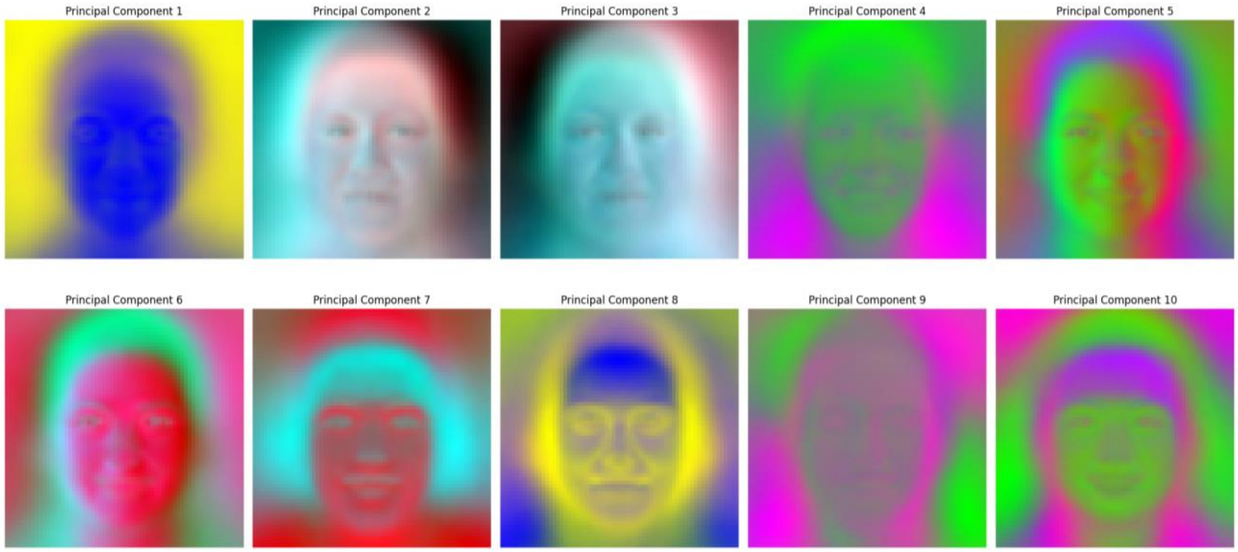
CS 464 Homework 2 - Görkem Kadir Solun – 22003214

PCA Question 1.1

```
Red Channel
For the first 10 components:
PVE values: [0.28929081 0.09373798 0.06788795 0.05859219 0.05419499 0.04384446
 0.02887794 0.02049209 0.01681582 0.01630701]
Sum of PVE values: 0.6900412482040855
Minimum component count to reach the threshold 0.7: 11
Green Channel
For the first 10 components:
PVE values: [0.32041101 0.08559319 0.08177433 0.05796603 0.04110711 0.03958217
 0.02568433 0.01855605 0.01663551 0.01604563]
Sum of PVE values: 0.70335535380908
Minimum component count to reach the threshold 0.7: 10
Blue Channel
For the first 10 components:
PVE values: [0.34357938 0.08854058 0.08477065 0.05722818 0.03805416 0.03354398
 0.02486367 0.01717688 0.01624892 0.01553017]
Sum of PVE values: 0.71953657404184
Minimum component count to reach the threshold 0.7: 9
```

The blue channel, with a proportion of variance explained (PVE) of 71.9537% achieved by the sum of the first ten components, surpasses the 70% threshold, requiring only nine components to meet the criterion, making it the most efficient channel due to its higher variance. Similarly, the green channel explains 70.3356% of the variance with ten components, just exceeding the threshold, indicating that ten components are sufficient for this channel. In contrast, the red channel achieves a PVE of 69.0041% with the sum of the first ten components, falling short of the 70% target, requiring 11 components to exceed the threshold, which makes it the least efficient channel. Thus, the red channel's requirement of 11 components establishes the minimum number needed to ensure that at least 70% of the variance is covered.

