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Starspots, stellar cycles and stellar flares: Lessons from solar dynamo models

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In this review, we discuss whether the present solar dynamo models can be extrapolated to explain various aspects of stellar activity. We begin with a summary of the following kinds of data for solar-like stars: (i) data pertaining to stellar cycles from Ca H/K emission over many years; (ii) X-ray data indicating hot coronal activity; (iii) starspot data (especially about giant polar spots); and (iv) data pertaining to stellar superflares. Then we describe the current status of solar dynamo modelling—giving an introduction to the flux transport dynamo model, the currently favoured model for the solar cycle. While an extrapolation of this model to solar-like stars can explain some aspects of observational data, some other aspects of the data still remain to be theoretically explained. It is not clear right now whether we need a different kind of dynamo mechanism for stars having giant starspots or producing very strong superflares.

sunspots, solar dynamo, solar cycle, stellar dynamo, stellar activity

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1 Introduction

In elementary textbooks on astrophysics [1,2], a star is usually modelled as a non-rotating, non-magnetic spherically symmetric object. It is the presence of rotation (especially differential rotation) and magnetic field that makes a real star a much more intriguing object, leading to many related phenomena which we collectively call 'stellar activity'. Although the study of stellar activity has become a thriving research field only within the last few decades, there is a long history of astronomers studying such activity of the Sun. It was Galileo [3] who first discovered the solar rotation in 1613 from the changing positions of sunspots on the solar disk. When Hale [4] discovered in 1908 that sunspots are regions of strong magnetic fields, it was the first discovery of magnetic fields in an astronomical system and ushered in a new era in astronomy with the realization that magnetic fields are ubiquitous in the astronomical universe. We now know that there is an intimate relation between the rotation and the magnetic field of a star. Rotation plays a key role in the dynamo process which presumably generates the magnetic field.

Since we can resolve the solar surface and observe the magnetic activities taking place there in considerable detail, solar astronomers have collected data for solar activity for more than a century. Large sunspots can have sizes of the order of 10000 km with magnetic field typically of strength 3000 G. Well before Hale's discovery of magnetic fields in sunspots, Schwabe [5] had noted in 1844 that the number of sunspots seen on the solar surface waxes and wanes periodically. The sunspot cycle is approximately 11 years and was recognized as the magnetic activity cycle of the Sun after Hale's discovery of magnetic fields in sunspots. A major development in solar astronomy was the realization in the 1940s that the solar corona is much hotter than the solar surface [6]. The hottest regions of the corona, where temperatures can be more than 2×10^6 K, usually are found to overlie sunspot complexes, indicating that magnetic fields play a crucial role in the heating of the corona [7]. Another dramatic manifestation of solar activity is the solar flare. First discovered by Carrington [8] in 1859, a large flare can release energy of the order of 10^{32} erg. The fact that flares occur in regions of complex magnetic fields around sunspots or decayed active regions clearly shows that a flare is also caused by the magnetic field and is another manifestation of the solar magnetic activity.

One intriguing question is whether other stars also have spots, activity cycles, coronae and flares. Since a normal star appears as an unresolved point of light even through the largest telescope, this question cannot be answered by direct observations. However, using ingenious techniques, astronomers have succeeded in gathering a large amount of information about stellar activity within the last few decades. It is found that some stars are much more magnetically active than the Sun. We have evidence for starspots much larger than the largest sunspots and stellar flares much more energetic than the most energetic solar flares.

Along with the observational study of solar activity, considerable amount of theoretical research has been carried out to understand the different manifestations of solar activity. Within the last few years, solar dynamo models have become sufficiently sophisticated and are used now to explain different aspects of solar activity. The main question we would like to discuss in the present review is whether the solar dynamo models can be extrapolated to other stars and explain their activities. As we shall see, some aspects of stellar activity can be explained readily by extrapolating solar dynamo models. However, it is not easy to explain very large starspots or very energetic flares by a straightforward extrapolation of the physics of the Sun. This raises the question whether dynamo action in some of these stars is of a qualitatively different nature from the solar dynamo. We still do not have a good answer to this question.

Let us begin with a disclaimer. The author of this review is not an expert on stellar activity and has only a limited knowledge of this subject. Still he has undertaken to write this review because he is not aware of any comprehensive review covering this increasingly important field of extrapolating solar dynamo models to explain different aspects of stellar activity. Some early monographs [9,10] had limited discussions on this subject and a review by Brun et al. [11] covered some aspects. Presumably even an incomplete and unsatisfactory survey of this important subject will be of help to many astronomers, until somebody more competent writes a better review. Within the last few years, there have been some conferences with the aim of bringing together the two communities working on solar activity and stellar activity. On the basis of his experience of attending a few such conferences (notably, IAU Symposium 273—*Physics of Sun and*

Star Spots [12]; IAU Symposium 286—*Comparative Magnetic Minima: Characterizing Quiet Times in the Sun and Stars* [13]), it became clear to the author that often there is a large communication gap between these communities. Hopefully a solar physicist not knowing much about stellar activity will get an idea from this review of solar-like activity phenomena in other stars. On the other hand, stellar astronomers not knowing much about the recent advances in solar dynamo theory will form an idea where the theoretical efforts stand now.

The next section summarizes the salient features of observational data related to stellar activity. Then sect. 3 gives an introduction to the flux transport dynamo model, the currently favoured theoretical model of the solar cycle. Afterwards we shall discuss in sect. 4 whether the flux transport dynamo model can be extrapolated to solar-like stars to model their activity. Our conclusions are summarized in sect. 5.

2 Observational data of stellar activity

The first indication that some stars are magnetically active came from observations in Calcium H/K lines. These lines form in the chromosphere somewhat above the stellar photosphere where the optical depth for these lines becomes ≈ 1. If this region has a temperature less than the photosphere, then we expect absorption lines in Ca H/K. However, if this region gets heated up due to the presence of the magnetic activity, then there can be an emission core. Research in the field of stellar activity began with the discovery by Eberhard and Schwarzschild [14] in 1913 that the spectra of some stars show emission in Ca H/K. It was later found by Wilson and Bappu [15] that the width of the Ca H/K lines is correlated with the absolute magnitude of the star—this correlation being now known as the *Wilson-Bappu effect*. Stellar chromospheric activity has been reviewed by Hall [16].

Before we start discussing the observational data in more detail, we would like to point out one important physical effect. The dynamo process which generates the magnetic activity in stars is powered by convection taking place inside the stars. So we expect the magnetic activity to be visible at the surface in those stars which have an outer shell of convection below their surfaces. This is the case for the late-type stars occupying the right side of the Hertzsprung-Russell (HR) diagram. On the other hand, the early-type stars occupying the left side of the HR diagram have convective cores. Even if the dynamo process takes place in the core, the magnetic field is unlikely to come out through the stable surrounding layers having high electrical conductivity. That is why we expect to find evidence for stellar activity primarily in the late-type stars. We shall see that this expectation is borne out by observational data.

What we present below is a very incomplete survey of a few selective aspects of stellar activity to which dynamo modellers should pay attention. The subject of asteroseismology, which may play an important role in stellar dynamo modelling in future, is outside the scope of this review (see ref. [11] for the current status of asteroseismology). We do not make any attempt of providing a comprehensive bibliography. Rather, the discussion is centred around a few key papers of historical importance.

2.1 Stellar activity cycles

If other solar-like stars also have activity cycles like the 11-year solar cycle, then we would expect the Ca H/K emission to vary with the stellar cycle. We need to monitor the Ca H/K emission from a star for several years in order to find out if the star has a cycle. In the 1960s Olin Wilson (of Wilson-Bappu effect fame) started an ambitious programme at Mount Wilson Observatory of monitoring Ca H/K emission from a large number of stars. After collecting data for several years, Wilson [17] reported the discovery of stellar cycles. The most comprehensive presentation of data from this project can be found in the paper by Baliunas et al. [18] published shortly after Wilson's death. Figure 1 is a panel from this paper showing the variation in Ca H/K emission with time for several stars. Many stars were found to have regular periods. Some stars showed more irregular variations in Ca H/K indicating the existence of grand minima as in the Sun (such as the Maunder minimum in the seventeenth century).

The Ca H/K emission averaged over a few years is a good index of a star's magnetic activity. Figure 2 taken from a paper by Vaughan and Preston [19] shows the averaged Ca H/K emission from many stars plotted against their spectral type. As we already pointed out, we expect visible manifestations of magnetic activity mainly from the late-type stars and this is what is seen in Figure 2. Very curiously, one sees a gap in Figure 2 between two bands of data points. This is known as the *Vaughan-Preston gap* and the theoretical reason behind it is still not fully understood.

