SABANCI UNIVERSITY, CS 302 Automata Theory, Spring 2023

Midterm Examination

QUESTION 1 *(50 pts)*

Name :

Surname :

Closed book and notes (of paper and electronic kind);
Calculators are not allowed and all phones must be switched off;
Duration: 60 minutes

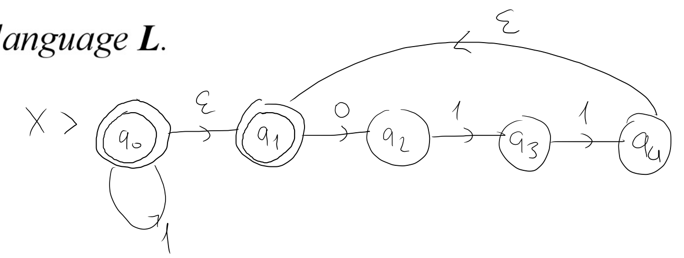
Consider the language $\mathbf{L} \subseteq \{0,1\}^*$ *where in each string of* **L** *every* **0** *is followed* ***precisely*** *by two* **1**'s.

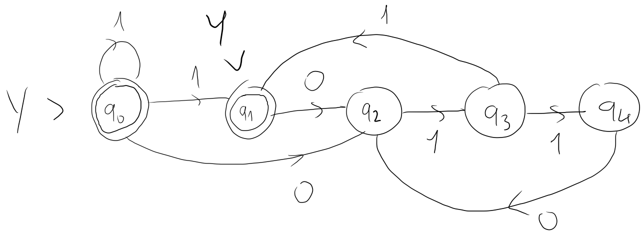
(a) *(15 pts) Write down a* ***regular expression E*** *corresponding to this language* **L**.

$1^* . (011)^*$

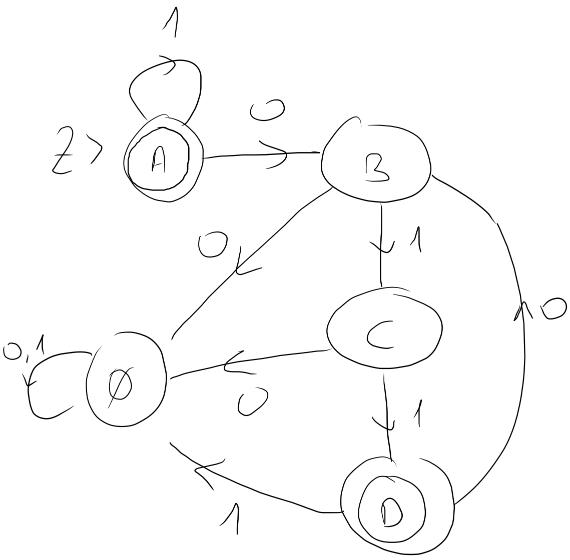
(b) *(35 pts) Sketch (i) an epsilon-NFA* **X**; *(ii) an NFA* **Y** *(without epsilon-transitions) ; (iii) a DFA* **Z** *and (iv) a* ***minimal state*** *DFA* **W** *that all accept the language* **L**.

b)



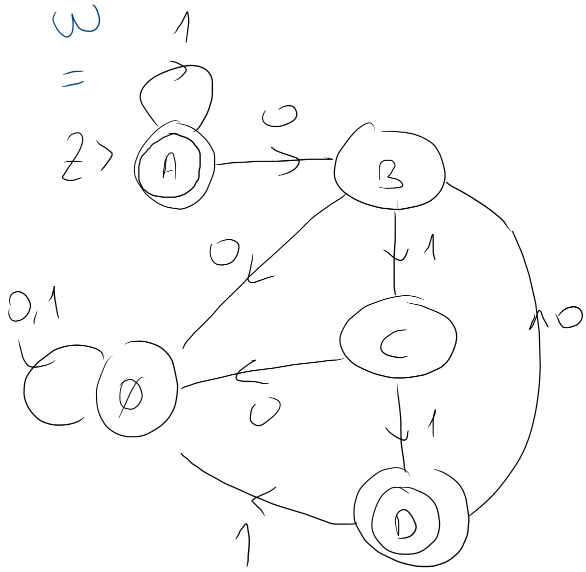


state	input	next state
> * q_0	0	q_2
	1	q_0, q_1
* q_1	0	q_2
	1	∅
q_2	0	∅
	1	q_3
q_3	0	∅
	1	q_1, q_4
q_4	0	q_2
	1	∅



q_1, q_4 *D	0	q_2 B
	1	∅
∅	0	∅
	1	∅

state	input	next state
> * q_0, q_1 A	0	q_2 B
	1	q_0, q_1 A
q_2 B	0	∅
	1	q_3 C
q_3 C	0	∅
	1	q_1, q_4 D



	B	C	Ø	A	D
B		1	2	0	0
C			1	0	0
Ø				0	0
A					1
D					

Z = W is already minimal.

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***QUESTION 2** (50 pts)*

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***(a)** (10 pts) For a **non-deterministic** finite automaton (NFA) **A** state the definition of the language **L(A)** accepted by **A** in terms of its **extended transition function** δE*

$L(A) = \{ w \in \Sigma^* \mid \delta E(Q_0, w) \cap F \neq \emptyset \}$ or $s \in L(A) \Leftrightarrow \delta E(Q_0, s) \cap F \neq \emptyset$

***(b)** (15 pts) State the **pumping lemma** for regular languages.*

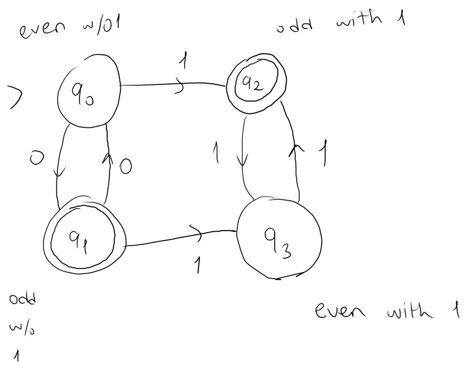
***(c)** (25 pts) Consider the languages L_1 and L_2 below:*

$L_1 = (\omega \in \{0,1\}^* | \omega = 0^n 1^m$; $n+m$ = *an* ***odd*** *number* ; n,m *nonnegative integers*)

$L_2 = (\omega \in \{0,1\}^* | \omega = 0^n 1^m$; $n > 3m$; n,m *nonnegative integers*)

*For each case **state** whether the language is a **regular** or an **irregular context-free language**. If it is regular exhibit an accepting NFA (or a regular expression), if it is not then exhibit a CFG that generates it.*

b) pumping length $n > 0$ s.t. $\forall w \in L$, $|w| \geq n$ and there is a decomposition $w = xyz$ where $|xy| \leq n$, $|y| > 0$ and $xy^iz \in L$ for all $i = 0, 1, 2, \ldots$



L_1 is regular

L_2: pumping length $n > 0$, $w = 0^{3n+1} 1^n \in L$, $|w| = 4n+1 \geq n$

$w = xyz$, $|xy| \leq n$, $|y| > 0$

$xy = 0^p$, $y = 0^q$, $z = 0^{3n+1-p} . 1^n$, $p \leq n$, $q > 0$ $(q \geq 1)$

for $i = 0$, $xy^iz = xz = 0^{3n+1-q} . 1^n \in L_2$, contradiction to PL,

L_2 is not regular.

$n > m$

$-q \leq -1 \Rightarrow 3n+1-q \leq 3n$