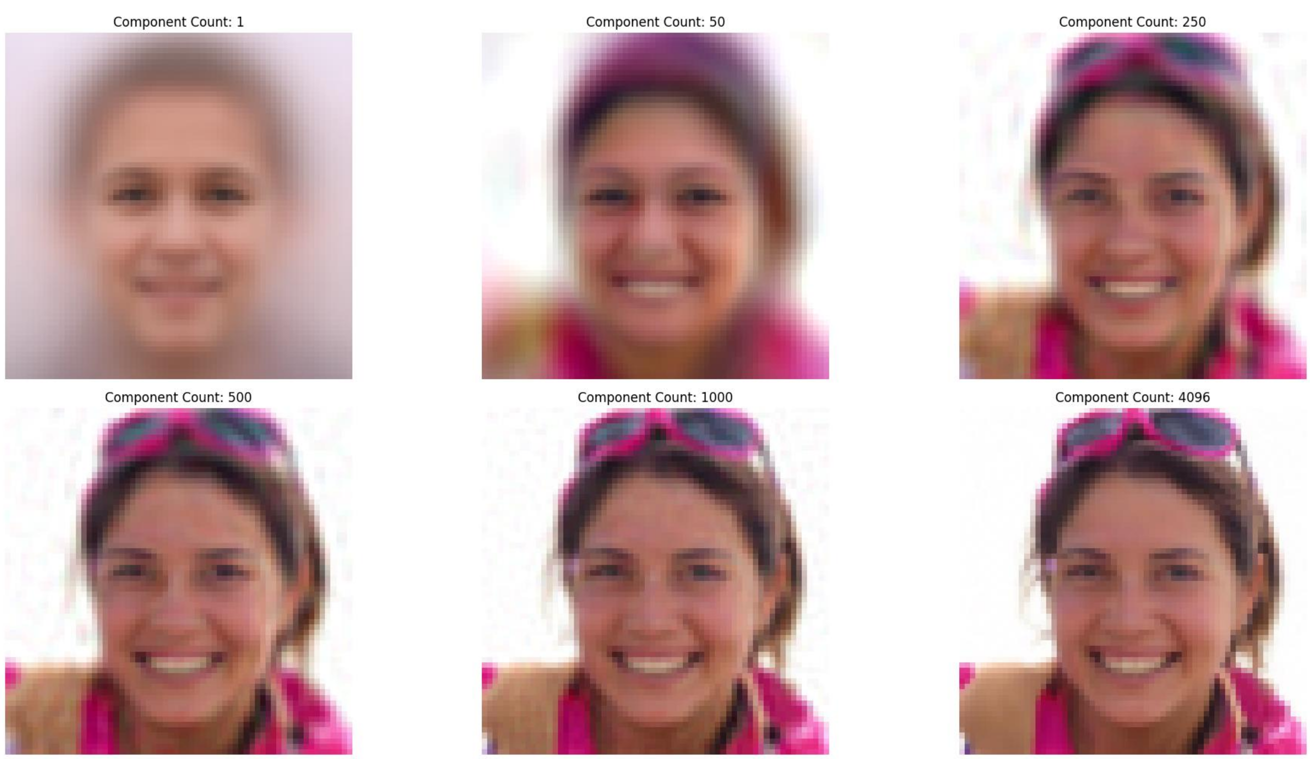
PCA Question 1.2



Every principal component (PC) captures a maximum variance in a specific direction, emphasizing the dataset's critical features. The predominant color patterns and most notable distinctions are represented by PC 1, which mainly shows subtle color intensity changes, such as gradients from yellow to blue. We can conclude that PC 8 is the opposite of PC 1. Definite distinctions and complex 3-D patterns, such as variations in particular regions and facial characteristics, are highlighted in PCs 2 and 3. PCs 4, 5, 9, and 10 emphasize intricate color transitions with less emphasis on general structure, concentrating on more delicate and localized patterns, such as nose or hair texture. In addition to representing hair and the bottom portion of the face, PCs 6 and 7 feature intriguing colors. By preventing rendering distortions from raw values exceeding acceptable ranges, min-max scaling normalizes eigenvector values to a range between 0 and 1, guaranteeing proper RGB image display and enhancing the clarity and comparability of color patterns among principal components. The red, green, and blue components are organized to create meaningful RGB graphics, illustrating how variances in the dataset are distributed across the spectrum and picturing relationships among colors, with red-green and blue-yellow contrasts highlighting fundamental color changes driving the variability.

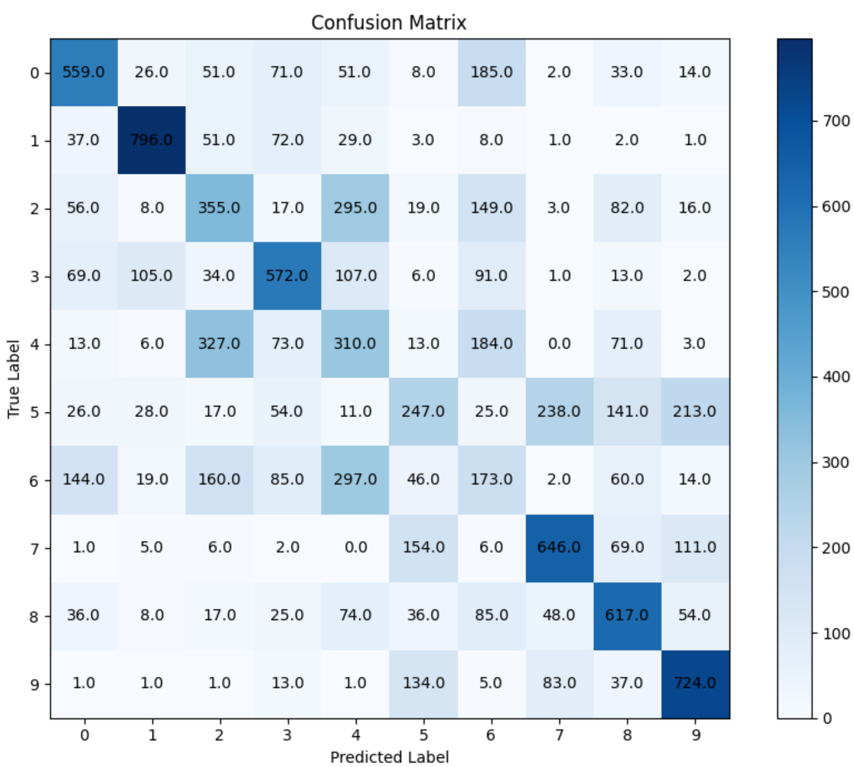
PCA Question 1.3

Original Image



Restoration using PCA involves projecting an image onto its eigenvectors and taking the first k eigenvectors. Eigenvector is also called PC. These eigenvectors correspond to the maximum variance in a direction in the dataset, with earlier components carrying more significant information as they are associated with higher eigenvalues. The choice of k, the number of principal components, controls the level of detail in the reconstructed image. A smaller k captures broader patterns, while a larger k incorporates finer details, achieving complete reconstruction when k equals the total number of original components, demonstrating how PCA efficiently reduces dimensionality while preserving essential information. This makes it a powerful tool for simplifying data while maintaining its meaningful structure and interpretability.

