Macroeconometrics
Fall 2021
Problem Set 1

- The problem set is due by **Sunday, October 24th at midnight** (the night before the class in the computer room on Monday, October 25th). Please send the material by e-mail to Alireza Aghaee Shahrbabaki, alireza.aghaee@phd.unibocconi.it).
- Work in groups of maximum 3 people.
- You are expected to structure the problem set in two parts. The first is an explanatory note in which you explain point by point what you are doing, showing graphs to clarify the points you want to make etc. and adding references to the Matlab functions you are using or that you created. The second is a directory (you can zip it) in which you put all the Matlab files needed to generate your results. Please create a single Matlab file with which it is possible to generate all the results in the explanatory note. Each group must send one explanatory note and one matlab folder to Alireza.

This problem set is a Matlab warm-up.

1. Generate 500 observations from an $AR(1)$ process Y_t with $E(Y_t) = 0$, $\phi = 0.7$ and the variance of the white noise forcing term $\sigma^2 = 0.6$ using the two methods below *(hint: the random number generator for iid normal is* `randn`*)*.
a. A `for` loop using the recursive structure of the $AR(1)$.
b. Using the function `filter`.
c. Check that when the forcing variables are the same, the output of the two approaches is the same (be careful with the starting conditions and with the random number generator).

2. Focus on the `for` approach and generate data from an AR(1) with $E(Y_t) \neq 0$. What happens if the starting condition you choose is far from the unconditional mean of the process? What would you do in order to make sure that the sample path is a "proper" realization of the process you want to simulate from?

3. Generate 500 observations from a $MA(1)$ process with $\theta = 0.5$ and the variance of the white noise forcing term $\sigma^2 = 0.4$, using the two methods below.
a. With a `for` loop using the recursive structure.
b. Using the function `filter`.

4. Write a function that generates T observations from an $ARMA(p, q)$ using the `for` loop approach.

The function must have the following inputs: 1) the number of observations, 2) the variance of the white noise forcing variables, 3) the coefficients of the AR and MA polynomials or the roots of the AR and MA polynomials (*hint: you may find the* `poly` *and* `roots` *functions useful*), and the realizations of the $ARMA(p, q)$ and of the white noise as output.

5. Compute the empirical distribution of the OLS estimator in the case of an $AR(1)$ with $\phi = 0.4$ and $T = 300$ (you are free to choose the variance of the innovation).

b. Costruct a t-test for the null hypothesis $H_0 : \phi = 0$, against a two-sided alternative $H_0 : \phi \neq 0$. How often do you reject the null at the 95% confidence level?

6. Compute the empirical distribution of the OLS estimator of the $AR(1)$ coefficient in the case in which the data generating process is $MA(1)$ with $\theta = 0.4$ and $T = 250$. What is the mean of the distribution? Do the same by lengthening the sample size. Does it converge to anything as $T \to \infty$? Discuss.

7. a. Compute the empirical distribution of the OLS estimator in the case of an $AR(1)$ with $\phi = 1$ and $T = 250$ (you are free to choose the variance of the innovation).

b. Construct a t-test for the null hypothesis $H_0 : \rho = \phi - 1 = 0,$ in a test regression : $\Delta y_t = \alpha + \rho y_{t-1} + \varepsilon_t$, against a two-sided alternative $H_0 : \rho \neq 0$. How often do you reject the null hypothesis at the 95% confidence level? Is the distribution symmetric? Discuss.

c. Compute now few percentiles of the empirical distribution of the t-test you generated at point b. and check that they are close to those simulated by Dickey and Fuller. (*hint: you can find additional details in Enders*).

8. a. Perform a Monte-Carlo in the case of a random walk with drift and $T = 250$ to study the performance of the Dickey-Fuller test.

b. Construct an F-test for the null hypothesis H_0 : there is unit root, against the alternative H_1 : there is no unit root. How often do you reject the null when it is true?

c. Generate now data from a deterministic time trend and perform a DF test. How often do you reject the null? (*hint: you can find additional details in Enders*).