If the dynamo process producing the magnetic activity depends on rotation, then we would expect more rapidly rotating stars to have stronger magnetic activity. This was established by Noyes et al. [20] in 1984. When they plotted the averaged Ca H/K data against rotation period, they found that there was statistically more Ca H/K emission from stars with shorter rotation period, but there was considerable scatter in the plot. Very intriguingly, when they plotted the averaged Ca H/K emission against the Rossby number (i.e. the ratio of the rotation period and the convective turnover time), the scatter was significantly reduced. Figure 3 reproduces a famous plot

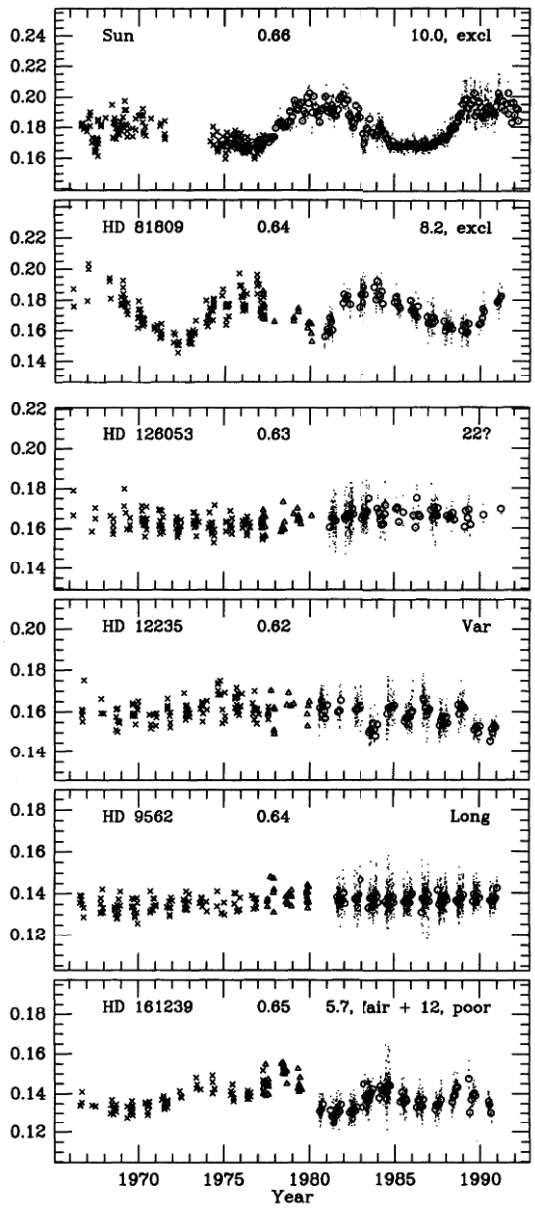

Figure 1 The variation of Ca H/K emission with time for the Sun and a few stars. From Baliunas et al. [18].

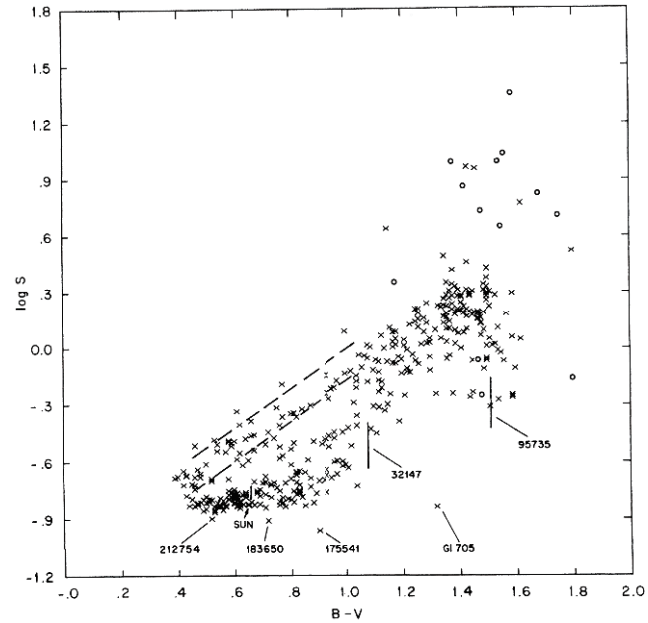

Figure 2 The averaged Ca H/K emission from many stars plotted against their spectral type. From Vaughan and Preston [19].

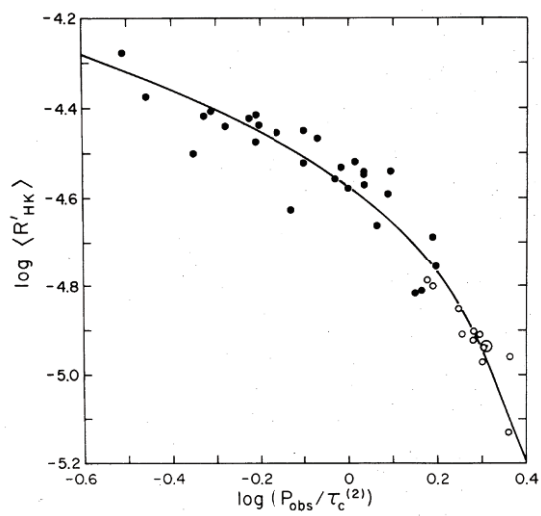

Figure 3 The averaged Ca H/K emission from many stars plotted against the Rossby number, defined as the ratio of the rotation period and the convective turnover time. From Noyes et al. [20].

from their paper. There is evidence that stellar rotations slow down with age [21]—presumably as a result of stars losing angular momentum through stellar wind, like what is believed to happen in the Sun. Though there is likely to be a spread in rotation periods when new stars are born, a longer rotation period (the right side of Figure 3) statistically implies an older star. So Figure 3 can be viewed as a plot showing that stellar activity decreases with stellar age.

If the Ca H/K emission from a star is roughly periodic, one can measure the stellar activity period and study its relation with the stellar rotation period [22,23]. Figure 4 taken from Noyes, Weiss and Vaughan [22] is a plot of inverse cycle period against the inverse of Rossby number. It is found that stars with shorter rotation periods tend to have shorter activity cycle periods. As we shall see later, this observational result has proved particularly challenging to explain with the flux transport dynamo model, although it could be explained very easily with the older $\alpha\Omega$ dynamo model [22].

2.2 Stellar coronae and X-ray emission

A hot gas having temperature of the order of a million degrees is expected to emit X-rays. The first X-ray images of the hot solar corona were obtained by Skylab in the early 1970s.

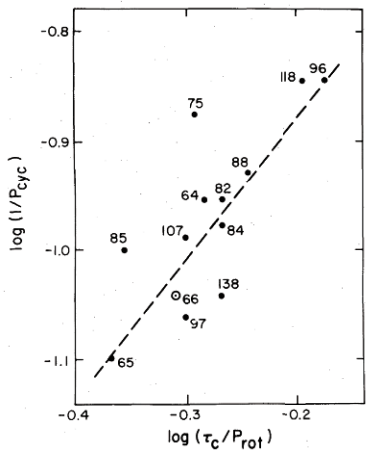

Figure 4 The plot of inverse cycle period against the inverse of Rossby number for several stars. From Noyes, Weiss and Vaughan [22].

More recent space missions like Yohkoh, SoHO, TRACE and Hinode have provided spectacular images of the solar corona obtained in X-rays or extreme ultraviolet. For other stars, we cannot expect to image their coronae. But, if they have hot coronae, then we can expect to detect X-ray emission from the stars.

The Einstein X-ray Observatory was able to detect X-ray emission from many stars: both early-type and late-type [24]. For the early-type stars, the strength of X-ray emission was found to be proportional to luminosity, suggesting that the X-ray emission from these stars was related to their overall structure and not to their magnetic activity. In the case of late-type stars, however, their X-ray brightness was found to be correlated with the Ca H/K emission [25], as seen in Figure 5. This clearly indicates that the X-ray emission in these stars is related to magnetic activity like the Ca H/K emission and presumably must be coming from the hot coronae.

Just as stars with low Rossby number (i.e. more rapidly rotating) have higher Ca H/K emission, they are expected to have more extensive coronae and stronger X-ray emission as well. This was actually found [26,27] as shown in Figure 6, where X-ray brightnesses of different late-type stars are plotted against their Rossby number. We find the data points distributed around a curve with only a modest scatter, as in the case of Figure 3, which was a similar plot with Ca H/K emission rather than X-ray emission.

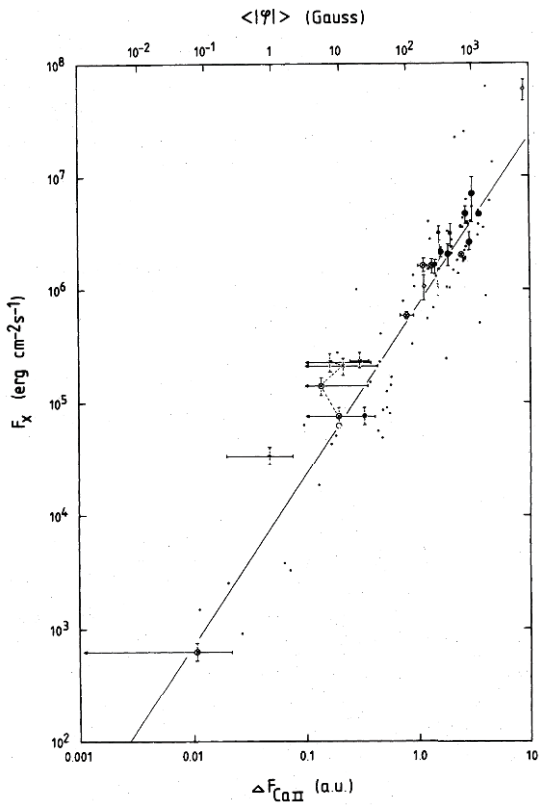

Figure 5 The correlation between the X-ray luminosity and the time-averaged emission in Ca H/K for several stars. From Schrijver, Dobson and Radick [25].

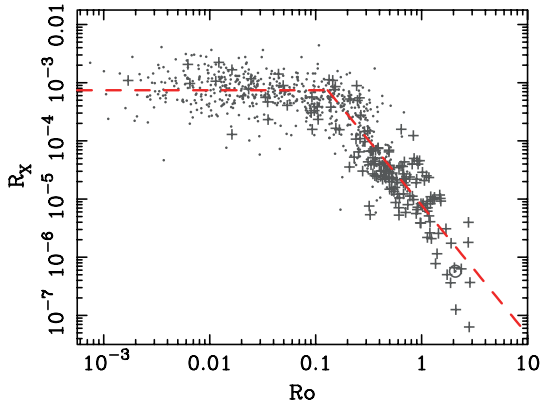

Figure 6 (Color online) The plot of X-ray luminosity against the Rossby number for several late-type stars. From Wright et al. [27].

