CS-302-HW-2

Gorkem YAR
27970 Gorkem

Question 1:

a) $\underbrace{(0+1)^* \cdot 1 \cdot (0+1)} + \underbrace{(0+1)^* \cdot 1 \cdot (0+1)}$

They are same since + operator means union we can simplify by erasing one of them

$\Rightarrow (0+1)^* \cdot 1 \cdot (0+1)$

b) $(\underbrace{((0^* \cdot 1^*) + 1)^*} (0+1)^*)^*$

$=$

$((\underbrace{0^* \cdot 1^*})^* + 1^*)^*$

$\Rightarrow ((0+1)^* + 1^*)^*$

$\Rightarrow ((0+1)+1)^*$

$\Rightarrow (0+1)^*$

$\Rightarrow ((0+1)^* (0+1)^*)^*$

$\Rightarrow ((0+1) + (0+1))^*$

$\Rightarrow \underline{\underline{(0+1)^*}}$

c) $(L+M^*)^* \Rightarrow (L^* + (M^*)^*)^* \Rightarrow \underline{(L^* + M^*)^*} \Rightarrow \underline{\underline{(L+M)^*}}$

d) $(L \cdot M^*)^*$ if $e \in L \Rightarrow (L \cdot M^*)^* = (L^* \cdot M^*)^*$

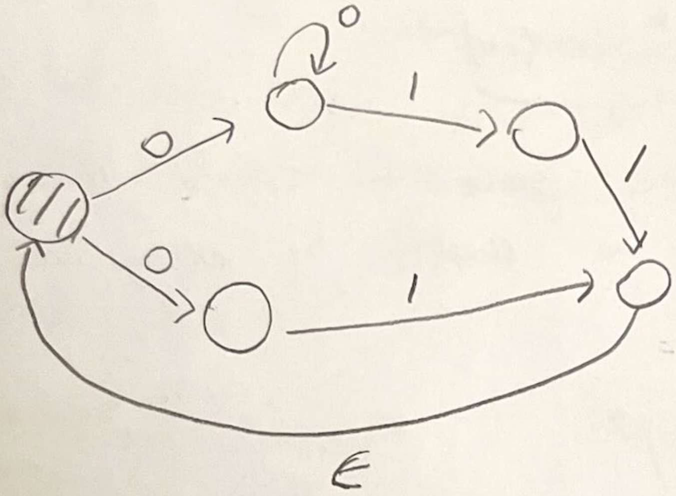
$= (L+M)^*$

if $e \notin L$ There is no simplification.

CS-302 HW2

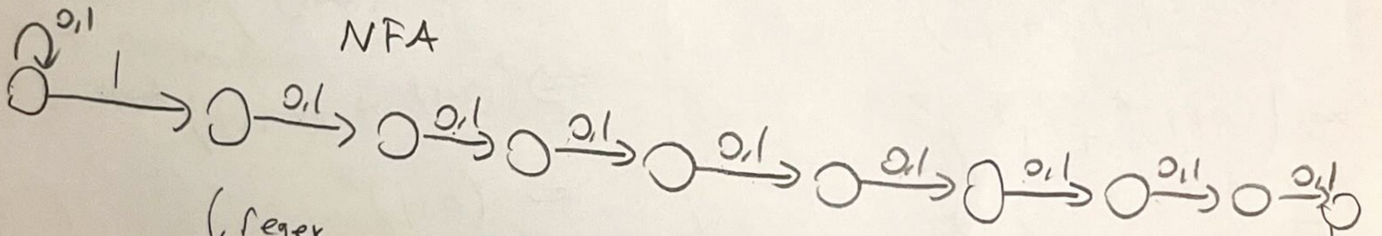
Gorkem YAR
27970
Gorkem

Question-2:

