Systematic Rewiring in Associative Neural Networks with Small-World Architecture*

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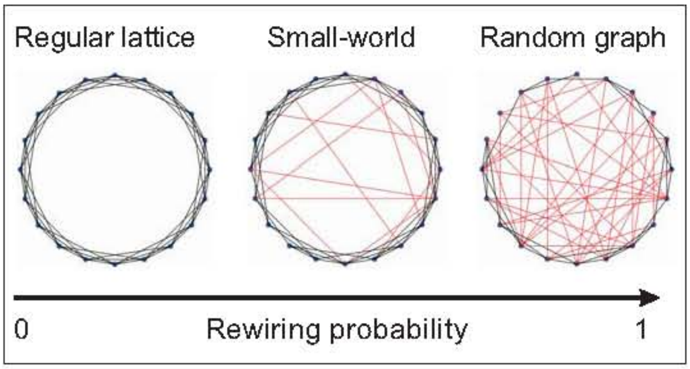
Abstract: It is a known fact that a small amount of randomly rewired connections greatly improves the performance of associative neural network with regular architecture, still preserving its attractive features such as local connectivity and small total connection length. In this paper we propose the systematic way of connection rewiring which further improves the associative properties of the network using the same amount of rewiring.

SMALL-WORLD

A sparse graph with *small-world* structure is characterized by:
- low average path length between vertices (like in random graphs)
- high clustering coefficient (like in regular lattices)

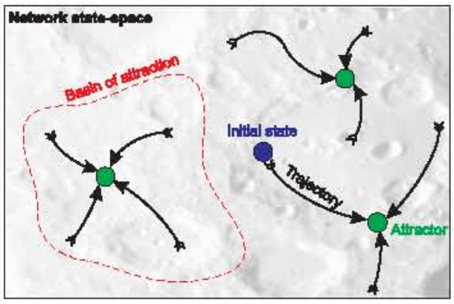
Small-world structures are found in:
- social networks ("six degrees of separation", [Milgram1967])
- the neural network of the worm *Caenorhabditis elegans* [Watts1998]
- power grid of the western United States [Watts1998], *etc.*

An analytical model for the small-world is proposed in [Watts1998] ⟹



SPARSE HOPFIELD NEURAL NETWORK

- Dynamical system
- Consists of a set of connected units (neurons)
- Interneuronal connections form weight matrix W
- If stable states (attractors) correspond to the memorized data
  => network functions as (auto) associative memory



Structure

Weight matrix (W)

Dynamics

$$s_{t+1} = \mathrm{sgn}(W s_t)$$

HOPFIELD NETWORK WITH SMALL-WORLD ARCHITECTURE

Factors affecting associative performance:
- number of interneuronal connections
- values of connection strength
- *location* of connections ⟹

For a certain number of connections in Hopfield network may hold [Bohland2001]:
- arranged locally (regular lattice) => exhibits NO associative properties
- small-world arrangement => exhibits associative properties
- random arrangement => exhibits associative properties

Advantages of the small-world architecture in sparse Hopfield networks:

- lower total connection length (comparing to random arrangement of connections), hence more hardware friendly
- more biologically feasible structure

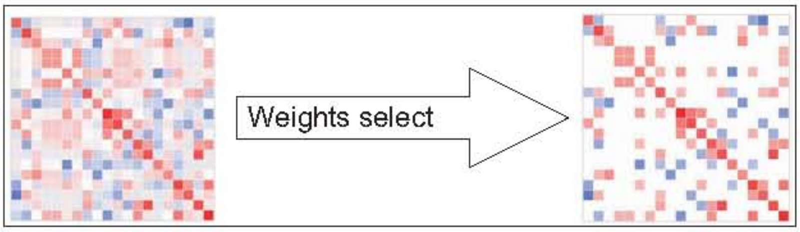
*This work was supported by INTAS Young Scientist Fellowship YSF 03-55-795

SYSTEMATIC APPROACH TO REWIRING

Objective: We are interested in obtaining a network with small-world structure and good associative properties

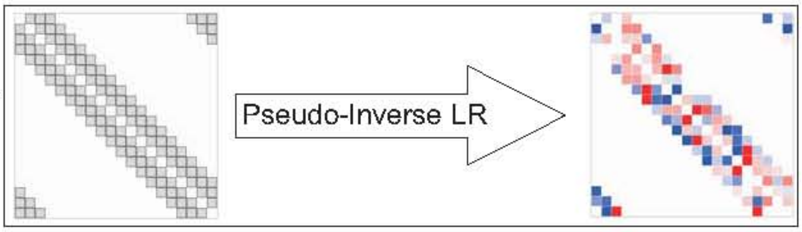
Q.: While constructing the small-world architecture, is it possible to do rewiring in some better way than just random?
A.: Yes. For this purpose we use the following fact [Sitchov1998]