Logistic Regression Question 2.1

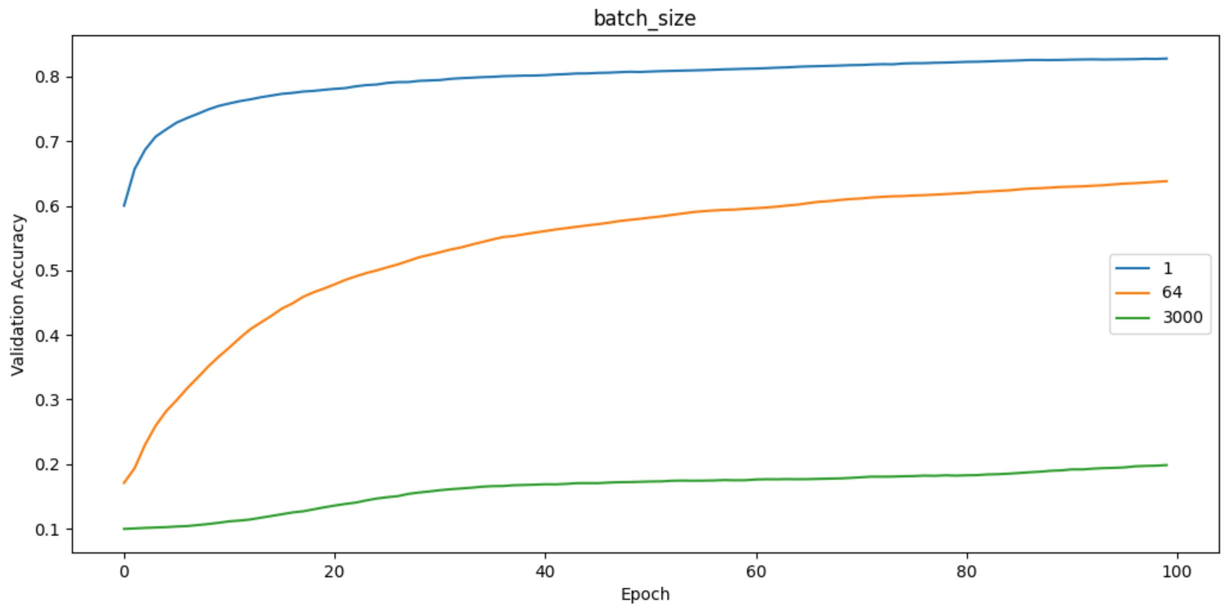


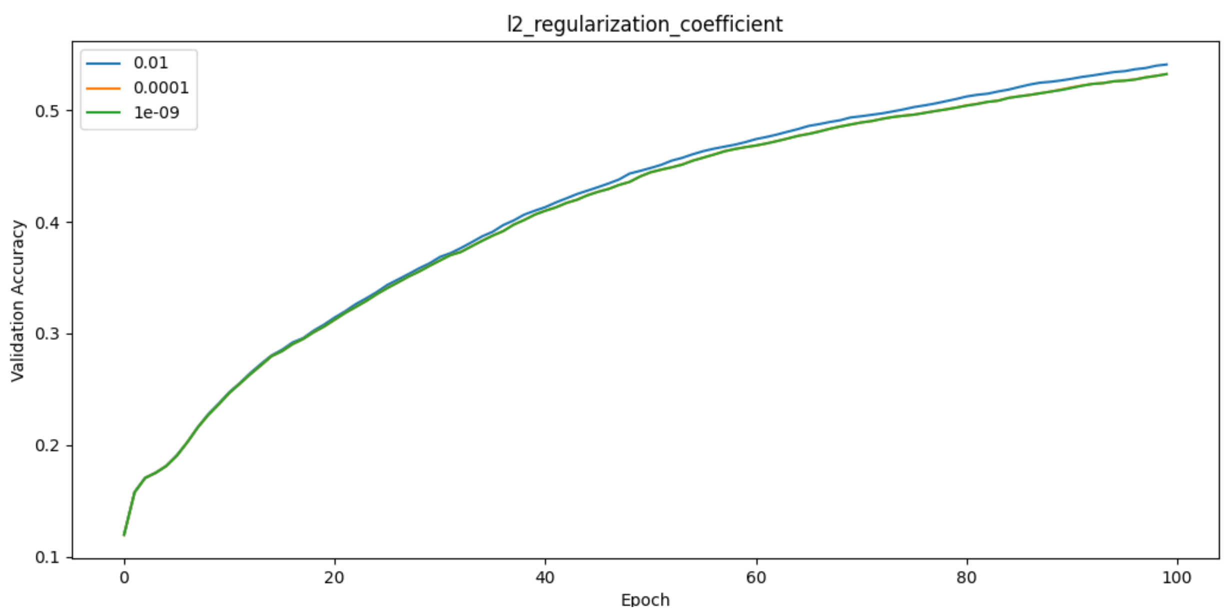

```
Epoch: 95, Validation Accuracy: 0.4953
Epoch: 96, Validation Accuracy: 0.4968
Epoch: 97, Validation Accuracy: 0.4981
Epoch: 98, Validation Accuracy: 0.5002
Epoch: 99, Validation Accuracy: 0.5015
Epoch: 100, Validation Accuracy: 0.5026
Test Accuracy: 0.4999
```

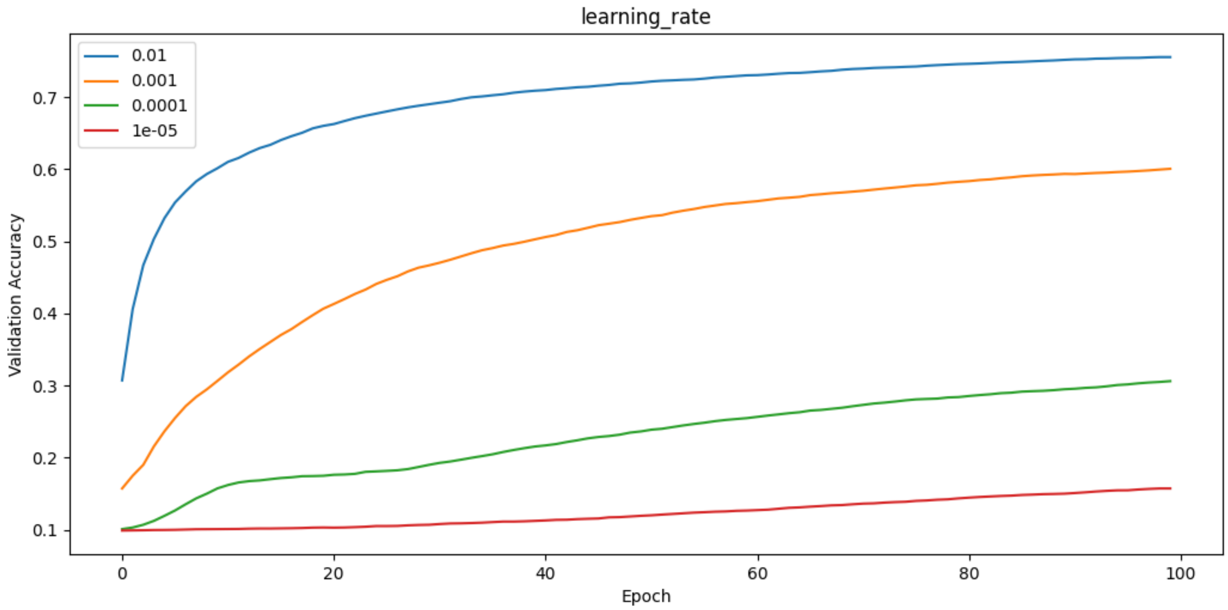
The model obtained a test accuracy of 49.99%. This suggests a mediocre performance overall, but the model consistently struggles to produce precise predictions in every class. Nevertheless, there are also significant variations in prediction accuracy among various classes. The model's configuration settings (a batch size of 200, regularization of 1e-4, and a learning rate of 0.0005) enable steady improvement in the validation dataset with each training epoch, ultimately leading to convergence. The model employs the conventional Gaussian normal initialization for its weights. Still, experiments suggest that alternative weight initialization techniques could yield better optimization results, indicating that improving initialization methods might enhance overall performance.

