Q1.

09.05.2018 temperature for every 3 hours.

time of day	t	temp
12 mid	0	17
3 am	1/8	13
6 am	2/8	15
9 am	3/8	21
12 noon	4/8	26
3 pm	5/8	28
6 pm	6/8	25
9 pm	7/8	21

$c_1 + c_2 cos2\pi(0) + c_3 sin2\pi(0) = 17$
$c_1 + c_2 cos2\pi(1/8) + c_3 sin2\pi(1/8) = 13$
$c_1 + c_2 cos2\pi(2/8) + c_3 sin2\pi(2/8) = 15$
$c_1 + c_2 cos2\pi(3/8) + c_3 sin2\pi(3/8) = 21$
$c_1 + c_2 cos2\pi(4/8) + c_3 sin2\pi(4/8) = 26$
$c_1 + c_2 cos2\pi(5/8) + c_3 sin2\pi(5/8) = 28$
$c_1 + c_2 cos2\pi(6/8) + c_3 sin2\pi(6/8) = 25$
$c_1 + c_2 cos2\pi(7/8) + c_3 sin2\pi(7/8) = 21$

$$\text{A}= \begin{bmatrix} 1 & cos0 & sin0 \\ 1 & cos\pi/4 & sin\pi/4 \\ 1 & cos\pi/2 & sin\pi/4 \\ 1 & cos3\pi/4 & sin\pi/4 \\ 1 & cos\pi/ & sin\pi \\ 1 & cos5\pi/4 & sin5\pi/4 \\ 1 & cos3\pi/2 & sin3\pi/2 \\ 1 & cos7\pi/4 & sin7\pi/4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & \sqrt{2}/2 & \sqrt{2}/2 \\ 1 & 0 & 1 \\ 1 & \text{-}\sqrt{2}/2 & \sqrt{2}/2 \\ 1 & -1 & 0 \\ 1 & \text{-}\sqrt{2}/2 & \text{-}\sqrt{2}/2 \\ 1 & 0 & -1 \\ 1 & \sqrt{2}/2 & \text{-}\sqrt{2}/2 \end{bmatrix} \quad \text{b}= \begin{bmatrix} 17 \\ 13 \\ 15 \\ 21 \\ 26 \\ 28 \\ 25 \\ 21 \end{bmatrix}$$

$\mathbf{A}^T$Ac=$\mathbf{A}^T$b

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} \text{c}_1 \\ \text{c}_2 \\ \text{c}_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \sqrt{2}/2 & 0 & -\sqrt{2}/2 & -1 & -\sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & \sqrt{2}/2 & 1 & \sqrt{2}/2 & 0 & -\sqrt{2}/2 & -1 & -\sqrt{2}/2 \end{bmatrix} \begin{bmatrix} 17 \\ 13 \\ 15 \\ 21 \\ 26 \\ 28 \\ 25 \\ 21 \end{bmatrix}$$

$$= \begin{bmatrix} 166 \\ -19.5 \\ -20.5 \end{bmatrix}$$

Then $(c_1, c_2, c_3) = (20.75, 4.875, -5.125)$

Our function becomes 20.75 - 4.875cos$2\pi t - 5.125 sin 2\pi t$
b - A$\overline{x} = r$

$$\begin{bmatrix}17\\13\\15\\21\\26\\28\\25\\21\end{bmatrix} - \begin{bmatrix}15.875\\13.678\\15.625\\20.573\\25.625\\27.821\\25.875\\20.926\end{bmatrix} = \begin{bmatrix}1.125\\-0.678\\-0.625\\0.427\\0.375\\0.179\\-0.875\\0.074\end{bmatrix} = \text{r}$$

Squares of the r matrices' elements=
1.265625
0.459684
0.390625
0.182329
0.140625
0.032041
0.765625
0.005476
Sum of these divided by 8 gives us 0.405253 and the square root of this value is 0.636 which is our RMSE.

Q2-1.

Linear systems' Jacobian matrix will exist of the same coefficients since in linear systems' equations we don't have more than 1 order equations so that when taking the derivative we will always get the coefficient of the unknowns.Then $\mathbf{A}^T$A will always be same which makes the method to converge to the solution in one iteration.

Q2-2.

$y_1 = c_1 x_1^{c_2}$
$y_2 = c_1 x_2^{c_2}$
$y_3 = c_1 x_3^{c_2}$

$r_1(x_1, y_1) = c_1 x_1^{c_2} - y_1$
$r_2(x_2, y_2) = c_1 x_2^{c_2} - y_2$
$r_3(x_3, y_3) = c_1 x_3^{c_2} - y_3$

$$\text{J=}\begin{bmatrix}\partial r_1/\partial c_1 & \partial r_1/\partial c_3\\ \partial r_2/\partial c_1 & \partial r_2/\partial c_2\\ \partial r_3/\partial c_1 & \partial r_3/\partial c_2\end{bmatrix} = \begin{bmatrix}\text{x}_1^{c_2} & \text{c}_2\text{c}_1 x_1^{c_2-1}\\ \text{x}_2^{c_2} & \text{c}_2\text{c}_1 x_2^{c_2-1}\\ \text{x}_3^{c_2} & \text{c}_2\text{c}_1 x_3^{c_2-1}\end{bmatrix}$$