Sabancı University, CS 302 AUTOMATA THEORY 17.1.2023

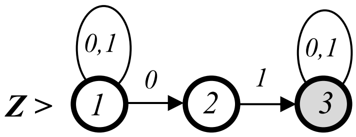
Answers to Final Examination

Answer 1 *(25 pts)*

(a) *(5 pts)*

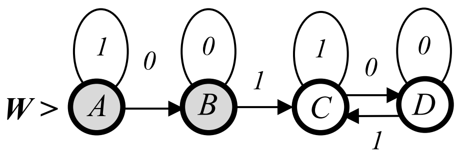
See the relevant slides.

(b) *(10 pts) We first construct the NFA* ***Z*** *that accepts the complement language* $\mathbf{L}^c$ *as shown below.*



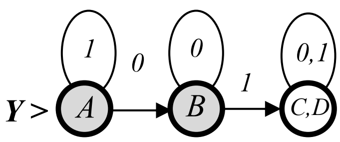
Using the table below we compute the DFA (hence NFA) ***W*** *that accepts* ***L***

state	*input*	*state'*
A:=1*	*0*	*B:=1,2*
*A**	*1*	*A*
*B**	*0*	*B*
*B**	*1*	*C:=1,3*
C	*0*	*D:=1,2,3*
C	*1*	*C*
D	*0*	*D*
D	*1*	*C*



(c) *(10 pts) Minimal state* ***Y*** *is computed using the table filling algorithm as below*

	C	*D*	*A*	*B*
C			*0*	*0*
D			*0*	*0*
A				*1*
B				



Answer 2 *(25 pts)*

(a) *(12 pts)*

$E = (0+1)^*. (0.0.0+0.0.1 + 0.1.0 + 1.0.0) .(0+1)^* + (1+0).(1+0) +1+0 + e$

(b) *(13 pts)*

The language L in question can be expressed as below :

$L = (w \in \{0,1\}^* \mid$ *if* $k := \#0\text{'s in } w$; $m := \#1\text{'s in } w$; *then* $|k-m| \leq 1$)

L is NOT a *regular* language as demonstrated by the pumping lemma (PL) as shown below :

Choose $w = 0^{n+1}1^n \in L$ where n is the PL constant then by the PL

$w = xyz$ *and* $|xy| \leq n$; $|y| = q > 0$; *and* $x y^j z \in L$ *for* $j=0, 1, 2, 3 \ldots$

Hence $xy = 0^p$, $p \leq n$ *and so for* $j=2$, $x y^2 z \in L$

But $x = 0^{p-q}$, $y = 0^q$ and $z = 0^{n+1-p} 1^n$ so $x y^2 z = 0^{p-q} 0^{2q} 0^{n+1-p} 1^n = 0^{q+n+1} 1^n \notin L$

since $| q+n+1 -n | = q+1 > 1$ and so PL is contradicted and L is not regular. and there is no regular expression E corresponding to L .

Answer 3 *(25 pts)*

(a) Given CFG : $S \rightarrow A \mid B$; $A \rightarrow aAb \mid C \mid e$; $C \rightarrow cCd \mid cC \mid e$; $B \rightarrow Bb$

convert into CNF

(i) Eliminate null productions

$S \rightarrow A \mid B$; $A \rightarrow aAb \mid C \mid ab$; $C \rightarrow cCd \mid cC \mid cd \mid c$; $B \rightarrow Bb$

(ii) Eliminate unit pairs

$S \rightarrow aAb \mid cCd \mid cC \mid cd \mid c \mid ab \mid Bb$; $A \rightarrow aAb \mid cCd \mid cC \mid cd \mid c \mid ab \mid$;

$C \rightarrow cCd \mid cC \mid cd \mid c$; $B \rightarrow Bb$

(iii) Eliminate the nongenerative variable B

$S \rightarrow aAb \mid cCd \mid cC \mid cd \mid c \mid ab$; $A \rightarrow aAb \mid cCd \mid cC \mid cd \mid c \mid ab \mid$;