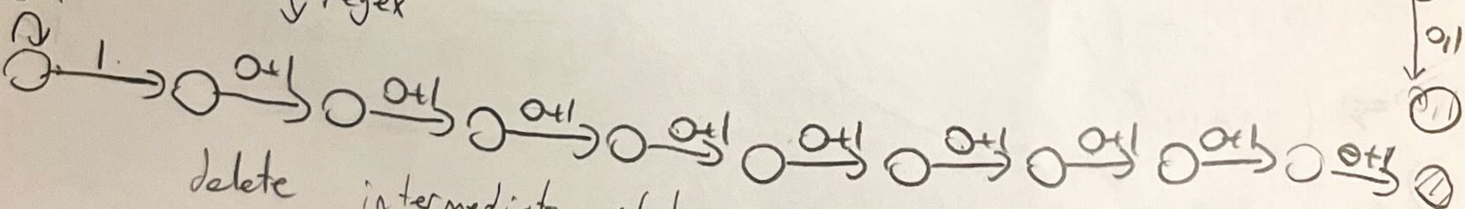


Question-3.b:

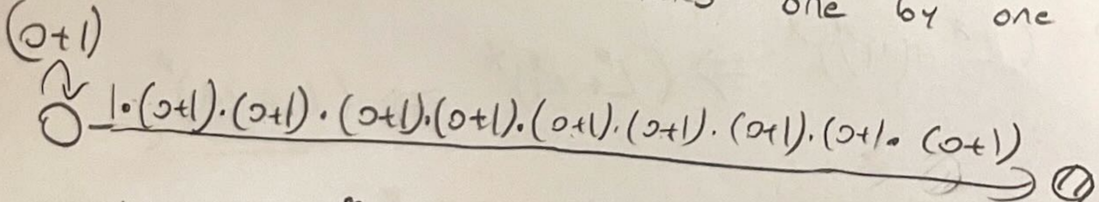
b) NFA



↓ regex



delete intermediate states one by one



result: $(0+1)^* \cdot \underset{\text{10th from right end}}{1} \cdot \underbrace{(0+1).(0+1).(0+1)(0+1).(0+1).(0+1).(0+1).(0+1).(0+1)}_{\text{9 times}}$

CS-302 HW2

Gorkem YAK
27970 Gorkem

Question 3.1.1

c) We can divide this regular expression into two parts:

1- the set of strings does not have any consecutive 1's

2- the set of strings that consist exactly one pair of consecutive 1's.

1- $(1+e).(0+01)^*$

2- $\underbrace{(0+10)^*}_{\text{Any string that does not include consecutive 1's and ends with 0.}}.1.1.\underbrace{(0+01)^*}_{\text{Any string that does not include consecutive 1's and starts with 0}}$

$$RE = (1+e).(0+01)^* + (0+10)^*.1.1.(0+01)^*$$

CS - 302 HW2

Görkem YAR
27970

Question 3.1.4

b) $\underbrace{(0^*1^*)^*}$ 000 $(0+1)^*$

$(0+1)^* \cdot 000 \cdot (0+1)^*$

A language that accepts strings that contains three consecutive zeros at least ones.

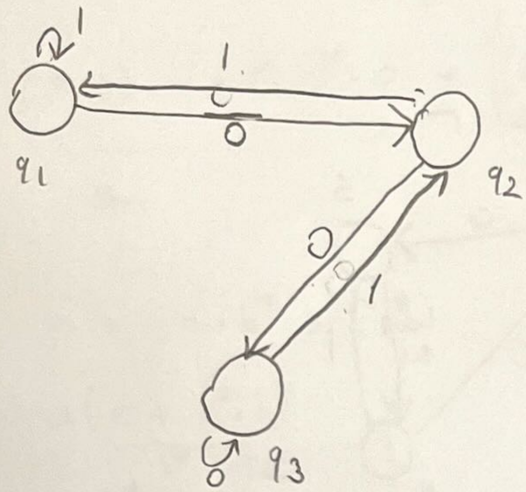
c) $(0+10)^* \cdot 1^*$

A language that does not include consecutive 1's before the last 0 element.

