Q1-1.

$$A= \begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

Subtract 1 x row1 from row2

$$= \begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

Subtract 1/2 x row 1 from row3

$$= \begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 1 & 3 \end{bmatrix}$$

Subtract 1/2 x row 2 from row3

$$U= \begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

For the L matrix, its identity matrix + 1/2 at (3,2) , 1 at (2,1) and 1/2 at (3,1) :

$$L= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 1/2 & 1/2 & 1 \end{bmatrix}$$

$$b= \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

Lc=b

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1/2 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

(c_1,c_2,c_3) = (2,2,4)

Ux = c

$$\begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$$

(x_1,x_2,x_3) = (1,-1,2)

Q1-2.

We don't need to change any rows according to the first columns elements.

A= $\begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix}$

Subtract 1 x row1 from row2

$= \begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 2 & 2 & 3 \end{bmatrix}$ P = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Subtract 1/2 x row1 from row3

$= \begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 1 & 3 \end{bmatrix}$

Subtract 1/2 x row2 from row3

U = $\begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$

P = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

L = $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1/2 & 1/2 & 1 \end{bmatrix}$

A = $\begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix}$

Lc=Pb

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

(c_1,c_2,c_3) = (2,2,4)

Ux=c

$$\begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$$

(x_1,x_2,x_3)=(1,-1,2)

Q1-3.

First find the exact solution, which is [x_1,x_2] = [1,0]

Backward error = $|| \, b - A\text{x}_a \, || \, \infty = || \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 2 & 4.01 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} || \, \infty$

$= || \begin{bmatrix} 0 \\ -0.01 \end{bmatrix} || \, \infty = 0.01$

$Forwarderror$ $||\text{x-x}_a \, || \, \infty = || \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} -1 \\ 1 \end{bmatrix} || \, \infty = || \begin{bmatrix} 2 \\ -1 \end{bmatrix} || \, \infty = 2$

Error magnification factor is (forward error/ ||x|| ∞)/(backward error / ||b|| ∞)
= (2/1)/(0.01/2) = 400

Q2-1.

(-1,0),(2,1),(3,1),(5,2)

$P_3(x)$
= 0*((x-2)(x-3)(x-5))/((-1-2)(-1-3)(-1-5))
+ 1*((x–1)(x-3)(x-5))/((2–1)(2-3)(2-5))
+ 1*((x–1)(x-2)(x-5))/((3–1)(3-2)(3-5))
+ 2*((x–1)(x-2)(x-3))/((5–1)(5-2)(5-3))

=(($x^2 - 2x - 3$)(x-5))/9 + (($x^2 - x - 2$)(x-5))/-8 + (($x^2 - x - 2$)(x-3))/18

=(3x^3-18x^2+33x+54)/72

$P_3(x)$ = x^3/24 - x^2/4 + 11x/24 + 3/4

Q2-2.

-1	0				
		$1/3$			
2	1		$-1/12$		
		0		$1/24$	$P(x)$ = 0 + 1/3(x+1) - 1/12(x+1)(x-2) + 1/24(x+1)(x-
3	1		$1/6$		
		$1/2$			
5	2				

2)(x-3)

=(8x+8-−2x^2+2x+4+x^3-4x^2+x+6)/24 =x^3/24 - x^2/4 + 11x/24 + 3/4

Q2-3.

x_1= -1 , x_2= 1 , x_3= 2 , a_1=y_1= 1 , a_2=y_2=1 , a_3=y_3=4
δ_1=x_2-x_1=2 , δ_2=x_3-x_2=1 , Δ_1=y_2-y_1=0 , Δ_2=y_2-y_1=3

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 6 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 9 \\ 0 \end{bmatrix}$$

(c_1,c_2,c_3)=(0,3/2,0)
d_1 = (c_2-c_1)/3δ_1 = 1/4
d_2 = (c_3-c_2)/3δ_2 = -1/2
b_1 = (Δ_1/δ_1) - ((δ_1/3)(2c_1+c_2)) = -1
b_2 = (Δ_2/δ_2) - ((δ_2/3)(2c_2+c_3)) = 2

$S_1(x)$ = 1-1(x+1)+0+1/4$(x+1)^3$ on [-1,1]
$S_2(x)$ = 1+2(x-1)+3/2$(x-1)^2$-1/2$(x-1)^3$ on [1,2]

Q2-4.

x(t)=1+6t^2+2t^3
x_1=1 , c_x=6 , d_x=2 , b_x=0
b_x=3$(x_2 - x_1)$=0 , $x_2 = x_1 = 1$
c_x=3$(x_3 - x_2) - b_x = 6$, x_3=3
d_x=$x_4 - x_1 - b_x - c_x = 2$, x_4=9

y(t)=1-t+t^3
y_1=1 , b_y=-1 , d_y=1 , c_y=0
b_y=3$(y_2 - y_1)$=1 , $y_1 = 1$, $y_2 = 2/3$
c_y=3$(y_3 - y_2) - b_y = 0$, y_3=1/3
d_y=$y_4 - y_1 - b_y - c_y = 1$, y_4=1

first end point = $(x_1, y_1) = (1, 1)$
control point = $(x_2, y_2) = (1, 2/3)$
control point = $(x_3, y_3) = (3, 1/3)$
last end point = $(x_4, y_4) = (9/1)$