

Student names: ... (please update)

Instructions: Update this file (or recreate a similar one, e.g. in Word) to prepare your answers to the questions. Feel free to add text, equations and figures as needed. Hand-written notes, e.g. for the development of equations, can also be included e.g. as pictures (from your cell phone or from a scanner). **This lab is graded.** and must be submitted before the **Deadline : 22-04-2020 23:59**. Please submit both the source file (*.doc/*.tex) and a pdf of your document, as well as all the used and updated Python functions in a single zipped file called **lab6_name1_name2_name3.zip** where name# are the team member's last names. **Please submit only one report per team!**

The file **lab#.py** is provided to run all exercises in Python. The list of exercises and their dependencies are shown in Figure 1. When a file is run, message logs will be printed to indicate information such as what is currently being run and what is left to be implemented. All warning messages are only present to guide you in the implementation, and can be deleted whenever the corresponding code has been implemented correctly.

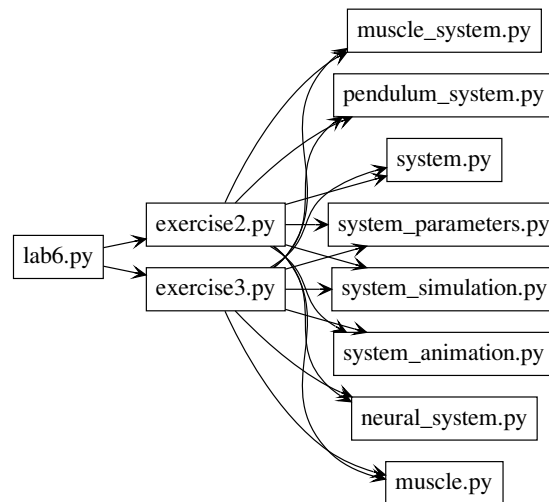


Figure 1: Exercise files dependencies. In this lab, you will be modifying **exercise2.py**, **exercise3.py** and optionally **pendulum_system.py**

Files to complete the exercises

- **lab6.py** : Main file
- **exercise2.py** : Main file to complete exercise 2
- **exercise3.py** : Main file to complete exercise 3
- **system_parameters.py** : Parameter class for Pendulum, Muscles and Neural Network (Create an instance and change properties using the instance. You do not have to modify the file)
- **muscle.py** : Muscle class (You do not have to modify the file)
- **system.py** : System class to combine different models like Pendulum, Muscles, Neural Network (You do not have to modify the file)
- **pendulum_system.py** : Contains the description of pendulum equation and Pendulum class. You can use the file to define perturbations in the pendulum.

- **muscle_system.py** : Class to combine two muscles (You do not have to modify the file)
- **neural_system.py** : Class to describe the neural network (You do not have to modify the file)
- **system_simulation.py** : Class to initialize all the systems, validate and to perform integration (You do not have to modify the file)
- **system_animation.py** : Class to produce animation of the systems after integration (You do not have to modify the file)

NOTE : 'You do not have to modify' does not mean you should not, it means it is not necessary to complete the exercises. But, **you are expected to look into each of these files and understand how everything works**. You are free to explore and change any file if you feel so.

Exercise 2 : Pendulum model with Muscles

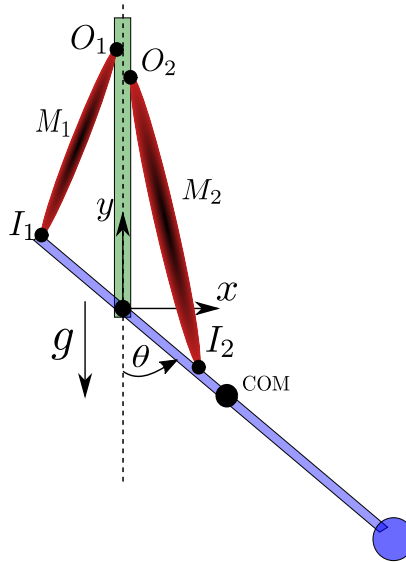


Figure 2: Pendulum with Antagonist Hill Muscles

The system is comprised of a physical pendulum described by equation 1 and a pair of antagonist muscles **M1** and **M2**. Muscle **M1** extends the pendulum (θ increases) and Muscle **M2** flexes the muscle (θ decreases).

Consider the system only for the pendulum range $\theta = [0, \pi]$

$$I\ddot{\theta} = -0.5 \cdot m \cdot g \cdot L \cdot \sin(\theta) \quad (1)$$

Where,

- I - Pendulum inertia about the pendulum pivot joint [$kg \cdot m^2$]
- θ - Pendulum angular position with the vertical [rad]
- $\ddot{\theta}$ - Pendulum angular acceleration [$rad \cdot s^{-2}$]
- m - Pendulum mass [kg]
- g - System gravity [$m \cdot s^{-2}$]
- L - Length of the pendulum [m]

Each muscle is modelled using the Hill-type equations that you are now familiar with. Muscles have two attachment points, one at the origin and the other at the insertion point. The origin points are denoted by $O_{1,2}$ and the insertion points by $I_{1,2}$. The two points of attachment dictate how the length of the muscle changes with respect to the change in position of the pendulum.

