_1$	Mean of Island 1 Payoff Signal	1.35
$\sigma^2_{\theta_1}$	Variance of Island 1 Payoff Signal	0.01
$\sigma^2_{\epsilon_1}$	Quality of Island 1 Payoff Signal	0.05
$\bar{\theta}_2$	Mean of Island 2 Payoff Signal	1.35
$\sigma^2_{\theta_2}$	Variance of Island 2 Payoff Signal	0.01
$\sigma^2_{\epsilon_2}$	Quality of Island 2 Payoff Signal	0.05
Realized Values		
θ_1	Island 1 Payoff Signal	1.25
θ_2	Island 2 Payoff Signal	1.25

an increase in γ increases λ_2 and decreases λ_1 as predicted by Proposition 3. Meanwhile, the fraction of all the traders that acquire information can increase or decrease based on γ_0. In a situation where γ_0 is low, however, an increase is more likely to increase information acquisition than decrease.

The second panel shows the price response. As shown in Table 2.5, the signal is 1.25 for both island 1 and island 2 traders. In other words, the expected payoff is equal for both islands conditional on the only piece of information available. Therefore, when price informativeness is ignored, there is no reason for the price to depend on γ. However, the price actually responds to changes in γ, since traders change how they perceive prices and their information acquisition behavior.

Panels 3 and 4 show how trader demands change. As expected, informed traders are on the buyers' side as the price is always below the signal. When γ is low, the price carries more information about θ_2. Island 2 uninformed traders interpret the low price as low θ_2 and sell the risky asset. Island 1 uninformed traders rely mostly on their prior, since price doesn't carry much information on θ_1, and buy the asset.

Panel 5 shows utilitarian welfare is lowest when the number of early and late traders is similar. Since trades are only transfers between agents, welfare is mainly determined by the resources spent on acquiring information. Indeed, welfare function closely resembles the total fraction of informed traders in Panel 1. Lastly, Panel 6 confirms Corollary 1: information about θ_2 stored in prices decreases with γ[9].

2.5.1 Comparison with a Noise Trader Economy

Here, we compare the properties of the model to a classical noise trader model as in Grossman and Stiglitz (1980). Specifically, we ask how does the equilibrium price and behavior of island 2 traders would change if island 1 traders were replaced by noise traders. We equate the ex-ante distribution of noise trader demand to the ex-ante realized distribution of island 1 trader demand in the full model at $\gamma = 0.5$ as well as the realized demand. Then, changing the parameters of the model, we ask how these two models differ in their responses. Figure 2 summarizes the differences in comparative statics with γ for both models.

2.5.2 Multiple Assets

The model generalizes naturally to settings with multiple risky assets. CARA utility functions imply that equilibrium objects (demand, price etc.) related to the risky asset i only depend on its own payoff, not the availability of other risky assets. Therefore, the solution to the multi-asset problem is identical to the single-asset problem. This setting allows us to do comparative statics on market variables, such as portfolio returns and cross-sectional price distributions. Since these variables are directly observable, the results here are crucial for testing the model. Here, we focus on a situation where each risky asset i is characterized by a $\theta_i = \theta_{1i} = \theta_{2i}$, although prior beliefs about the risky assets are identical, realized information θ_i is uniformly distributed between 1 and 1.5.

[9]The comparative statics of the model where the signal turns out to be better than expected can be found in Figure 4.2 in Appendix.

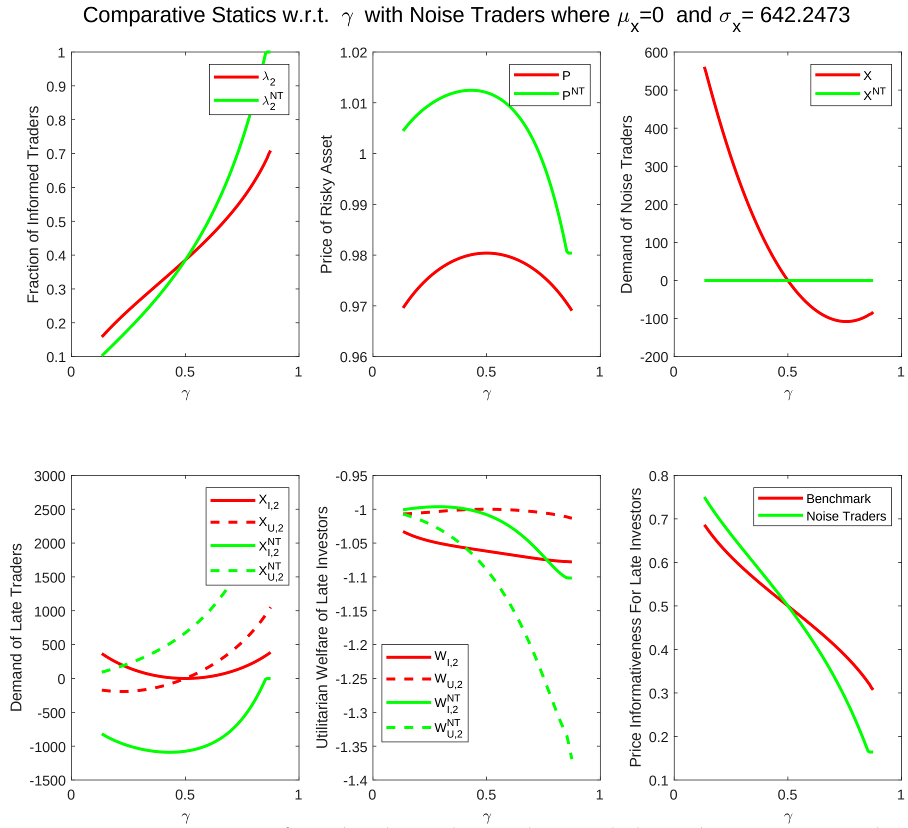

Figure 2: Comparative statics for the benchmark model and 'Noise Trader' model with respect to γ