2.3 Starspot imaging

Can we map spots on the surface of a star which we are unable to resolve? The very ingenious technique of Doppler imaging, pioneered by Vogt and Penrod [28] in the early 1980s, has now made this possible. Figure 7 explains this technique. If a star is rotating around its axis, the part moving towards us will have spectral lines blue-shifted, whereas the part moving away will have them red-shifted. Since the star is not resolved by the telescope, the net result is the broadening of the spectral line. Now suppose there is a large starspot. When it is in the part moving towards us, there is less contribution to the blue-shifted part of the spectral line. As a result,

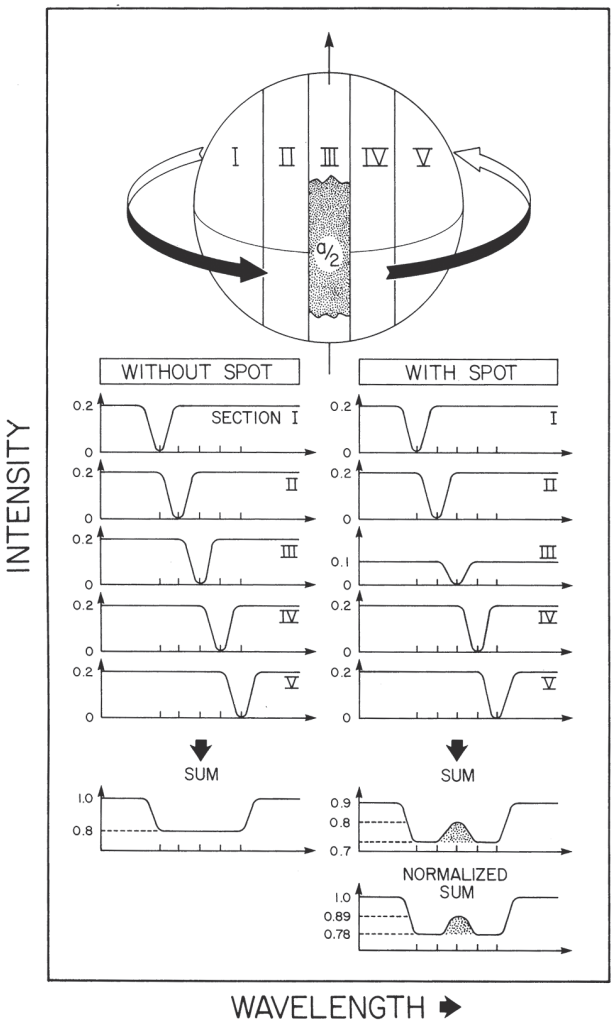

Figure 7 A schematic illustration to explain the Doppler imaging technique. The spectra from the five parts of a rotating star are indicated for the two cases: (i) when there is no spot, and (ii) when there is a spot in the middle. The spectrum of the unresolved star is the sum of the contributions from all these different parts. From Vogt and Penrod [28].

there will be a bump in the blue-shifted part of the spectral line. As the starspot moves across the surface of the star and goes to the part moving away, the bump will move towards the red-shifted part of the spectral line. By analyzing the movement of the bump across the spectral line, it is possible to figure out the size and the location (i.e. the latitude) of the starspot.

Through this Doppler imaging technique, many stars have been found to have very large starspots. What is more remarkable, these giant starspots are often found in the polar regions of stars—especially in the case of rapidly rotating stars. Figure 8 shows a giant polar starspot mapped by Strassmeier [29]. We shall later discuss the possible reasons behind these starspots appearing near the poles, in contrast to sunspots which usually appear within 40° of the solar equator. The field of observational study of starspots has really

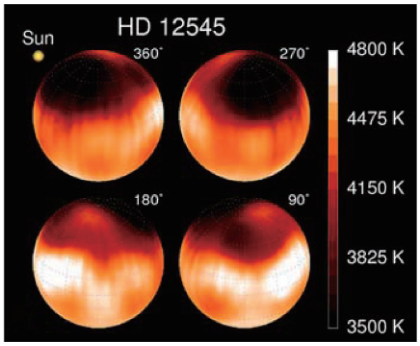

Figure 8 (Color online) A giant polar starspot mapped by the Doppler imaging technique. This is the colour version (given in ref. [33]) of the original figure from Strassmeier [29].

blossomed in the last few years. The evolution and decay of giant starspots have been studied [30]. It has also become possible to do magnetic field measurements of starspots through the Zeeman-Doppler technique [31]. The readers are referred to the comprehensive reviews by Berdyugina [32] and Strassmeier [33] for an account of the field of starspots, whereas the subject of magnetic field measurement of late-type stars is reviewed by Reiners [34].

Differential rotation plays a crucial role in the dynamo process. It has been possible to estimate the surface differential rotation of some stars from the study of starspots [35]. Another intriguing aspect of starspots which should be a challenge for dynamo modellers is the so-called 'flip-flop': the observational indication that some stars have starspots separated by 180° in longitude which alternate in strength [36,37].

2.4 Stellar flares

The most powerful solar flares release energy of the order of at most 10^{32} erg. Curiously, the first solar flare recorded by any human being, the Carrington flare of 1859 [8], has so far remained the most powerful flare recorded and presumably had an energy of such magnitude. Even at its peak, such a flare would not increase the overall brightness of the Sun by more than 1%. It will be extremely hard to detect such a flare in a distant star. Only when the stellar flare is a much more energetic superflare ($\approx 10^{33}$-10^{34} erg) and leads to an appreciable temporary increase in the luminosity of a star, we have a good chance of detecting it. Since there is no way of knowing in advance when a stellar flare is going to take place, the first detections of stellar flares were serendipitous detections when a superflare occurred while a star was being observed [38].

A systematic study of stellar flares became possible only after the launch of the Kepler mission aimed at discovering exoplanets. This mission continuously monitors the brightness of about 145000 stars in a fixed field of view. Analyz-

ing the data of this mission, Maehara et al. [39] reported the discovery of 365 superflares. Figure 9 shows the brightness variations of two stars in which superflares were seen. Initially it was thought that the occurrence of a superflare required a close binary companion, like a 'hot Jupiter' (i.e. a nearby massive planet). However, it was found that 14 of these superflares took place in slowly rotating stars, which do not have any close companions.

Although the number 365 of stellar superflares is not very large to do a completely reliable statistical analysis, one can still study the occurrence statistics. Figure 10 taken from Shibata et al. [40] shows the occurrence statistics of these superflares in the same plot with the occurrence statistics of solar flares as well as microflares and nanoflares occuring on the Sun. All these different kinds of flares seem to obey the same power law to a good approximation.

The most powerful solar flare in recent times was the 1989 flare which caused a 8-hour power blackout in Quebec (the regions around the geomagnetic pole being affected the maximum). The 1859 Carrington flare has been estimated to be about three times more powerful than the 1989 flare [41]. These flares involved energy of order 10^{32} erg. If one believes the occurrence statistics based on 14 superflares that occurred in slowly rotating solar-like stars, then a flare of energy 10^{34} erg is expected in 800 years and a flare of energy

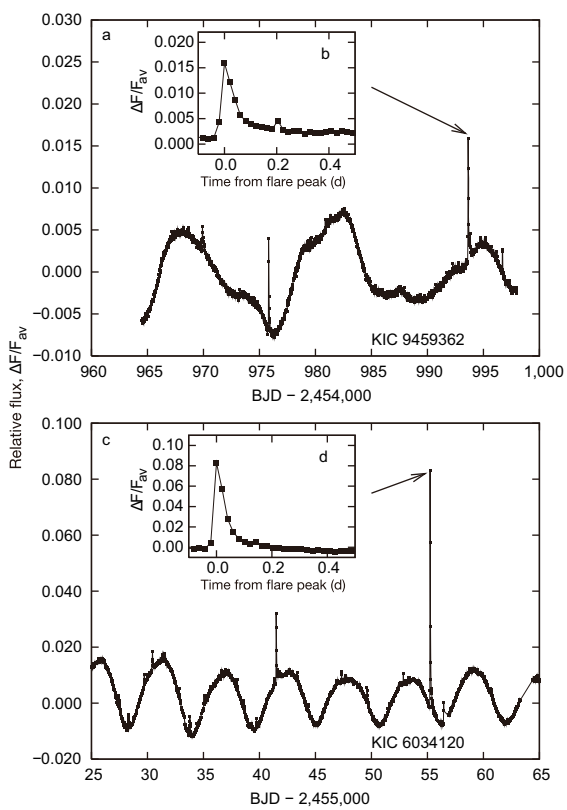

Figure 9 The brightness variations of two stars in which superflares were seen. From Maehara et al. [39].

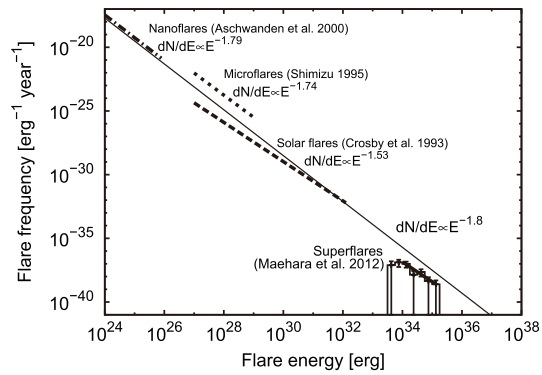

Figure 10 The frequency of various kinds of flares plotted against the flare energy. From Shibata et al. [40].

10^{35} erg in 5000 years [39]. If flares of such strength do occur and affect the Earth, it will have a disastrous effect on our current technology-dominated human civilization. Hence it is a very important question whether such superflares can occur on our Sun. We shall come back to this question later.

3 The flux transport dynamo model of the solar cycle

After summarizing the salient features of stellar magnetic activity, we now introduce the flux transport dynamo model of the solar cycle, before coming to the question of providing theoretical explanations for different aspects of stellar activity in the next section. It is the nonlinear interaction between the magnetic field and the velocity field within the solar convection zone which sustains the solar magnetic field and produces the solar cycle. One of the remarkable developments in solar physics within the last few decades has been helioseismology, which has provided a huge amount of information about large-scale flows in the solar convection zone such as the differential rotation and the meridional circulation. It is the lack of such detailed information about the flow fields inside stars which hampers the development of stellar dynamo models.