The active and passive forces produced by the muscle are transmitted to the pendulum via the tendons. In order to apply this force on to the pendulum, we need to compute the moment based on the attachments of the muscle.

Using the laws of sines and cosines, we can derive the length of muscle and moment arm as below. The reference to the paper can be found here [Reference](#),

$$L_2 = \sqrt{a_1^2 + a_2^2 + 2 \cdot a_1 \cdot a_2 \cdot \cos(\theta)} \quad (2)$$

$$h_2 = \frac{a_1 \cdot a_2 \cdot \sin(\theta)}{L_2} \quad (3)$$

Where,

- L_2 : Length of muscle 2
- a_1 : Distance between muscle 2 origin and pendulum origin ($|O_2C|$)
- a_2 : Distance between muscle 2 insertion and pendulum origin ($|I_2C|$)
- h_2 : Moment arm of the muscle

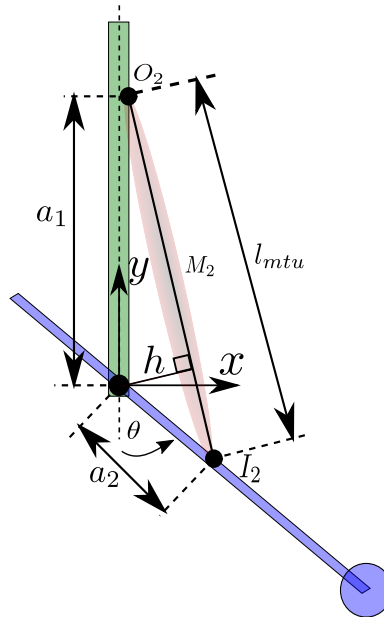


Figure 3: Computation of muscle length and moment arm

Equation 2 can be extended to the Muscle 2 in similar way. Thus, the final torque applied by the muscle on to the pendulum is given by,

$$\tau = F \cdot h \quad (4)$$

NOTE : τ can also be computed as the cross product of the force vector and radial vector. This method is used in the code to compute the torques and there by bypassing the moment arm computation.

Where,

- τ : Torque [$N \cdot m$]
- F : Muscle Tendon Force [N]
- h : Muscle Moment Arm [m]

In this exercise, the following states of the system are integrated over time,

$$X = \begin{bmatrix} \theta & \dot{\theta} & A_1 & l_{CE1} & A_2 & l_{CE2} \end{bmatrix} \quad (5)$$

Where,

- θ : Angular position of the pendulum [rad]
- $\dot{\theta}$: Angular velocity of the pendulum [rad/s]
- A_1 : Activation of muscle 1 with a range between [0.05, 1]. 0.05 corresponds to base line stimulation and 1 corresponds to maximal stimulation.
- l_{CE1} : Length of contractile element of muscle 1
- A_2 : Activation of muscle 2 with a range between [0.05, 1]. 0.05 corresponds to base line stimulation and 1 corresponds to maximal stimulation.
- l_{CE2} : Length of contractile element of muscle 2

To complete this exercise you will make use of the following files, `exercise2.py`, `system_parameters.py`, `muscle.py`, `system.py`, `pendulum_system.py`, `muscle_system.py`, `system_simulation.py`

2a. For the given default set of attachment points, compute and plot the muscle length and moment arm as a function of θ between $[\pi/4, 3\pi/4]$ using equations in **eqn:2** and discuss how it influences the pendulum resting position and the torques muscles can apply at different joint angles. You are free to implement this code by yourself as it does not have any other dependencies.

2b. Using simple activation wave forms (example : sine or square waves) applied to muscles (use `system_simulation.py::add_muscle_stimulations` method in **exercise2.py**), try to obtain a limit cycle behavior for the pendulum. Use relevant plots to prove the limit cycle behavior. Explain and show the activations wave forms you used. Use `pendulum_system.py::PendulumSystem::pendulum_system` function to perturb the model.

2c. Explore the relationship between stimulation frequency with the resulting pendulum's behavior. Report your inferences for a low and high frequency condition.

Exercise 3 : Neural network driven pendulum model with muscles

In this exercise, the goal is to drive the above system (Fig 2) with a symmetric four-neuron oscillator network. The network is based on Brown's half-center model with fatigue mechanism. Here we use the leaky-integrate and fire neurons for modelling the network. Figure 4 shows the network structure and the complete system.