3 Dynamic Model

In this section, we integrate a close variant of the static model in Section 2 into a neoclassical growth model with firm heterogeneity. We analyze how the degree of misallocation (of inputs) depends on the information production in the stock markets.

3.1 Environment

Preferences There is a measure one of stock traders who live one period and a measure one of infinitely lived households.

At the start of each period, newborn stock traders receive a liquidity shock and a $\gamma \in (0, 1)$ fraction of them ('day traders') need to consume early while the rest ('night traders') consume after the production is finalized. Stock traders have CARA period utility functions, that is, utility from consuming an amount W is given by $\nu(W) = -e^{-aW}$.

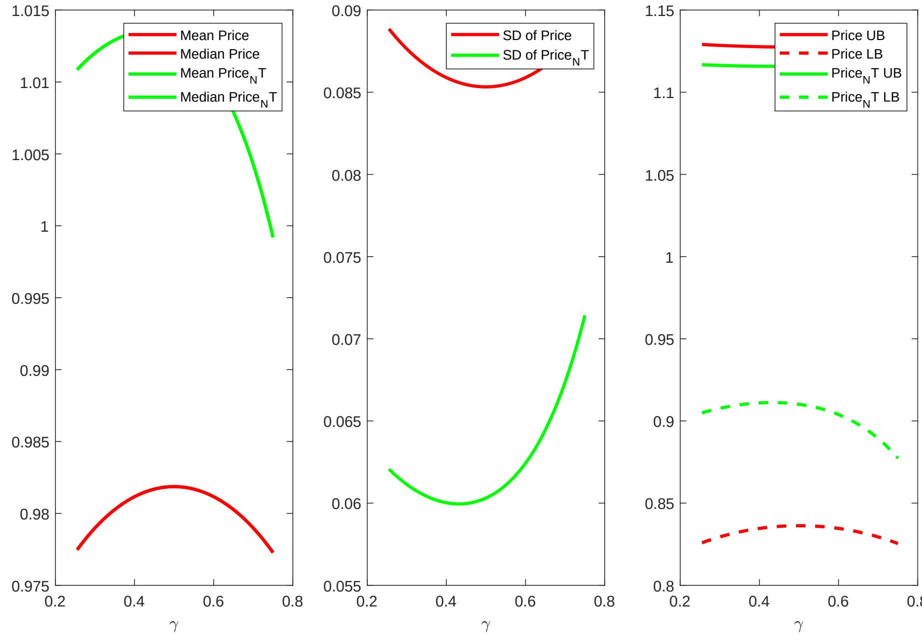

Figure 3: Comparative statics for the model with many risky assets with respect to γ, where assets differ on their payoffs

The representative household has CRRA utility function with inter-temporal elasticity of substitution parameter η and discounts the future with β.

Technology There is a measure one of firms (indexed by i) with the production function $z_{in}f(K_i)$ where z_{in} is the idiosyncratic productivity of firm i and K_i is the capital used. z_{in} are assumed to be *iid* across assets.

Endowments The firms are owned by stock traders who can freely trade the shares of the firms among themselves. The outstanding share amount of each firm is normalized to 1. The households are not allowed to hold shares[10]. The day traders sell their stocks to incoming traders before the production is finished to be able to consume early. Each stock loses a z_{id}/z_{in} fraction of its value when sold prematurely. Thus, z_{id}, is a measure of how much resale risk is associated with stock i. We assume z_{id} are *iid* across assets.

Since the stock traders are short lived, there is no capital trade between the stock traders and the households. Total capital holdings of the stock traders are assumed to be 0. All the capital is held by the households and rented to a hedge fund for a fixed rental rate r. The hedge fund allocates the rented capital across firms to maximize its return[11].

[10]The allocation of the stocks across stock traders is irrelevant for aggregate quantities, since CARA utility functions give rise to linear policy functions.

[11]Here, the households face no risk since hedge fund offers a deterministic interest rate for the capital.

