

Question-1

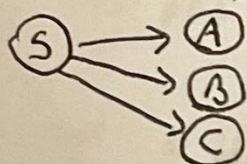
a)  $G_1 = (V, \Sigma, R, S)$ ,  $V = \{S, A, B, C, D, E\}$ ,  $\Sigma = \{a, b, c\}$   
only  $E$  is nullable in  $G$  After removing the null transitions.

$S \rightarrow AE | EB | A | B | C$ ;  $A \rightarrow aA | a$ ;  $B \rightarrow Bb | b$ ;  $C \rightarrow Cc$   
 $D \rightarrow aCb | a | b | c$ ;  $E \rightarrow aEb | ab$

b)  $G_2 = (V, \Sigma, R, S)$ ,  $V = \{S, A, B, C, D, E\}$ ,  $\Sigma = \{a, b, c\}$   
There is only unitary transition in  $S$ .

$S \rightarrow AE | EB | aA | a | Bb | b | Cc$ ;  $A \rightarrow aA | a$ ;  $B \rightarrow Bb | b$ ;  $C \rightarrow Cc$   
 $D \rightarrow aCb | a | b | c$ ;  $E \rightarrow aEb | ab$

The graph representing unitary transition



c)  $\bar{I}$  will firstly eliminate nongenerating then  $\bar{I}$  will eliminate nonreachable.  
In  $G_2$ ,  $C$  is not generating since it can never terminate. We need to eliminate the transitions that include  $C$ .

$G_{2,1} = (V, \Sigma, R, S)$ ,  $V = \{S, A, B, D, E\}$ ,  $\Sigma = \{a, b, c\}$   
 $S \rightarrow AE | EB | aA | a | Bb | b$ ;  $A \rightarrow aA | a$ ;  $B \rightarrow Bb | b$   
 $D \rightarrow a | b | c$ ;  $E \rightarrow aEb | ab$

Now, we need to eliminate non reachable the only non-reachable variable is  $D$ .

$G_3 = (V, \Sigma, R, S)$ ,  $V = \{S, A, B, E\}$ ,  $\Sigma = \{a, b\}$   
 $S \rightarrow AE | EB | aA | a | Bb | b$ ;  $A \rightarrow aA | a$ ;  $B \rightarrow Bb | b$   
 $E \rightarrow aEb | ab$

d) CNF for  $G_3$ , we need to introduce new transitions to replace terminal symbols.  $T_a \rightarrow a$ ;  $T_b \rightarrow b$

$S \rightarrow AE | EB | T_a A | a | B T_b | b$ ;  $A \rightarrow T_a A | a$ ;  $B \rightarrow B T_b | b$ ;  $E \rightarrow T_a E T_b | T_a T_b$   
\*  $S, A$ , and  $B$  already in the Chomsky Normal form for the  $E$  we need to introduce a new transition  $P$  to make it CNF.  $P \rightarrow E T_b$

Overall all transitions:

$S \rightarrow AE | EB | T_a A | B T_b | a | b$ ;  $A \rightarrow T_a A | a$ ;  $B \rightarrow B T_b | b$ ;  $E \rightarrow T_a P | T_a T_b$ ;  $P \rightarrow E T_b$

e) The language where all of the  $a$ 's comes before all of the  $b$ 's.  
 $L(CNF) = \{S E a^i b^j \mid i \geq 0, j \geq 0 \text{ and not } (i=0 \text{ and } j=0)\}$



## Question-2

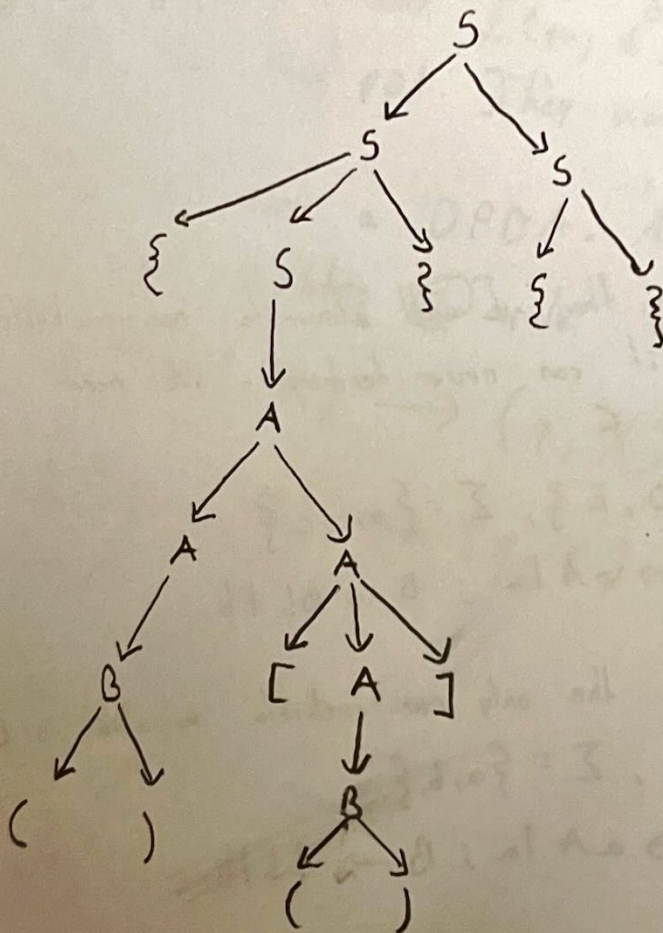
a)  $G$  is a CFG.  $G = (V, T, P, S)$   
 $V = \{S, A, B\}$ ,  $T = \{(), \{\}, [, ]\}$   
 $P$ :

$$S \rightarrow \{S\} \mid SS \mid A \mid \{\}$$

$$A \rightarrow [A] \mid AA \mid B \mid []$$

$$B \rightarrow (B) \mid BB \mid ()$$

b) Parse tree of  $\{() [()] \} \{ \}$





## Exercise

6.4.1

a) This is not a DPDA. In DPDA, the following properties should hold:

$$\textcircled{1} |L(q, a, X)| \leq 1 \text{ where } q \in Q, a \in \Sigma, X \in \Gamma$$

$$\textcircled{2} \text{ if } |L(q, a, X)| = 1 \text{ then } |L(q, e, X)| = 0.$$

There are productions that violates  $\textcircled{2}$  rule.

