Q1

for MoM:
$\int_0^1 xf(x)dx = \int_0^1 x\theta x^{\theta-1}dx = \theta\int_0^1 x\theta dx = \theta(\frac{x^{\theta-1}}{\theta+1})$ for x 1 to 0

$\theta(\frac{1^{\theta+1}}{\theta+1}) = \frac{\theta}{\theta+1} = m_1$
$\theta = \frac{m}{1-m}$

m=E(x) , mean
for X we have 0.65 as mean.If we plug 0.65 as m in the theta function we will get 1.85 as the MoM estimator.

for MLE:
$f(x) = \theta x^{\theta-1}$
$lnf(x) = ln\theta + (\theta-1)lnx$
$lnf(x) = \sum_{i=1}^{n}(ln\theta + (\theta-1)lnx_i)$
$lnf(x) = nln\theta + (\theta-1)\sum_{i=1}^{n}(lnx_i)$

$\frac{\partial lnf(x)}{\partial\theta} = \frac{n}{\theta} + \sum_{i=1}^{n}(ln_i)$= 0

$\theta = \frac{-n}{\sum_{i=1}^{n}(lnx_i)}$

If we plug X samples' values to this function we would get$\theta = -1.97$ as the MLE estimator.

Q2

For the inverse function of f(x) = $2.4x^{1.4}$
$f^{-1}(x) = (\frac{x}{2.4})^{1.4}$
From our sample set, the values we would get are:
0.054 , 0.143 , 0.214 , 0.253

Q3

Variance of the estimators gets smaller when N increases which is expected since an increase in sample size when taking samples from same interval(P list only contains elements from the inverse function with interval 0¡x¡1) will make the variance smaller.
Mean converges to the expected value as N increases since while increasing sample size average value converges to the mean with more and more elements are added.
With the increase of N we can much clearly see which values the estimators converge to.Which are roughly 0.4 and 0.1.
I would prefer the Method of Moments on this experiments since even at small sample sizes , variance is much smaller than what we would get in Maximum Likelihood Method.