Information Both z_{id} and the z_{in} consist of two parts:

$$\begin{aligned} z_{id} &= \theta_{id} + \epsilon_{id} \quad \theta_{id} \sim \mathcal{N}(\bar{\theta}_{id}, \sigma^2_{\theta_{id}}) \quad \& \quad \epsilon_{id} \sim \mathcal{N}(0, \sigma^2_{\epsilon_{id}}) \\ z_{in} &= \theta_{in} + \epsilon_{in} \quad \theta_{in} \sim \mathcal{N}(\bar{\theta}_{in}, \sigma^2_{\theta_{in}}) \quad \& \quad \epsilon_{in} \sim \mathcal{N}(0, \sigma^2_{\epsilon_{in}}) \end{aligned} \tag{11}$$

where

$$\theta_{ikt} = \tilde{\theta}_{ik}(1-\rho_k) + \rho_k z_{ik(t-1)} + v_{ikt}$$

and $v_{idt}, v_{int}, \epsilon_{idt}, \epsilon_{int}$ are assumed to be jointly **independent**. Thus

$$\begin{aligned} \bar{\theta}_{ikt} &= \tilde{\theta}_{ik}(1-\rho_k) + z_{ik(t-1)} \\ \sigma_{\theta^2_{ikt}} &= \sigma v^2_{ikt} \end{aligned} \tag{12}$$

For each stock i, traders can pay a cost $c_i(\lambda_i k)$ to learn the realizations for θ_{id} and θ_{in}[12] while ϵ_{id} and ϵ_{in} cannot be learned. We denote with λ_{ik} the fraction of $k \in \{d, n\}$ traders that are informed about stock i and assume $c_i(\lambda_{ik})$ is convex in λ_{ik}.

The hedge fund doesn't have access to the information technology. Thus, similar to the traders who chose not to pay the cost, the hedge fund infers each θ_i by observing the stock market prices.

3.2 Recursive Competitive Equilibrium

3.2.1 Portfolio Choice Problem

After receiving the liquidity shock, for each stock, the traders first decide whether to pay the cost $c_k(\lambda_{ik})$ to get information and then decide on a portfolio of assets. After the informational decision, the portfolio choice problem of a night trader who is endowed

The hedge fund faces the idiosyncratic risk of each firm. Because the productivity shocks are independent, the hedge fund can perfectly pool this risk and face no aggregate risk.

[12] Since the only random part in θ_{id} and θ_{in} are v_{id} and v_{in} learning $theta$s is equivalent to learning vs.

with $\bar{k}$ amount of capital and $\bar{X}_i$ amount of stock i becomes

$$\begin{aligned}&\max_{k,\{X_{in}\}_{i\in(0,1)}} E\Big[-exp\Big[-a[(1+r)k+\int_i X_{in}\big(z_{in}f(K_i)-rK_i-p_i\big)]\Big]\Big]\\&s.t.\ k+\int_i P_iX_{in}=\bar{k}+\int_i P_i\bar{X}_i\end{aligned} \tag{13}$$

and the portfolio choice problem of a day trader becomes

$$\begin{aligned}&\max_{k,\{X_{id}\}_{i\in(0,1)}} E\Big[-exp\Big[-a[(1+r)k+\int_i X_{id}\big((z_{in}-z_{id})f(K_i)-rK_i-p_i\big)]\Big]\Big]\\&s.t.\ k+\int_i P_iX_{id}=\bar{k}+\int_i P_i\bar{X}_i\end{aligned} \tag{14}$$

where k and $\{X_i\}_{i\in(0,1)}$ are capital and stock demands of the traders.

Given the information structure, we can write the demand for risky asset by the informed and uninformed traders as follows:

$$\begin{aligned}X_{id}^{U*}&=\frac{E[z_{in}-z_{id}|p_i]f(K_i)-rK_i-(1+r)p_i}{aV[z_{in}-z_{id}|p_i]f^2(K_i)},\\X_{id}^{I*}&=\frac{(\theta_{in}-\theta_{id})f(K_i)-rK_i-(1+r)p_i}{a(\sigma^2_{\epsilon_{in}}+\sigma^2_{\epsilon_{id}})f^2(K_i)}\\X_{in}^{U*}&=\frac{E[z_{in}|p_i]f(K_i)-rK_i-(1+r)p_i}{aV[z_{in}|p_i]f^2(K_i)},\\X_{in}^{I*}&=\frac{(\theta_{in})f(K_i)-rK_i-(1+r)p_i}{a\sigma^2_{\epsilon_{in}}f^2(K_i)}\end{aligned} \tag{15}$$

3.2.2 Information Acquisition Problem

Each trader decides whether to acquire information about each stock i, by comparing the cost of acquiring information with the decrease in the consumption variance. Define $\psi^k(\lambda_{ik})$ as

$$\psi^k(\lambda_{ik})\equiv e^{ac_k(\lambda_{ik})}\sqrt{\frac{Var[z_{ik}|\theta_{ik}]}{Var[z_{ik}|p_i]}} \tag{16}$$

for $k \in \{d, n\}$.

Corollary 4. *$\psi^k(\lambda)$ is monotone in each λ_{ik}. Therefore,*
(i) If $\psi^k(\lambda_i) > 1 \ \forall \lambda_{ik} \in [0,1]$, all k traders become informed, i.e. $\lambda^*_{ik} = 1$.
(ii) If $\psi^k(\lambda_i) < 1 \ \forall \lambda_{ik} \in [0,1]$, no k traders become informed, i.e. $\lambda^*_{ik} = 0$.
(iii) Otherwise, λ^*_{ik} as a function of λ_{i-k} is given by $\psi^k(\lambda^*_{ik}) = 1$.

