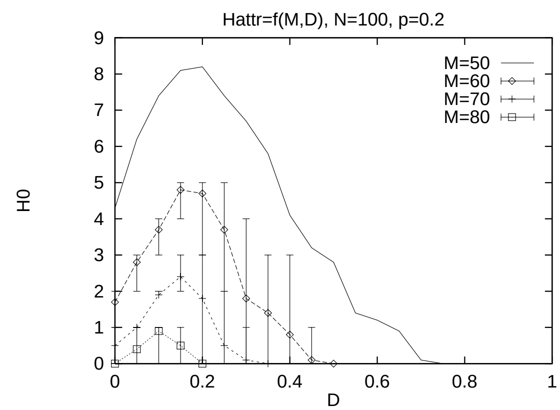
The optimal value of self-connection

Dmitry O. Gorodnichy
Department of Computing Science, University of Alberta, Canada

Purpose. The fact that reducing self-connections improves the performance of the autoassociative networks built by the pseudo-inverse learning rule is known already for quite a while [Kanter&Sompolinsky 1986, Gorodnichy&Reznik 1997]. In particular, the following properties of the network related to this phenomena have already been obtained. If weight matrix $\mathbf{C}$ of the network is calculated as $\mathbf{C} = \mathbf{V}\mathbf{V}^+$, where $\mathbf{V}^+ \doteq (\mathbf{V}^T\mathbf{V})^{-1}\mathbf{V}^T$ is the *pseudoinverse* of matrix $\mathbf{V}$ and $\mathbf{V}$ is the matrix made of column prototype vectors, then decreasing of self-connection weights by $C_{ii} := D \cdot C_{ii}$, $(0 < D < 1)$, restores the balance of weight $\frac{\langle C_{ii}\rangle}{\langle |C_{ij}|\rangle}$ and allows the network to escape local minima. It has also been proved that, although the decrease of D increases the direct attraction radius of the network, it also increases the number of spurious dynamic attractors. Thus, it has been concluded that the optimal value of the coefficient of self-connection reduction D lies somewhere in the range (0;0.5). However, the exact value has not been obtained yet. This paper gives an explicit answer to the question what is the optimal value of the self-connection reduction. It also shows how the indirect attraction radius increases with the decrease of D.

Method. Using the approach of [Kanter&Sompolinsky 1986], we prove that the reduction of the self-connection decreases the number of spurious static attractors, while keeping the prototypes as stable states of the network. Since theoretical calculation of the indirect attraction radius is known to be NP-hard, extensive Monte-Carlo simulations have been carried out in order to find it. In these simulations, random vectors, where the probability p of a neuron being in state $+1$ is the same for all neurons in all prototypes, are used. Then the analysis is done as for applicability of the obtained results for the case of non-random prototypes.



Results. One of the obtained results is shown in the figure above. The figure shows the average values of the indirect attraction radius HO as a function of the coefficient D for the network of size $N = 100$ and different numbers M of stored prototypes. The minimal and maximal values of the attraction radius are drawn as vertical bars. The obtained results provide the evidence for choosing the optimal value D.

New or breakthrough aspect of work. The results presented in the paper are obtained in the author's dissertation "Investigation and Design of High Performance Fully Connected Neural Networks" defended in 1997 at the IMMS of the Ukrainian Academy of Sciences, Kiev, Ukraine. The dissertation is written in Russian and these results are therefore not known outside of the x-USSR. We believe that making these results available for English speaking research community would promote the development of autoassociative neural networks.

Conclusions. For the autoassociative memory designed with the pseudo-inverse learning rule, the optimal value of self-connection reduction $C_{ii} = D \cdot C_{ii}$ is $D = 0.15$. The memory with self-connections built in this way exhibits the retrieval of $70\%N$ prototypes from $2\%N$ noise. When implementing this memory, the update flow technique described in the paper should be used in order to handle efficiently the cycles which may occur.

1. I. Kanter and H. Sampolinsky. Associative recall of memory without errors, *Phys. Rev. A* vol 35, pp. 380-392, 1987.

2. D.O. Gorodnichy and A.M. Reznik. Increasing Attraction of Pseudo-Inverse Autoassociative Networks, *Neural Processing Letters*, volume 5, issue 2, pp. 123-127, 1997.