$C \rightarrow cCd \mid cC \mid cd \mid c$

(iv) Replace terminals by nonterminals

$S \rightarrow XAY \mid UCV \mid UC \mid UV \mid c \mid XY$; $A \rightarrow XAY \mid UCV \mid UC \mid UV \mid c \mid XY$;

$C \rightarrow UCV \mid UC \mid UV \mid c$; $X \rightarrow a$; $Y \rightarrow b$; $U \rightarrow c$; $V \rightarrow d$;

,

(iv) Reduce triple nonterminals

$S \to WY \mid ZV \mid UC \mid UV \mid c \mid XY$; $A \to WY \mid ZV \mid UC \mid UV \mid c \mid XY$;

$C \to ZV \mid UC \mid UV \mid c$; $X \to a$; $Y \to b$; $U \to c$; $V \to d$;

$W \to XA$; $Z \to UC$

(b) $S \Rightarrow WY \Rightarrow XAY \Rightarrow aAY \Rightarrow aWYY \Rightarrow aXAYY \Rightarrow aaAYY \Rightarrow aaZVYY$

$\Rightarrow aaUCVYY \Rightarrow aacCVYY \Rightarrow aaccVYY \Rightarrow aaccdYY \Rightarrow aaccdbY \Rightarrow$

aaccdbb

(c) $L = \{ a^n c^m d^k b^n \mid n,m,k \geq 0 ,\ n+m+k > 0 ,\ m \geq k \}$

Answer 4 (25 pts)

(a) *(5 pts)* L_1 *is a CFL generated by* $G = (\{S,A,B,C\},\{a,b,c\}, R, S)$ *where* R *is given as below.*

$S \to AC$; $A \to aAb \mid B$; $B \to aB \mid a$; $C \to cC \mid c$

(b) *(10 pts)* L_2 *is not a CFL as proved by the PL below.*

Given $n > 0$ *choose* $z = a^{n+1}b^{n+1}c^n \in L_2$ *and* $|z| = 3n+2 > n$

Then by PL $z = uvwxy$, $|vwx| \leq n$, $|vx| = q > 0$ *and* $uv^jwx^jy \in L_2$ *for* $j=0,1,2,3,\ldots$

But (i) $vwx = a^k$ *or* $= b^k$ *or (ii)* $= c^k$ *where* $k \leq n$ *. If (i) prevails then for* $j=0$

$uwy = a^{n+1-q}\, b^{n+1}\, c^n$ *or* $= a^{n+1}\, b^{n+1-q}\, c^n$ *both not in* L_2 *since* $n+1-q \neq n+1$ *else if*

(ii) prevails then for $j=3$ *namely,* $uv^3wx^3y = a^{n+1}\, b^{n+1}\, c^{n+2q}$ *not in* L_2 *since* $n+2q > n+1$.

Other two possibilities are (iii) $vwx = a^i b^j$ *or* $= b^i c^j$ *with* $i+j \leq n$ *which again leads to conclusion that either* uwy *is not in* L_2 *when* $vwx = a^i\, b^j$ *; or when* $vwx = b^i c^j$ *with* $i,j > 0$ *;* uwy *is not in* L_2 *. Hence PL is contradicted and* L_2 *is not a CFL .*

(c) *(10 pts)*

Label	*Condition*	*TM*
$M >$	-	$R_{\{a, b ,c,\#\}}$. A
A	$\sigma = a$	x .$R_{\{b ,c,\#\}}$. B
	else	h_{NO}
B	$\sigma = b$	x . $R_{\{a,c,\#\}}$. C
	else	h_{NO}
C	$\sigma = c$	x . $R_{\{a,b, \#\}}$ .D
	else	h_{NO}
D	$\sigma = \#$	$L_{\#}$. E
	else	h_{NO}
E	$\sigma = a$	x .$R_{\{b,c,\#\}}$ .F
	$\sigma = \#$	h_{YES}
	else	h_{NO}
F	$\sigma = b$	x . $R_{\{a,c, \#\}}$. G
	else	h_{NO}
G	$\sigma = c$	x . $R_{\{a,b, \#\}}$ .H
	else	h_{NO}
H	$\sigma = \#$	$L_{\#}$ .E
	else	h_{NO}