Registration of Images with Geometric Distortions

ARDESHIR GOSHTASBY, MEMBER, IEEE

Abstract—**A technique for registration of images with geometric distortions is described. This technique uses two surface splines to represent the *X*-component and the *Y*-component of a mapping function. A mapping function is described in such a way that it would map corresponding control points in the images exactly on top of each other and map other points in the images by interpolation using information about local geometric distortion between the images.**

Keywords—**Image registration, geometric distortion, surface splines, satellite images, mapping functions.**

I. Introduction

IMAGE registration is the process of overlaying two images of the same scene. This process is often required in analysis of multiple images of the same scene. Traditionally, image registration has been a two-step process. In the first step, the positions of a number of corresponding points (known as the control points) are determined in the images and in the second step, this correspondence is used to estimate a mapping function that can overlay the rest of the points in the images.

Determination of corresponding control points in the images is a very difficult and important step in the registration process. Therefore, in the past, more effort has been spent on determining ways to locate control points in the images as accurately as possible and establishing correspondence between them. Davis and Kenue [1] used window centers as control points and by template matching established correspondence between the points. Stockman *et al.* [2] used line intersections as control points and by a clustering technique determined correspondence between the points. Goshtasby and Page [3] used centers of gravity of closed-boundary regions as the control points, and by a relaxation process, determined corresponding control points in the images.

So far, very little attention has been given toward determination of appropriate mapping functions for image registration. Many techniques in the past [4]–[6] have used polynomial mapping functions to register images. Parameters of the polynomials were determined by requiring that the polynomials overlay the control points as closely as possible. This was accomplished by minimizing the sum of squared errors in the overlaying process.

These image registration techniques have implicitly assumed some combination of the following constraints: 1) The scene is flat. 2) The area to be imaged is small compared to the distance of the camera to the scene. 3) Viewing angle is the same when the images are obtained. 4) The camera is a pin-hole camera (a perfect camera). 5) The effect of atmospheric turbulence or other distortions in the images is negligible.

In a given pair of images any combinations of the above constraints could be violated. Images obtained from different viewing angles from a 3-D scene will have local geometric distortions that depend on the local 3-D structure of the scene. Sensor nonlinearity and atmospheric turbulence could also contribute to local geometric distortion between the images.

The problem with using polynomial mapping functions for image registration is that the least-squares technique, which is used to determine the parameters of the polynomials, averages a local geometric distortion equally all over the image. A mapping function should use information about local geometric distortion to register local areas of the images and should not use that information to register all image areas identically. In this paper, in Section II, we formulate image registration as a surface-fitting problem in such a way that the obtained surfaces represent the components of a mapping function and characterize local geometric distortion between the images. An example exhibiting the given surface-fitting techniques is shown in Section III. In Section IV, the past and the proposed image-registration techniques are compared.

II. Surface Fitting as a Method For Determining Mapping Functions

Given positions of n corresponding control points in the images $([(x_i, y_i), (X_i, Y_i)], i = 1, \cdots, n)$, we would like to determine functions f and g that would register the images by mapping corresponding control points in the images exactly on top of each other $(X_i = f(x_i, y_i), Y_i = g(x_i, y_i), i = 1, \cdots, n)$ and map other points in the images by interpolation. We can restate this problem in another form also. Given two sets of 3-D points $\{(x_i, y_i, X_i), i = 1, \cdots, n\}$ and $\{(x_i, y_i, Y_i), i = 1, \cdots, n\}$, we would like to determine two smooth surfaces $X = f(x, y)$ and $Y = g(x, y)$ that would, correspondingly, pass through the two sets of points. In the following, we will refer to the image with coordinates (x, y) as the reference image and the image with coordinates (X, Y) as the sensed image.

Imagine having an infinite thin plate and adding point loads to the plate at positions determined by the control points in the reference image so that the plate deflects at the control points and takes on values equal to the X-com-

Manuscript received September 4, 1986; revised July 10, 1987. This work was supported in part by a grant from the Center For Robotics and Manufacturing Systems, University of Kentucky.
The author is with the Department of Computer Science, University of Kentucky, Lexington, KY 40506-0027.
IEEE Log Number 8717812.

