CS-302
HW-5

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Question-1:

a) The language that accepts a's and b's where all of the a's comes before b's and number of a's is greater than number of b's.

$L(m) = \{ a^m b^n \mid m > n \}$

b)

aab

$\delta(q_0, aab, Z_0) \rightarrow (q, Z_0)$
$\Rightarrow \delta(q, ab, Z_0) \rightarrow (q, aZ_0)$
$\Rightarrow \delta(q, b, a) \rightarrow (q, e)$

Accepted

abb

$\delta(q_0, abb, Z_0) \rightarrow (q, Z_0)$
$\delta(q, bb, Z_0) = ?\ (q, e)$

not accepted by the language no rule to further continue

aba

$\delta(q_0, aba, Z_0) \rightarrow (q, Z_0)$
$\delta(q, ba, Z_0) = ?\ (q, e)$

not accepted by the language no rule to further continue

c) aaabb

$\delta(q_0, a, Z_0) \rightarrow (q, Z_0)$
$\delta(q, a, Z_0) \rightarrow (q, aZ_0)$
$\delta(q, a, a) \rightarrow (q, aa)$
$\delta(q, b, a) \rightarrow (q, e)$
$\delta(q, b, a) \rightarrow (q, e)$

L(P) Acceptance

aaab

$\delta(q_0, a, Z_0) \rightarrow (q, Z_0)$
$\delta(q, a, Z_0) \rightarrow (q, aZ_0)$
$\delta(q, a, a) \rightarrow (q, a, a)$
$\delta(q, b, a) \rightarrow (q, e)$

L(P) Acceptance

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Question - 2

a) $P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$, P is a PDA

$Q = \{q_0, q_1\}$, $\Sigma = \{a, b\}$, $\Gamma = \{a, Z_0\}$, $q_0 = q_0$, $Z_0 = Z_0$, $F = \{\}$

$\delta(q_0, a, Z_0) \rightarrow (q_0, aZ_0)$

$\delta(q_0, a, a) \rightarrow (q_0, aa)$

$\delta(q_0, b, Z_0) \rightarrow (q_1, Z_0)$

$\delta(q_0, b, a) \rightarrow (q_1, e)$

$\delta(q_1, b, a) \rightarrow (q_1, e)$

$\delta(q_1, b, Z_0) \rightarrow (q_1, Z_0)$

$\delta(q_1, e, Z_0) \rightarrow (q_1, e) \longrightarrow N(P)$ Acceptance

$\delta(q_0, e, Z_0) \rightarrow (q_1, Z_0)$

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b) $P = (Q, \Sigma, \Gamma, q_0, Z_0, F)$, P is a PDA

$Q = \{q_0, q_1, q_2\}$, $\Sigma = \{a, b, c\}$, $\Gamma = \{a, Z_0\}$, $q_0 = q_0$, $Z_0 = Z_0$, $F = \{\}$

$\delta(q_0, a, Z_0) \rightarrow (q_0, aZ_0)$; $\delta(q_0, e, Z_0) \rightarrow (q_0, e) \rightarrow N(P)$ Acceptance

$\delta(q_0, a, a) \rightarrow (q_0, aa)$

$\delta(q_0, c, Z_0) \rightarrow (q_1, Z_0)$

$\delta(q_0, c, a) \rightarrow (q_1, a)$

$\delta(q_0, b, a) \rightarrow (q_2, e)$

$\delta(q_1, c, Z_0) \rightarrow (q_1, Z_0)$

$\delta(q_1, c, a) \rightarrow (q_1, a)$ $\qquad \delta(q_1, e, Z_0) \rightarrow (q_1, e) \rightarrow N(P)$ Acceptance

$\delta(q_1, b, a) \rightarrow (q_2, e)$

$\delta(q_2, b, a) \rightarrow (q_2, e)$

$\delta(q_2, e, Z_0) \rightarrow (q_2, e) \rightarrow N(P)$ Acceptance

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Question-2

d) To solve this, I will form a CFG and convert it to a PDA

G is a CFG $\rightarrow$ $G = (V, T, P, S)$

$V = \{S, A, B\}$

$T = \{a, b, c\}$ $\quad S = \{S\}$

$P: S \rightarrow AB \mid e$

$P: A \rightarrow aAb \mid ab$

$P: B \rightarrow bBc \mid bc$

P is the equivalent PDA that accepts G, $P = \{Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q_0, q, f\}$, $\Sigma = T$, $\Gamma = V \cup T \cup \{Z_0\}$, $F = \{f\}$

