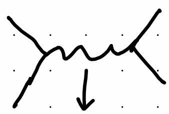
UNIVERSALITY

if so one could think of building a kind of gauge theory like QED but with massive vectors. → "opening up" the vertex

to have universality

$e^+ e^- \to \mu^+ \mu^-$



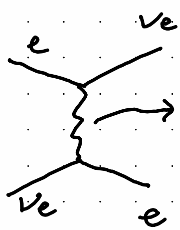
$\frac{1}{k^2}$

propagator $\frac{i}{p^2 - M^2}$ → if $p^2 \ll M^2 \to \frac{1}{M^2}$; if $p^2 \gg M^2 \to \frac{1}{p^2}$

⇓

the Fermi theory is the low limit energy of the real theory

$e^- + \nu_e \to e^- + \nu_e$



$W^{+/-}$ to have charge conservation

So now we would couple a leptonic CHARGED current

$$\mathcal{L}_{INT} = (J_\mu W^\mu + h.c.)\, g$$

W^μ must be complex field (to be charged)

$$\mathcal{L} = \mathcal{L}_{DIRAC} + \mathcal{L}_{INT} + \mathcal{L}_{KIN} + \mathcal{L}_{MASS}$$

$\hookrightarrow m^2 W^+_\mu W^-_\mu$

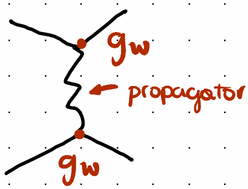
we need to impose coherence of the full UV theory with the low energy EFT one the mass term could even be ok if the gauge interaction would be abelian.

W + Higgs

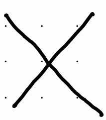
W massive

FERMI

Using a Higgs model we could give the mass. If the gauge interaction is non-abelian then the only way is through SSB and Higgs way.



$$g_W^2 \frac{i}{q^2 - M_W^2} \underset{|q^2| << M_W^2}{\sim} -\frac{g_W^2}{M_W^2}$$



$$-\frac{G_F}{\sqrt{2}} \bar{\psi} \gamma^\mu (1-\gamma_5) \psi \cdot \bar{\psi} \gamma^\mu (1-\gamma_5) \psi$$

WE MATCH THE TWO THEORIES

$$\frac{g_W^2}{M_W^2} \simeq \frac{G_F}{\sqrt{2}}$$

at low energy there is no way of measuring just g_W or M_W

let's see what we would get for g_W

$$g_W = \left(\frac{G_F}{\sqrt{2}} M_W^2 \right)^{1/2} \simeq \left[\frac{1.166 \cdot 10^{-5} \ 6.4 \cdot 10^3}{2} \right]^{1/2} =$$

$$= \sqrt{3.6 \cdot 10^{-1}} \approx 0.2$$

compare this with

$$\alpha = \frac{1}{137} = \frac{e^2}{4\pi} \rightarrow \sqrt{\frac{4\pi}{137}} = e \sim \sqrt{0.1} = 0.3$$

<u>AMAZING</u>! SIMILAR TO WEAK INTERACTIONS

VERY SUGGESTIVE COINCIDENCE!

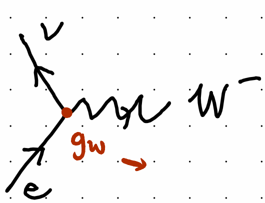
MAYBE THE WEAK INTERACTIONS ARE "WEAK" NOT BECAUSE OF THE GAUGE COUPLING BUT BECAUSE THEY ARE SB, THE LARGE WEAK BOSON MASS MAKES THEM WEAKER THAN QED AND SHORT RANGE!

NOW THE ISSUE IS

W^+, W^- ? $[U(1)]^2$ or $SU(2)$

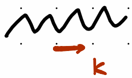
but if $SU(2)$ then one has a third Boson! → $SU(2)$ has 3 generators

Let's first see what happens for $\nu e \rightarrow \nu e$.



