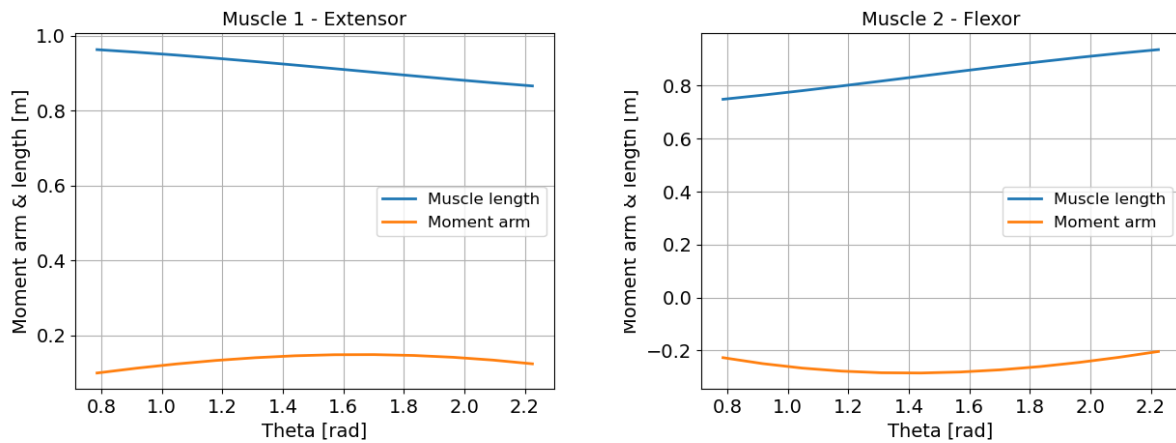


Student names: Jon Märki, Jérôme Savary, Gabriel Tornare

Exercise 2 : Pendulum model with Muscles

2a. For the given default set of attachment points, compute and plot the muscle length and moment arm as a function of θ between $[\pi/4, 3\pi/4]$ using previous equations and discuss how it influences the pendulum resting position and the torques muscles can apply at different joint angles. You are free to implement this code by yourself as it does not have any other dependencies.

The result of the measures based on the given equations can be observed in figure 1. By changing the angle of the arm, we also change the lengths of both muscles, therefore the force they can apply, and finally therefore the torque, which also varies with the moment arm. The resting position == the resting angle is the one at which both muscles, given their respective lengths and stimulations, apply the same opposed torques = no resulting torque.



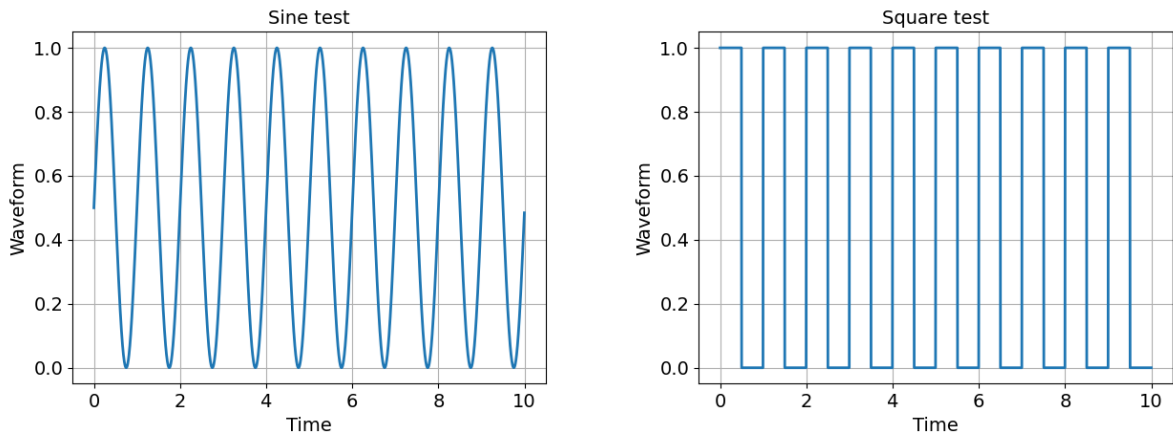
(a) Result of experiment 2a on muscle 1.

(b) Result of experiment on muscle 2.

Figure 1: Result of experiment 2a.