$\delta(q_0, e, Z_0) \rightarrow (q, SZ_0)$

$\delta(q, e, S) \rightarrow (q, AB)$

$\delta(q, e, S) \rightarrow (q, e)$

$\delta(q, e, A) \rightarrow (q, aAb)$

$\delta(q, e, A) \rightarrow (q, ab)$

$\delta(q, e, B) \rightarrow (q, bBc)$

$\delta(q, e, B) \rightarrow (q, bc)$

$\delta(q, a, a) \rightarrow (q, e)$

$\delta(q, b, b) \rightarrow (q, e)$

$\delta(q, c, c) \rightarrow (q, e)$

$\delta(q, e, Z_0) \rightarrow (f, Z_0)$ $\rightarrow$ L(P) Acceptance

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Question-2

e) For this question, I will construct a CFG, then convert it to a PDA. G is a CFG and P is the equivalent PDA for G.

$G = (V, T, P, S)$

$V = \{S\}$, $T = \{a, b\}$, $S = \{S\}$

$P: S \rightarrow aSb \mid aSbb \mid e$

$P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q_0, q_1, f\}$, $\Sigma = T$, $\Gamma = V \cup T \cup \{Z_0\}$, $F = \{f\}$, $q_0 = q_0$, $Z_0 = Z_0$

$\delta(q_0, e, Z_0) \rightarrow (q, SZ_0)$

$\delta(q, e, S) \rightarrow (q, aSb)$

$\delta(q, e, S) \rightarrow (q, aSbb)$

$\delta(q, e, S) \rightarrow (q, e)$

$\delta(q, a, a) \rightarrow (q, e)$

$\delta(q, b, b) \rightarrow (q, e)$

$\delta(q, e, Z_0) \rightarrow (f, Z_0) \longrightarrow L(P)$ Acceptance

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Question-2

f) To solve this, I will form a CFG and convert it to a PDA. G is a CFG.

$G = (V, T, P, S)$

$V = \{S, A\}$, $T = \{a, b, c\}$, $S = \{S\}$

$S \to aSc \mid aAc$

$A \to aAb \mid ab$

P is the equivalent PDA for G. $P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$.

$Q = (q_0, q, f)$, $\Sigma = T$, $\Gamma = V \cup T \cup \{z_0\}$, $q_0 = q_0$, $z_0 = z_0$, $F = \{f\}$

$\delta(q_0, e, z_0) \to (q, Sz_0)$

$\delta(q, e, S) \to (q, aSc)$

$\delta(q, e, S) \to (q, aAc)$

$\delta(q, e, A) \to (q, aAb)$

$\delta(q, e, A) \to (q, ab)$

$\delta(q, a, a) \to (q, e)$

$\delta(q, b, b) \to (q, e)$

$\delta(q, c, c) \to (q, e)$

$\delta(q, e, z_0) \to (f, z_0) \to L(P)$ Acceptance

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Question-3:

a) $P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q_0, f\}$, $\Sigma = \{a, b\}$, $\Gamma = \{a, Z_0\}$, $q_0 = q_0$, $Z_0 = Z_0$, $F = \{f\}$

$\delta(q_0, a, Z_0) \to (q_0, aZ_0)$

$\delta(q_0, b, Z_0) \to (f, Z_0) \to$ L(P) Acceptance ; $\delta(f, a, Z_0) \to (f, Z_0)$

$\delta(q_0, a, a) \to (q_0, aa)$; $\delta(f, b, Z_0) \to (f, Z_0)$

$\delta(q_0, b, a) \to (q_0, e)$

b) $P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F\}$

$Q = \{q_0, q\}$, $\Sigma = \{a, b\}$, $\Gamma = \{a, Z_0\}$, $q_0 = q_0$, $Z_0 = Z_0$, $F = \{\}$

$\delta(q_0, a, Z_0) \to (q_0, aZ_0)$; $\delta(q, a, Z_0) \to (q, Z_0)$

$\delta(q_0, b, Z_0) \to (q, Z_0)$; $\delta(q, b, Z_0) \to (q, Z_0)$

$\delta(q_0, a, a) \to (q_0, aa)$; $\delta(q, e, Z_0) \to (q, e) \to$ N(P) Acceptance

$\delta(q_0, b, a) \to (q_0, e)$

Question-4) Exercise 6.2.6

a) Currently, the PDA in the 6.1.1 accepts into L(P) sense.
I will convert it to $N(P_1) = L(P)$.