3.1 The toroidal and the poloidal magnetic fields of the Sun

Sunspots often appear in pairs at approximately the same solar latitude. When Hale et al. [42] discovered in 1919 that the two sunspots of a typical pair have opposite magnetic polarity (the polarity sense being opposite in the two hemispheres), it became clear that there must be some toroidal magnetic flux system underneath the solar surface from which magnetic strands rise to the solar surface to produce the bipolar sunspots. We, therefore, assume the sunspots to be a proxy for the toroidal field. With the discovery of the much

weaker magnetic field near the Sun's polar region by Babcock and Babcock [43], it was established that the Sun has a poloidal magnetic field as well. Now we know that the polar field appears weak (about 10 G) only in low-resolution magnetograms, but is actually concentrated inside magnetic flux bundles to strength of order 1000 G [44]. The theoretical understanding of why the magnetic field at the solar surface appears intermittent comes the study of magnetoconvection, first pioneered by Chandrasekhar [45]. Further work by Weiss [46] and others established that the interaction with convection makes magnetic field confined within flux concentrations. It is believed that the magnetic field exists in the form of magnetic flux tubes throughout the solar convection zone.

In a pathbreaking work in 1955, Parker [47] developed the scenario that the toroidal and the poloidal magnetic fields of the Sun sustain each other. Although we now believe that some important modifications of Parker's ideas are needed, the overall scenario of the toroidal and poloidal fields sustaining each other gets support from the observational data of the solar polar magnetic fields. Figure 11 shows the time variation of the magnetic fields at the two poles of the Sun, along with the sunspot number plotted below. It is seen that the sunspot number, which is an indication of the strength of the toroidal component, becomes maximum around the time when the polar field (the manifestation of the poloidal component) is close to zero. On the other hand, the polar fields are strongest when the sunspot number is close to zero.

3.2 The generation and the dynamics of the toroidal field

In order to have the kind of oscillation between the toroidal and poloidal fields seen in Figure 11, we need processes to generate the toroidal field from the poloidal field and to generate the poloidal field from the toroidal field. Since we are dealing with a high magnetic Reynolds number situation and the magnetic field is approximately frozen in the plasma (see, for example, Choudhuri [49]), differential rotation is expected to stretch out the poloidal field to produce the toroidal field. Since helioseismology has discovered that the region of strong differential rotation (known as the tachocline) is concentrated near the bottom of the convection zone [50], we

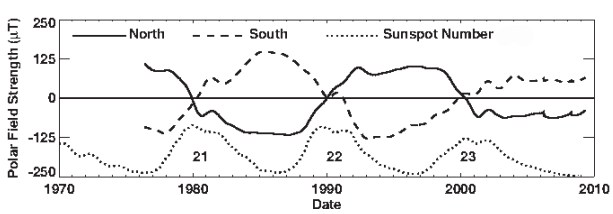

Figure 11 The evolution of the Sun's polar fields with time, with the sunspot number plotted at the bottom. From Hathaway [48].

expect the strong toroidal field to be generated there. Interaction with convection presumably keeps this toroidal field concentrated within toroidal magnetic flux tubes. Parker [51] proposed the idea of magnetic buoyancy that the magnetic pressure inside the flux tube may make it expand and cause a decrease in density, making the flux tube buoyant. It is found that the magnetic buoyancy is particularly destabilizing within the convection zone, but is suppressed to a large extent below its bottom [52,53]. Since the toroidal flux tube is created by the differential rotation of the tachocline exactly at the bottom of the convection zone, a part of it may come within the convection zone, become buoyant and rise through the convection zone to produce the bipolar sunspots, whereas other parts may remain anchored slightly below the bottom of the convection zone. In order to understand how the bipolar sunspots form, one then has to study the dynamics of the part of the flux tube that has come within the convection zone, which can be done with the help of the thin flux tube equation [54,55].

Although the two sunspots in a bipolar pair appear approximately at the same latitude, a more careful study shows that the leading sunspot is statistically found slightly closer to the solar equator. The tilts of the bipolar sunspot pairs are found to increase with latitude—a result known as Joy's law, after Hale's collaborator Joy who established this law in their pioneering study of sunspot pairs [42]. Presumably this tilt is produced by the action of the Coriolis force on the rising flux tube. Choudhuri and Gilman [56] and Choudhuri [57] were the first to study the effect of the Coriolis force on magnetic buoyancy and found that the Coriolis force plays a much more important role in this problem than recognized hitherto. If we assume the magnetic energy to be in equipartition with the kinetic energy of convection, then the magnetic field inside the flux tube at the bottom of the convection zone cannot be larger than 10^4 G. Flux tubes with such magnetic field strengths are diverted to rise parallel to the rotation axis and emerge at high latitudes. As we shall discuss later, this result is likely to have important implications for polar starspots. Only if the magnetic field inside flux tubes is as strong as 10^5 G, the flux tubes can come out radially, making the appearance of sunspots at low latitudes possible. D'Silva and Choudhuri [58] also found that they could fit Joy's law with their simulation only if the magnetic field inside the flux tubes at the bottom of the convection zone was of order 10^5 G. Soon confirmed by other authors [59,60], this result puts an important constraint on the magnetic field inside the Sun and imposes a constraint on possible dynamo mechanisms, as we shall see. Although some effects have been postulated that can suppress the Coriolis force [61,62], it is not clear if these effects would be important in the interior of the Sun and the initial magnetic field inside rising flux tubes presumably

has be of order 10^5 G in order to match observations.

3.3 The generation of the poloidal field

If we start from a poloidal field, we have discussed how the differential rotation can stretch it out to create the toroidal field in the tachocline and then how parts of this toroidal field can rise in the form of flux tubes to produce the bipolar sunspots. In order to have the magnetic cycle encapsulated in Figure 11, we now need a mechanism for producing the poloidal field from the toroidal field. The early idea due to Parker [47] and then elaborated by Steenbeck, Krause and Rädler [63] was that the turbulence within the Sun's convection zone would involve helical fluid motions due to the presence of the Coriolis force arising out of the solar rotation and that this helical turbulence would twist the toroidal field to produce the poloidal field. This mechanism, christened as the α-effect, can work only if the toroidal field is not too strong so that it can be twisted by turbulence. When the flux tube simulations of sunspot formation described in sect. 3.2 suggested that the toroidal field at the bottom of the convection zone is much stronger than the equipartition value, it became clear that the α-effect could not twist such a strong field.

Another alternative mechanism for the generation of the poloidal field was proposed by Babcock [64] and Leighton [65]. Since the two opposite-polarity sunspots in a bipolar pair form at slightly different latitudes, they pointed out that the decay of these sunspots would cause magnetic flux of opposite polarities to be spread out at slightly different latitudes, giving rise to a poloidal field. So we can view a bipolar sunspot pair as a conduit for converting the toroidal field to the poloidal field. It forms due to the buoyant rise of the toroidal field and we get the poloidal field after its decay. Now, one requirement of solar dynamo models is that we should have something like a dynamo wave propagating equatorward, in order to explain the the appearance of sunspots at increasingly lower latitudes with the progress of the cycle (as encapsulated by the well-known butterfly diagram). If the toroidal field is produced by the differential rotation as mapped by helioseismology and the poloidal field is produced by the Babcock-Leighton mechanism, then it is found that the dynamo wave would propagate poleward, creating sunspots at higher latitudes with the progress of the solar cycle. So we need something else to make the theory fit with the observations.

We believe that this additional something is provided by the meridional circulation of the Sun. It has been known for some time that there is a poleward flow of plasma at the solar surface having an amplitude of order 20 m s^{-1}. Since we do not expect the plasma to pile up at the poles, there has to be a return flow underneath the Sun's surface to bring back the plasma to the equatorial region. This meridional circulation presumably arises from turbulent stresses within the Sun's convection zone. So we expect this circulation to be confined within the convection zone and the most plausible assumption is that the return flow towards the equator is located at the bottom of the convection zone. Choudhuri, Schüssler and Dikpati [66] showed that a dynamo model with this type of meridional circulation can explain the appearance of sunspots at lower latitudes with the progress of the solar cycle. The type of dynamo model in which the poloidal field is generated by the Babcock-Leighton mechanism and the meridional circulation plays a crucial role is called the *flux transport dynamo model*. It may be mentioned that the diffuse magnetic field outside active regions migrates poleward with the solar cycle. This is believed to be caused by advection due to the poleward meridional circulation near the surface [67-70]. This behaviour of the poloidal field at the solar surface automatically comes out in flux transport dynamo models. We now summarize the basic features of this model through a cartoon.

3.4 The whole picture

Figure 12 is a cartoon encapsulating how the solar dynamo operates. If you understand this cartoon, then you would have got the central idea of the flux transport dynamo! The toroidal field is produced in the tachocline by the differential rotation stretching out the poloidal field. Then this toroidal field rises due to magnetic buoyancy to produce bipolar sunspots at the solar surface, where the poloidal field is generated by the Babcock-Leighton mechanism from these bipolar sunspots. The poloidal field so generated is carried by the meridional circulation first to the polar region and then underneath the

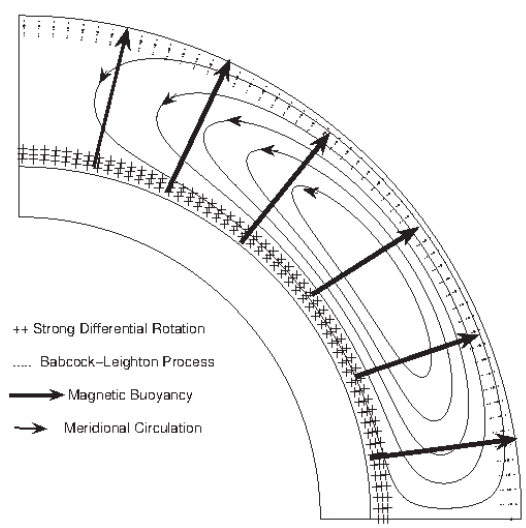

Figure 12 A schematic cartoon explaining the basic principles of the flux transport dynamo.

surface to the tachocline to be stretched by the differential rotation—thus completing the cycle. Since the meridional circulation, as indicated by the streamlines sketched in Figure 12, is equatorward at the bottom of the convection zone, the toroidal field generated there is advected equatorward, such that sunspots appear at increasingly lower latitudes with the progress of the solar cycle. Although the basic idea of the flux transport dynamo can be found in an early paper by Wang, Sheeley and Nash [71], the first 2D models of the flux transport dynamo were constructed in the mid-1990s by Choudhuri, Schüssler and Dikpati [66] and Durney [72].

