1.1.1 Boolean Algebra

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\begin{aligned}
x \text{ or } y &= \bar{x}y + x\bar{y} + xy \\
&= \bar{x}y + x(y + \bar{y}) \\
&= x + \bar{x}y \\
&= x + \overline{x + \bar{y}} \\
&= \overline{\overline{x + \overline{(x + y)}}} \\
&= \overline{\bar{x}(x + \bar{y})} \qquad \text{where } \overline{x + y} = \bar{x}\bar{y} \\
&= \overline{x\bar{x} + \bar{x}\bar{y}} \\
&= \overline{\bar{x}\bar{y}} \\
&= \overline{\overline{x + y}} \\
&= x + y \\
\text{if } y \text{ then } x &= \bar{x}\bar{y} + x\bar{y} + xy \\
&= \bar{x}\bar{y} + x(y + \bar{y}) \\
&= x + \bar{x}\bar{y} \\
&= x + \overline{x + y} \\
&= \overline{\overline{x + \overline{x + y}}} \\
&= \overline{\bar{x}(x + y)} \\
&= \overline{x\bar{x} + \bar{x}y} \\
&= \overline{\bar{x}y} \\
&= x + \bar{y} \\
\text{if } x \text{ then } y &= \bar{x}\bar{y} + \bar{x}y + xy \\
&= \bar{x}\bar{y} + (x + \bar{x})y \\
&= \bar{x}\bar{y} + y \\
&= \overline{x + y} + y \\
&= \overline{\overline{\overline{x + y} + y}} \\
&= \overline{(x + y)\bar{y}} \\
&= \overline{x\bar{y} + y\bar{y}} \\
&= \overline{x\bar{y}} \\
&= \bar{x} + y \\
x \text{ nand } y &= \bar{x}\bar{y} + \bar{x}y + x\bar{y} \\
&= \bar{y}(x + \bar{x}) + \bar{x}y \\
&= \bar{y} + \bar{x}y \\
&= \bar{x} + \bar{y} \qquad \text{where } x + y = x + \bar{x}y \\
&= \overline{\overline{\bar{x} + \bar{y}}} \\
&= \overline{xy}
\end{aligned}
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