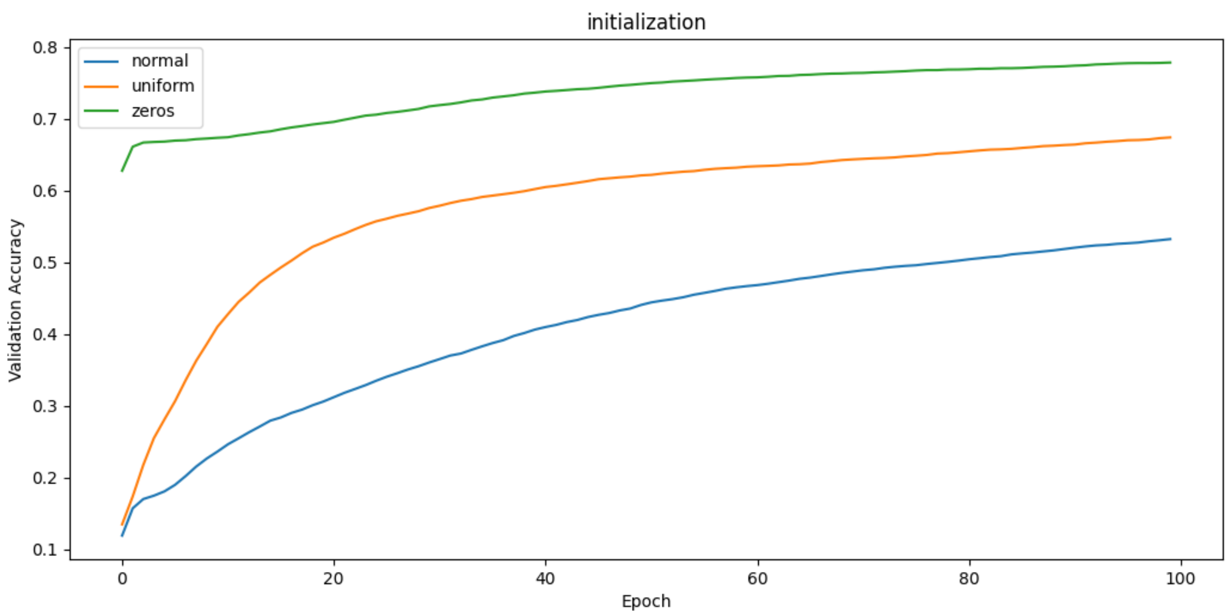
According to the confusion matrix, classes 3, 8, 7, and 9 are predicted more accurately, as evidenced by their diagonal solid values in the matrix. On the other hand, classes 2, 4, 5, and 6 demonstrate a need for greater accuracy, indicating that the model might struggle to perform well on unseen data because its regularization strength is too weak. Regularization is a technique to prevent overfitting, where the model becomes overly specialized to the training data and fails to generalize to new, unseen datasets. With low regularization, the model may not adequately control complexity, especially when dealing with complex datasets, leading to a higher likelihood of overfitting and reduced performance on new data.

Logistic Regression Question 2.2










```
Epoch: 100, Validation Accuracy: 0.7784
Best Parameters:
batch_size: 1
learning_rate: 0.01
l2_regularization_coefficient: 0.01
initialization: zeros
```

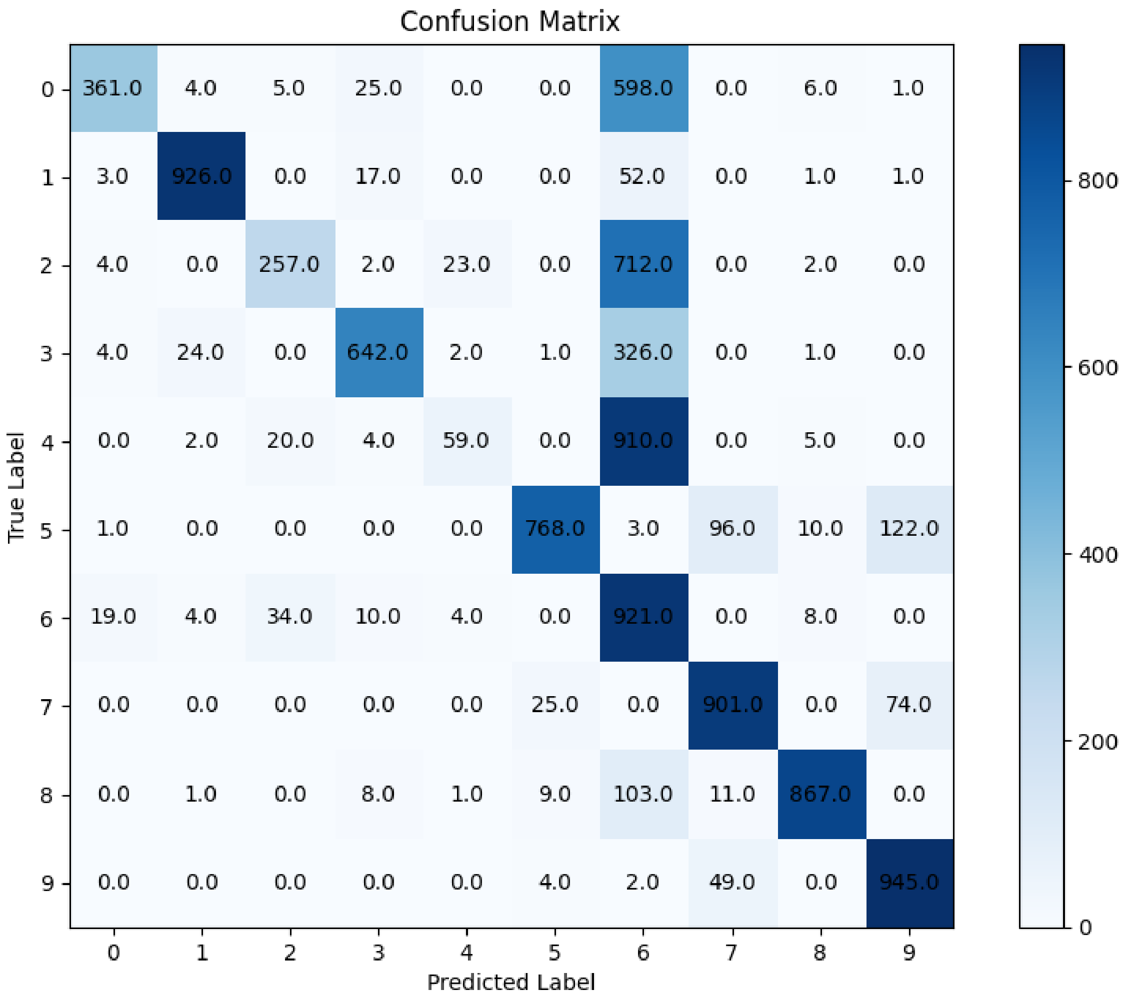
The first is how changes in batch size affect model accuracy. Small batch sizes (1) yield higher accuracy (about 80%) and a faster convergence rate. The model can learn more complex patterns given the default parameters since smaller batches enable more frequent weight updates. Nonetheless, batch size 64 shows respectable accuracy and convergence and stops at roughly 65%. The learning is not good, and only 25% accuracy is achieved in a batch of 3000. Thus, huge batch sizes may hinder the model's ability to capture the dataset's features and diminish its performance.

The second point examines the model's accuracy concerning various weight initialization techniques. Zero, uniform distribution, and Gaussian distribution are examples of initialization techniques. The findings demonstrate that, within the first epochs, the uniform initialization performs better than the others in terms of accuracy. While it reaches an optimal point eventually, the Gaussian initialization requires more iterations to do so. Notably, zero initiation produces the highest final accuracy after the epochs, at roughly 75%. This implies that zero initialization is the best. These results demonstrate the importance of using an appropriate starter.