So far we have avoided getting into equations. For those readers who wish to see the equations, the central equations of the flux transport dynamo theory are now shown. In spherical coordinates, we write the magnetic field as:

$$\boldsymbol{B} = B(r,\theta)\boldsymbol{e}_\phi + \nabla \times [A(r,\theta)\boldsymbol{e}_\phi], \tag{1}$$

where $B(r,\theta)$ is the toroidal component and $A(r,\theta)$ gives the poloidal component. We can write the velocity field as $\boldsymbol{v} + r\sin\theta\,\Omega(r,\theta)\boldsymbol{e}_\phi$, where $\Omega(r,\theta)$ is the angular velocity in the interior of the Sun and $\boldsymbol{v}$ is the velocity of meridional circulation having components in r and θ directions. Then the main equations telling us how the poloidal and the toroidal fields evolve with time are

$$\frac{\partial A}{\partial t} + \frac{1}{s}(\boldsymbol{v}.\nabla)(sA) = \lambda_{\mathrm{T}}\left(\nabla^2 - \frac{1}{s^2}\right)A + S(r,\theta;B), \tag{2}$$

$$\frac{\partial B}{\partial t} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rv_rB) + \frac{\partial}{\partial\theta}(v_\theta B)\right] = \lambda_{\mathrm{T}}\left(\nabla^2 - \frac{1}{s^2}\right)B + s(\boldsymbol{B}_p.\nabla)\Omega + \frac{1}{r}\frac{\mathrm{d}\lambda_{\mathrm{T}}}{\mathrm{d}r}\frac{\partial}{\partial r}(rB), \tag{3}$$

where $s = r\sin\theta$ and λ_{T} is the turbulent diffusivity inside the convection zone. The source term $S(r,\theta;B)$ in eq. (2) is responsible for the generation of the poloidal field and is often taken as $S(r,\theta;B) = \alpha B$ in many dynamo models. We should point out that eqs. (2) and (3) are mean field equations obtained by averaging over the turbulence in the convection zone and describe the mean behaviour of the average magnetic field. Since eqs. (2) and (3) are coupled partial differential equations, nothing much can be done analytically. Our research group in IISc Bangalore has developed a numerical code *Surya* for studying the flux transport dynamo problem by solving these equations [73,74][1)].

Although the flux transport dynamo has succeeded in explaining many aspects of the observational data pertaining to solar cycles, many big uncertainties remain. The model outlined above is of kinematic nature and the various flow fields have to be specified in order to construct a model. While the differential rotation has been pinned down by helioseismology [50], the nature of the meridional circulation deep down in the convection zone remains uncertain [75,76]. However, it is now realized that flux transport dynamos may work even with more complicated meridional circulations than what is indicated in Figure 12 [77]. Several comprehensive reviews [78-80] may be recommended to readers desirous of learning more about the current status of the flux transport dynamo model.

3.5 Modelling irregularities of activity cycles

The solar cycle is only approximately periodic. Not only the Sun, the other stars also show irregularities in their cycle, which is evident from Figure 1. The most notable feature of the irregularities is the grand minima, like the Maunder minimum of the Sun during 1640-1715 when sunspots were seldom seen and several cycles went missing. In the time series of Ca H/K emission from many solar-like stars, one finds evidence for grand minima.

We make a few remarks about the recent works on modelling the irregularities of the solar cycle, since these works presumably have important implications for stellar cycles. Although certain aspects of the irregularities are explained best as arising out of nonlinearities in the dynamo equations [81], the sustained irregularities of the solar cycle are more likely caused by stochastic processes [82]. The Babcock-Leighton mechanism for generating the poloidal field depends on the tilts of bipolar sunspot pairs. Although the average tilt is given by Joy's law, one finds quiet a lot of scatter around it—presumably caused by the effect of turbulence on rising flux tubes [83]. Assuming that the randomness in the Babcock-Leighton mechanism arising out of this scatter in tilt angles is the main source of irregularities of the solar cycle, Choudhuri, Chatterjee and Jiang [84] made a prediction for the strength of the present solar cycle 24 before its onset. This turned out to be the first successful prediction of a solar cycle from a theoretical model, justifying the physics used in the model [85]. One important aspect of the irregularities is the so-called Waldmeier effect: the observation that the strengths of solar cycles are anti-correlated with their rise times. It has been possible to explain this effect by invoking fluctuations in the meridional circulation [86]. The Ca H/K data presented by Baliunas et al. [18] show evidence for the Waldmeier effect in several stars, indicating that these stars also must be having flux transport dynamos inside them [86]. Choudhuri and Karak [87] developed a model of grand minima, assuming that they are produced by combined fluctuations in the Babcock-Leighton mechanism and the meridional circulation [88]. The theoretical efforts for modelling irregularities have been reviewed by Choudhuri [89].

1) The code *Surya* and a detailed guide for using it can be sent to anybody who sends a request to arnab@physics.iisc.ernet.in.

4 The extrapolation of the flux transport dynamo to stars

After summarizing the main features of the flux transport dynamo model, which has been so successful in explaining different aspects of the solar cycle, we come to the question whether this model can be extrapolated to other stars to explain various features of their magnetic activity. As should be clear from the discussions of the previous section, we need to specify the differential rotation and the meridional circulation in order to construct a model of the flux transport dynamo. In the case of the Sun, we have got quite a lot of information about these large-scale flows from helioseismology. One of the main reasons behind the success of the recent solar dynamo models is that we can use these results of helioseismology as inputs to our solar dynamo models. We do not have such data about the differential rotation and the meridional circulation inside other stars—except some data about differential rotation at the surface for a few stars [35]. So the first big hurdle for constructing models of stellar dynamos is that we have to figure out the nature of these large-scale flows inside different stars from purely theoretical considerations.

4.1 Large-scale flows inside stars

In a pioneering study in 1963, Kippenhahn [90] showed that an anisotropic viscosity can give rise to large-scale flows inside a rotating star. If the radial viscosity is larger, then that causes slower rotation at the equatorial region. In order to have faster rotation near the equator, which is the case for the Sun, we need to have larger horizontal viscosity. Within the Sun's convection zone, viscosity is mainly provided by turbulence, compared to which the molecular viscosity is negligible. The turbulent viscosity within the convection zone is certainly expected to be anisotropic due to two factors: (i) gravity makes the radial direction special; and (ii) rotation makes the polar direction special. To get a physical picture of how the large-scale flows are induced by turbulent transport mechanisms inside the convection zone of a rotating stars, see the very clear reviews by Kitchatinov [91,92] on this complex subject.

In order to compute large-scale flows inside stars, we can follow one of the two possible approaches. The first approach is to do a direct numerical simulation of convection in a rotating star from first principles. Such simulations have shown the occurrence of differential rotation and meridional circulation. The second approach is to first calculate the various components of turbulent viscosity from a mixing length theory of convective turbulence and then to use these in a mean field model of stellar hydrodynamics to compute the large-scale flows inside the stars. This second approach was pioneered by Kitchatinov and Rüdiger [93].

Once we have the large-scale flows inside a star, we can substitute these in the equations of the flux transport dynamo and obtain a model of the stellar dynamo. Jouve, Brown and Brun [94] constructed stellar dynamo models by following the first approach of computing the large-scale flows through direct numerical simulations. Constructing such models is computationally demanding and it is difficult to explore the parameter space extensively by following this approach. On the other hand, Karak, Kitchatinov and Choudhuri ([95], hereafter KKC) followed the second approach of computing the large-scale flows from the mean field model of Kitchatinov and Olemskoy [96] and then constructing stellar dynamo models. In this approach, it is possible to explore the parameter space more extensively.

4.2 Comparing stellar dynamo models with observations

Figure 13 shows the differential rotations computed by KKC [95] for stars of mass $1M_\odot$ rotating with different rotation periods. For the slowly rotating case with a period of 30 d (close to the solar rotation period of 27 d), the angular velocity is constant over cones in the convection zone, similar to what is found for the Sun from helioseismology. On the other hand, for the rapidly rotating case with a period of 1 d, the angular velocity tends to be constant over cylinders. The meridional circulation is also computed and is found to be weaker for faster rotators. It may be noted that, when the rotation is very slow and its effect on convection negligible, the radial part of viscosity becomes dominant due to the primarily up-down nature of the convective motions and the differential rotation changes over to an anti-solar pattern with slower rotation near the equator, as confirmed by recent simulations [97]. This case is not covered in Figure 13.