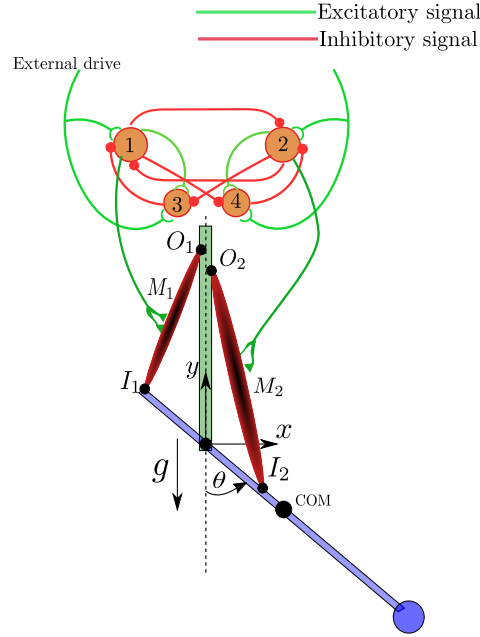


Figure 4: Pendulum with Antagonist Hill Muscles Driven Half Center Neural Network.

Since each leaky-integrate and fire neuron comprises of one first order differential equation, the states to be integrated now increases by four(one state per neuron). The states are,

$$X = [\theta \quad \dot{\theta} \quad A_1 \quad l_{CE1} \quad A_2 \quad l_{CE2} \quad m_1 \quad m_2 \quad m_3 \quad m_4] \quad (6)$$

Where,

- m_1 : Membrane potential of neuron 1
- m_2 : Membrane potential of neuron 2
- m_3 : Membrane potential of neuron 3
- m_4 : Membrane potential of neuron 4

To complete this exercise, additionally you will have to use `neural_system.py` and `exercise3.py`

3a. Find a set of weights and time constants for the neural network that produces oscillations to drive the pendulum into a limit cycle behavior. Plot the output of the network and the phase plot of the pendulum

3b. As seen in the course, apply an external drive to the individual neurons and explain how the system is affected. Show plots for low [0] and high [1] external drives. To add external drive to the network you can use the method

`system_simulation.py::add_external_inputs_to_network`

3c. [Open Question] What are the limitations of the half center model in producing alternating patterns to control the pendulum? What would be the effect of sensory feedback on this model? (No plots required)

Appendix I : Hill muscle model equations

Name	Equation	Comment
F_{mtu}	$F_m = F_{se} = F_{ce} + F_{pe} - F_{be}$	Force of generated by the MTU
F_{se}	$F_{se} = \begin{cases} F_{max} \cdot (\frac{\varepsilon}{\varepsilon_{ref}})^2, & \text{if } \varepsilon > 0 \\ 0, & \text{otherwise} \end{cases}$	Force generated by the tendon
F_{pe}	$F_{pe} = \begin{cases} F_{max} \cdot (\frac{l_{ce}-l_{opt}}{l_{opt}w})^2 \cdot f_v(v_{ce}), & \text{if } l_{ce} > l_{opt} \\ 0, & \text{otherwise} \end{cases}$	Force of the parallel element preventing muscle overextension
F_{be}	$F_{be} = \begin{cases} F_{max} \cdot (\frac{l_{opt}-w-l_{ce}}{l_{opt}w/2})^2, & \text{if } l_{ce} \leq l_{opt} \cdot (1 - w) \\ 0, & \text{otherwise} \end{cases}$	Force of the parallel element preventing muscle to collapse
F_{ce}	$F_{ce} = A \cdot F_{max} \cdot f_l(l_{ce}) \cdot f_v(v_{ce})$	Force generated by the muscle (ce)
A	$\frac{dA}{dt} = \tau \cdot (S(t) - A)$	Muscle activation is equal to the integral of the input signal I(t). Biologically, the input signal can be viewed as the normalized frequency of neuronal spike that reaches the muscle (i.e. restricted to the [0.05; 1] interval). The activation is thus also limited to this interval.
$f_l(l_{ce})$	$f_l(l_{ce}) = \exp(c \frac{l_{ce}-l_{opt}}{l_{opt}w} ^3)$	Muscle force - muscle length relationship function (function of the muscle length (lce))
$f_v(v_{ce})$	$f_v(v_{ce}) = \begin{cases} \frac{v_{max}-v_{ce}}{v_{max}+k \cdot v_{max}}, & \text{if } v_{ce} < 0 \\ N + (N - 1) \frac{v_{max}-v_{ce}}{7.56 \cdot K \cdot v_{ce}-v_{max}}, & \text{otherwise} \end{cases}$	Muscle force - muscle velocity relationship function (function of the muscle velocity (vce))
$v_{ce}(f_v)$	$v_{ce}(f_v) = \begin{cases} v_{max} \cdot \frac{1-f_v}{1+f_v \cdot K}, & \text{if } f_v < 1.0 \\ v_{max} \cdot \frac{f_v-1}{7.56 \cdot K \cdot (f_v-N)+1-N}, & \text{otherwise} \end{cases}$	Inverse of the muscle force - velocity function. This function is used for the resolution of the muscles equations.
l_{ce}	$\int v_{ce} \cdot dt$	Muscle (ce) length
l_{se}	$l_{se} = l_{mtu} - l_{ce}$	Tendon (se) length
l_{mtu}	$l_{mtu} = l_{opt} + l_{slack} + \delta l_{mtu}$	MTU length