Proof. Follows Lemma 3. □

3.2.3 Problem of the Household and the Hedge Fund

The representative household solves

$$\begin{aligned} &V(K,k) = \max_{k'} u(k(1+r(K)) - k') + \beta V(K', k') \\ &K' = G(K) \end{aligned} \tag{17}$$

where $V(.)$ is the value function, β is the discount factor, k is the individual capital holdings and K is the aggregate capital holdings. $G(.)$ determines how the household forms expectations over the future path of the aggregate capital. The household has the classical Euler Equation:

$$u'(k(1+r(K)) - k') = \beta u'(k'(1+r(K')) - k^{''}) \tag{18}$$

The hedge fund allocates the capital to the firms to maximize expected profits, such that

$$E[z_{in}|p] f'(K_i) = r \quad \forall i \tag{19}$$

3.2.4 Market Clearing

Let's define λ_{id} as the fraction of day traders that pay the cost to be informed and λ_{in} as the fraction of night traders that pay the cost to be informed. We denote $\lambda_i \equiv \{\lambda_{id}, \lambda_{in}\}$. Market clearing condition for stock i is

$$\gamma \Big[\lambda_{id} X^I_{id} + (1-\lambda_{id}) X^U_{id} \Big] + (1-\gamma) \Big[\lambda_{in} X^I_{in} + (1-\lambda_{in}) X^U_{in} \Big] = 0 \tag{20}$$

The capital market clearing condition is

$$\int_i K_i = K + \bar{K} \tag{21}$$

where K is the total capital held by households and $\bar{K}$ by the traders.

3.2.5 Equilibrium

Definition $V, r, k', \{K_i, X_{id}^I, X_{id}^U, X_{in}^I, X_{in}^U, \lambda_{id}, \lambda_{in}, \phi_{i0}, \phi_{id}, \phi_{in}, p_i\}_{i\in(0,1)}$ constitute a Recursive Linear Rational Expectations Equilibrium such that
(i)k' solves (18).
(ii)V solves (17).
(iii) $p_i = \phi_{i0} + \phi_{id}\theta_{id} + \phi_{in}\theta_{in}$
(iv) ϕ_{i0}, ϕ_{id}, and ϕ_{in} solve (20).
(v) $X_{id}^I, X_{id}^U, X_{in}^I$, and X_{in}^U solve (15).
(vi) r solves (21).
(vii) $\lambda_{id}, \lambda_{in}$ are given by Corollary 4.

3.3 Discussion of the Model Ingredients

The model is similar to a neoclassical growth model with heterogeneous firms. The main difference is in the problem of the hedge fund. In an environment with readily available information, the hedge fund would allocate the capital across firms according to

$$\theta_{in} f'(K_i) = r \quad \forall i \tag{22}$$

in which case, the sole 'misallocation' would be due to ϵ_{in}, which is inevitable. In our setting, the hedge fund can only rely on the stock prices p_i to infer θ_{in}. In terms of allocation of capital across firms, the stock liquidity is irrelevant. However, a high price could stem from a high θ_{in} or a low θ_{id}. In other words, stocks may be priced higher due to higher long term value or higher short term liquidity. Thus, compared to the benchmark in (22), firms with lower (higher) than expected θ_{id} shocks are allocated more (less) capital. In summary, the existence of day traders prevent prices from perfectly revealing θ_{in}.

The discrepancy between the model's allocation and the benchmark allocation in (22) depends on how close $E[z_{in}|p]$ is to θ_{in}, that is, how informative the prices are for inferring θ_{in}. Given that we restrict attention to linear pricing equilibria where $p_i = \phi_{i0} + \phi_{id}\theta_{id} + \phi_{in}\theta_{in}$, informativeness solely depend on ϕ_{in}/ϕ_{id} ratios of each stock. This ratio depends on two components: (1) the fraction of informed night traders to informed day traders and (2) how aggressively informed night traders trade based off their information relative to informed day traders. The first component is simply $\frac{(1-\gamma)\lambda_{in}}{\gamma\lambda_{id}}$. Given the information, the sole uncertainty faced by the traders are the ϵ_is. Thus, the second component is $\frac{\sigma^2_{\epsilon_{in}}+\sigma^2_{\epsilon_{id}}}{\sigma^2_{\epsilon_{in}}}$[13].

3.4 Liquidity Needs and Misallocation

We now analyze how the degree of misallocation changes with the fraction of day traders. Specifically, we look at (1) the 'output gap', which is the output difference between the perfect information case and our model of inference from prices and (2) the 'maximum output gap', which is the output difference between the perfect information case and a no-information case where capital is allocated across all firms equally.

Figure 3.4 presents the results of a comparative static of steady-state equilibria where we change the fraction of day traders in the economy. The pricing ratio declines as γ increases, similar to the result in Section 2.5. The top right panel shows that night traders respond by acquiring more information, as expected. Surprisingly, day traders also acquire more information in this parametrization. A larger fraction of day traders masks the information about the fundamental payoff which day traders also care about. In a way, increasing day trader activity can mask the information for themselves for some parameters. The input allocation depends on how much can be learned from the price regarding the fundamental payoff. Following the deteriorating information, the output gap increases with a decline in the output. Thus, the model with a larger number of day traders has larger misallocation even though agents spend more on information acquisition in steady-state.