0196-2892/88/0100-0060$01.00 © 1988 IEEE

ponent of the corresponding control point in the sensed image. The surface obtained in this manner represents the X-component mapping function because the surface value at a control point in the reference image is equal to the X-component of the corresponding control point in the sensed image. Given a point in the reference image, we can use this surface to determine the X-component of the same point in the sensed image. If we obtain a surface to represent the Y-component mapping function similarly, then the obtained surfaces represent the two components of the mapping function required to register the images.

A mapping function determined in this manner would be sensitive to local geometric distortion between the images because the value of a surface at a point is determined by the nearby point loads and the influence of a point load on a point decreases as the distance of the point to the point load increases. This property assures that local geometric distortion between the images will not be averaged equally all over the image but rather it would be used locally in the registration process.

The surface of an infinite plate obtained under the imposition of point loads is known as the surface spline and its equation is given in [7]

$$f(x, y) = a_0 + a_1 x + a_2 y + \sum_{i=1}^{n} F_i r_i^2 \ln r_i^2 \tag{1}$$

where n is the number of point loads, $r_i^2 = (x - x_i)^2 + (y - y_i)^2$, (x_i, y_i) is the position of the i^{th} point load (in our case, the position of the i^{th} control point in the reference image), and $f(x, y)$ is the surface value or the elevation of the surface at point (x, y) (in our case the X-component of the point corresponding to (x, y) in the sensed image). The obtained surface represents the X-component mapping function. The surface representing the Y-component mapping function is determined similarly. Parameters $a_0, a_1, a_2, F_i, i = 1, \cdots, n$ of (1) are determined by substituting the coordinates of corresponding control points into (1) and solving the obtained system of linear equations.

$$\begin{cases} \sum_{i=1}^{n} F_i = 0 \\ \sum_{i=1}^{n} x_i F_i = 0 \\ \sum_{i=1}^{n} y_i F_i = 0 \\ f(x_1, y_1) = a_0 + a_1 x_1 + a_2 y_1 \\ \qquad + \sum_{i=1}^{n} F_i r_{i1}^2 \ln r_{i1}^2 \\ \vdots \\ f(x_n, y_n) = a_0 + a_1 x_n + a_2 y_n \\ \qquad + \sum_{i=1}^{n} F_i r_{in}^2 \ln r_{in}^2. \end{cases} \tag{2}$$

In Section III, we solve this system of equations to determine the surface spline that passes through a set of generated points in 3-D, and in Section IV, we use this surface to represent the components of a mapping function for image registration.

III. An Example

Suppose that seven points, along with their elevations as shown in Fig. 1(a), are available. To determine the surface spline that passes through these points, we substitute the coordinates and the elevations of the points into system (2) for $x_i, y_i, f(x_i, y_i), i = 1, \cdots, 7$, and solve the obtained system of equations for a_0, a_1, a_2, and F_i, $i = 1, 7$. Doing so, we obtain $a_0 = 0.005288$, $a_1 = 0.003588$, $a_2 = 0.009618$, $F_1 = 0.011928$, $F_2 = -0.009627$, $F_3 = 0.035677$, $F_4 = -0.079525$, $F_5 = 0.009886$, $F_6 = 0.014471$, and $F_7 = -0.001306$. Substituting these parameter values back into (1) and computing and rounding $f(x, y)$ at $x = 1, \cdots, 20$ and $y = 1, \cdots, 20$, we obtain the array of Fig. 1(b).

If we let the positions of points in Fig. 1(a) represent the positions of control points in the reference image, and the third component of the points represent the X-component of the corresponding control points in the sensed image, then Fig. 1(b) is in fact the X-component mapping function that maps the sensed image into the reference image by the nearest-neighbor rule. An entry (x, y) of the array in Fig. 1(b) is the X-component of the same point in the sensed image. If we obtain another surface to represent the Y-component mapping function in the same manner, then we would be able to register the underlying images using these component mapping functions.

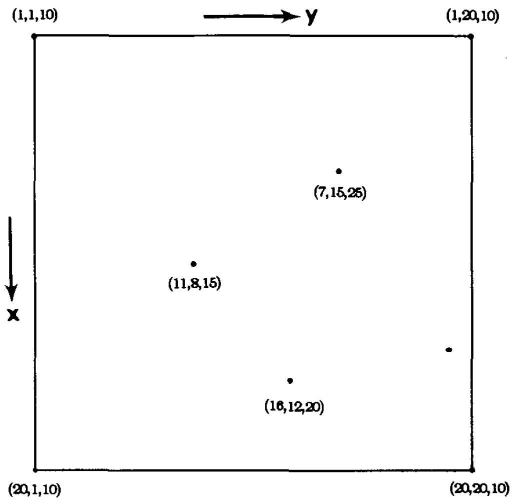
IV. Results

To exhibit the practicality of the proposed image registration technique, the following three experiments were carried out.

