PHSA0030 Mini Project - Brazilian nuts simulation using a Monte Carlo approach

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Abstract

The formal report should contain: an abstract; the background to the project; a summary of the work done (though not in a narrative form); the results and a discussion of their significance; any conclusions that you have drawn, and possibly suggestions for future work, or where you would take the project given more time; and a bibliography.

1 Background

Empirical observations show that if a container of a mix of nuts of different sizes is shaken thoroughly, the larger pieces (such as the brazil nut kernels) tend to rise to the top. In this project we will be using a Monte Carlo method is an effort to simulate this physical situation. More specifically, we will be using the Metropolis approach as described in the course from which this assignment is part of.

1.1 Monte Carlo simulation in a nutshell

In our simulation we will start with a distribution of elements, select a random element and displace it a random distance in a random direction. If the movement overall lowers the energy of the system we will accept it as valid. But in order to avoid local minima we will introduce a probability for an energetically *worse* movement to be done. The probability used is $p = e^{\frac{\Delta E}{k_B T}}$ Where the energy change is ΔE, thus the probability decreases as the energy change increases. The probability also increases as the temperature increases, which can be observed in nature.

2 Summary

In the simulation we assumed that the brazilian nuts were two dimensional disks and the regular nuts as disks of half of the radius of the disks representing the brazilian nuts. The box has a limited horizontal width but we assume it to be infinitely tall.

2.1 Comment on the tools used

For this project, Python 3.7 was used, along with the libraries numpy for the data structures and calculations and matplotlib for plotting and graphics. Additionally, Jupyter notebooks were used as logbooks detailing the progress and stages of the project.

2.2 Initial configuration of the system

As per the guidelines of the assignment, the initial configuration of the system was a brazilian nut at the bottom and then regular nuts spread randomly in the x coordinate in increasingly high positions.

2.3 Monte Carlo method

The Montecarlo approached was implemented as follows: First a particle in the system is chosen at random. Then a step size is calculated at random based on the value for the largest nearest neighbor of the system. The angle of the displacement is also chosen randomly. If the movement is energetically better (i.e. $\Delta y < 0$) it is accepted, otherwise it is checked against a probability $p = e^{\frac{\Delta E}{k_B T}}$. Then the move is tested for collisions with the walls or with other particles. If the move fulfilled all these tests it is be taken as *accepted* and the system is updated. The change in energy achieved by this move is accumulated in some ΔE variable that keeps track of the change in energy achieved every 1000 attempts. As the particles become incrementally closer to each other and to the bottom of the bottom of the box, both the value for the largest nearest neighbor drops and the frequency of accepted moves drop substantially. This results eventually results in ΔE not reaching an arbitrary threshold and thus th system being considered *settled*. From there it can be tested whether the big nuts are on top and otherwise the system may be *shaken*(i.e. all the y values multiplied by 2) and the process started all over.

3 Results

3.1 Relation between n_{nut} and n_{shakes}

Evidently, the increase of the number of nuts in the system n_{nut} resulted in an overall increase in the number of shakes n_{shakes} needed in order to take the brazilian nut to the top.

However, a factor that was highly relevant in order to judge the value of the investigations presented in the following subsections was the standard deviation σ_{shakes} between consecutive simulations of the same configurations. This value was revealed to decrease as n_{nut} increases. Thus a high value for n_{nut} is preferred in order to minimize the uncertainity in our results. However, the computational times needed to simulate systems with very large n_{nut} values quickly start to become non-trivial.

3.2 Effects of $r_{brazilian}/r$

3.3 Study of the energy

3.4 Temperature and p

4 Future work

- 61cm ramp
- stack of textbooks
- ball
- meter stick
- tape

- timer
- Logger Pro
- motion sensor

$$K = U_g \tag{1}$$
$$mgh = \frac{1}{2}mv^2 \tag{2}$$
$$(10)(.225) = \frac{1}{2}v^2$$
$$4.5 = v^2$$
$$2.12\frac{m}{s} = v \tag{3}$$

5 Analysis

This is the most important part of the lab report; it is where you analyze the data. In this section you will interpret your results. You need to look at your data and decide if the hypothesis was supported or contradicted by your data.

Your discussion should include the following at a minimum. [1] What is the relationship between your measurements and your final results? [2] What trends were observable? [3] What can you conclude from the graphs that you made? [4] How did the independent variables affect the dependent variables? (For example, did an increase in a given measured (independent) variable result in an increase or decrease in the associated calculated (dependent) variable?)

Then describe how your experimental results substantiate/agree with the theory. (This is not a single statement that your results agree or disagree with theory.) When comparison values are available, discuss the agreement using either uncertainty and/or percent differences. This leads into the discussion of the sources of error. Your discussion should include the calculation of averages and standard deviation to be able to describe precision of experiment. All data points should be plotted $\pm$ two standard deviations and compared with theoretical data to interpret accuracy. It is ok to say that your results were inconclusive. It is important to cite all possible sources of error and state specifically how you believed they affected the collected data. If you get a result or an uncertainty that is ridiculous (or just really big/small), show that you have noticed and thought about it, not just copied a number from your calculator and moved on.

For example, when rolling a ball down a ramp, you may not have taken in account the effects of rolling friction or the fact that some gravitational potential energy is converted into rotational kinetic energy. Both of these the move cause the overall time for the ball to roll down the ramp to increase.

6 Conclusion

The conclusion should connect to the introduction and re-state the relevance and importance of the experiment. Its a nice touch to sometimes make historical connections in this part of the report as well. It is always good to end on a note stating the importance of your findings, the connections to other topics in physics and science, and opportunities for future

extensions/research/experiments in the subject. Remember to report your results with correct units and uncertainties, for example $g = 9.7 \pm 0.2 m \cdot s^{-2}$.

References

[1] Cite your first source here

[2] Cite another source