[13] See Definition 2.4 for more details.

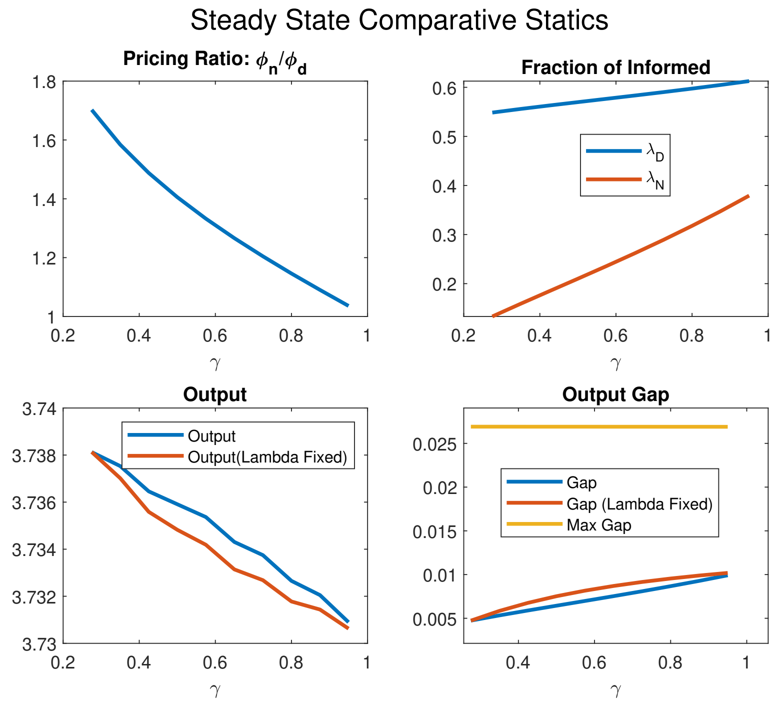

Figure 4: Comparative statics for the full model with respect to γ, where assets differ on their payoffs

The model structure also helps us disentangle the contribution of changing trading behavior and the changing information acquisition behavior. The bottom panels also show what the output response would be if information acquisition did not change. Shutting this channel amplifies the output drop by not letting the traders respond to the deteriorating information.

4 Conclusion

Recessions are characterized by decreased investment and productivity, increased misallocation of capital, and information production activities. We suggest an increase in the concern for asset liquidity may be behind all these patterns. We build a model of stock trading with costly information acquisition where noise in prices is endogenously determined. Then we introduce this stock trading model into a neoclassical growth model where the real sector observes the stock markets to learn about investment opportunities.

The model can simultaneously generate an increase in information acquisition and an increase in input misallocation by a shock to the number of traders that value asset liquidity. It also allows separating the decline in output due to costs of information acquisition and increased misallocation of capital across firms.

The model provides a mapping between the evolution of structural parameters and stock prices' behavior over time. A next step would be to use the model to quantify these channels' contribution to the observed recessions.

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Appendix

4.1 Proofs

Lemma 3. *Under the distributional assumptions,*

$$\frac{E[V(W_{Ij}^{\lambda k})|P_\lambda]}{E[V(W_{Uj}^{\lambda k})|P_\lambda]} = e^{ac_k(\lambda_k)}\sqrt{\frac{Var[u_k|\theta_k]}{Var[u_k|P_\lambda]}} \equiv \psi^k(\lambda) \tag{23}$$

for $k \in \{1,2\}$ *and* $j \in [0,1]$.

Proof. The proof adapts the corresponding proof in Grossman and Stiglitz (1980) to this environment. Expected value of being informed for investor j that consumes in period k can be written as

$$E[V(W_{Ij}^{\lambda k})|P_\lambda] = E[e^{-aW_{Ij}^{\lambda k}}] = -\exp\left[-a\Big[E[W_{Ij}^{\lambda k}|\theta] - \frac{a}{2}Var[W_{Ij}^{\lambda k}|\theta]\Big]\right] \tag{24}$$

Using Equation 5, we can write

$$E[W_{Ij}^{\lambda k}|\theta] = R(W_{0j} - c^k(\lambda_k)) + \frac{\big(E[u^k|\theta] - RP_\lambda\big)^2}{a\sigma_{\epsilon_k}^2} \tag{25}$$

$$Var[W_{Ij}^{\lambda k}|\theta] = \frac{\big(E[u^k|\theta] - RP_\lambda\big)^2}{a^2\sigma_{\epsilon_k}^2} \tag{26}$$

since W_{0j} and P_λ are not random given θ. Thus, we can rewrite Equation 24 as

$$
\begin{aligned}
E[V(W_{Ij}^{\lambda k})|P_\lambda] &= -\exp\left[-aR(W_{0j}-c^k(\lambda_k)) - \frac{\left(E[u^k|\theta]-RP_\lambda\right)^2}{2\sigma_{\epsilon_k}^2}\right] \\
&= -\exp\left[-aR(W_{0j}-c^k(\lambda_k))\right]E\left[\exp\left(\frac{-1}{2\sigma_{\epsilon_k}^2}\left(E[u^k|\theta]-RP_\lambda\right)^2\right)|P_\lambda\right]
\end{aligned} \tag{27}
$$