The third point shows the impact of various learning rates on the model's accuracy: 0.01, 1e-3, 1e-4, and 1e-5. A higher learning rate of 0.01 produces the highest final accuracy of roughly 75% and accelerates convergence. The lowest learning rate of 1e-5 performs the worst, and smaller learning rates lead to slower learning and poorer end performance. Accordingly, more excellent learning rates must be carefully chosen to prevent problems like overshooting minima or creating instability in the training process, even though they might speed up learning and increase ultimate accuracy.

The fourth point shows how different regularization coefficients change accuracy. Interestingly, the differences between the tested regularization values are insignificant, with all configurations leading to similar accuracy levels of around 50%. The model's performance in this scenario is relatively insensitive to the regularization parameter within this range. Despite the minimal impact observed, regularization still plays a crucial role in preventing overfitting by penalizing large weights, which helps the model generalize better to unseen data.

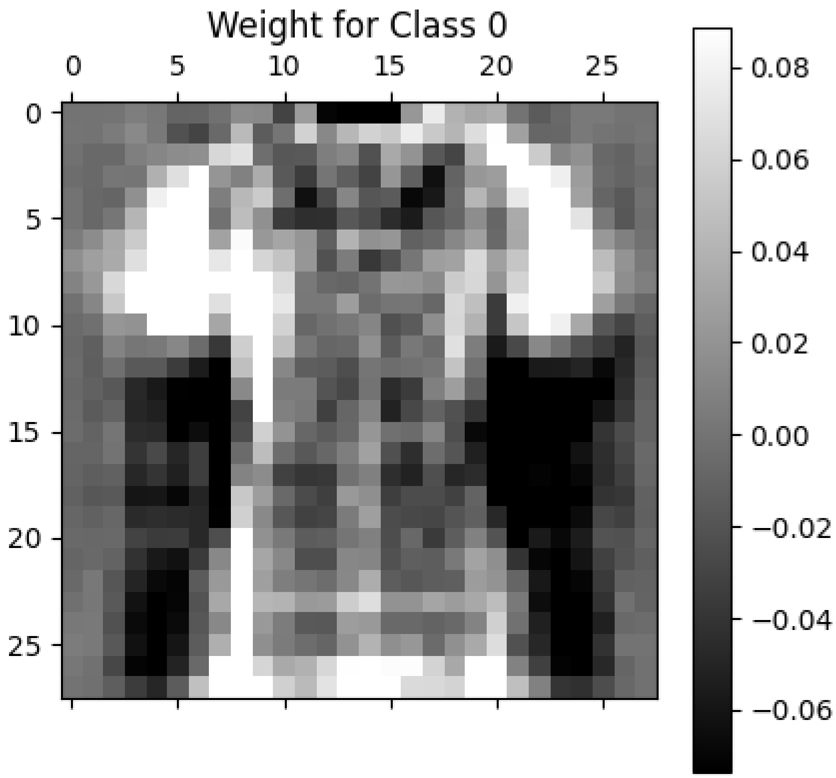
Logistic Regression Question 2.3

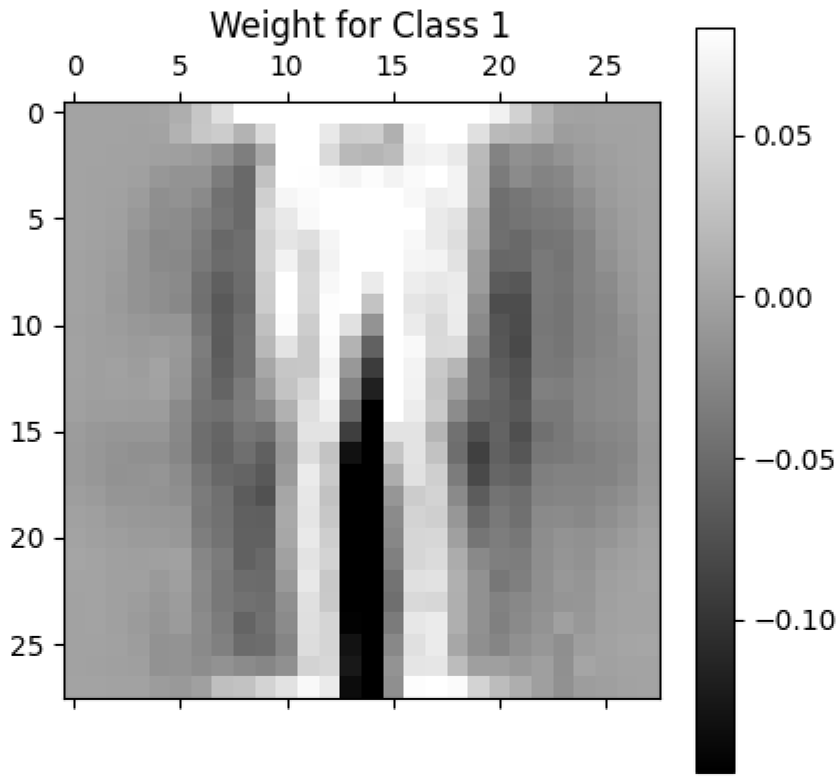


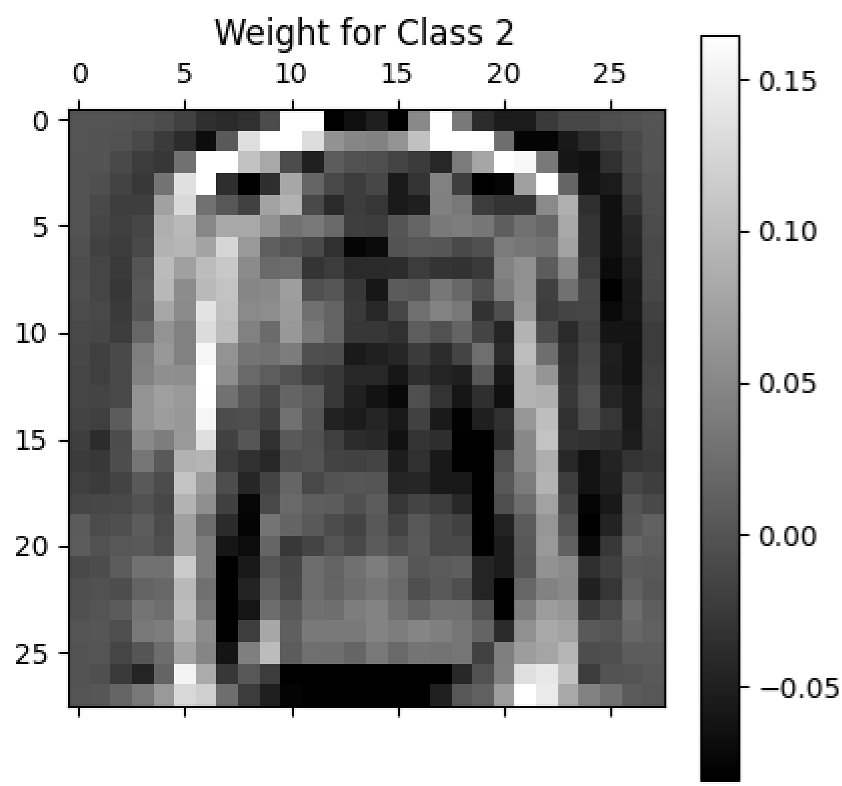

```
Epoch: 96, Validation Accuracy: 0.6739
Epoch: 97, Validation Accuracy: 0.6739
Epoch: 98, Validation Accuracy: 0.6739
Epoch: 99, Validation Accuracy: 0.6739
Epoch: 100, Validation Accuracy: 0.6739
Test Accuracy: 0.6647
```