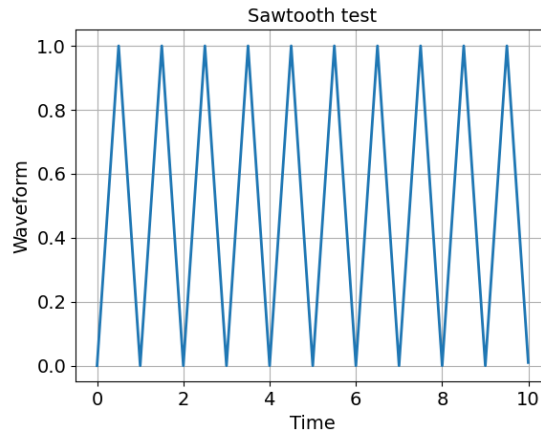
2b. Using simple activation wave forms (example : sine or square waves) applied to muscles (use `system_simulation.py::add_muscle_stimulations` method in `exercise2.py`), try to obtain a limit cycle behavior for the pendulum. Use relevant plots to prove the limit cycle behavior. Explain and show the activations wave forms you used. Use `pendulum_system.py::PendulumSystem::pendulum_system` function to perturb the model.

For this part, we inputted the three different signals at two different frequencies (1 and 10 [Hz]). Our 1[Hz] signals can be visualised on Figure 2.



(a) $1[\text{Hz}]$ Sine-shaped stimulation input signal.

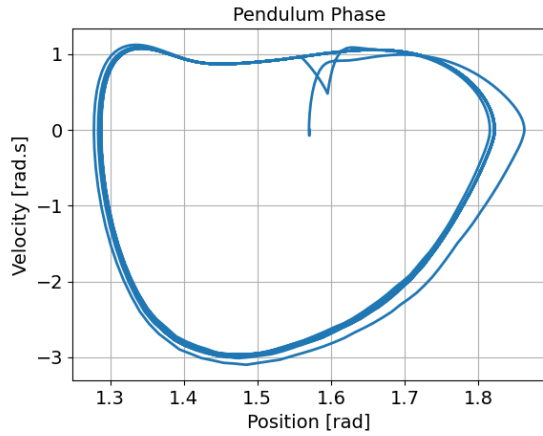
(b) $1[\text{Hz}]$ Square-shaped stimulation input signal.



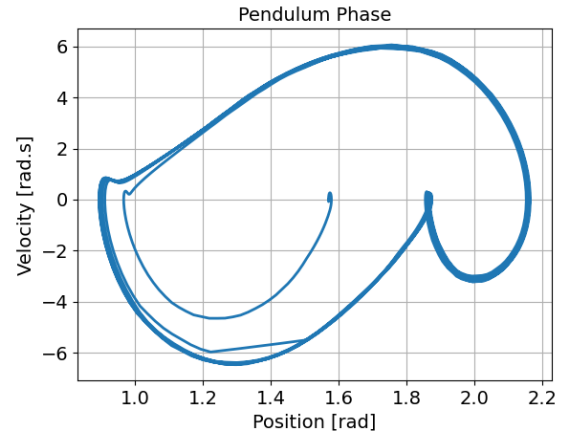
(c) $1[\text{Hz}]$ Sawtooth-shaped stimulation input signal.

Figure 2: $1[\text{Hz}]$ signals for exercise 2b.

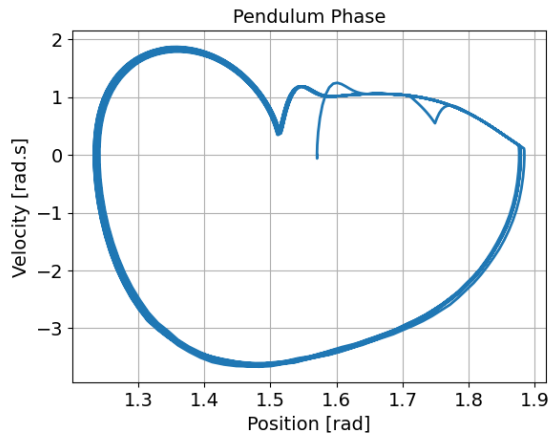
The result of the measures based on the given equations can be observed in figure 3 on page 3. We can observe from these measures that the pendulum reaches a limit circle which is about in the same order of magnitude for the three signals, and the measurements from the sine and sawtooth-shaped signals share more similarities than the ones from the square-shaped one (logically, since they are more similar). Each stimulation signal has two phase pendulum phases measured: one when the stimulation signals for the flexor and extensor muscles are in phase (left), the other when they are in counterphase (right)



