Geometric Deep Learning

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1 Lab 5

1.1 Motivation

It is considered a problem of constructing a phase trajectory of a video file using tensor-based singular spectrum analysis method. Usually, phase trajectory of a time series is constructed using SSA method that is applied to a matrix with two indices [1, 2]. In this work it is proposed to apply tensor-based SSA method that constructs a trajectory matrix that has 3 indices, then apply HOSVD method for extracting principal components.

1.2 Problem statement

Given a sequence of images that composes a gray-scale video. Let us consider this as a time series system. Each time series is a sequence of a particular pixel brightness:

$$F^{(k)} = (f_j^{(k)})_{j=0}^{N-1}, \quad k = 1, \dots, hw,$$

where h is the video height, w is the video width, hw is a number of signals with length N.

The purpose is to construct a phase trajectory of the time series system and make a forecast $\hat{\mathbf{X}}'$ using tensor-based SSA method and HOSVD.

1.3 Problem solution

It is proposed to solve a problem with tensor-based singular spectrum analysis and make a forecast using HOSVD.

1st step: Embedding

Let L be a window length, $1 < L < N$. The embedding procedure forms $K = N - L + 1$ lagged vectors for every time series:

$$X_l^{(k)} = (f_{l-1}^{(k)}, \dots, f_{l+L-2}^{(k)})^\top.$$

The trajectory matrix of the time series system $F^{(1)}, \dots, F^{(hw)}$ is a tensor that has a form

$$\hat{\mathbf{X}} = (\mathbf{X}^{(1)}, \dots, \mathbf{X}^{(hw)}),$$

where $\mathbf{X}^{(k)} = (X_l^{(k)})_{l=0}^{K}$.

2nd step: HOSVD

Truncated HOSVD is performed:

$$\hat{\mathbf{X}} = \mathbf{S} \times_1 U_1 \times_2 U_2 \times_3 U_3,$$

where $\mathbf{S}$ is a core tensor, U_1, U_2, U_3 are matrices with unitary columns containing a basis of the left singular vectors corresponding to the nonzero singular values of the standard factor-k flattening $X_{(k)}$ of X for $k = 1, 2, 3$.

The HOSVD of $\hat{\mathbf{X}}$ can be represent as

$$\hat{\mathbf{X}} = \hat{\mathbf{X}}_1 + \dots + \hat{\mathbf{X}}_d,$$

where $\hat{\mathbf{X}}_i$ has rank equal to 1.

3rd step: Grouping

The grouping procedure partitions the set of indices $\{1, \ldots, d\}$ into m disjoint subsets $I_1, \ldots, I_m$. Let $I = i_1, \ldots, i_p$. Then the resultant matrix $\hat{\mathbf{X}}$ corresponding to the group I is defined as $\hat{\mathbf{X}}_I = \hat{\mathbf{X}}_{i_1} + \cdots + \hat{\mathbf{X}}_{i_p}$. Thus, we have the grouped decomposition:

$$\hat{\mathbf{X}} = \hat{\mathbf{X}}_{I_1} + \cdots + \hat{\mathbf{X}}_{I_m}.$$

4th step: Diagonal averaging

The last step is in a sense opposite to the first step and transforms each matrix of the grouped decomposition into a system of new (reconstructed) series of length N by hankelization-like procedure.

1.4 Code analysis

Code for HOSVD decomposition was taken from GitHub repository[1]. Code of the computational experiment can be found in the repository[2].

1.5 Experiment

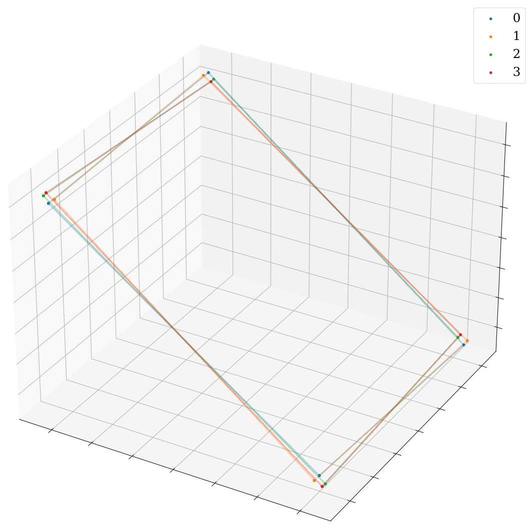

Рис. 1: Some description

Fig. 1 plots phase trajectories of 4 pixels of the basic video.

Fig. 1 plots phase trajectories of 1000 pixels of the video.

[1]https://github.com/hottbox/hottbox

[2]https://github.com/gorpinich-m/Math-methods-of-forecasting

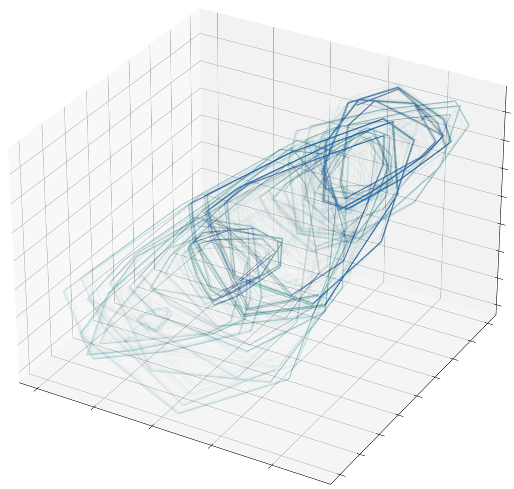

Рис. 2: Some description

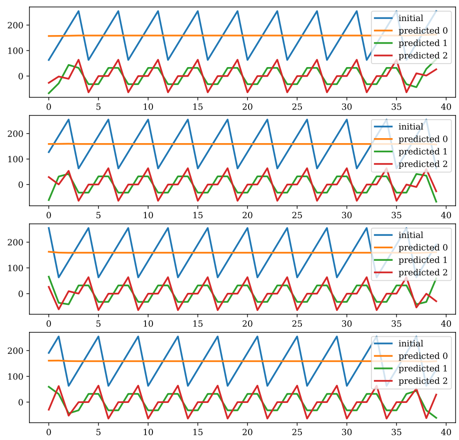

Рис. 3: Some description

Fig. 3 plots SSA decomposition of the basic video. We can see that the first component is a trend and other 2 components depict periodicity ans seasonality.

Fig. 4 plots SSA decomposition of the more complex video.

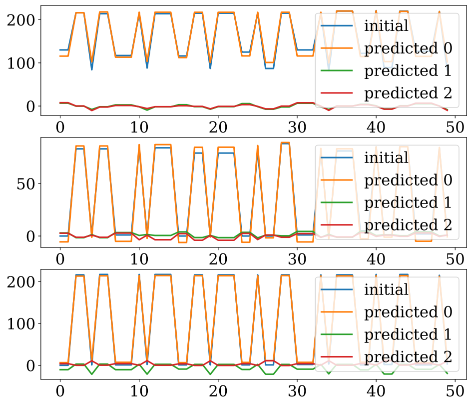

Рис. 4: Some description

Список литературы

[1] N. Golyandina и D. Stepanov. “SSA-based approaches to analysis and forecast of multidimensional time series”. В: *Proceedings of the 5th St.Petersburg Workshop on Simulation (2005)* (2005), с. 293—298.

[2] Карина Равилевна Усманова и др. “Аппроксимация фазовой траектории квазипериодических сигналов методом сферической регрессии”. В: *Вестник Московского университета. Серия 15: Вычислительная математика и кибернетика* 4 (2020), с. 40—46.