A. Experiment 1

In Experiment 1, two satellite images as shown in Fig. 2 were used. Fig. 2(a) is a band 4 Landsat Thematic Mapper (TM) image acquired on October 18, 1982 from an area over Crawford County, Michigan. Fig. 2(b) is a band 4 Landsat MultiSpectral Scanner (MSS) image from about the same area acquired on June 17, 1980. The resolution of the MSS image is 80 m and the resolution of the TM image is 60 m. The original resolution of the TM image was 30 m but was reduced by half so that the two images, when of the same size, would cover about the same area of the scene. Both images are of size 256 × 256.

To determine corresponding control points in the images, the images were segmented and closed-boundary regions were extracted from the images by a process described in [8]. Thirty-one corresponding regions were located in the images. Then centers of gravity of corresponding regions were used as corresponding control points to estimate the registration mapping function. Since these images were in good condition, the geometric distortion between them was locally negligible and we could

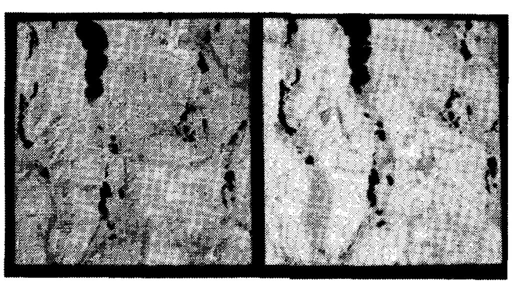


(a)

	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20
01	10	11	13	14	15	16	16	17	18	18	18	19	18	18	18	17	16	14	12	10
02	11	12	14	15	16	16	17	18	19	19	20	20	20	20	19	19	18	16	14	12
03	12	13	14	15	16	17	18	19	19	20	21	21	21	21	21	20	19	18	16	14
04	12	14	15	16	16	17	18	19	20	21	21	22	22	23	23	22	21	20	18	16
05	13	14	15	16	16	17	18	19	20	21	22	23	23	24	24	23	22	21	20	18
06	13	14	15	15	16	17	18	19	19	21	22	23	24	24	25	24	23	22	21	19
07	13	14	15	15	16	16	17	18	19	20	21	23	24	25	25	25	24	23	22	20
08	13	14	14	15	15	16	16	17	18	20	21	22	24	25	25	25	24	23	22	20
09	13	13	14	14	15	15	15	16	17	19	20	22	23	24	25	25	24	24	22	21
10	13	13	14	14	14	14	15	15	17	18	20	21	23	24	24	25	24	23	22	21
11	13	13	14	14	14	14	14	15	16	18	19	21	22	23	24	24	24	23	22	20
12	12	13	14	14	14	14	14	15	16	18	19	21	22	23	23	24	23	23	22	20
13	12	13	14	14	14	14	15	16	17	18	19	21	22	22	23	23	23	22	21	20
14	12	13	14	14	15	15	15	16	17	18	19	20	21	22	22	22	22	21	20	19
15	12	13	14	14	15	15	16	17	17	18	19	20	21	22	22	22	21	20	19	18
16	12	13	14	15	15	16	16	17	18	19	19	20	21	21	21	21	20	19	18	17
17	12	13	14	15	15	16	16	17	18	18	19	20	20	20	20	20	19	18	17	15
18	12	13	14	15	15	16	16	17	18	18	19	19	19	19	19	19	18	17	16	14
19	11	12	13	14	15	15	16	17	17	18	18	18	18	18	18	17	16	15	14	12
20	10	11	12	13	14	15	15	16	16	17	17	17	17	17	16	16	15	14	12	10

(b)

Fig. 1. (a) Seven points with their coordinates. The third component of a point is the elevation of the point. (b) Surface spline obtained by requiring the surface pass through the points.



(a) (b)

Fig. 2. Images used for registration in Experiment 1. (a) The TM image. (b) The MSS image.

use the centers of gravity of corresponding regions as corresponding points. However, if two images have locally significant geometric distortions, then centers of gravity of corresponding regions may not correspond to each other and they cannot be used as control points. In that case, features invariant to geometric distortion such as line intersections and line end points should be used.