c)  $L(q_0, 0, 0)$  and  $L(q_0, \epsilon, 0)$  are both productions to rule for this PDA. They violate the  $\textcircled{2}$  rule.

e) This is not a DPDA. As I stated the rules in part a) this PDA violates  $\textcircled{2}$  rule as well.

$$L(q, 1, X) \rightarrow (q, XX)$$

$$L(q, e, X) \rightarrow (q, \epsilon)$$

} Violation of the  $\textcircled{2}$  rule.



## Exercise 6.4.2

a)  $P$  is a DPDA

$$P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$$

$$Q = \{q_0, q_1, q_2, f_1, f_2\}; \Sigma = \{0, 1\}$$

$$\Gamma = \{0, z_0\}$$

$$F = \{f_1\}$$

$$\delta(q_0, e, z_0) = (f_1, z_0)$$

$$\delta(f_1, 0, z_0) = (q_1, 0z_0)$$

$$\delta(f_1, 1, z_0) = (f_2, z_0)$$

$$\delta(q_1, 0, 0) = (q_1, 00)$$

$$\delta(q_1, 1, 0) = (q_2, e)$$

$$\delta(q_2, 1, 0) = (q_2, e)$$

$$\delta(q_2, e, z_0) = (f_2, z_0)$$

$$\delta(f_2, 1, z_0) = (f_2, z_0)$$

b)  $P$  is a DPDA

$$P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$$

$$Q = \{q_0, q_1, N\}, \Sigma = \{0, 1\}, \Gamma = \{0, z_0\}, F = \{q_0, q_1\}$$

$$\delta(q_0, 0, z_0) = (q_0, 0z_0)$$

$$\delta(q_0, 0, 0) = (q_0, 00)$$

$$\delta(q_0, 1, z_0) = (N, z_0)$$

$$\delta(q_0, 1, 0) = (q_1, e)$$

$$\delta(q_1, 1, 0) = (q_1, e)$$

$$\delta(q_1, 1, z_0) = (N, z_0)$$

$$\delta(N, 1, z_0) = (N, z_0)$$



## Exercise 6.4.2

c)  $P$  is a OPDA

$$P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$$

$$Q = \{q_0, q_1, q_2, q_3, f_1, f_2, f_3\}$$

$$\Sigma = \{0, 1\}$$

$$\Gamma = \{z_0, 0, 1\}, F = \{f_1, f_2, f_3\}$$

$$\delta(q_0, \epsilon, z_0) \rightarrow (f_1, z_0)$$

$$\delta(f_1, 1, z_0) \rightarrow (f_2, 1z_0)$$

$$\delta(f_1, 0, z_0) \rightarrow (q_1, 0z_0)$$

$$\delta(q_1, 0, 0) \rightarrow (f_1, 00)$$

$$\delta(f_1, 0, 0) \rightarrow (q_1, 00)$$

$$\delta(f_1, 1, 0) \rightarrow (q_2, 0)$$

$$\delta(q_1, 1, 0) \rightarrow (q_2, 0)$$

$$\delta(q_2, 1, 0) \rightarrow (q_2, 0)$$

$$\delta(q_2, 0, 0) \rightarrow (q_3, \epsilon)$$

$$\delta(q_3, 0, 0) \rightarrow (q_3, \epsilon)$$

$$\delta(q_3, \epsilon, z_0) \rightarrow (f_3, z_0)$$

$$\delta(q_3, \epsilon, z_0) \rightarrow (f_3, z_0)$$

$$\delta(q_3, \epsilon, z_0) \rightarrow (f_3, z_0)$$

$$\delta(q_3, \epsilon, z_0) \rightarrow (f_3, z_0)$$

$$\delta(q_3, \epsilon, z_0) \rightarrow (f_3, z_0)$$

$$\delta(q_3, \epsilon, z_0) \rightarrow (f_3, z_0)$$

$$\delta(q_3, \epsilon, z_0) \rightarrow (f_3, z_0)$$

$$\delta(q_3, \epsilon, z_0) \rightarrow (f_3, z_0)$$

$$\delta(q_3, \epsilon, z_0) \rightarrow (f_3, z_0)$$

$$\delta(q_3, \epsilon, z_0) \rightarrow (f_3, z_0)$$

$$\delta(q_3, \epsilon, z_0) \rightarrow (f_3, z_0)$$

$$\delta(q_3, \epsilon, z_0) \rightarrow (f_3, z_0)$$

$$\delta(q_3, \epsilon, z_0) \rightarrow (f_3, z_0)$$

$$\delta(q_3, \epsilon, z_0) \rightarrow (f_3, z_0)$$

only one

$$\delta(f_2, 1, 1) \rightarrow (f_2, 11)$$

Accept or reject strings without 1.

final state acceptance