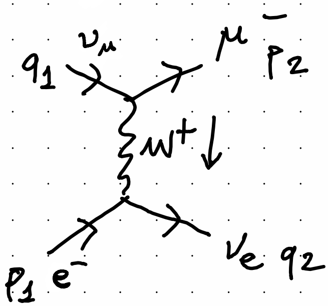
$$-i\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{1/2} \gamma_\mu (1-\gamma_5)$$

Then the intermediate vector would have



$$\frac{-i\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{M^2}\right)}{q^2-M^2}$$

propagator of a massive gauge boson



$$k = p_2 - q_1 = p_1 - q_2$$

and now we try to compute the full amplitude

$$i\mathcal{M} = \frac{i G_F M_W^2}{\sqrt{2}} \bar{u}(q_2)\gamma^\mu(1-\gamma_5)u(p_1) \cdot \frac{-i\left(g^{\mu\nu} - k_\mu k_\nu / M_W^2\right)}{k^2-M_W^2} \cdot \bar{u}(p_2)\gamma^\nu(1-\gamma_5)u(q_1)$$

The contribution from $k_\mu k_\nu$ is suppressed by the mass of the charged fermions $\Rightarrow$ neglect it.
(masses are 0)

$$= \frac{G_F M_W^2}{\sqrt{2}(k^2-M_W^2)}\left\{\bar{u}(q_2)\gamma^\mu(1-\gamma_5)u(p_1) \cdot \bar{u}(p_2)\gamma_\mu(1-\gamma_5)u(q_1)\right\}$$

THIS IS THE SAME AS BEFORE

we neglect m_μ and m_e with respect to M_W and E.

$$\frac{d\sigma}{d\Omega} \sim \frac{G_F^2 \, s}{2\pi^2} \frac{1}{\left[1 + s/2\,(1-\cos\theta)/M_W^2\right]^2}$$

result of the Fermi theory with a correction $\longrightarrow$ if $s \ll M_W^2$ we go back to the 4 fermion interaction

there is an extra $k^2 = (p_1 - q_2)^2 = t = -\frac{s}{2}(1-\cos\theta)$

$$\sigma = \frac{G_F^2 \, s}{\pi} \frac{1}{1 + s/M_W^2}$$

which for $m_e, m_\mu \ll E \ll M_W$ $\qquad 2mE = s$

$$\sigma = \frac{G_F^2}{\pi} s \left(1 - \frac{s}{M_W^2}\right) \sim \frac{G_F^2 s}{\pi}$$

for $E \gg M_W$

$$\sigma \sim \frac{G_F^2 M_W^2}{\pi}$$

IT DOES NOT GROW WITH ENERGY!

the correction acts like a suppression of σ at large energies and everything works

THIS IS OK!

So is this good enough?

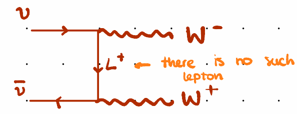
W^+, W^- with a mass.

Let's what happens if we calculate

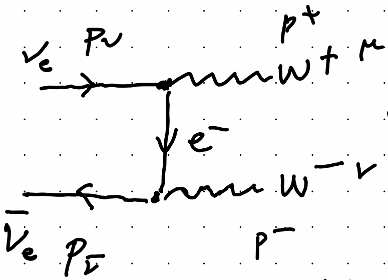
→ spoiler : unitarity violation

$\nu_e \bar{\nu}_e \to W^+ W^-$

i cannot have any other diagram



something more with photons ?



$\downarrow P = p_\nu - p^+ = p^- - p_{\bar{\nu}}$

How does this amplitude behave at High energies?

We can study the longitudinal polarizations

the W has 3 polarizations, 2 transversal and 1 longitudinal

if $p^\mu = (E, \vec{p})$

$$\epsilon_L^\mu = \frac{(|\vec{p}|, E\hat{p})}{M_W} \overset{\text{for } E \gg M_W}{=} \frac{p_\mu}{E_W} \qquad \epsilon^\mu \cdot p_\mu = 0$$

to keep $\epsilon^\mu p_\mu = 0$

if $p_\mu = (M, 0, 0, 0)$
$\Rightarrow \epsilon^{1,2,3} = (0, \vec{\epsilon}_{1,2,3})$

This polarization grows with energy.

