Q1.a

Three point centered difference formula for $f\prime\prime(x) =$

$$\frac{f(x-h) - 2f(x) + f(x+h)}{h^2}$$

So if we plug in the values of h and f(x) we'd get

$$\frac{cos(x-0.01) - 2cosx + cos(x+0.01)}{0.0001}$$

$10^4(cos(-0.01) - 2 + cos(0.01))=$
$10^4(0.9999500004166653 - 2 + 0.9999500004166653)=$
$f\prime\prime(0) = -0.9999916666947328$
Error is $-1 - (-0.9999916666947328) = -0.0000083333052672$

Q1.b

$4f(x+h) = 4f(x) + 4hf\prime(x) + \frac{4h^2}{2}f\prime\prime(x) + \frac{4h^3}{6}f'''(x) + \frac{4h^4}{24}f''''(c_1)$
$f(x-2h) = f(x) - 2hf\prime(x) + \frac{4h^2}{2}f\prime\prime(x) - \frac{8h^3}{6}f\prime\prime\prime(x) + \frac{16h^4}{24}f''''(c_2)$

Subtract 2nd equation from the 1st one.

$4f(x+h) - f(x-2h) = 3f(x) + 6hf\prime(x) + \frac{12h^3}{6}f\prime\prime\prime(x) + \frac{4h^4}{24}f\prime\prime\prime\prime(c_1) - \frac{16h^4}{24}f\prime\prime\prime\prime(c_2)$

$f\prime(x) = \frac{4f(x+h) - f(x-2h) - 3f(x)}{6h} - \frac{h^2 f\prime\prime\prime(x)}{3} - \frac{h^4 f\prime\prime\prime\prime(c_1)}{6} + \frac{2h^4 f\prime\prime\prime\prime(c_2)}{3}$

Error term is $-\frac{h^2 f\prime\prime\prime(x)}{3} - \frac{h^4 f\prime\prime\prime\prime(c_1)}{6} + \frac{2h^4 f\prime\prime\prime\prime(c_2)}{3}$

Q2.a

Composite Trapezoid Rule:
$\int_a^b f(x)dx = \frac{h}{2}\ [f(a) + f(b) + 2(\sum_{i=1}^{m-1} f(x_i))]$, h=(b-a)/m

for m=1

$\int_0^{\pi/2} cosxdx = \frac{b-a}{2m}[cos0 + cos\frac{\pi}{2} + 2(\sum_{i=1}^{0} f(x_i)\ ]$

$= \dfrac{\pi}{4}$ (1+0+0)

$= \dfrac{\pi}{4} = 0.78539816339$
And the error is 1 - 0.78539816339 = 0.214640783661

for m=2

$\int_0^{\pi/2} cosxdx = \frac{b-a}{2m}[cos0 + cos\frac{\pi}{2} + 2(\sum_{i=1}^{1} f(x_i)\ ]$

$= \frac{\pi}{8}$ (1+0+2(cos(π/4))

$= \frac{\pi}{8}$ (1+$\sqrt{2}$)

$= \frac{\pi}{8} + \frac{\pi\sqrt{2}}{8} = 0.9480594489474579$

And the error is 1 - 0.948059448947457 = 0.051940551052543

for m=4

$\int_0^{\pi/2} cosxdx = \frac{b-a}{2m}[cos0 + cos\frac{\pi}{2} + 2(\sum_{i=1}^{4} f(x_i)\ ]$

$= \frac{\pi}{16}$ (1+0+2(cos($\pi/8) + cos(2\pi/8) + cos(3\pi/8)$))

$= \frac{\pi}{16}$ (1+2($\sqrt{2\sqrt{2}}/2 + \sqrt{2}/2 + \sqrt{2-\sqrt{2}}$))

$= \frac{\pi}{16}$ (1+$\sqrt{2+\sqrt{2}} + \sqrt{2} + \sqrt{2-\sqrt{2}}$) = 0.987115799762

And the error is 1 - 0.987115799762 = 0.012884200238

Q2.b

Composite Simpson's Rule:
$\int_a^b f(x)dx = \frac{h}{3}$ [y$_0 + y_{2m}$ + 4($\sum_{i=1}^{m} y_{2i-1}$ + 2($\sum_{i=1}^{m-1} y_{2i}$)] , h=(b-a)/2m

for m=1
$\int_0^{\pi/2} cosxdx = \frac{\pi}{12}$ [cos(0) +cos(π/2) + 4($\sum_{i=1}^{1} y_{2i-1}$ + 2($\sum_{i=1}^{0} y_{2i}$)]

$\int_0^{\pi/2} cosxdx = \frac{\pi}{12}$ [cos(0) +cos(π/2) + 4(cos(π/4))]

$\frac{\pi}{12}$ $[1 + 0 + 4(\sqrt{2}/2)]$
$= (\pi/12) + (\pi\sqrt{2})/6 = 1.00227987748$

And the error is 1 - 1.00227987748 = -0.00227987748
for m=2
$\int_0^{\pi/2} cosxdx = \frac{\pi}{24}$ [cos(0) +cos(π/2) + 4($\sum_{i=1}^{2} y_{2i-1}$ + 2($\sum_{i=1}^{1} y_{2i}$)]

$\int_0^{\pi/2} cosxdx = \frac{\pi}{24}$ [1 + 0 +4($cos(\pi/8) + cos(3\pi/8)) + 2cos(\pi/8)$]

$\frac{\pi}{24}$ $[1 + 0 + 4(\frac{\sqrt{2+\sqrt{2}}}{2} + \frac{\sqrt{2-\sqrt{2}}}{2} + 2(\frac{\sqrt{2}}{2})$

$= \pi/24(1 + 2\sqrt{2+\sqrt{2}} + 2\sqrt{2-\sqrt{2}} + \sqrt{2}) = 1.00013458373669$

And the error is 1 - 1.00013458373669 = -0.00013458373669
for m=4
$\int_0^{\pi/2} cosxdx = \frac{\pi}{48}$ [cos(0) +cos(π/2) + $4(\sum_{i=1}^{4} y_{2i-1}$ + $2(\sum_{i=1}^{3} y_{2i})]$

$\int_0^{\pi/2} cosxdx = \frac{\pi}{48}$ [cos(0) +cos(π/2) + 4(cos(π/16) $+ cos(3\pi/16) + cos(5\pi/16) + cos(7\pi/16))$ + $2(cos(\pi/8) + cos(2\pi/8) + cos(3\pi/8]$

$= \frac{\pi}{48} [1 + 4(\frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} + \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2} + \frac{\sqrt{2-\sqrt{2+\sqrt{2}}}}{2} + \frac{\sqrt{2-\sqrt{2-\sqrt{2}}}}{2} + \frac{\sqrt{2-\sqrt{2+\sqrt{2}}}}{2}) + 2(\frac{\sqrt{2+\sqrt{2}}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2-\sqrt{2}}}{2})] = 0.985632973662$

And the error is 1 - 0.985632973662 = 0.014367026338

Q2.c

Trapezoid rule for $\int_0^3$x$^2 lnx$ dx
for m=32, result = 6.894560766905436
for m=64, result = 6.889270017551926
for m=128, result = 6.8879500617240925

Q2.d

Simpson's rule for $\int_0^3$x$^2 lnx$ dx
for m=32, result = 6.887477432374826
for m=64, result = 6.58557606071753
for m=128, result = 6.734987796577359