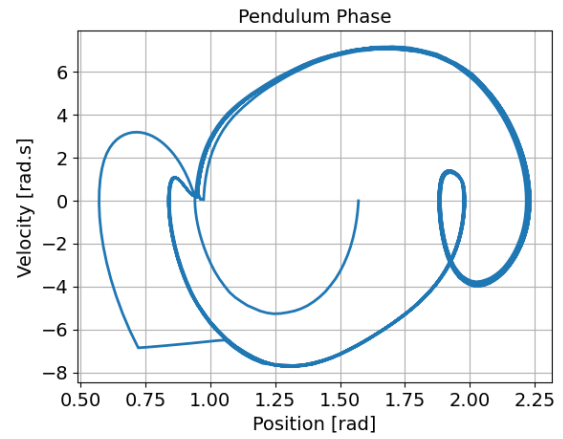
(a) Phase diagram of the pendulum with the 1[Hz] phased square-shaped stimulation signal.



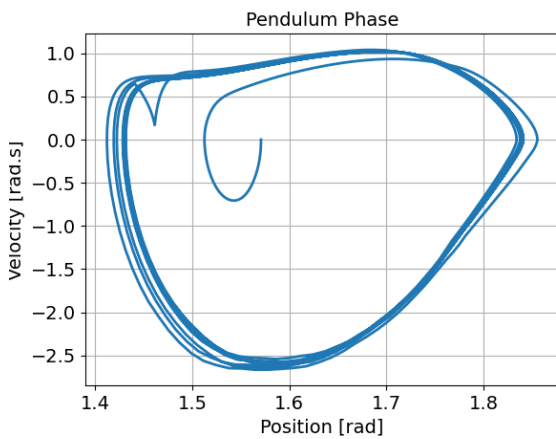
(b) Phase diagram of the pendulum with the 1[Hz] counterphased sine-shaped stimulation signal.



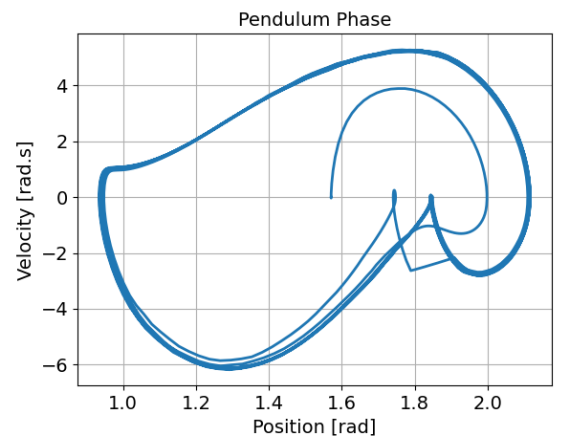
(c) Phase diagram of the pendulum with the 1[Hz] phased square-shaped stimulation signal.



(d) Phase diagram of the pendulum with the 1[Hz] counterphased square-shaped stimulation signal.



(e) Phase diagram of the pendulum with the 1[Hz] phased square-shaped stimulation signal.