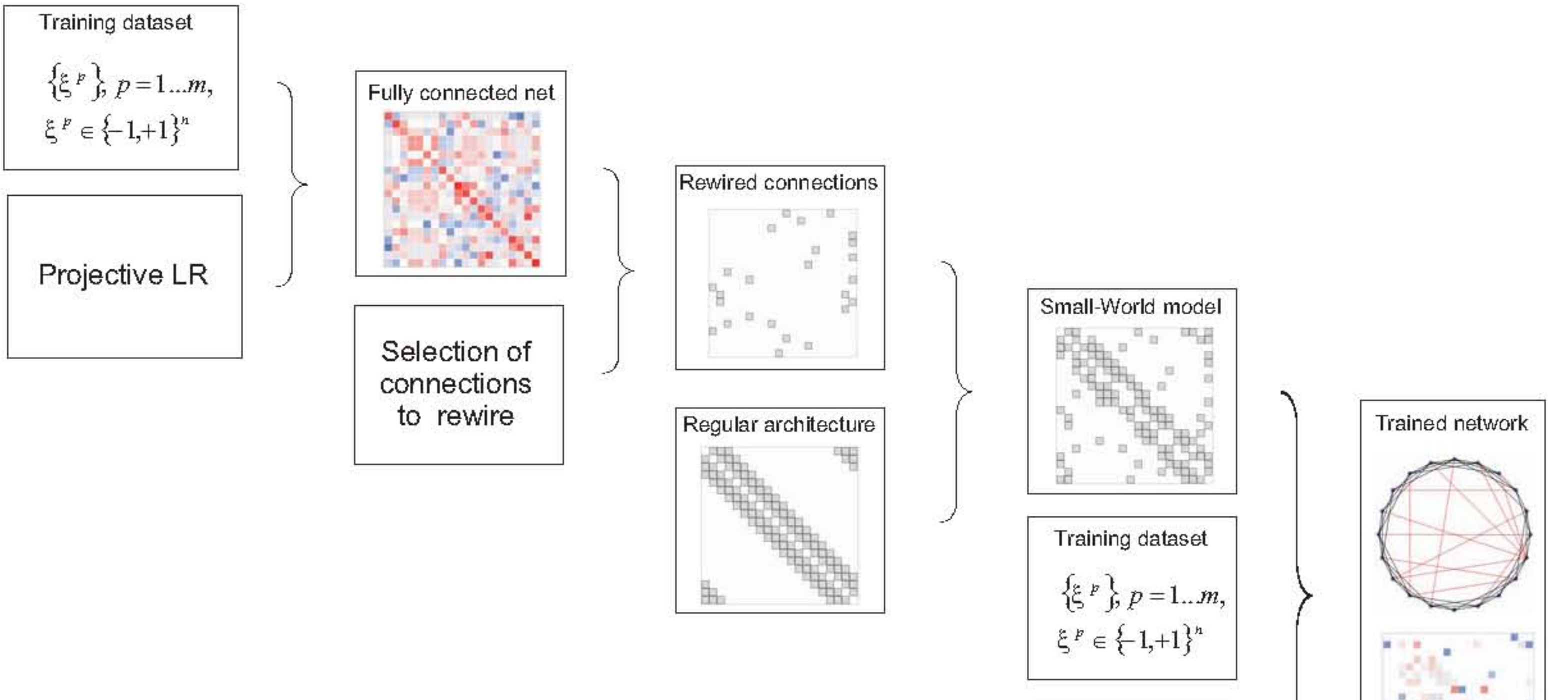
Fully connected Hopfield network still performs well as associative memory even after the removal of 80% of neuron connections with the least absolute values.

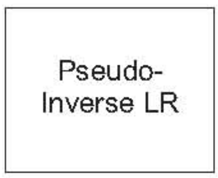


Solution: The location of the remaining connections is apparently of great importance for the associative properties of the network and reflects some hidden interrelationships in stored data - let's use it for the rewiring procedure!

Once network architecture is known, we use *Pseudoinverse Learning Algorithm* [Brucoli1995] to find the weight values. This algorithm provides quick calculation of weight matrix for any network architecture

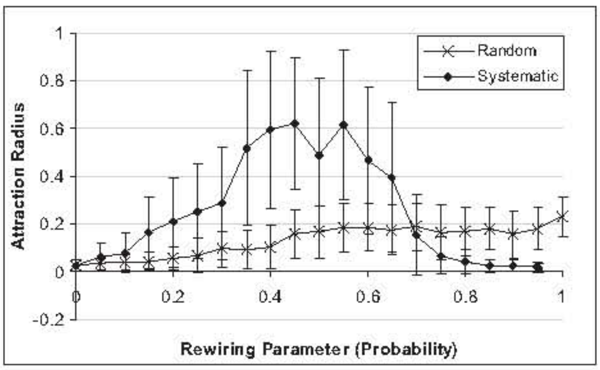


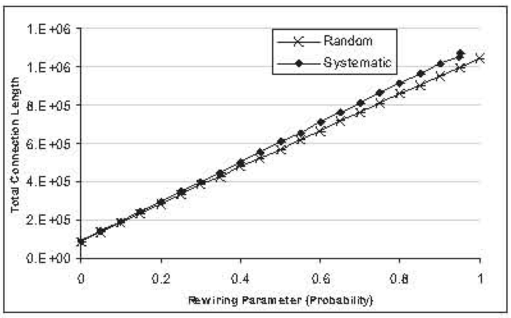




EXPERIMENTAL RESULTS

Results are averaged over 10 random datasets. The size of the data set is $m = 10$ and its dimension is $n = 400$. We tested the network with neuron neighbourhood size $k = 20$ (connection dencity $r = 0.05$)





KEY REFERENCES

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