Homework-7

Görkem YAR
27970

Question-1

* To convert a CFG to a DFA we need to do following steps:

→ Each nonterminal variable should correspond to a state

→ Non-terminal symbols that can end the string (empty string) correspond to a final state

→ For each production rule A → aB (right linear) or A → Ba (left linear) we can create a transition from state A to B with label a.

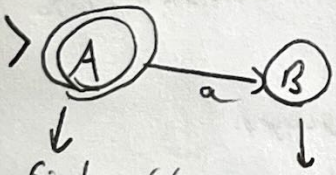
a) Right Linear

We have a right linear CFG if all the transitions in the form A → aB or A → ε

Example

A → aB | ε

Convert



Final state non final state

The resulting FSM has finite number of fields (# number of non terminal variables) and finite set of symbols (alphabet). We have also finite set of productions (state transitions). So, we can create a DFA for this CFG.

* Since DFA's correspond to regular languages the right linear CFGs generate regular languages.

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Görkem YAK
27970

Question-1

b) Left-Linear:

if we have a left linear CFG in the form $A \rightarrow Ba \mid \epsilon$

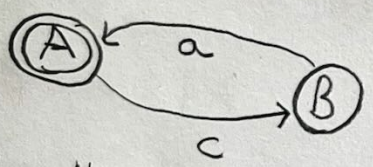
Again, we can convert it to a DFA.

DFA corresponding to this grammar would be the reverse of the right linear grammar correspond to it.

Ex:

CFG: $A \rightarrow Ba \mid \epsilon$

$B \rightarrow Ac$



* we reversed the transition ways.

The left linear transitions can be reversed as a right linear transition. Resulting finite state machine can be converted into a DFA which will generate regular languages.

* Left-Linear CFGs generate regular languages.

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Görkem YAR
27970 ~~Görkem~~

Problem-2

Exercise 7.1.3

G is a CFG

$G = (V, T, P, S)$

$V = \{S, A, B\}$

$T = \{0, 1\}$

P: $S \to 0A0 \mid 1B1 \mid BB$

$A \to C$

$B \to S \mid A$

$C \to S \mid \epsilon$

* Eliminate epsilon transitions.

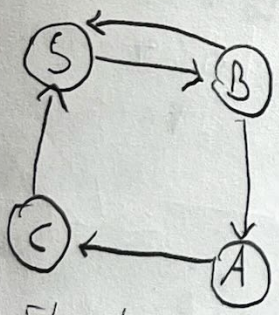
$S \to 0A0 \mid 1B1 \mid BB \mid 00 \mid 11 \mid B$

$A \to C$

$B \to S \mid A$

$C \to S$

* Eliminate unit transitions.



digraph for unit transitions

$S \to 0A0 \mid 1B1 \mid BB \mid 00 \mid 11$

$A \to 0A0 \mid 1B1 \mid BB \mid 00 \mid 11$

$B \to 0A0 \mid 1B1 \mid BB \mid 00 \mid 11$

$C \to 0A0 \mid 1B1 \mid BB \mid 00 \mid 11$

* Eliminate non-useful transitions.
* there is no non generating symbol.
* C is not reachable, eliminate C.

P: $S \to 0A0 \mid 1B1 \mid BB \mid 00 \mid 11$

$A \to 0A0 \mid 1B1 \mid BB \mid 00 \mid 11$

$B \to 0A0 \mid 1B1 \mid BB \mid 00 \mid 11$

* introduce new non terminal variables for CNF form

zero $\to 0$ one $\to 1$

P: $S \to$ zero A zero | one B one | BB | zero zero | one one

$A \to$ zero A zero | one B one | BB | zero zero | one one

$B \to$ zero A zero | one B one | BB | zero zero | one one

→ next page

Gokcan YAR
27970 ~~Gokcan~~

★ Introduce new variables for the productions size >2.

D → A zero
E → B one
zero → 0
one → 1

P: S → zero D | one E | BB | zero zero | one one
A → zero D | one E | BB | zero zero | one one
B → zero D | one E | BB | zero zero | one one

} CNF

Problem 2
Exercise 7.1.4:

G is a CFG, G = (V, T, P, S)
V = {S, A, B}, T = {0, 1}

P: S → AAA | B
A → aA | B
B → ∈

★ Eliminate epsilon transitons
S → AAA | AA | A | B
A → aA | a | B

★ Eliminate unit transitions. B is eliminated it is not generating
S → AAA | AA | aA | a
A → aA | a

★ The language is already useful.