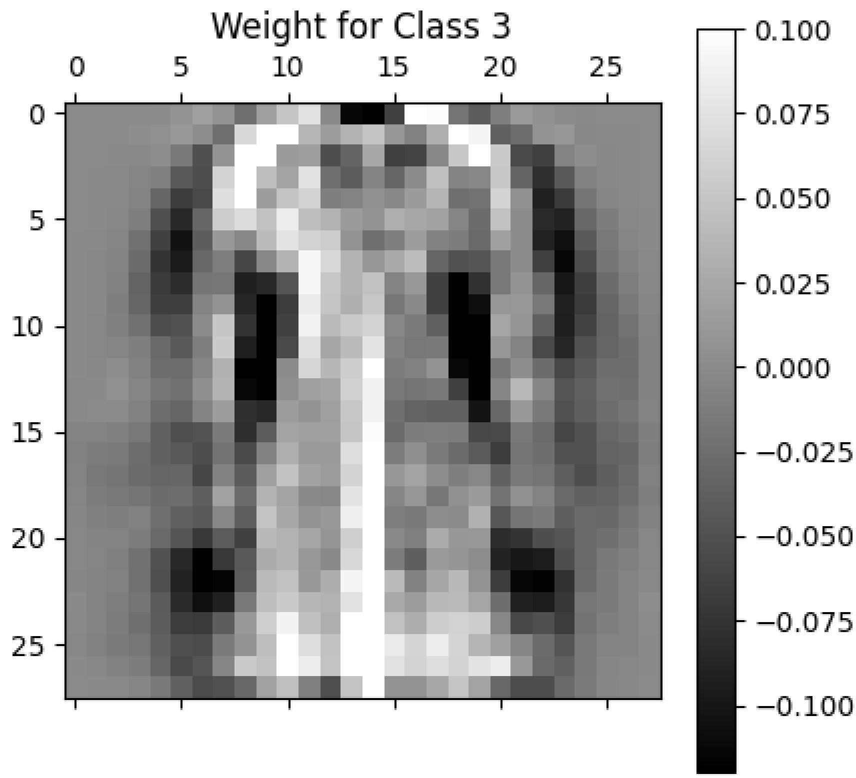
Interestingly, combining the best parameters results in a slightly reduced performance of 66%, while the maximum validation accuracy of the parameters tested is around 75%.