KKC [95] constructed dynamo models of $1M_\odot$ mass stars rotating with different angular speeds by inserting the differential rotation and the meridional circulation computed from the mean field model into the dynamo eqs. (2) and (3). In order to make sure that the dynamo solutions do not grow indefinitely, it is necessary to include a quenching. They took the source function appearing in eq. (2) to have the form

$$S(r,\theta;B) = \frac{\alpha(r,\theta)}{1+(B(r_t,\theta)/B_0)^2}B(r_t,\theta), \tag{4}$$

where $B(r_t,\theta)$ is the toroidal field at the bottom of the convection zone ($r = r_t$) and the coefficient $\alpha(r,\theta)$ is assumed to be concentrated near the surface, to account for the Babcock-Leighton mechanism in which the toroidal field from the bottom of the convection zone rises to the surface to produce a poloidal field at the surface. This coefficient, although denoted by the symbol α, has a physical origin completely different from the traditional α-effect. The quenching factor

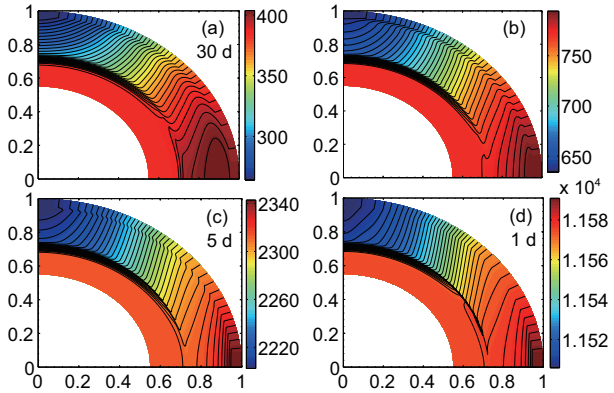

Figure 13 (Color online) The angular velocity profiles of $1M_\odot$ stars rotating with rotation periods of (a) 30 d, (b) 15 d, (c) 5 d, and (d) 1 d, as computed from a mean field model. From Karak, Kitchatinov and Choudhuri (KKC) [95].

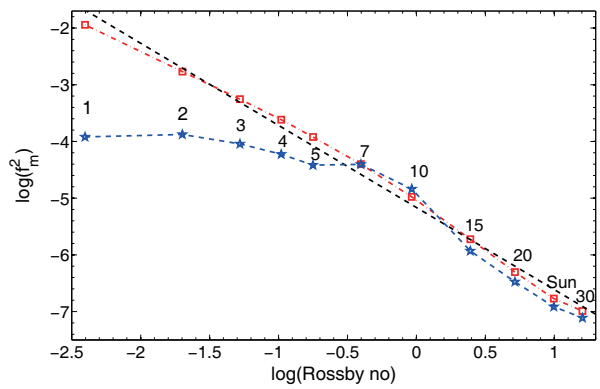

Figure 14 (Color online) The theoretically computed f_m^2 as a function of the Rossby number. The blue curve corresponds to the case in which we assume a saturation of the Babcock-Leighton mechanism for fast rotators, whereas the red curve corresponds to the case without such a saturation. From KKC [95].

appearing in the denominator ensures that the source term becomes very small when the toroidal field $B(r_t, \theta)$ is much larger than B_0. Hence the dynamo is found to saturate with $B(r_t, \theta)$ hovering around a value not much larger than B_0. The total toroidal flux through the convection zone can be written as $fB_0R_\odot^2$. KKC calculated f from their dynamo model and the mean f_m of its unsigned value averaged over the cycle was taken as a measure of the toroidal flux generated in a particular situation. Since Ca H/K or X-ray emission presumably arises from energy generated due to magnetic reconnection between two flux systems, we may naively expect these emissions to be proportional to f_m^2. Figure 14 taken from KKC shows how f_m^2 varies with the Rossby number in the theoretical model. If we assume that there is some mechanism which saturates the Babcock-Leighton mechanism for fast rotations, then we get a theoretical curve which agrees with Figure 3 (Ca H/K emission plot) or Figure 6 (X-ray emission plot) remarkably well.

While we were happy that we (KKC) could model the increase of emission with lower Rossby number, we failed to explain the observed increase of cycle period with rotation period, as indicated in Figure 4. We reproduce the theoretical plot from KKC in Figure 15, showing the increase of the cycle period with decreasing rotation period, contradicting the observational data. This results from the fact that the meridional circulation becomes weaker in faster rotating stars according to the mean field hydrodynamic model which we had used in order to calculate the meridional circulation. Since the period of the flux transport dynamo depends on the time scale of meridional circulation, a faster rotating star (with shorter rotation period) gives rise to weaker meridional circulation and therefore longer cycle period. Jouve, Brown and Brun [94], who computed the meridional circulation from direct numerical simulations, also found the same difficulty. An

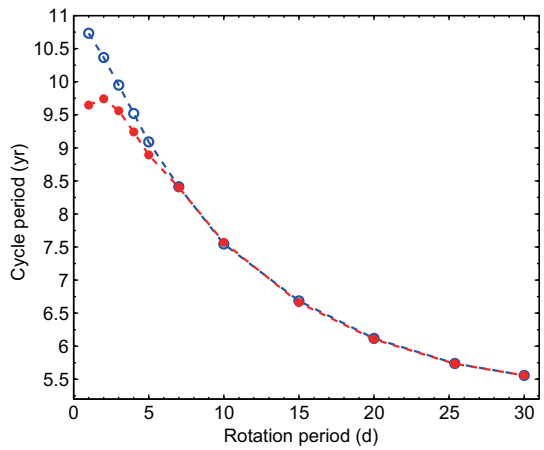

Figure 15 (Color online) The variation of the cycle period with the rotation period according to the theoretical model of KKC [95].

intriguing question is whether there is a flaw in our understanding of the meridional circulation and whether it could be stronger for more highly rotating stars, which would solve this problem. Interestingly, in the traditional $\alpha\Omega$ dynamo model, the cycle frequency goes as the square root of ($\alpha\times$ gradient of angular velocity) (see ref. [49], p. 360). While the flux transport dynamo model fails to explain the observed relation between the rotation period and the cycle period, this relation can be explained easily in the traditional $\alpha\Omega$ dynamo model on assuming that α and/or the gradient of angular velocity increase with increasing rotation frequency, as pointed out by Noyes, Weiss and Vaughan [22]. This raises the question whether the nature of the stellar dynamo changes in stars rotating very fast for which the meridional circulation will be weak and we may have an $\alpha\Omega$ dynamo instead of a flux transport dynamo. These questions remain to be addressed by future research.

It should be clear from our brief discussion of the physics of the flux transport dynamo in sect. 3 that the bottom of the convection zone plays an important role in the dynamo process. It is there that the toroidal field is produced by the strong differential rotation and then a part of it remains stored below the bottom where the magnetic buoyancy is suppressed. Now, stars having mass less than about $0.4M_\odot$ (of spectral type later than M3-3.5) are supposed to be fully convective, without a bottom below which the toroidal field can be stored. Whether the usual flux transport dynamo can operate in such a star is an important question. Recently, Wright and Drake [98] pointed out that X-ray emission from some fully convective stars satisfy the relation between X-ray luminosity and Rossby number that we see in Figure 6, suggesting that these stars also have dynamos similar to other late-type stars. Although there have been some works on dynamo action in fully convective stars [99-101], our understanding of this subject is still very incomplete.

4.3 Large sunspots and strong flares

As mentioned in sects. 2.3 and 2.4, we have evidence for starspots much larger than the largest sunspots and stellar superflares much stronger than the strongest solar flares. One important question is whether such large starspots and strong stellar flares require physical mechanisms different from what are operative in the Sun, indicating that the nature of the dynamo also may be somewhat different. It is difficult to answer this question at the present time because we have very little understanding of what determines the sizes of sunspots or fluxes in active regions. Since we believe that some amount of magnetic flux broken from the toroidal flux system stored at the bottom of the solar convection zone rises to the surface to produce sunspots and active regions, presumably their sizes depend on the nature of the instabilities that break up the toroidal flux at the bottom of the convection zone (see the review by Fan [102]). Our understanding of the storage and breakup process of the toroidal flux is very poor at present. The best we can do is to try to estimate maximum possible sizes of sunspots based on some 'reasonable' assumptions.

As we pointed out in sect. 2.3, large starspots in rapidly rotating stars are often found near the polar region. Schüssler and Solanki [103] provided an explanation for this by extrapolating the results of Choudhuri and Gilman [56], who studied the effect of the Coriolis force on magnetic buoyancy (see also ref. [104]). As discussed in sect. 3.2, Choudhuri and Gilman [56] found that, when the Coriolis force wins over magnetic buoyancy, the magnetic flux rising due to magnetic buoyancy is diverted by the Coriolis force to rise parallel to the rotation axis and emerge at high latitudes. Presumably this is what happens in rapidly rotating stars, causing the rising flux to emerge at polar latitudes to create polar starspots. Isik, Schmitt and Schüssler [105] combined a dynamo model with magnetic buoyancy calculations to study the distribution of starspots over the stellar surface.

Since we know of stellar flares much stronger than the strongest solar flares recorded so far, we now come to the question whether these require different physics or whether it is possible for such strong flares to occur on the Sun also. Since the largest known solar flares (of energy of order 10^{32} erg) have been known to cause serious disruptions in human activities (especially on electrical and electronic appliances), significantly stronger solar flares will certainly cause havoc and disrupt our lives in major ways. Figure 16 taken from Shibata et al. [40] plots the energy of flares against the spot group area (or, equivalently, magnetic flux) with which these flares have been associated. Both solar flares (lower left region of the figure) and stellar flares (upper right region) are shown. Note that only stellar flares much more energetic than solar flares can be observed by us. However, it appears from the figure that there might be a continuity between the solar flares and stellar superflares. Figure 16 seems to suggest that superflares of energy 10^{35} erg would be associated with spot groups having magnetic flux of about 10^{24} Mx, whereas the largest sunspots carry flux not more than 10^{23} Mx. Whether such superflares can occur on the Sun then basically hinges on the question whether we can have active regions with such flux. Pushing the various parameters connected with the dynamo generation of the toroidal field to their extreme values, Shibata et al. [40] concluded that this is not entirely impossible, although we are not sure whether the extreme values

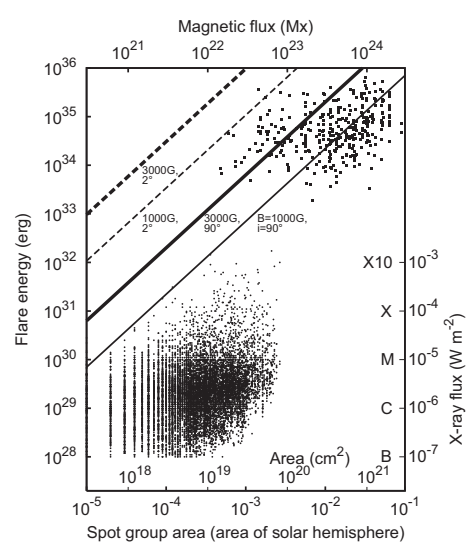

Figure 16 Flare energies plotted against areas (or equivalently magnetic fluxes) of the active regions with which the flares were associated. Both solar flares (lower left) and stellar superflares (upper right) are shown. From Shibata et al. [40].

assumed in the estimate are completely justified. If these are justified, then we have to conclude that the occurrence of such superflares may not require dynamo action qualitatively different from the solar dynamo and, at the same time, we cannot rule out such superflares on our Sun occurring with extremely low frequency—such as a superflare of energy 10^{35} erg in 5000 years as estimated by Maehara et al. [39]. It is also possible that large fluctuations in dynamo parameters may push a star to an extreme temporary phase when superflares are more likely [106].