$P_1 = (\{q, p, \underset{\text{new state}}{t}\}, \{0, 1\}, \{Z_0, X\}, \delta, q, Z_0, \{\})$

$\delta(q, 0, Z_0) \to (q, XZ_0)$
$\delta(q, 0, X) \to (q, XX)$
$\delta(q, 1, X) \to (q, X)$
$\delta(q, \epsilon, X) \to (p, \epsilon)$
$\delta(p, \epsilon, X) \to (p, \epsilon)$
$\delta(p, 1, X) \to (p, XX)$
$\delta(p, 1, Z_0) \to (p, \epsilon)$

old transitions

★ We actually do not need the last for transitions. P have already a N(P) acceptance with this transition.

$\delta(p, e, X) \to (t, X)$
$\delta(p, e, Z_0) \to (t, Z_0)$

transfer to the new state

$\delta(t, e, X) \to (t, e)$ — pop until it is empty

$\delta(t, e, Z_0) \to (t, e)$ — Accept $N(P_1)$ sense

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Question 4)

Exercise. 6.2.6:

b) The PDA P also have $N(P)$ acceptance. Now, I will convert it to $L(P_2) = N(P)$

$P_2 = (\{q, q_0, P\}, \{0, 1\}, \{Z_0, x\}, \delta, q, Z_0, \{P\})$

q_0 → new state

$\delta(q, e, Z_0) \rightarrow (q_0, Z_{00} Z_0)$ → new Z_{00} transition

$\delta(q_0, 0, Z_{00}) \rightarrow (q_0, x Z_{00})$

$\delta(q_0, 0, x) \rightarrow (q_0, xx)$

$\delta(q_0, 1, x) \rightarrow (q_0, x)$

$\delta(q_0, \epsilon, x) \rightarrow (P, \epsilon)$

$\delta(P, \epsilon, x) \rightarrow (P, \epsilon)$

$\delta(P, 1, x) \rightarrow (P, xx)$

$\delta(P, 1, Z_{00}) \rightarrow (P, Z_0)$

we do not need to add a transition from q_0 to final states since P is already a final state

$\delta(P, e, Z_0) \rightarrow (P, Z_0)$ → empty transition to final state

Exercise : 6.3.2

P is a PDA

$P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q_0, q, f\}$, $\Sigma = \{a, b\}$, $\Gamma = \{A, S, a, b, Z_0\}$, $q_0 = q_0$, $Z_0 = Z_0$, $F = \{f\}$

$\delta(q_0, e, Z_0) \rightarrow (q, S Z_0)$

$\delta(q, e, S) \rightarrow (q, aAA)$

$\delta(q, e, A) \rightarrow (q, aS)$

$\delta(q, e, A) \rightarrow (q, bS)$

$\delta(q, e, A) \rightarrow (q, a)$

$\delta(q, a, a) \rightarrow (q, e)$

$\delta(q, b, b) \rightarrow (q, e)$

$\delta(q, e, Z_0) \rightarrow (f, Z_0)$ → $L(P)$ Acceptance

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Question-4:

Exercise 6.3.4:

PDA $P = (\{q, p\}, \{0,1\}, \{Z_0, X\}, \delta, q, Z_0, \{p\})$

Convert it to CFG.

CFG $G = (V, T, P, S)$

$T = \{0,1\}$, $V = \{[qZ_0q], [qZ_0p], [pZ_0q], [pZ_0p], [qXq], [qXp], [pXq], [pXp], S\}$

$S = \{[qZ_0q], [qZ_0p]\}$

1. $\delta(q, 0, Z_0) \to (q, XZ_0)$

$[qZ_0q] \to 0[qXq][qZ_0q] \mid 0[qXp][pZ_0q]$

$[qZ_0p] \to 0[qXq][qZ_0p] \mid 0[qXp][pZ_0p]$

2. $\delta(q, 0, X) \to (q, XX)$

$[qXq] \to 0[qXq][qXq] \mid 0[qXp][pXq]$

$[qXp] \to 0[qXq][qXp] \mid 0[qXp][pXp]$

3. $\delta(q, 1, X) \to (q, X)$

$[qXq] \to 1[qXq]$

$[qXp] \to 1[qXp]$

4. $\delta(q, \epsilon, X) \to (p, \epsilon)$

$[qXq] \to e$

$[qXp] \to e$

5. $\delta(p, \epsilon, X) \to (p, \epsilon)$

$[pXq] \to e$

$[pXp] \to e$

6. $\delta(p, 1, X) \to (p, XX)$

$[pXq] \to 1[pXq][qXq] \mid 1[pXp][pXq]$

$[pXp] \to 1[pXp][pXp] \mid 1[pXq][qXp]$

7. $\delta(p, 1, Z_0) \to (p, \epsilon)$

$[pZ_0p] \to 1$

$[pZ_0q] \to 1$