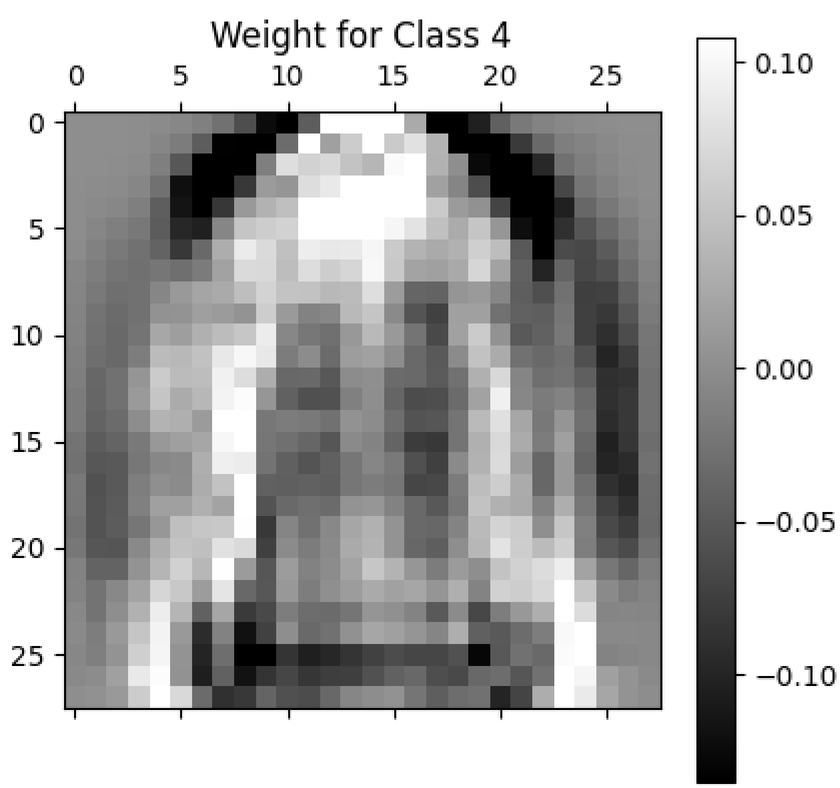
Logistic Regression Question 2.4

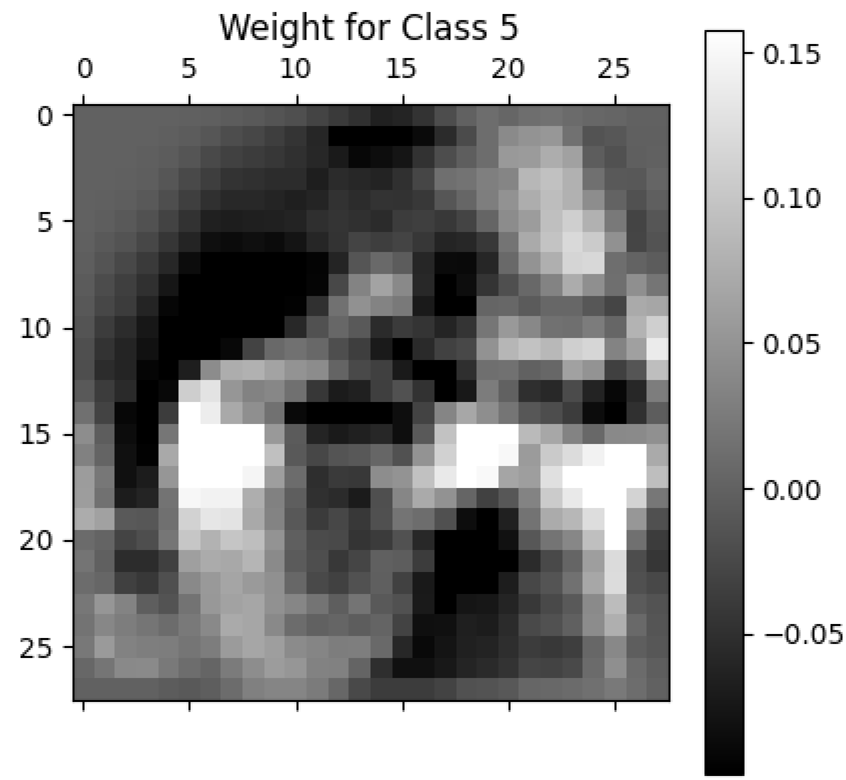


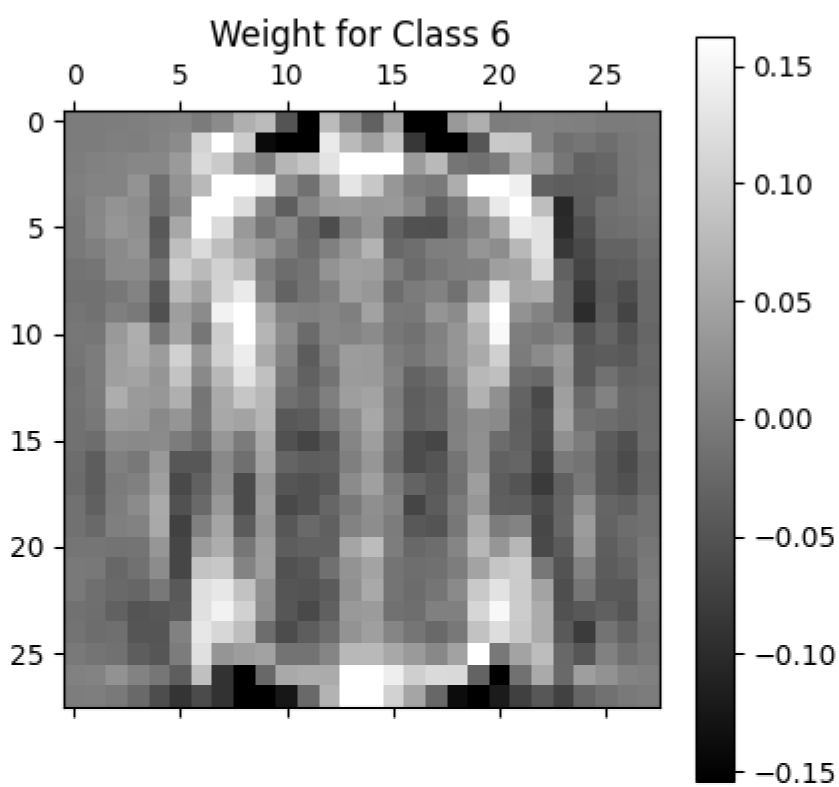

Weight for Class 1
0
5
10
15
20
25
0.05
0.00
−0.05
−0.10


Weight for Class 2
0
5
10
15
20
25
0.15
0.10
0.05
0.00
−0.05

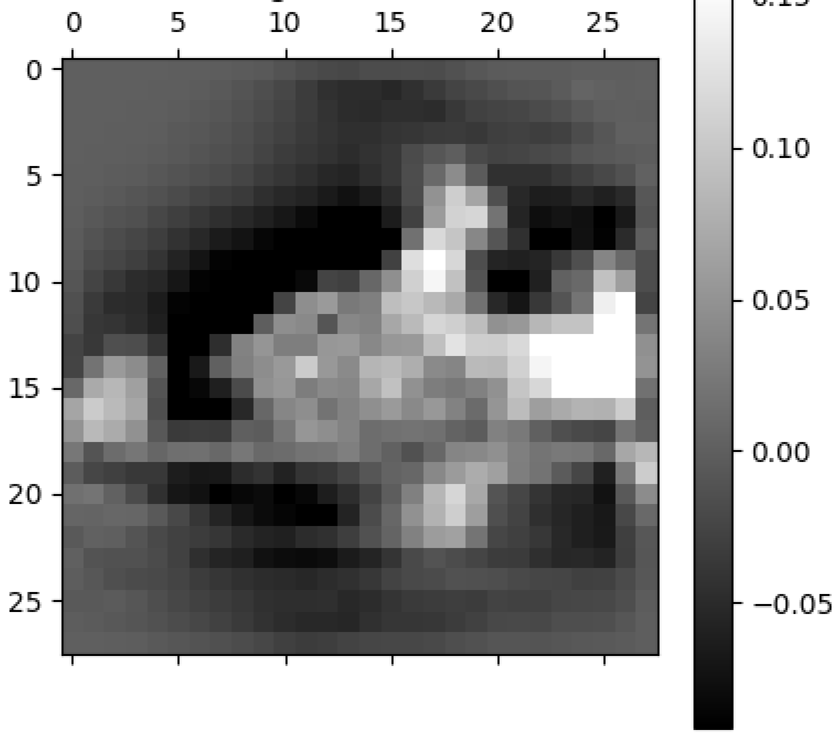

Weight for Class 3
0.100
0.075
0.050
0.025
0.000
−0.025
−0.050
−0.075
−0.100


Weight for Class 4
0.10
0.05
0.00
−0.05
−0.10

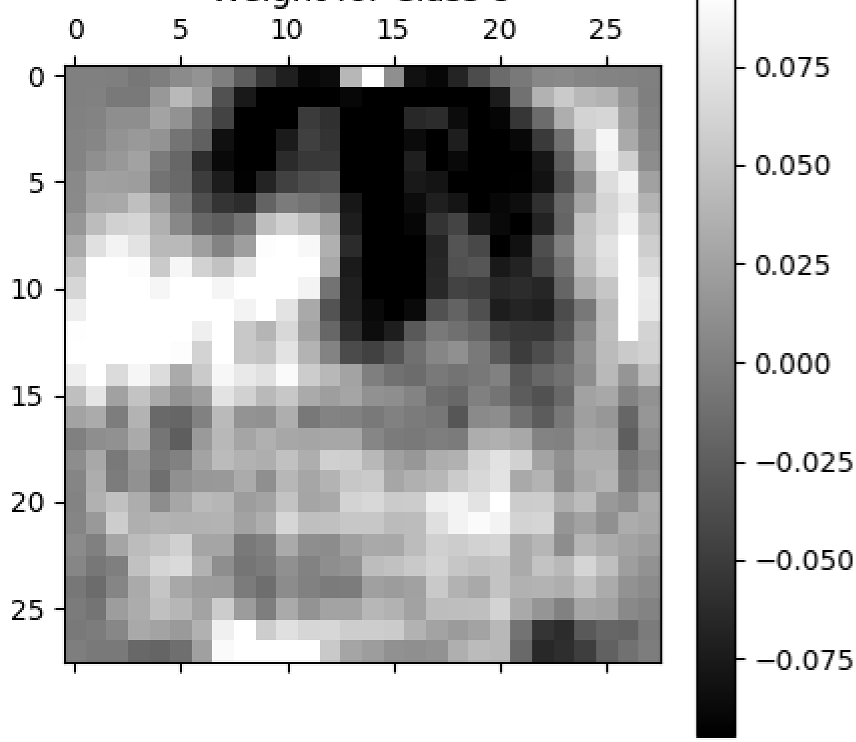

Weight for Class 5

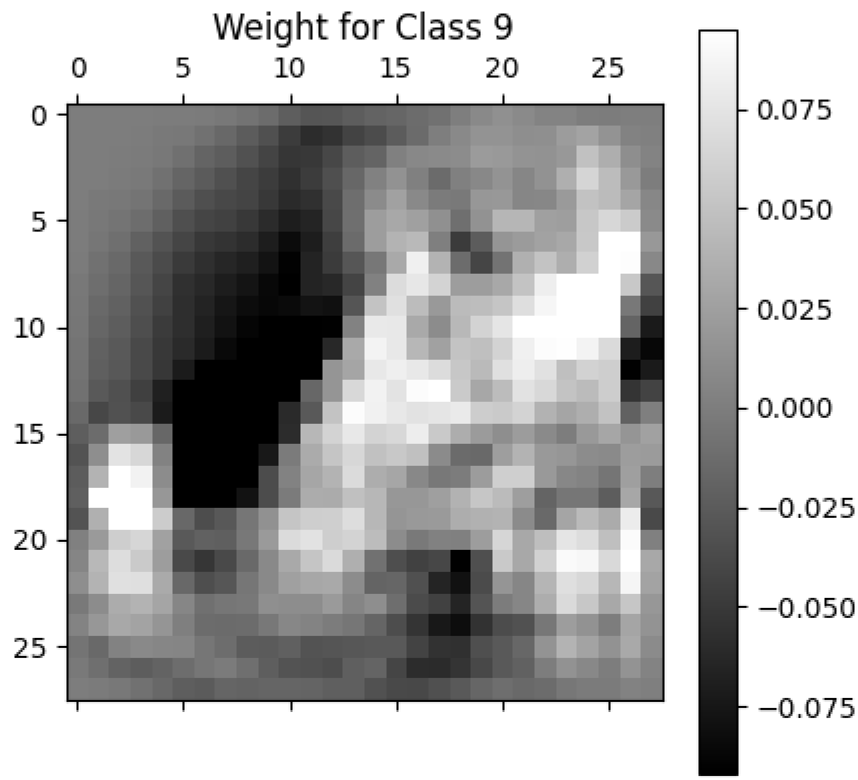

Weight for Class 6

Weight for Class 7



Weight for Class 8





Each weight map visualization highlights the traits necessary for differentiating one class from another, helping the model make classification judgments by capturing each class's distinctive qualities. The color bars indicate that various classes have different weight values, potentially reflecting the varying complexity or difficulty in categorizing those classes. For instance, certain classes might have more distinctive traits than others, while others might share qualities that make them more challenging to differentiate. In every class, we can see different clothing, such as shoes, t-shirts, or pants. The weight maps highlight the significance of regions for categorization and assist in visualizing the model's emphasis on the salient characteristics that set these categories apart. It is expected that the weight maps may appear hazy. By punishing abnormally high weight values, L2 regularization helps avoid overfitting and improves the model's ability to generalize to new data, which may have caused this blur in the image. Additionally, improper scaling or normalizing weights to fit into the image range can lead to loss of contrast, making the image appear less sharp. The classification of that class is greatly aided by bright regions, which correlate more with input features. Darker areas, on the other hand, indicate negative weights, emphasizing characteristics that actively lower the input's chance of falling into that class. Grey areas indicate little impact on the class's classification. These regions stand for neutrality when the associated pixels don't strongly support or disagree with the classification choice.

Logistic Regression Question 2.5

```
Metrics for the Best Model:
   Precision  Recall  F1 Score  F2 Score
0   0.920918   0.361  0.518678  0.410974
1   0.963580   0.926  0.944416  0.933280
2   0.813291   0.257  0.390578  0.297729
3   0.906780   0.642  0.751756  0.681818
4   0.662921   0.059  0.108356  0.072145