Note that since $m_\nu = 0$ $m_e \neq 0$ gauge invariance must be (spont.) broken.

$$iM = \frac{G_F M_W^2}{\sqrt{2}} \bar{v}(p_{\bar{\nu}}) \gamma^\nu (1-\gamma_5) \frac{i(\not{P}+m)}{P^2 - m^2} \gamma_\mu (1-\gamma_5) u(p_\nu)$$

$$\epsilon_+^{*\mu}(p^+) \, \epsilon_-^{*\nu}(p^-)$$

$$\begin{cases} \epsilon_+^{*\mu} \sim \frac{p^+}{M_W} \\ \epsilon_-^{*\mu} \sim \frac{p^-}{M_W} \end{cases}$$

in the massless limit the longitudinal polarization disappears while the transversal ones stay → since the massless theory has no problems, any inconsistencies are related to the longitudinal polarization

$$P = p_\nu - p_+ = p_- - p_{\bar{\nu}} \quad \checkmark$$

$$\mathcal{M} = \frac{G_F M_W^2}{\sqrt{2}} \bar{v} \frac{\cancel{p}_-}{M_W} (1-\gamma_5) \frac{\cancel{p}+m}{p^2-m^2} \frac{\cancel{p}_+}{M_W} (1-\gamma_5) u$$

$$= \frac{G_F}{\sqrt{2}} \bar{v} \cancel{p}_- (1-\gamma_5) \frac{\cancel{p}+m}{p^2-m^2} \cancel{p}_+ (1-\gamma_5) u$$

$$\cancel{p}_\nu u(p_\nu) = 0$$

$$\bar{v}(p_{\bar{\nu}}) \cancel{p}_{\bar{\nu}} = 0$$

$$\begin{cases} \cancel{p}_+ \to \cancel{p}_+ - \cancel{p}_\nu = -\cancel{p} \\ p_- \to \cancel{p}_- - p_{\bar{\nu}} = \cancel{p} \end{cases}$$

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \bar{v} \cancel{p} (1-\gamma_5) \boxed{\frac{\cancel{p}\cancel{p}}{p^2}} (1-\gamma_5) u$$

$\boxed{\frac{\cancel{p}\cancel{p}}{p^2}} \to 1$

$m = 0$

$$\boxed{\mathcal{M} = i G_F \sqrt{2} \, \bar{v} \cancel{p} (1-\gamma_5) u}$$

not so good

because i have a fermion propagator

$$|M|^2 = 2 G_F^2 \, tr\left[\cancel{P}(1-\gamma_5)\cancel{p}_\nu (1+\gamma_5)\cancel{P} \, p_{\bar\nu}\right]$$

$$= 4 G_F^2 \, tr\left[(1-\gamma_5)\cancel{p}_\nu \, \cancel{P} \, \cancel{p}_{\bar\nu} \, P\right]$$

$$= 16 G_F^2 \left[2 \, p_\nu \cdot P \; p_{\bar\nu}\cdot P - p_\nu \cdot p_{\bar\nu} \, P^2\right]$$

$$p_\nu + p_{\bar\nu} = p^+ + p^- \qquad (p_\nu - p_+)^2 = (p^- - p_{\bar\nu})^2 = t$$

$$M_W^2 - 2 p_\nu \cdot p_+ = M_W^2 - 2 p^- p_{\bar\nu} = t .$$

$$= 16 G_F^2 \left[-2 \underbrace{p_\nu \cdot p^+}_{(t - M_W^2)} \cdot \underbrace{p_{\bar\nu}\cdot p^-}_{\frac{M_W^2 - t}{2}} - \frac{s}{2} t\right]$$

$$= -8 G_F^2 \left[(t - M_W^2)^2 + s \cdot t\right]$$

$$\simeq -8 G_F^2 \left[-\frac{s^2}{2}\right] \simeq 4 G_F^2 s^2$$

$$\begin{cases} s \\ t = -\frac{s}{2} \\ u = -\frac{s}{2} \end{cases}$$

$$\sigma(\nu \bar\nu \to W_L^+ W_L^-) \longrightarrow \frac{G_F^2 s}{3\pi}$$

which violates unitarity → we cannot have a theory with just $W^\pm$

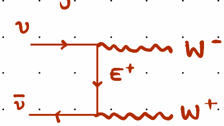
↓

2 possible solutions

↳ 1) change the matter content

add a paralepton to have (E^+, E^-, ν_e)