Now define

$$
h_\lambda^k := Var(E[u^k|\theta]|P_\lambda)
$$
$$
z_\lambda^k := \frac{E[u^k|\theta]-RP_\lambda}{\sqrt{h_\lambda^k}}
$$

Now we can rewrite Equation 27 as

$$
E[V(W_{Ij}^{\lambda k})|P_\lambda] = e^{ac^k(\lambda_k)}V(RW_{0j})E\left[exp\left(\frac{-h_\lambda^k}{2\sigma_{\epsilon_k}^2}(z_\lambda^k)^2\right)|P_\lambda\right] \tag{28}
$$

Since P_λ is a linear function of θ, conditional on P_λ, $E[u^k|\theta]$ is normally distributed. Therefore, $(z_\lambda^k)^2$ is distributed with Chi-squared. Hence, moment generating function of $(z_\lambda^k)^2$ has the form

$$
E[e^{-t(z_\lambda^k)^2}|P_\lambda] = \frac{1}{\sqrt{1+2t}}exp\left(\frac{-t(E[z_\lambda^k|P_\lambda])^2}{1+2t}\right) \tag{29}
$$

Also, by definition, $Var[u_k|P_\lambda] = \sigma_{\epsilon_k}^2 + h_\lambda^k$. Thus we can simplify the second term in Equation 28 as

$$
\begin{aligned}
E\left[exp\left(\frac{-h_\lambda^k}{2\sigma_{\epsilon_k}^2}(z_\lambda^k)^2\right)|P_\lambda\right] &= \frac{1}{\sqrt{1+\frac{h_\lambda^k}{\sigma_{\epsilon_k}^2}}}exp\left(\frac{-E[u^k|\theta]-RP_\lambda)^2}{2(h_\lambda^k+\sigma_{\epsilon_k}^2)}\right) \\
&= \sqrt{\frac{Var[u_k|\theta]}{Var[u_k|P_\lambda]}}exp\left(\frac{-E[u^k|\theta]-RP_\lambda)^2}{2Var[u_k|P_\lambda]}\right)
\end{aligned} \tag{30}
$$

Using (30) we can rewrite (28) as

$$E[V(W_{Ij}^{\lambda k})|P_\lambda] = e^{ac^k(\lambda_k)}V(RW_{0j})\sqrt{\frac{Var[u_k|\theta]}{Var[u_k|P_\lambda]}}exp\left(\frac{-E[u^k|\theta]-RP_\lambda)^2}{2Var[u_k|P_\lambda]}\right) \tag{31}$$

Using similar steps, we can also write

$$E[V(W_{Uj}^{\lambda k})|P_\lambda] = V(RW_{0j})exp\left(\frac{-E[u^k|\theta]-RP_\lambda)^2}{2Var[u_k|P_\lambda]}\right) \tag{32}$$

The result immediately follows. □

Lemma 4. *Given the distributional assumptions, there exists a market price for a given λ with the form*

$$P_\lambda(\theta) = \alpha_{0\lambda} + \alpha_{0\lambda}\left(\theta_2 + \frac{\gamma\lambda_1\sigma_{\epsilon_2}^2}{(1-\gamma)\lambda_2\sigma_{\epsilon_1}^2}\theta_1\right) \tag{33}$$

where $\alpha_{0\lambda}$ and $\alpha_{0\lambda}$ are real numbers that possibly depend on λ but not on θ_1 or θ_2.

Proof. Conjecture a linear price function for a given λ:

$$P_\lambda(\theta) = \alpha_{0\lambda} + \alpha_{1\lambda}\theta_1 + \alpha_{0\lambda}\theta_2 1 \tag{34}$$

Then, the signal uninformed period i investor will use from observing the price can be drawn from

$$\theta_i = \frac{P_\lambda - \alpha_{0\lambda} - \alpha_{k\lambda}\theta_k}{\alpha_{i\lambda}} \tag{35}$$

where i and k denote opposite periods. Since prior distribution is normal and the signal is a linear function of a normally distributed random variable, posterior distribution for u_i will also be normal with mean and variance:

$$E[u_i|P_\lambda] = \frac{\frac{\bar{\theta}_i}{\sigma^2_{\theta_i}} + \frac{1}{\sigma^2_{\theta_k}}\left(\frac{\alpha_{i\lambda}}{\alpha_{k\lambda}}\right)^2 \frac{P_\lambda - \alpha_{0\lambda} - \alpha_{k\lambda}\bar{\theta}_k}{\alpha_{i\lambda}}}{\frac{1}{\sigma^2_{\theta_i}} + \frac{1}{\sigma^2_{\theta_k}}\left(\frac{\alpha_{i\lambda}}{\alpha_{k\lambda}}\right)^2} \tag{36}$$

$$Var[u_i|P_\lambda] = \sigma^2_{\epsilon_i} + \frac{1}{\frac{1}{\sigma^2_{\theta_i}} + \frac{1}{\sigma^2_{\theta_k}}\left(\frac{\alpha_{i\lambda}}{\alpha_{k\lambda}}\right)^2} \tag{37}$$