5 Conclusion

In the last few years a huge amount of data about magnetic activity of solar-like stars have come. We have also witnessed remarkable developments in solar dynamo modelling. To the best of our knowledge, this is the first attempt to review the question whether the current solar dynamo models can be extrapolated to model magnetic activities of other solar-like stars.

Stellar magnetism is a vast field—magnetic fields playing important roles in the star formation process as well as in the final phase of stellar collapse (see refs. [107,108]). It may be mentioned that a modified version of our solar dynamo code has been used to study the role of magnetic fields in accreting neutron stars [109,110]. In this review, however, we restrict ourselves only to late-type solar-like stars in their main sequence, having convection zones just below their surfaces. Such stars are found to have activity cycles, coronae, spots and flares just like the Sun. Very intriguingly, some of them have spots much larger than sunspots and flares much stronger than solar flares.

A particularly successful model of the solar cycle is the flux transport dynamo model, in which the poloidal field is generated by the Babcock-Leighton mechanism and the meridional circulation plays an important role. A very pertinent question is whether other solar-like stars also have such flux transport dynamos operating inside them. We have seen that such dynamo models can explain the relation between the activity-related emission (in Ca H/K or in X-ray) and the Rossby number, giving us confidence that we are probably on the right track. We have pointed out that Ca H/K emission of several stars indicate the Waldmeier effect, which has been explained as arising out of the fluctuations of the meridional circulation [86]. This presumably indicates that these stars also have meridional circulation with fluctuations. However, we should keep in mind that the flux transport dynamo model failed to explain the observed relation between the rotation period and the cycle period, which could be explained easily by the earlier $\alpha\Omega$ dynamo model [22]. This raises the question whether the meridional circulation becomes very weak in rapidly rotating stars and the dynamo changes over to an $\alpha\Omega$ dynamo from a flux transport dynamo. In this connection, note also that the cycle of chromospheric Ca H/K emission may not always indicate the magnetic cycle [111]. We know that for rapidly rotating stars polar regions will dominate the activity, as the Coriolis force will deflect the magnetic flux rising due to magnetic buoyancy to polar regions [56,103]. However, we are not sure whether very large starspots and very strong stellar superflares can be explained by extrapolating solar models or some different kinds of dynamo models are needed. The accompanying question is whether we should expect to see much larger sunspots or much stronger solar flares than the ones we have so far seen. There is no doubt that many solar-like stars have flux transport dynamos like the Sun. But is this some kind of universal dynamo model that can account for magnetic activity of all stars—including those which have very large spots and very strong flares? Further research is definitely needed to answer this question.

While summarizing the flux transport dynamo model in sect. 3, we have restricted ourselves to 2D kinematic models. While these models have been very successful in explaining different aspects of solar activity, these models have obvious limitations and there are efforts under way to go beyond these simple models. For example, magnetic buoyancy leading to the Babcock-Leighton mechanism is an inherently 3D process and cannot be captured realistically in 2D models [112]. There are now attempts of constructing 3D kinematic models [113-115]. However, we ultimately need to go beyond kinematic models and develop fully dynamical 3D models. Some exploratory studies have produced striking initial results [116,117]. There is no doubt that such developments will have a big impact on stellar dynamo research in future and this will remain a very active research field for years to come.

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1 D. Maoz, *Astrophysics in a Nutshell* (Princeton University Press, Princeton, 2007).
2 A. R. Choudhuri, *Astrophysics for Physicists* (Cambridge University Press, Cambridge, 2010).
3 G. Galileo, *History and Demonstration Concerning Sunspots* (work based on the Three Letters on Sunspots, Roma, 1613).
4 G. E. Hale, Astrophys. J. **28**, 315 (1908).
5 S. H. Schwabe, Astron. Nachr. **21**, 233 (1844).
6 B. Edlén, Zeits. f. Astroph. **22**, 30 (1943).
7 B. C. Low, Sci. China-Phys. Mech. Astron. **58**, 015201 (2015).
8 R. C. Carrington, Mon. Not. R. Astron. Soc. **20**, 13 (1859).
9 C. J. Schrijver, and C. Zwaan, *Solar and Stellar Magnetic Activity*