we get another diagram

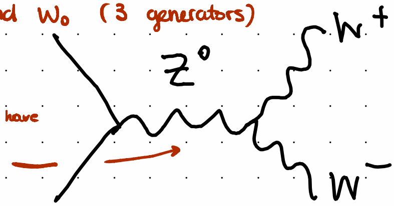


the only constrain is ← $m_E < 600$ GeV

second solution: add another boson

W^+, W^- and W_0 (3 generators)

the kinetic term in the lagrangian will have a self interaction term



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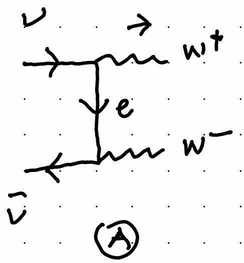
and again this additional contribution cancels the problem

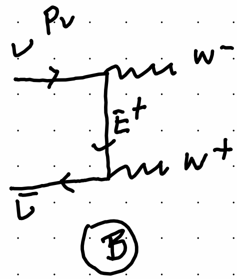
EXERCISE WITH PARALEPTON

EXERCISE: Verify that the existence of a paralepton

ie $\begin{pmatrix} E^+ \\ \nu \\ e^- \end{pmatrix}$ is enough to restore unitarity

$$p_\nu + p_{\bar\nu} = p^+ + p^-$$





$$p = p_\nu - p_+ = p^- - p_{\bar\nu}$$

$$\bar{p} = p_\nu - p_- = p^+ - p_{\bar\nu}$$

A

$$\boxed{\mathcal{M}_A = G_F\sqrt{2}\ \bar{v}\ \not{p}\,(1-\gamma_5)\, u}$$

B

$$\mathcal{M}_B = -\frac{G_F M_W^2}{\sqrt{2}}\ \bar{v}\ \frac{\not{p}_+}{M_W}\,(1-\gamma_5)\,\frac{\bar{\not{p}}+M}{\bar{p}^2-M^2}\ \frac{\not{p}_-}{M_W}\,(1-\gamma_5)\,u$$

$$= -\frac{G_F}{\sqrt{2}}\ \bar{v}\ \not{p}_+\,(1-\gamma_5)\,\frac{\bar{\not{p}}}{\bar{p}^2}\ \not{p}_-\,(1-\gamma_5)\,u$$

$$= +\frac{G_F}{\sqrt{2}}\ \bar{v}\ \bar{\not{p}}\,(1-\gamma_5)\,\frac{\bar{\not{p}}\bar{\not{p}}}{\bar{p}^2}\qquad (1-\gamma_5)\,u$$

$$\boxed{\mathcal{M}_B = G_F\sqrt{2}\ \bar{v}\ \bar{\not{p}}\,(1-\gamma_5)\ u}$$

$$\mathcal{M}_A + \mathcal{M}_B =$$

$$= G_F \sqrt{2}\; \bar{v}(p_{\bar{\nu}}) \left(\not{p} + \not{\bar{p}}\right)(1-\gamma_5)\, u(p_{\bar{\nu}})$$

$$\not{P}_{TOT}$$

$$\downarrow$$

$$\emptyset$$

IT CANCELS!

So the one diagram with W^+ and W^- leads to unitarity violations. We need to extend either the particle content or change the nature of the gauge interaction from abelian to non-abelian. We have no evidence for a PARALEPTON.

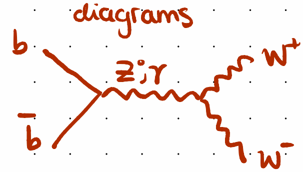
EXERCISE : W decay

COMPUTE THE DECAY WIDTH OF A W, also in the case it's polarized.

this is why all matter is in doublets

an argument like this tells us we need to see the top

we need the two diagrams



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