Plugging Equations 36 and 37 into the market clearing condition in 3, we can rewrite as

$$\gamma\Bigg[\lambda_1\Big[\frac{\theta_1 - RP_\lambda}{a\sigma^2_{\epsilon_1}}\Big] + (1-\lambda_1)\Big[\frac{\frac{\bar{\theta}_1}{\sigma^2_{\theta_1}} + \frac{1}{\sigma^2_{\theta_2}}\left(\frac{\alpha_{1\lambda}}{\alpha_{2\lambda}}\right)^2\left(\frac{P_\lambda - \alpha_{0\lambda} - \alpha_{2\lambda}\bar{\theta}_2}{\alpha_{1\lambda}}\right) - RP_\lambda\left(\frac{1}{\sigma^2_{\theta_1}} + \frac{1}{\sigma^2_{\theta_2}}\left(\frac{\alpha_{1\lambda}}{\alpha_{2\lambda}}\right)^2\right)}{a + a\sigma^2_{\epsilon_1}\left(\frac{1}{\sigma^2_{\theta_1}} + \frac{1}{\sigma^2_{\theta_2}}\left(\frac{\alpha_{1\lambda}}{\alpha_{2\lambda}}\right)^2\right)}\Big]\Bigg]$$
$$+ (1-\gamma)\Bigg[\lambda_2\Big[\frac{\theta_2 - RP_\lambda}{a\sigma^2_{\epsilon_2}}\Big] + (1-\lambda_2)\Big[\frac{\frac{\bar{\theta}_2}{\sigma^2_{\theta_2}} + \frac{1}{\sigma^2_{\theta_1}}\left(\frac{\alpha_{2\lambda}}{\alpha_{1\lambda}}\right)^2\left(\frac{P_\lambda - \alpha_{0\lambda} - \alpha_{1\lambda}\bar{\theta}_1}{\alpha_{2\lambda}}\right) - RP_\lambda\left(\frac{1}{\sigma^2_{\theta_2}} + \frac{1}{\sigma^2_{\theta_1}}\left(\frac{\alpha_{2\lambda}}{\alpha_{1\lambda}}\right)^2\right)}{a + a\sigma^2_{\epsilon_2}\left(\frac{1}{\sigma^2_{\theta_2}} + \frac{1}{\sigma^2_{\theta_1}}\left(\frac{\alpha_{2\lambda}}{\alpha_{1\lambda}}\right)^2\right)}\Big]\Bigg] = 0 \tag{38}$$

From Equation 38, after a basic but tedious algebra P_λ can be left alone. From there, coefficient of θ_1 becomes $\alpha_{1\lambda}$, coefficient of θ_2 becomes $\alpha_{2\lambda}$ and the rest becomes $\alpha_{0\lambda}$.

Lastly, it is trivial to see from Equation 38 that

$$\alpha_{1\lambda} = \frac{\gamma\lambda_1\sigma^2_{\epsilon_2}}{(1-\gamma)\lambda_2\sigma^2_{\epsilon_1}}\alpha_{2\lambda} \tag{39}$$

Hence the expression in the Lemma follows. □

Proposition 3. *In interior equilibria, fraction of informed period* 1 *investors* λ_1 *increases as*
(1) a*,* C_1*,* C_2*,* γ *and* $\sigma^2_{\epsilon_2}$ *decreases*
(2) $\sigma^2_{\theta_2}$ *increases.*

Proof. Equating $\psi^k(\lambda)$ and taking the square of both sides, we can write

$$e^{2ac_1(\lambda_1)}\sigma_{\epsilon_1}^2 = \sigma_{\epsilon_1}^2 + \frac{1}{\frac{1}{\sigma_{\theta_1}^2} + \frac{1}{\sigma_{\theta_2}^2}\left(\frac{\gamma\lambda_1\sigma_{\epsilon_2}^2}{(1-\gamma)\lambda_2\sigma_{\epsilon_1}^2}\right)^2} \tag{40}$$

and

$$e^{2ac_2(\lambda_2)}\sigma_{\epsilon_2}^2 = \sigma_{\epsilon_2}^2 + \frac{1}{\frac{1}{\sigma_{\theta_2}^2} + \frac{1}{\sigma_{\theta_1}^2}\left(\frac{\gamma\lambda_1\sigma_{\epsilon_2}^2}{(1-\gamma)\lambda_2\sigma_{\epsilon_1}^2}\right)^{-2}} \tag{41}$$

Further algebra yields $\lambda_1(\lambda_2)$ and $\lambda_2(\lambda_1)$:

$$\lambda_2 = \lambda_1 \frac{\gamma\sigma_{\epsilon_2}^2}{(1-\gamma)\sigma_{\epsilon_1}\sigma_{\theta_2}} \frac{1}{\sqrt{\frac{1}{e^{2ac_1(\lambda_1)}-1} - \frac{\sigma_{\epsilon_1}^2}{\sigma_{\theta_1}^2}}} \tag{42}$$

and

$$\lambda_1 = \lambda_2 \frac{(1-\gamma)\sigma_{\epsilon_1}^2}{\gamma\sigma_{\epsilon_2}\sigma_{\theta_1}} \frac{1}{\sqrt{\frac{1}{e^{2ac_2(\lambda_2)}-1} - \frac{\sigma_{\epsilon_2}^2}{\sigma_{\theta_2}^2}}} \tag{43}$$