CS-302 HW2 Görkem YAR

27970 Görkem

Question 3.2.1



a)

$R_{11}^{0} = 1+e$

$R_{22}^{0} = e$

$R_{12}^{0} = 0$

$R_{21}^{0} = 1$

$R_{33}^{0} = 0+e$

$R_{31}^{0} = \phi$

$R_{13}^{0} = \phi$

$R_{23}^{0} = 0$

$R_{32}^{0} = 1$

b)

$R_{11}^{1} = R_{11}^{0} + R_{11}^{0}\cdot(R_{11}^{0})^{*}\cdot R_{11} = (1+e)+(1+e)\cdot(1+e)^{*}\cdot(1+e) = 1^{*}$

$R_{22}^{1} = R_{22}^{0} + R_{21}^{0}\cdot(R_{11}^{0})^{*}\cdot R_{12}^{0} = e + 1\cdot(1+e)^{*}\cdot 0 = e+1\cdot 1^{*}\cdot 0$

$R_{12}^{1} = R_{12}^{0} + R_{11}^{0}\cdot(R_{11}^{0})^{*}\cdot R_{12}^{0} = 0+(1+e)\cdot(1+e)^{*}\cdot 0 = 1^{*}\cdot 0$

$R_{21}^{1} = R_{21}^{0} + R_{21}^{0}\cdot(R_{11}^{0})^{*}\cdot R_{11}^{0} = 1 + 1\cdot(1+e)^{*}\cdot(1+e) = 1\cdot 1^{*}$

$R_{33}^{1} = R_{33}^{0} + R_{31}^{0}\cdot(R_{11}^{0})^{*}\cdot R_{13}^{0} = 0+e$

$R_{31}^{1} = R_{31}^{0} + R_{31}^{0}\cdot(R_{11}^{0})^{*}\cdot R_{11}^{0} = \phi + \phi = \phi$

$R_{13}^{1} = R_{13}^{0} + R_{11}^{0}(R_{11}^{0})^{*}\cdot R_{13}^{0} = \phi + \phi = \phi$

$R_{23}^{1} = R_{23}^{0} + R_{21}^{0}\cdot(R_{11}^{0})^{*}\cdot R_{13}^{0} = 0$

$R_{32}^{1} = R_{32}^{0} + R_{31}^{0}\cdot(R_{11}^{0})^{*}\cdot R_{12}^{0} = 1$

c)

$R_{11}^{2} = R_{11}^{1} + R_{12}^{1}\cdot(R_{22}^{1})^{*}\cdot R_{21} = 1^{*} + 1^{*}\cdot 0\cdot(1\cdot 1^{*}\cdot 0)^{*}\cdot 1\cdot 1^{*}$

$R_{22}^{2} = R_{22}^{1} + R_{22}^{1}\cdot(R_{22}^{1})^{*}\cdot R_{22} = (1\cdot 1^{*}\cdot 0)^{*}$

$R_{12}^{2} = R_{12}^{1} + R_{12}^{1}\cdot(R_{22}^{1})^{*}\cdot R_{22}^{1} = 1^{*}\cdot 0 + (1^{*}\cdot 0)\cdot(1\cdot 1^{*}\cdot 0)^{*} = (1^{*}\cdot 0)\cdot(1\cdot 1^{*}\cdot 0)^{*}$

$R_{21}^{2} = R_{21}^{1} + R_{22}^{1}\cdot(R_{22}^{1})^{*}\cdot R_{21}^{1} = 1\cdot 1^{*} + (1\cdot 1\cdot 0)^{*}\cdot 1\cdot 1^{*} = (1\cdot 1\cdot 0)^{*}\cdot 1\cdot 1^{*}$

$R_{33}^{2} = R_{33}^{1} + R_{32}^{1}\cdot(R_{22}^{1})^{*}\cdot R_{23}^{1} = (0+e) + 1\cdot(1\cdot 1^{*}\cdot 0)^{*}\cdot 0$

$R_{31}^{2} = R_{31}^{1} + R_{32}^{1}\cdot(R_{22}^{1})^{*}\cdot R_{21}^{1} = 1\cdot(1\cdot 1^{*}\cdot 0)^{*}\cdot 1\cdot 1^{*}$

$R_{13}^{2} = R_{13}^{1} + R_{12}^{1}\cdot(R_{22}^{1})^{*}\cdot R_{23}^{1} = (1^{*}\cdot 0)\cdot(1\cdot 1^{*}\cdot 0)^{*}\cdot 0$

$R_{23}^{2} = R_{23}^{1} + R_{22}^{1}\cdot(R_{22}^{1})^{*}\cdot R_{23}^{1} = 0 + (1\cdot 1^{*}\cdot 0)^{*}\cdot 0 = (1\cdot 1^{*}\cdot 0)^{*}\cdot 0$