5   0.951673   0.768  0.850028  0.798835
6   0.253929   0.921  0.398098  0.603776
7   0.852412   0.901  0.876033  0.890844
8   0.963333   0.867  0.912632  0.884694
9   0.826772   0.945  0.881941  0.918724
```

	Precision	Recall	F1 Score	F2 Score
0	0.920918	0.361	0.518678	0.410974
1	0.96358	0.926	0.944416	0.93328
2	0.813291	0.257	0.390578	0.297729
3	0.90678	0.642	0.751756	0.681818
4	0.662921	0.059	0.108356	0.072145
5	0.951673	0.768	0.850028	0.798835
6	0.253929	0.921	0.398098	0.603776
7	0.852412	0.901	0.876033	0.890844
8	0.963333	0.867	0.912632	0.884694
9	0.826772	0.945	0.881941	0.918724

Training with the optimal hyperparameters:

Batch size: 1

Weight initialization: Zeros

Learning rate: 0.01

L2 regularization parameter: 0.01

The model's performance was evaluated using a confusion matrix, and a test accuracy of 66.47% was achieved under these settings.

The model performs exceptionally well in certain classes, such as 1, 7, and 9, which demonstrate high precision and recall, while other classes, like 4, show significant misclassification issues with lower precision and recall. Particularly challenging is class 6, where the model has varying precision (0.25) and recall (0.92) scores. Class 4 shows poor separability, with a precision of 0.66 and recall of 0.05, as many instances are misclassified into class 5, likely due to overlapping features between classes or insufficient representation of critical features in the dataset.

While these parameter values performed best individually with other parameters set to their default values, the overall optimality of the model remains uncertain due to the unknown specifics of those default settings. While combining these parameters resulted in the highest mean validation score, models with alternative parameter settings achieved better accuracy than the initially considered ‘best’ parameters.

The model demonstrated strong performance for classes 1, 7, and 9, achieving high precision, recall, and F-measure scores, including F1 and F2. This indicates that the model was effective at correctly identifying these classes with minimal errors and capturing a balance between precision and recall. However, its performance was notably weaker for classes 0, 2, 4, and 6, which exhibited low precision and recall. This suggests that the model had difficulty distinguishing these classes from others, as reflected by significant misclassifications observed in the confusion matrix. These results highlight variability in the model's ability to classify different classes accurately.