Lastly, using (42) and (43), we can derive

$$\frac{\sigma_{\epsilon_1}\sigma_{\epsilon_2}}{\sigma_{\theta_1}\sigma_{\theta_2}} = \sqrt{\frac{1}{e^{2ac_1(\lambda_1)}-1} - \frac{\sigma_{\epsilon_1}^2}{\sigma_{\theta_1}^2}}\sqrt{\frac{1}{e^{2ac_2(\lambda_2)}-1} - \frac{\sigma_{\epsilon_2}^2}{\sigma_{\theta_2}^2}} \tag{44}$$

Comparative static results can be derived from impossibility results, i.e. whether a certain direction of the movement in the parameters and the equilibrium object can be compatible with the equations (42), (43) and (44). Below is a table that summarizes which directions can be compatible with the equations and which equations the other directions violate.

	Direction of λ_1, λ_2			
Pararameter	+,+	-,-	+,-	-,+
γ	(44)	✓	(43)	(42)
c_1	(44)	✓	(42)	(43)
c_2	(44)	✓	(43)	(42)
$\sigma^2_{\epsilon_1}$	(44)	✓	✓	(43)
$\sigma^2_{\epsilon_2}$	(44)	✓	(42)	✓
$\sigma^2_{\theta_1}$	✓	(44)	(43)	✓
$\sigma^2_{\theta_2}$	✓	(44)	✓	(42)

Table 2: The table shows which -if any- equations would not be satisfied if the parameter in the row were to increase and the equilibrium objects λ_1 and λ_2 were to move as designated in the column.

The results follow from the table.

□

4.2 Comparative Statics Cont.

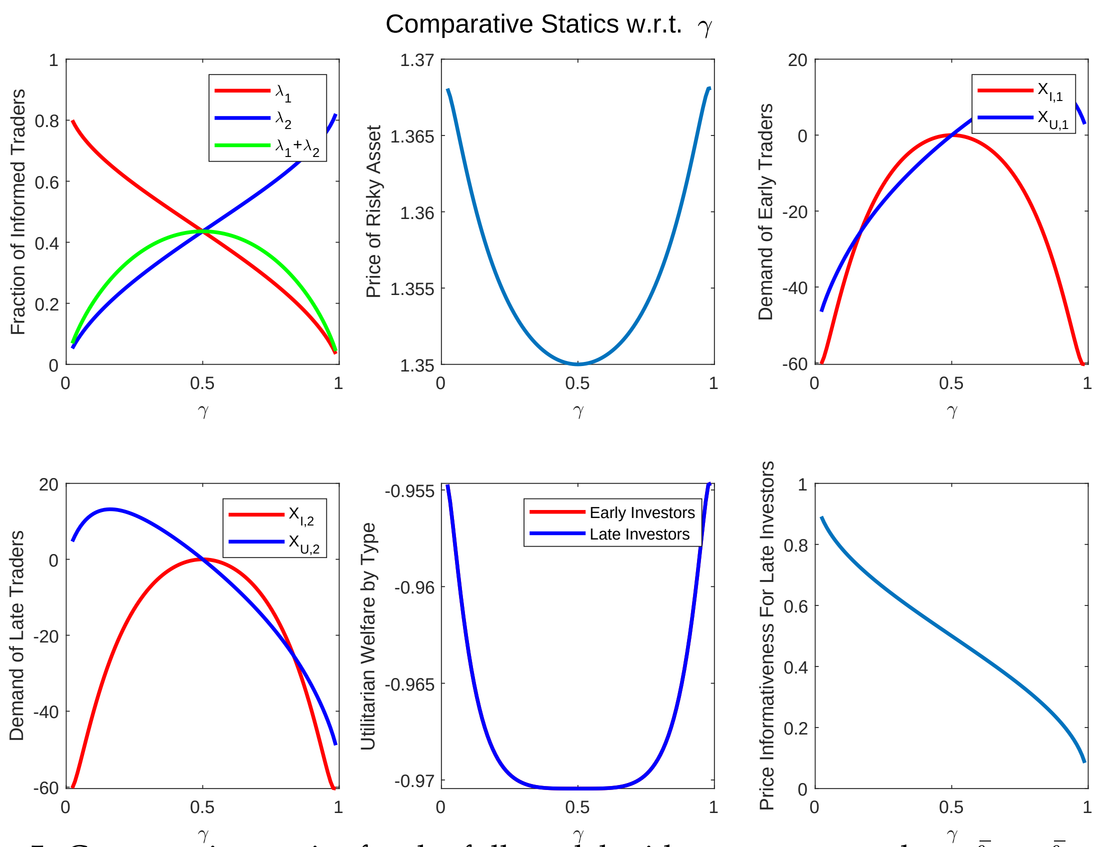

Figure 5: Comparative statics for the full model with respect to γ, where $\bar{\theta}_1 = \bar{\theta}_2 = 1.25$ and $\theta_1 = \theta_2 = 1.35$.