The coordinates of the corresponding control points were used in the procedure of Section II to determine the surface splines. The surfaces that were obtained represent the X-component and the Y-component mapping functions. Using the obtained mapping function, the image of Fig. 2(b) was resampled and overlaid with the image of Fig. 2(a). This is shown in Fig. 3(a).

To compare the proposed technique with the polynomial mapping function with the least-squares technique, the following polynomials of degree two were used as the mapping function.

$$\begin{cases} X = a_0 + a_1 x + a_2 y + a_3 xy + a_4 x^2 + a_5 y^2 \\ Y = b_0 + b_1 x + b_2 y + b_3 xy + b_4 x^2 + b_5 y^2. \end{cases} \quad (3)$$

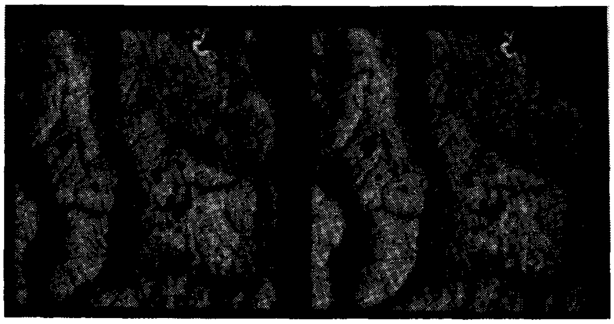
Then, using the corresponding control points in the images, the parameters of the mapping function were determined by minimizing the sum of squared errors

$$\begin{aligned} E = \sum_{i=1}^{n} \Big\{ \big[X_i - (a_0 + a_1 x_i + a_2 y_i \\ + a_3 x_i y_i + a_4 x_i^2 + a_5 y_i^2) \big]^2 \\ + \big[Y_i - (b_0 + b_1 x_i + b_2 y_i \\ + b_3 x_i y_i + b_4 x_i^2 + b_5 y_i^2) \big]^2 \Big\}. \end{aligned}$$

This process results in a system of 12 linear equations (see [8] for more details) that can be solved to determine the registration parameters. Using this technique, the registration result of Fig. 3(b) was obtained. Although at some places the registration is good, there is serious misregistration at the upper left and mid-upper areas of the images. The rms error at the control points is 1.3 pixels for Fig. 3(b). The rms error for the same points is zero for Fig. 3(a) because the obtained mapping function maps corresponding control points in the images exactly on top of each other. Using 20 of the 31 control points to register the images, and using the remaining 11 control points to evaluate the registration accuracy, we obtained rms error equal to 0.43 pixels for the surface splines method.

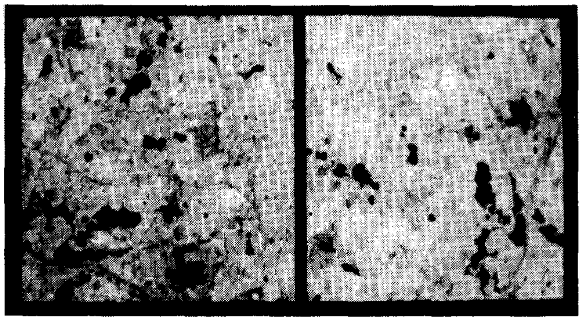
B. Experiment 2

In Experiment 2, band 7 of a Landsat MultiSpectral Scanner (MSS2) image acquired on August 27, 1980 from an area over Pontiac, Michigan, and band 3 of a Thematic Mapper Simulation (TMS) image aquired on August 19, 1981 from about the same area were used. Fig. 4(a) and 4(b) shows the MSS2 and the TMS images, respectively. The TMS image is a simulated Thematic Mapper image that has been taken by an aircraft and contains a considerable amount of distortion, especially near its borders.



(a) (b)

Fig. 3. Registration of images of Fig. 2. (a) Using the proposed surface-fitting technique. (b) Using polynomial mapping function and the least-squares technique. Note the poor registration obtained by the least-squares technique in the upper-left and mid-upper areas of (b).



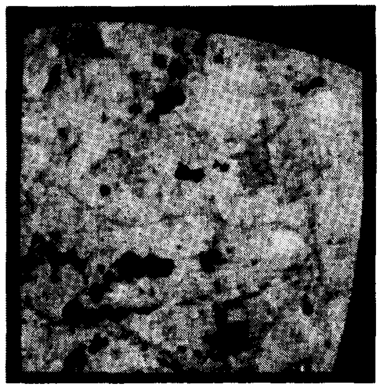
(a) (b)

Fig. 4. The images used for registration in Experiment 2. (a) The MSS2 image. (b) The TMS image.