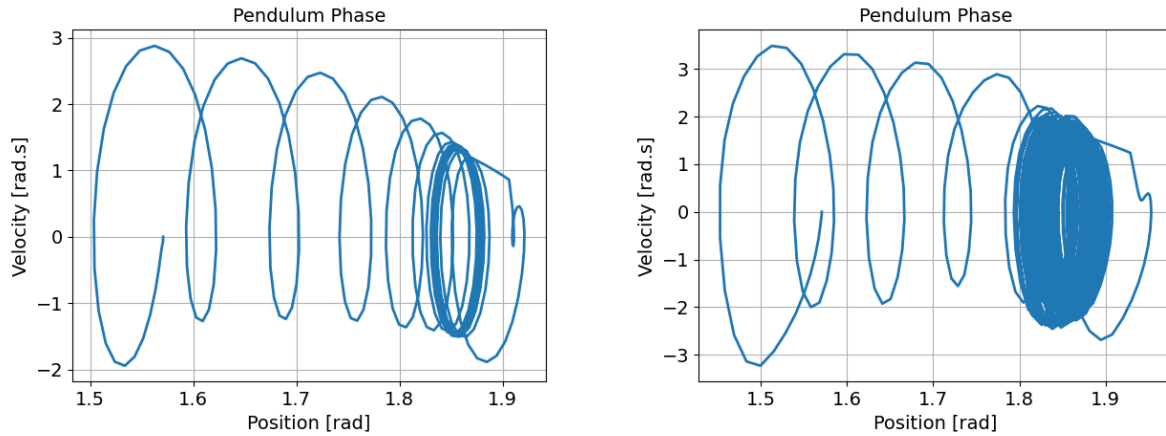


(f) Phase diagram of the pendulum with the 1[Hz] counterphased sawtooth-shaped stimulation signal.

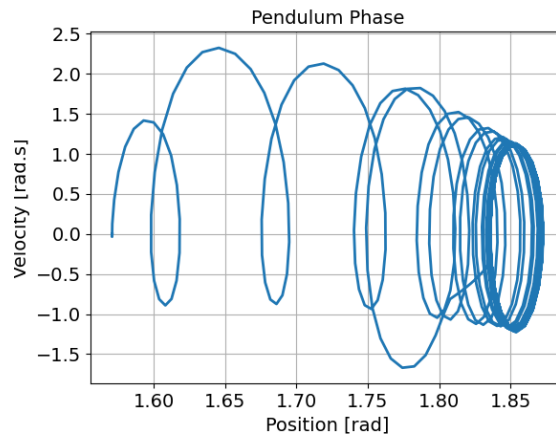
Figure 3: Measurements of experiment 2b.

2c. Explore the relationship between stimulation frequency with the resulting pendulum's behavior. Report your inferences for a low and high frequency condition.

When we increase the frequency to 10[Hz], the limit circle tends to be much smaller and less clearly defined (less stable). We can suppose that the muscle is reached a state of "crispation" and has the time constant (inertia) of the system is too high to be able to follow precisely the stimulation variation (see Figure 4). We also observe that the system takes more cycles to reach it.



(a) Position-velocity behaviour with the Sine-shaped stimulation signal. (b) Position-velocity behaviour with the square-shaped stimulation signal.



(c) Position-velocity behaviour with the sawtooth-shaped stimulation signal.

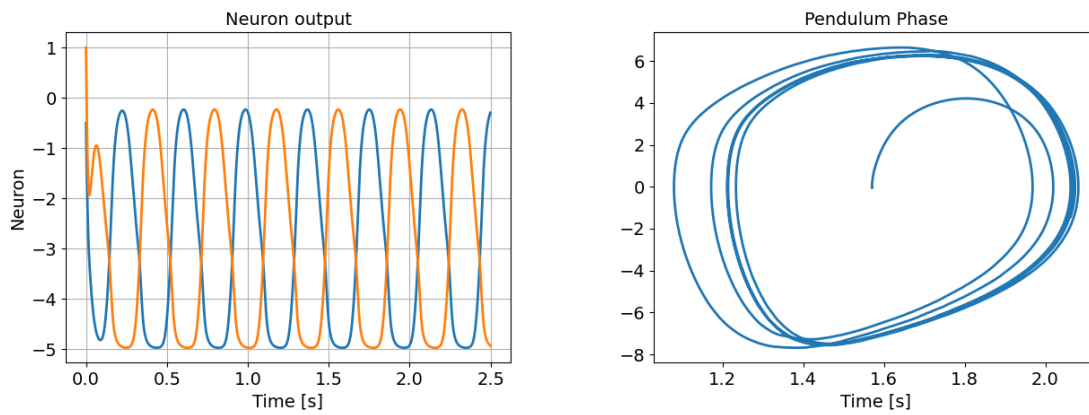
Figure 4: Position-velocity behaviour of the system for each 10[Hz] signal. NOTE: for those three simulations, the flexor and extensor stimulation signals are counterphased.