★ To convert CNF, introduce new non terminal variables for CNF form
D → a
S → AAA | AA | DA | a
A → DA | a

★ Introduce new variables for the productions size >2.
C → AA
D → a
S → AC | AA | DA | a
A → DA | a

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27970

Problem-2

Exercise 7.2.1

b) Let n be the constant number such that if is equal to $2^{|M|+1}$

Pick $w = a^n b^n c^n$ that accepted by this language. By pumping Lemma $w = uvwxy$ where $|vwx| \leq n$ and $|vx| > 0$.

Pumping Lemma says that $uv^i w x^i y \in L$ for all $i \geq 0$

There are five cases for this example.

(1) $|vwx| = a^k$

$|vx| = p > 0$

if $i = 0$

$a^{n-p} b^n c^n$ does not hold. $p > 0$

$n - p \not\geq n$

not correct

(2) $|vwx| = b^k$

$|vx| = p > 0$

if $i = 0$

$a^n b^{n-p} c^n$ does not hold $p > 0$

$n - p \not\geq n$

not correct

(3) $|vwx| = c^k$

$|vx| = p > 0$

if $i = 2$

$a^n b^n c^{n+p}$ does not hold $p > 0$

$n \not\geq n+p$

not correct

(4) $|vwx| = a^t . b^k$

$|vx| = p > 0$

$l + m = p$

if $i = 0$

$a^{n-l} . b^{n-m} . c^n$

$n-l$ and $n-m < n$

so this is not accepted by language

(5)

$|vwx| = b^t c^k$

$|vx| = p > 0$

$l + m = p$

if $i = 2$

$a^n . b^{n+l} . c^{n+m}$

since $n+m > n$ this is not accepted by language.

★ By pumping lemma this is not CFL

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Gürkan YAR
27970 ~~Gürkan~~

Problem - 2

Exercise 7.2.1

c) Pumping length $n > 0$ pick a prime p such that $p \geq n$

$z = a^p \in L$

$z = uvwxy$ st

① $|vwx| \leq n$ ② $|vx| > 0$ ③ $uv^iwx^iy \in L$ for all $i \geq 0$

$|vx| = q$

$uv^iwx^iy = a^{p + (i-1)\cdot q}$

if we pick i as $p+1$ then

$a^{p + p \cdot q} = a^{p(q+1)} \in L$ should hold

but $p \cdot (q+1)$ is not prime

★ By Contradiction to PL L is not a CFL.

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Gürkan Yılmaz
27970

Problem. 2

Exercise 7.4.3

b) baaab

{S,C}				
{S,A,C}	{S,C}			
∅	{S,A,C}	{B}		
{S,A}	{B}	{B}	{S,C}	
{B}	{A,C}	{A,C}	{A,C}	{B}
b	a	a	a	b

$X_{11} = \{B\}$ $X_{12} = \{S, A\}$

$X_{22} = \{A, C\}$ $X_{23} = \{B\}$

$X_{33} = \{A, C\}$ $X_{34} = \{B\}$

$X_{44} = \{A, C\}$ $X_{45} = \{S, C\}$

$X_{55} = \{B\}$

$X_{13} = \emptyset$ $X_{14} = \{S, A, C\}$

$X_{24} = \{S, A, C\}$ $X_{25} = \{S, C\}$

$X_{35} = \{B\}$ $X_{15} = \{S, C\}$

Since S is part of the X_{15} baaab is in the language.

c) aabab

{S,C}				
{S,A,C}	{B}			
{B}	{B}	{S,C}		
{B}	{S,C}	{S,A}	{S,C}	
{A,C}	{A,C}	{B}	{A,C}	{B}

$X_{11} = \{A, C\}$ $X_{12} = \{B\}$

$X_{22} = \{A, C\}$ $X_{13} = \{B\}$

$X_{33} = \{B\}$ $X_{23} = \{S, C\}$

$X_{44} = \{A, C\}$ $X_{34} = \{S, A\}$ $X_{24} = \{B\}$

$X_{55} = \{B\}$ $X_{45} = \{S, C\}$ $X_{35} = \{S, C\}$

$X_{14} = \{S, A, C\}$

$X_{25} = \{B\}$ $X_{15} = \{S, C\}$

Since S is part of the X_{15} aabab is in the language.