

Q1:

Graph (a)

- 1) Maximum degree of concurrency: 8
- 2) Critical path length: 4 nodes 3 length
- 3) Maximum speedup: $\frac{15}{4} = 3.75$
- 4) Minimum number of processors for max speed up = 8
- 5) Maximum speed up with 4 threads = $\frac{15}{5} = 3$

Graph (b)

- 1) Maximum degree of concurrency = 8
- 2) Critical path length = 4 nodes 3 length
- 3) Maximum speed up = $\frac{15}{4} = 3.75$
- 4) Minimum number of processor for max speed up = 8
- 5) Maximum speed up with 4 threads = $\frac{15}{5} = 3$

Graph (c)

- 1) Maximum degree of concurrency: 8
- 2) Critical path length = 7 nodes 6 length
- 3) Maximum speed-up = $\frac{14}{7} = 2$
- 4) Minimum number of processors for max speed up = 3
- 5) Maximum speed up with 4 threads = $\frac{14}{7} = 2$

Graph (d)

- 1) Maximum degree of concurrency: 8
- 2) Critical path length = 8 nodes 7 length
- 3) Maximum speed up = $\frac{15}{8} = 1.75$
- 4) Minimum Number of processor for max speed up = 2
- 5) Maximum speed up with 4 threads = $\frac{15}{8} = 1.75$

Q2:

revised code

```
int temp;
for (j=1; j < n; j++) {
    temp = C[j]
    for (i=1; i < n; i++) {
        C[j][i] = C[j-1][i-1] + temp
    }
}
```

* improvement-2: Using a temp variable to store $C[j]$. Instead of accessing it from memory multiple times, we put it in a register. This reduced memory accesses, (Locality in registers) ↷

* improvement-1: change loop order to improve spatial locality. In the previous version the code access the matrix column by column. Now the access is done by row by row. As a result, the cache miss rate will reduce. (Cache Locality) ↷

Q3: Using Amdahl's law:

Serial fraction $1-r$; parallel fraction r ; T_s represents serial time
 $T_p(P) = (1-r) \cdot T_s + \frac{r \cdot T_s}{P}$ T_p represents parallel time
 Amdahl's law for speedup: P

$$= T_s / T_p(P)$$

$$= \frac{T_s}{(1-r) \cdot T_s + \frac{r \cdot T_s}{P}} = \frac{1}{(1-r) + \frac{r}{P}}$$

if we make $\frac{r}{P}$ to small compared to $1-r$ then it can be negligible.

$$\lim_{P \rightarrow \infty} \frac{1}{(1-r) + \frac{r}{P}} = \frac{1}{1-r} //$$

speedup maximum

Q4:

a) Latency: The time it takes for the data to travel from memory to its destination. It is the measurement of delay and unit of latency is seconds.

Bandwidth: The maximum rate for the data to transfer from memory. it can be measured as \rightarrow capacity bit/seconds or byte/seconds.

* Low latency means faster response, high bandwidth means more data can be transmitted at once.

b) Spatial Locality: It means use of data elements within relatively close in memory (Location). if a particular memory accessed nearby locations will be accessed as well.

Temporal Locality: Refers to the reuse of specific data within a small time interval.

Q5: 10,6 Tera flops = $10,6 \cdot 10^{12}$ floating point operations per second
720 GB/s = $\frac{720}{4} \cdot 10^9 = 180 \cdot 10^9$ floats per second from global memory
→ size of (float)

$$\frac{10,6 \cdot 10^{12}}{180 \cdot 10^9} = 58,89 \text{ floating point operations per float.}$$

on average 59 operations.

Q6: a) $50.000 / 2048 = 24,414 \rightarrow \boxed{25}$ blocks each block has 1024 threads

b) Each warp in CUDA is 32 threads each block has 1024 threads
so each block has $1024 / 32 = 32$ warps
total warps = $25 \cdot 32 = 800$ warps

c) $1024 \cdot 25 = 25600$ threads in total.

d) Let's consider the following threads with block numbers:

blockIdx.x = 24

blockDim.x = 1024

id of the last block's first thread $\rightarrow 49.152$

$50000 - 49152 = 848$ is the threadIdx of the first thread that does not do any job.

block = 25 (last block)

warp = $\lceil 848 / 32 \rceil = 26$ is the warp number

e) The control diverges for the threads that higher idx or warp number in line 5 and 7.

Also, the first 848 threads of the last block worps that are 26 or higher diverge in line 7 as well.
first 26 warps less than 26

Q7:

a) Using Amdahl's law: The unparallelisable part of a program determines the maximum speed up.

Serial	Fraction	40%
parallel	Fraction	60%

$$T_p(P) = \%40 \cdot T_s + \frac{\%60 \cdot T_s}{P}$$

$$\text{Speed up} = \frac{T_s}{\frac{T_s \cdot 2}{5} + \frac{3 \cdot T_s}{5 \cdot P}} = \frac{1}{\frac{2}{5} + \frac{3}{5 \cdot P}}$$

$$\text{if } \lim_{P \rightarrow \infty} = \frac{5}{2} = 2.5 \text{ times speed up}$$

b) theoretically maximum speed up with P processor is P . However, in practice the maximum speed up can exceed P (we call this superlinearity).
examples
- N queens problem: parallel version can have less work since the solution might appear in one parallel block much quicker than doing all serial work.
- Also, the parallel executions can improve cache-hit ratios this is called resource-based superlinearity.

$$c) E = \frac{S}{P} = \frac{T_s}{P \cdot T_p} = \frac{n}{P \cdot (n/P + \log P)} = \frac{n}{n + P \log P}$$

d) $E = \Theta(1)$ to be cost optimal. The cost optimality can also be written in the form $P \cdot T_p = \Theta(T_s)$.

$$\Rightarrow P \left(\frac{n}{P} + \log P \right) = \Theta(n)$$

$$= n + P \log P = \Theta(n)$$

$$\Rightarrow P \log P = \Theta(n)$$

$$\Rightarrow P \log P \leq kn$$

$$\frac{n}{\log n} (\log n - \log \log n) \rightarrow \boxed{P = \frac{n}{\log n}} \text{ by substitution}$$

$$\text{to make } \frac{n - \log \log n}{\log n} \leq k \cdot n \text{ holds}$$