(Cambridge University Press, Cambridge, 2000).
10 J. H. Thomas, and N. O. Weiss, *Sunspots and Starspots* (Cambridge University Press, Cambridge, 2008).
11 A. S. Brun, R. A. García, G. Houdek, D. Nandy, and M. Pinsonneault, Space Sci. Rev. **196**, 303 (2015).
12 D. P. Choudhary, and K. G. Strassmeier, *Physics of Sun and Star Spots—IAU Symposium 273* (Cambridge University Press, Cambridge, 2011).
13 C. H. Mandrini, and D. F. Webb, *Comparative Magnetc Minima: Characterizing Quiet Times in the Sun and Stars—IAU Symposium 286* (Cambridge University Press, Cambridge, 2012).
14 G. Eberhard, and K. Schwarzschild, Astrophys. J. **38**, 292 (1913).
15 O. C. Wilson, and M. K. V. Bappu, Astrophys. J. **125**, 661 (1957).
16 J. C. Hall, Living Rev. Solar Phys. **5**, 2 (2008).
17 O. C. Wilson, Astrophys. J. **226**, 379 (1976).
18 S. L. Baliunas, R. A. Donahue, W. H. Soon, J. H. Horne, J. Frazer, L. Woodard-Eklund, M. Bradford, L. M. Rao, O. C. Wilson, Q. Zhang, W. Bennett, J. Briggs, S. M. Carroll, D. K. Duncan, D. Figueroa, H. H. Lanning, T. Misch, J. Mueller, R. W. Noyes, D. Poppe, A. C. Porter, C. R. Robinson, J. Russell, J. C. Shelton, T. Soyumer, A. H. Vaughan, and J. H. Whitney, Astrophys. J. **438**, 269 (1995).
19 A. H. Vaughan, and G. W. Preston, Publ. Astron. Soc. Pacific **92**, 385 (1980).
20 R. W. Noyes, L. W. Hartmann, S. L. Baliunas, D. K. Duncan, and A. H. Vaughan, Astrophys. J. **279**, 763 (1984).
21 A. Skumanich, Astrophys. J. **171**, 565 (1972).
22 R. W. Noyes, N. O. Weiss, and A. H. Vaughan, Astrophys. J. **287**, 769 (1984).
23 S. H. Saar, and A. Brandenburg, Astrophys. J. **524**, 295 (1999).
24 R. Pallavicini, L. Golub, R. Rosner, G. S. Vaiana, T. Ayres, and J. L. Linsky, Astrophys. J. **248**, 279 (1981).
25 C. J. Schrijver, A. K. Dobson, and R. R. Radick, Astrophys. J. **258**, 432 (1992).
26 N. Pizzolato, A. Maggio, G. Micela, S. Sciortino, and P. Ventura, Astron. Astrophys. **397**, 147 (2003).
27 N. J. Wright, J. J. Drake, E. E. Mamajek, and G. W. Henry, Astrophys. J. **743**, 48 (2011).
28 S. S. Vogt, and G. D. Penrod, Publ. Astron. Soc. Pacific **95**, 565 (1983).
29 K. G. Strassmeier, Astron. Astrophys. **347**, 225 (1999).
30 A. Künstler, T. A. Carroll, and K. G. Strassmeier, Astron. Astrophys. **578**, 101 (2015).
31 J.-F. Donati, M. Semel, B. D. Carter, D. E. Rees, and A. Collier Cameron, Mon. Not. R. Astron. Soc. **291**, 658 (1997).
32 S. V. Berdyugina, Living Rev. Solar Phys. **2**, 8 (2005).
33 K. G. Strassmeier, Ann. Rev. Astron. Astrophys. **17**, 251 (2009).
34 A. Reiners, Living Rev. Solar Phys. **8**, 1 (2012).
35 J. R. Barnes, A. Collier Cameron, J.-F. Donati, D. J. James, S. C. Marsden, and P. Petit, Mon. Not. R. Astron. Soc. **357**, L1 (2005).
36 S. V. Berdyugina, and I. Tuominen, Astron. Astrophys. **336**, L25 (1998).
37 D. Elstner, and H. Korhonen, Astron. Nachr. **326**, 278 (2005).
38 B. E. Schaefer, J. R. King, and C. P. Deliyannis, Astrophys. J. **529**, 1026 (2000).
39 H. Maehara, T. Shibayama, S. Notsu, Y. Notsu, T. Nagao, S. Kusaba, S. Honda, D. Nogami, and K. Shibata, Nature **485**, 478 (2012).
40 K. Shibata, H. Isobe, A. Hillier, A. R. Choudhuri, H. Maehara, T. T. Ishii, T. Shibayama, S. Notsu, Y. Notsu, T. Nagao, S. Honda, and D. Nogami, Publ. Astron. Soc. Japan **65**, 49 (2013).
41 B. T. Tsurutani, W. D. Gonzalez, G. S. Lakhina, and S. Alex, J. Geophys. Res. A **108**, 1268 (2003).
42 G. E. Hale, F. Ellerman, S. B. Nicholson, and A. H. Joy, Astrophys. J. **49**, 153 (1919).
43 H. W. Babcock, and H. D. Babcock, Astrophys. J. **121**, 349 (1955).
44 S. Tsuneta, K. Ichimoto, Y. Katsukawa, B. W. Lites, K. Matsuzaki, S. Nagata, D. Orozco Suárez, T. Shimizu, M. Shimojo, R. A. Shine, Y. Suematsu, T. K. Suzuki, T. D. Tarbell, and A. M. Title, Astrophys. J. **688**, 1374 (2008).
45 S. Chandrasekhar, Phil. Mag. **43**, 501 (1952).
46 N. O. Weiss, J. Fluid Mech. **108**, 247 (1981).
47 E. N. Parker, Astrophys. J. **122**, 293 (1955).
48 D. H. Hathaway, Living Rev. Solar Phys. **7**, 1 (2010).
49 A. R. Choudhuri, *The Physics of Fluids and Plasmas: An Introduction for Astrophysicists* (Cambridge University Press, Cambridge, 1998).
50 J. Schou, H. M. Antia, S. Basu, R. S. Bogart, R. I. Bush, S. M. Chitre, J. Christensen-Dalsgaard, M. P. Di Mauro, W. A. Dziembowski, A. Eff-Darwich, D. O. Gough, D. A. Haber, J. T. Hoeksema, R. Howe, S. G. Korzennik, A. G. Kosovichev, R. M. Larsen, F. P. Pijpers, P. H. Scherrer, T. Sekii, T. D. Tarbell, A. M. Title, M. J. Thompson, and J. Toomre, Astrophys. J. **505**, 390 (1998).
51 E. N. Parker, Astrophys. J. **121**, 491 (1955).
52 E. N. Parker, Astrophys. J. **198**, 205 (1975).
53 F. Moreno-Insertis, Astron. Astrophys. **122**, 241 (1983).
54 H. C. Spruit, Astron. Astrophys. **98**, 155 (1981).
55 A. R. Choudhuri, Astron. Astrophys. **239**, 335 (1990).
56 A. R. Choudhuri, and P. A. Gilman, Astrophys. J. **316**, 788 (1987).
57 A. R. Choudhuri, Solar Phys. **123**, 217 (1989).
58 S. D'Silva, and A. R. Choudhuri, Astron. Astrophys. **272**, 621 (1993).
59 Y. Fan, G. H. Fisher, and E. E. DeLuca, Astrophys. J. **405**, 390 (1993).
60 P. Caligari, F. Moreno-Insertis, and M. Schüssler, Astrophys. J. **441**, 886 (1995).
61 A. R. Choudhuri, and S. D'Silva, Astron. Astrophys. **239**, 326 (1990).
62 S. Z. D'Silva, and A. R. Choudhuri, Solar Phys. **136**, 201 (1991).
63 M. Steenbeck, F. Krause, and K.-H. Rädler, Z. Naturforsch. **21a**, 1285 (1966).
64 H. W. Babcock, Astrophys. J. **133**, 572 (1961).
65 R. B. Leighton, Astrophys. J. **156**, 1 (1969).
66 A. R. Choudhuri, M Schüssler, and M Dikpati, Astron. Astrophys. **303**, L29 (1995).
67 Y.-M. Wang, A. G. Nash, and N. R. Sheeley, Astrophys. J. **347**, 529 (1989).
68 M. Dikpati, and A. R. Choudhuri, Astron. Astrophys. **291**, 975 (1994).
69 M. Dikpati, and A. R. Choudhuri, Solar Phys. **161**, 9 (1995).
70 A. R. Choudhuri, and M. Dikpati, Solar Phys. **184**, 61 (1999).
71 Y.-M. Wang, N. R. Sheeley, and A. G. Nash, Astrophys. J. **383**, 431 (1991).
72 B. R. Durney, Solar Phys. **160**, 213 (1995).
73 D. Nandy, and A. R. Choudhuri, Science **296**, 1671 (2002).
74 P. Chatterjee, D. Nandy, and A. R. Choudhuri, Astron. Astrophys. **427**, 1019 (2004).
75 J. Zhao, R. S. Bogart, A. G. Kosovichev, T. L. Duvall, and T. Hartlep, Astrophys. J. **774**, L29 (2013).
76 S. P. Rajaguru, and H. M. Antia, Astrophys. J. **813**, 114 (2015).
77 G. Hazra, B. B. Karak, and A. R. Choudhuri, Astrophys. J. **782**, 93 (2014).
78 A. R. Choudhuri, Pramana **77**, 77 (2011).
79 P. Charbonneau, Ann. Rev. Astron. Astrophys. **52**, 251 (2014).
80 B. B. Karak, J. Jiang, M. S. Miesch, P. Charbonneau, and A. R. Choudhuri, Space Sci. Rev. **186**, 561 (2014).
81 P. Charbonneau, G. Beaubien, and C. St-Jean, Astrophys. J. **658**, 657 (2007).
82 A. R. Choudhuri, Astron. Astrophys. **253**, 277 (1992).
83 D. Longcope, and A. R. Choudhuri, Solar Phys. **205**, 63 (2002).
84 A. R. Choudhuri, P. Chatterjee, and J. Jiang, Phys. Rev. Lett. **98**, 131103 (2007).
85 J. Jiang, P. Chatterjee, and A. R. Choudhuri, Mon. Not. R. Astron. Soc. **381**, 1527 (2007).

86 B. B. Karak, and A. R. Choudhuri, Mon. Not. R. Astron. Soc. **410**, 1503 (2011).
87 A. R. Choudhuri, and B. B. Karak, Phys. Rev. Lett. **109**, 171103 (2012).
88 B. B. Karak, and A. R. Choudhuri, Res. Astron. Astrophys. **13**, 1339 (2013).
89 A. R. Choudhuri, Indian J. Phys. **88**, 877 (2014).
90 R. Kippenhahn, Astrophys. J. **137**, 664 (1963).
91 L. L. Kitchatinov, in *Proceedings of the First Asia-Pacific Solar Physics Meeting*, edited by A. R. Choudhuri, and D. Banerjee (Indian Institute of Astrophysics, Bangalore, 2011), p. 71.
92 L. L. Kitchatinov, in *Solar and Astrophysical Dynamos and Magnetic Activity—IAU Symposium 294*, edited by A. G. Kosovichev, E. de Gouveia Dal Pino, and Y. Yan (IAU, Beijing, 2013), p. 399.
93 L. L. Kitchatinov, and G. Rüdiger, Astron. Astrophys. **299**, 446 (1995).
94 L. Jouve, B. P. Brown, and A. S. Brun, Astron. Astrophys. **509**, 32 (2010).
95 B. B. Karak, L. L. Kitchatinov, and A. R. Choudhuri, Astrophys. J. **791**, 59 (2014).
96 L. L. Kitchatinov, and S. V. Olemskoy, Mon. Not. R. Astron. Soc. **411**, 1059 (2011).
97 B. B. Karak, P. J. Käpylä, M. J. Käpylä, A. Brandenburg, N. Olspert, and J. Pelt, Astron. Astrophys. **576**, 26 (2015).
98 N. J. Wright, and J. J. Drake, Nature **535**, 526 (2016).
99 B. R. Durney, D. S. De Young, and I. W. Roxburgh, Solar Phys. **145**, 207 (1993).
100 W. Dobler, M. Stix, and A. Brandenburg, Astrophys. J. **638**, 336 (2006).
101 M. K. Browning, Astrophys. J. **676**, 1262 (2008).
102 Y. Fan, Living Rev. Solar Phys. **1**, 1 (2004).
103 M. Schüssler, and S. K. Solanki, Astron. Astrophys. **264**, L13 (1992).
104 M. Schüssler, P. Caligari, A. Ferriz-Mas, S. K. Solanki, and M. Stix, Astron. Astrophys. **314**, 503 (1996).
105 E. Isik, D. Schmitt, and M. Schüssler, Astron. Astrophys. **528**, 135 (2011).
106 L. L. Kitchatinov, and S. V. Olemskoy, Mon. Not. R. Astron. Soc. **459**, 4353 (2016).
107 L. Mestel, *Stellar Magnetism* (Clarendon Press, Oxford, 1999).
108 J.-F. Donati, and J. D. Landstreet, Ann. Rev. Astron. Astrophys. **47**, 333 (2009).
109 A. R. Choudhuri, and S. Konar, Mon. Not. R. Astron. Soc. **332**, 933 (2002).
110 S. Konar, and A. R. Choudhuri, Mon. Not. R. Astron. Soc. **348**, 661 (2004).
111 V. See, M. Jardine, A. A. Vidotto, J.-F. Donati, S. Boro Saikia, J. Bouvier, R. Fares, C. P. Folsom, S. G. Gregory, G. Hussain, S. V. Jeffers, S. C. Marsden, J. Morin, C. Moutou, J. D. do Nascimento Jr., P. Petit, and I. A. Waite, Mon. Not. R. Astron. Soc. **462**, 4442 (2016).
112 A. R. Choudhuri, and G. Hazra, Adv. Space Res. **58**, 1560 (2016).
113 A. R. Yeates, and A. Muñoz-Jaramillo, Mon. Not. R. Astron. Soc. **436**, 3366 (2013).
114 M. S. Miesch, and M. Dikpati, Astrophys. J. **785**, L8 (2014).
115 G. Hazra, A. R. Choudhuri, and M. S. Miesch, arXiv: 1610.02726.
116 B. P. Brown, M. K. Browning, A. S. Brun, M. S. Miesch, and J. Toomre, Astrophys. J. **711**, 424 (2010).
117 M. Ghizaru, P. Charbonneau, and P. K. Smolarkiewicz, Astrophys. J. **715**, L133 (2010).