

# Networks

## An Introduction

M. E. J. Newman

*University of Michigan*  
*and*  
*Santa Fe Institute*

**OXFORD**  
UNIVERSITY PRESS

# CONTENTS

<b>Preface</b>	<b>x</b>
<b>1 Introduction</b>	<b>1</b>
<b>I The empirical study of networks</b>	<b>15</b>
<b>2 Technological networks</b>	<b>17</b>
2.1 The Internet .....	18
2.2 The telephone network .....	28
2.3 Power grids .....	31
2.4 Transportation networks .....	32
2.5 Delivery and distribution networks .....	33
<b>3 Social networks</b>	<b>36</b>
3.1 The empirical study of social networks .....	36
3.2 Interviews and questionnaires .....	39
3.3 Direct observation .....	46
3.4 Data from archival or third-party records .....	47
3.5 Affiliation networks .....	53
3.6 The small-world experiment .....	54
3.7 Snowball sampling, contact tracing, and random walks	58
<b>4 Networks of information</b>	<b>63</b>
4.1 The World Wide Web .....	63
4.2 Citation networks .....	67
4.3 Other information networks .....	72
<b>5 Biological networks</b>	<b>78</b>
5.1 Biochemical networks .....	78
5.2 Neural networks .....	94
5.3 Ecological networks .....	99

<b>II</b>	<b>Fundamentals of network theory</b>	<b>107</b>
<b>6</b>	<b>Mathematics of networks</b>	<b>109</b>
6.1	Networks and their representation .....	109
6.2	The adjacency matrix .....	110
6.3	Weighted networks .....	112
6.4	Directed networks .....	114
6.5	Hypergraphs .....	122
6.6	Bipartite networks .....	123
6.7	Trees .....	127
6.8	Planar networks .....	129
6.9	Degree .....	133
6.10	Paths .....	136
6.11	Components .....	142
6.12	Independent paths, connectivity, and cut sets .....	145
6.13	The graph Laplacian .....	152
6.14	Random walks .....	157
<b>7</b>	<b>Measures and metrics</b>	<b>168</b>
7.1	Degree centrality .....	168
7.2	Eigenvector centrality .....	169
7.3	Katz centrality .....	172
7.4	PageRank .....	175
7.5	Hubs and authorities .....	178
7.6	Closeness centrality .....	181
7.7	Betweenness centrality .....	185
7.8	Groups of vertices .....	193
7.9	Transitivity .....	198
7.10	Reciprocity .....	204
7.11	Signed edges and structural balance .....	206
7.12	Similarity .....	211
7.13	Homophily and assortative mixing .....	220
<b>8</b>	<b>The large-scale structure of networks</b>	<b>235</b>
8.1	Components .....	235
8.2	Shortest paths and the small-world effect .....	241
8.3	Degree distributions .....	243
8.4	Power laws and scale-free networks .....	247
8.5	Distributions of other centrality measures .....	261
8.6	Clustering coefficients .....	262

8.7	Assortative mixing .....	266
<b>III Computer algorithms</b>		<b>273</b>
<b>9</b>	<b>Basic concepts of algorithms</b>	<b>275</b>
9.1	Running time and computational complexity .....	278
9.2	Storing network data .....	282
9.3	The adjacency matrix .....	283
9.4	The adjacency list .....	286
9.5	Trees .....	290
9.6	Other network representations .....	298
9.7	Heaps .....	301
<b>10</b>	<b>Fundamental network algorithms</b>	<b>308</b>
10.1	Algorithms for degrees and degree distributions .....	308
10.2	Clustering coefficients .....	310
10.3	Shortest paths and breadth-first search .....	315
10.4	Shortest paths in networks with varying edge lengths .....	329
10.5	Maximum flows and minimum cuts .....	333
<b>11</b>	<b>Matrix algorithms and graph partitioning</b>	<b>345</b>
11.1	Leading eigenvectors and eigenvector centrality .....	345
11.2	Dividing networks into clusters .....	354
11.3	Graph partitioning .....	358
11.4	The Kernighan—Lin algorithm .....	360
11.5	Spectral partitioning .....	364
11.6	Community detection .....	371
11.7	Simple modularity maximization .....	373
11.8	Spectral modularity maximization .....	375
11.9	Division into more than two groups .....	378
11.10	Other modularity maximization methods .....	380
11.11	Other algorithms for community detection .....	382
<b>IV Network models</b>		<b>395</b>
<b>12</b>	<b>Random graphs</b>	<b>397</b>
12.1	Random graphs .....	398
12.2	Mean number of edges and mean degree .....	400
12.3	Degree distribution .....	401

## CONTENTS

12.4	Clustering coefficient	402
12.5	Giant component	403
12.6	Small components	408
12.7	Path lengths	419
12.8	Problems with the random graph .....	423
13	Random graphs with general degree distributions	428
13.1	Generating functions	429
13.2	The configuration model	434
13.3	Excess degree distribution	445
13.4	Clustering coefficient	449
13.5	Generating functions for degree distributions .....	450
13.6	Number of second neighbors of a vertex .....	451
13.7	Generating functions for the small components .....	456
13.8	Giant component	460
13.9	Size distribution for small components .....	465
13.10	Power-law degree distributions .....	470
13.11	Directed random graphs .....	473
14	Models of network formation	486
14.1	Preferential attachment	487
14.2	The model of Barabási and Albert .....	500
14.3	Further properties of preferential attachment models .....	503
14.4	Extensions of preferential attachment models .....	514
14.5	Vertex copying models	534
14.6	Network optimization models	541
15	Other network models	552
15.1	The small-world model	552
15.2	Exponential random graphs	565
V	Processes an networks	589
16	Percolation and network resilience	591
16.1	Percolation	592
16.2	Uniform random removal of vertices .....	594
16.3	Non-uniform removal of vertices .....	609
16.4	Percolation in real-world networks .....	615
16.5	Computer algorithms for percolation .....	616

<b>17 Epidemics on networks</b>	<b>627</b>
17.1 Models of the spread of disease .....	627
17.2 The SI model .....	628
17.3 The SIR model .....	631
17.4 The SIS model .....	636
17.5 The SIRS model .....	637
17.6 Epidemic models on networks .....	639
17.7 Late-time properties of epidemics on networks .....	640
17.8 Late-time properties of the SIR model .....	642
17.9 Time-dependent properties of epidemics on networks .....	648
17.10 Time-dependent properties of the SI model .....	648
17.11 Time-dependent properties of the SIR model .....	661
17.12 Time-dependent properties of the SIS model .....	669
<b>18 Dynamical systems on networks</b>	<b>676</b>
18.1 Dynamical systems .....	677
18.2 Dynamics on networks .....	686
18.3 Dynamics with more than one variable per vertex .....	695
18.4 Synchronization .....	701
<b>19 Network search</b>	<b>705</b>
19.1 Web search .....	705
19.2 Searching distributed databases .....	709
19.3 Message passing .....	713
<b>References</b>	<b>727</b>
<b>Index</b>	<b>740</b>