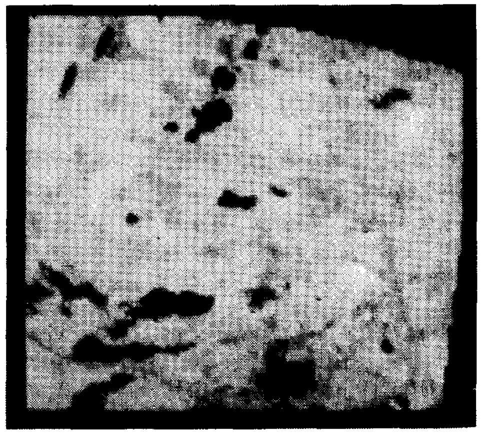
The centers of gravity of 14 corresponding regions were used as corresponding control points to register the images. Using the surface splines, the registration result shown in Fig. 5(a) was obtained with no registration error at the control points. Using ten of the control points to register the images, and the other four control points to evaluate the registration accuracy, we obtained rms error equal to 0.63 pixels. Using the 14 control points in the least-squares technique with a polynomial of degree two, we obtained a registration result of Fig. 5(b) with a rms error equal to 1.9 pixels. As can be seen from Fig. 5(b), the least-squares technique fails to register the images in areas near the image borders where there is larger distortion.

C. Experiment 3

In Experiment 3, we prepared two images in the laboratory with significantly large geometric distortions. On a rubber sheet that had nonuniform thickness we hand-printed a few words as shown in Fig. 6(a). Then the rubber sheet was pulled in various directions by different magnitude forces applied on the boundary of the sheet. This is shown in Fig. 6(b). The amount of stretch made to the rubber sheet at a given point depends on the thickness of the sheet and also on the forces applied to the sheet. Since the thickness of the rubber sheet and the

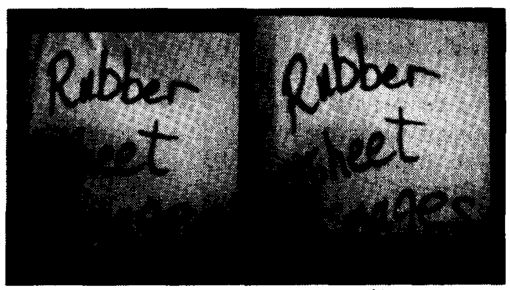


(a)



(b)

Fig. 5. Registration of images of Fig. 4 using (a) the surface splines and (b) the least-squares with polynomials of degree two. Image (b) is copied from [8].

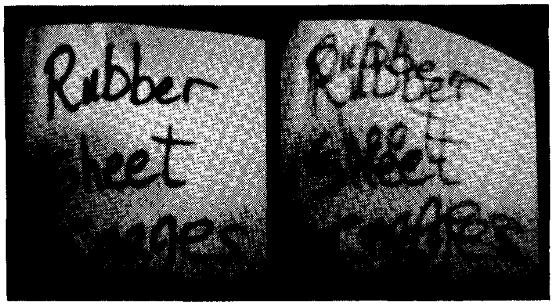


(a) (b)

Fig. 6. Data used in Experiment 3. (a) Original rubber sheet. (b) Rubber sheet after being stretched by different forces on its boundary.

forces applied to the sheet varies from point to point on the sheet, the amount of distortion made to the sheet varies locally from point to point.

Since the images have local geometric distortions, the centers of gravity of corresponding regions do not correspond to each other and they cannot be used as corresponding control points. Line intersections and line end points, however, remain invariant under nonlinear distortions and can be used as control points for image registra-



(a) (b)

Fig. 7. Registration using the surface splines. (b) Registration using the least-squares with polynomials of degree two.

tion. We segmented the images by a simple thresholding process and then thinned the segmented images. Next, 32 corresponding line intersections and line end points were selected from the images and were applied to the process of Section II. Doing so, we obtained the registration result shown in Fig. 7(a). Again, corresponding control points in the images fall exactly on top of each other after the registration. Using 20 of the control points for registering the images, and using the remaining 12 control points for measuring the registration accuracy, we obtained a rms error equal to 1.1 pixels. Carrying out the same process this time using the least-squares technique with polynomials of degree two, we obtained the registration result of Fig. 7(b) with a rms error equal to 19.7 pixels. This registration is by no means satisfactory.