$R_{32}^{2} = R_{32}^{1} + R_{32}^{1}\cdot(R_{22}^{1})^{*}\cdot R_{22}^{1} = 1 + 1\cdot(1\cdot 1^{*}\cdot 0)^{*} = 1\cdot(1\cdot 1^{*}\cdot 0)^{*}$

$\star$ $R_{22}^{1}\cdot(R_{22}^{1})^{*} = (e+1\cdot 1^{*}\cdot 0)\cdot(e+1\cdot 1^{*}\cdot 0)^{*}$

$\Rightarrow (e+1\cdot 1^{*}\cdot 0)^{*} = (R_{22}^{1})^{*}$

using this fact

d) The language for the automation is

$$R_{13}^{3} = R_{13}^{2} + R_{13}^{2} \cdot (R_{33}^{2})^{*} \cdot R_{33}^{2}$$

$$\Rightarrow \underbrace{(1^{*} \cdot 0) \cdot (1 \cdot 1^{*} \cdot 0)^{*} \cdot 0}_{a} + \underbrace{\left((1^{*} \cdot 0) \cdot (1 \cdot 1^{*} \cdot 0)^{*} \cdot 0\right)}_{a} \cdot \underbrace{\left((0+e) + 1 \cdot (1 \cdot 1^{*} \cdot 0)^{*} \cdot 0\right)^{*}}_{b} \cdot$$

$$\hookrightarrow \underbrace{\left((0+e) + 1 \cdot (1 \cdot 1^{*} \cdot 0)^{*} \cdot 0\right)}_{b}$$

$$\Rightarrow a + a \cdot b^{*} \cdot b$$

$$\Rightarrow a(\underbrace{e + b^{*} \cdot b}_{b^{*}})$$

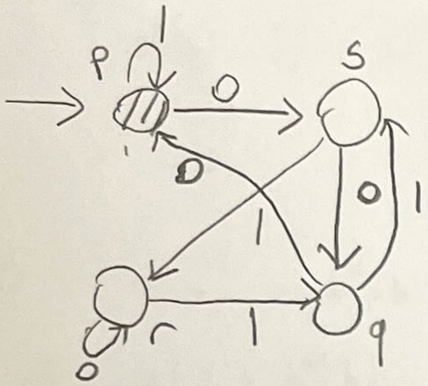
$$\Rightarrow a \cdot b^{*}$$

$$\Rightarrow \underbrace{(1^{*} \cdot 0) \cdot (1 \cdot 1^{*} 0)^{*} \cdot 0 \cdot \left((0+e) + 1 \cdot (1 \cdot 1^{*} \cdot 0)^{*} \cdot 0\right)^{*}}_{\text{The regular expression that accepts the language.}}$$

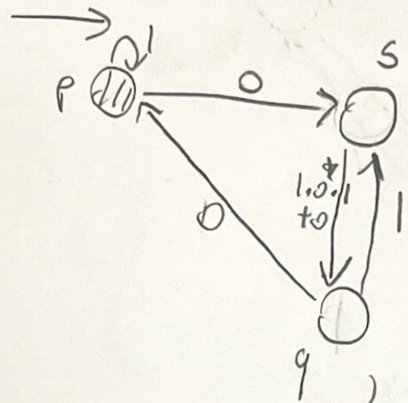
CS-302 HW2

Görkem YAR
27970 ~~Görkem~~

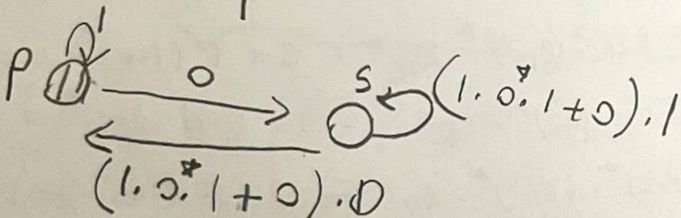
Question 3.2.3



eliminate r



eliminate q



$$\left(1+\left(1^*\cdot 0\cdot\left(\left((1\cdot 0^*\cdot 1)+0\right)\cdot 1\right)^*\cdot(1\cdot 0^*\cdot 1+0)\cdot 0\right)\right)^*$$