Exercise 3 : Neural network driven pendulum model with muscles

3a. Find a set of weights and time constants for the neural network that produces oscillations to drive the pendulum into a limit cycle behavior. Plot the output of the network and the phase plot of the pendulum

The selected parameters were taken from Lecture 4 on Neural Networks, slide 85:

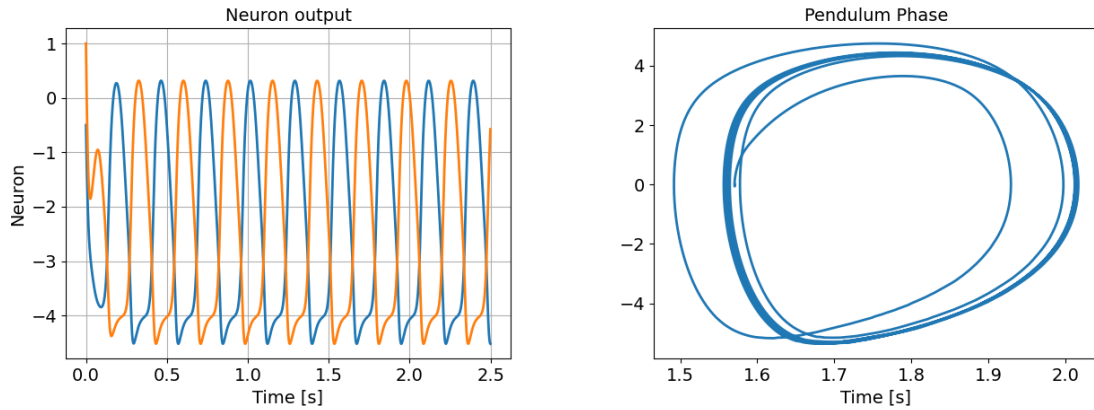
$$Weights = \begin{pmatrix} 0 & -5 & -5 & 0 \\ -5 & 0 & 0 & -5 \\ 5 & -5 & 0 & 0 \\ -5 & 5 & 0 & 0 \end{pmatrix}$$



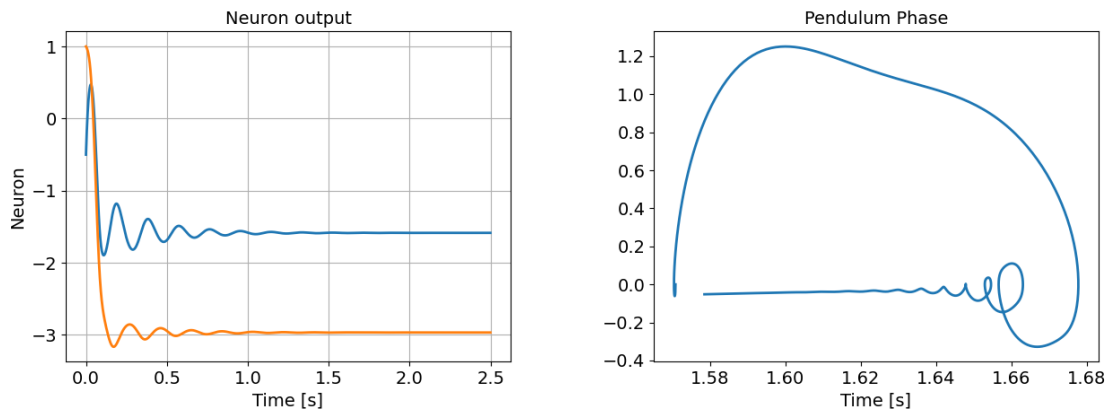
(a) Neuron output of the network without any external drive (b) Pendulum phase with our given parameters and no external drive.

Figure 5: Measurements on the neural network with our weight matrix

3b. As seen in the course, apply an external drive to the individual neurons and explain how the system is affected. Show plots for low [0] and high [1] external drives. To add external drive to the network you can use the method `system_simulation.py::add_external_inputs_to_network`



(a) Neuron output of the network with an external drive gain of 1. (b) Pendulum phase with an external drive gain of 1.



(c) Neuron output of the network with an external drive gain of 6. (d) Pendulum phase with an external drive gain of 6.

Figure 6: Measurements on the neural network with two different external drives.

3c. [Open Question] What are the limitations of the half center model in producing alternating patterns to control the pendulum? What would be the effect of sensory feedback on this model? (No plots required)

As we've observed in the experiment, a too high external drive can stop the generation of alternating patterns. Therefore, a sensory feedback high and long enough could "short-circuit" the neural system and stop the actuation (e.g. a high load on the motors) and render the robot motionless. It is also sensible to the starting position of the arm, and also the actuation symmetry is forced (we cannot stimulate each muscle independently).