As the geometric distortion between the images increases, the surface-fitting technique proposed in this paper performs better than the least-squares technique, because in the least-squares technique a mapping function is determined by a process that averages a local distortion to the entire image. The surface fitting approach, on the other hand, determines a mapping function by a weighted method that takes into consideration the local geometric distortion between the images.

Computationally, the surface-fitting approach is more expensive than the least-squares approach. In the least-squares approach, with polynomials of degree two, we need to solve a system of 12 equations to find parameters a_i, b_i, $i = 0, \cdots, 5$ [8]. In the surface-fitting approach, the number of equations to be solved depends on the number of available control points. If n control points are given from each image, then it is required to solve a system of $n + 3$ linear equations to determine each component of the mapping function. For images of Fig. 2 we need to solve a system of 12 equations for the least-squares approach but need to solve two systems of 34 equations each, for the surface-fitting approach, to determine the registration mapping function.

V. Conclusion

A new approach to digital image registration was described above that uses two surface splines as the X-component and the Y-component mapping functions. The surfaces were obtained by requiring that the surface splines pass through 3-D points obtained by the corresponding control points in the images. These surfaces characterize local geometric distortion between the images and, therefore, are especially effective in registering large images, images obtained from different viewing angles, and images obtained by a sensor with nonlinear characteristics.

Although the proposed technique requires more computation time than the previously used least-squares technique, it provides a better accuracy, especially when the images have local geometric distortions.

Acknowledgment

The author would like to thank the Center for Remote Sensing, Michigan State University for providing the satellite images for this study. This work was supported in part by a grant from the Center for Robotics and Manufacturing Systems, University of Kentucky.

References

[1] W. A. Davis and S. K. Kenue, "Automatic selection of control points for the registration of digital images," in *Proc. 4th Int. Joint Conf. Pattern Recognition*, pp. 936-938, 1978.

[2] G. C. Stockman, S. Kopstein, and S. Benett, "Matching images to models for registration and object detection via clustering," *IEEE Trans. Pattern Anal. Machine Intell.*, vol. PAMI-4, no. 3, pp. 229-241, 1982.

[3] A. Goshtasby and C. V. Page, "Image matching by a probabilistic relaxation labeling process," in *Proc. 7th Int. Conf. Pattern Recognition*, pp. 307-309, 1984.

[4] D. G. Leckie, "Use of polynomial transformations for registration of airborne digital line scan images," in *14th Int. Symp. Remote Sensing Environment*, pp. 635-641, 1980.

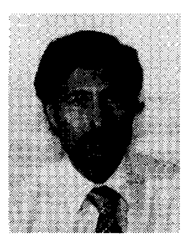
[5] M. L. Nack, "Rectification and registration of digital images and the effect of cloud detection," in *Proc. Machine Processing of Remotely Sensed Data*, pp. 12-23, 1977.

[6] P. Van Wie and M. Stein, "A Landsat digital image rectification system," *IEEE Trans. Geosci. Electron.*, vol. GE-15, no. 3, pp. 130-137, 1977.

[7] R. L. Harder and R. N. Desmarais, "Interpolation using surface splines," *J. Aircraft*, vol. 9, no. 2, pp. 189-191, 1972.

[8] A. Goshtasby, G. C. Stockman, and C. V. Page, "A region-based approach to digital image registration with subpixel accuracy," *IEEE Trans. Geosci. Remote Sensing*, vol. GE-24, no. 3, pp. 390-399, 1986.

*



Ardeshir Goshtasby (S'81-M'83) received the B.E. degree in electronics engineering from the University of Tokyo, Japan, in 1974, the M.S. degree in computer science from the University of Kentucky, Lexington, in 1975, and the Ph.D. degree in computer science from Michigan State University, East Lansing, in 1983.

He joined the staff of the Computer Science Department, University of Kentucky, in 1983, where he is presently an Assistant Professor of Computer Science. His main areas of research are computer vision, image processing, computer graphics, and remote sensing. He is the co-recipient of the William Fischer Memorial Award for the best contributed paper in the 17th International Symposium on Remote Sensing of Environment, 1983.

Dr. Goshtasby is a member of the Pattern Recognition Society, the IEEE Computer Society, and the Association of Computing Machinery.