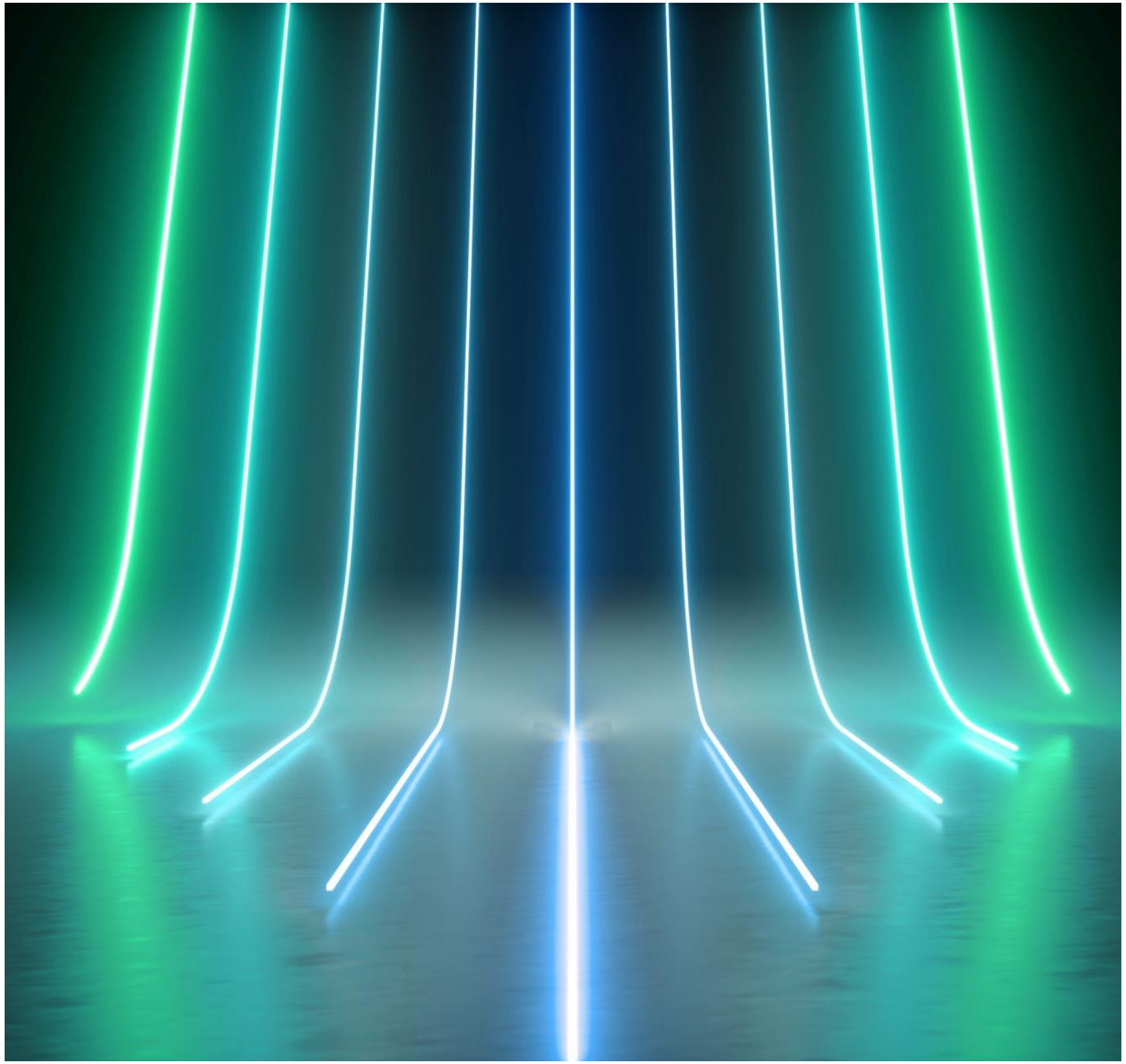
Parametric Circuits and Variational Methods



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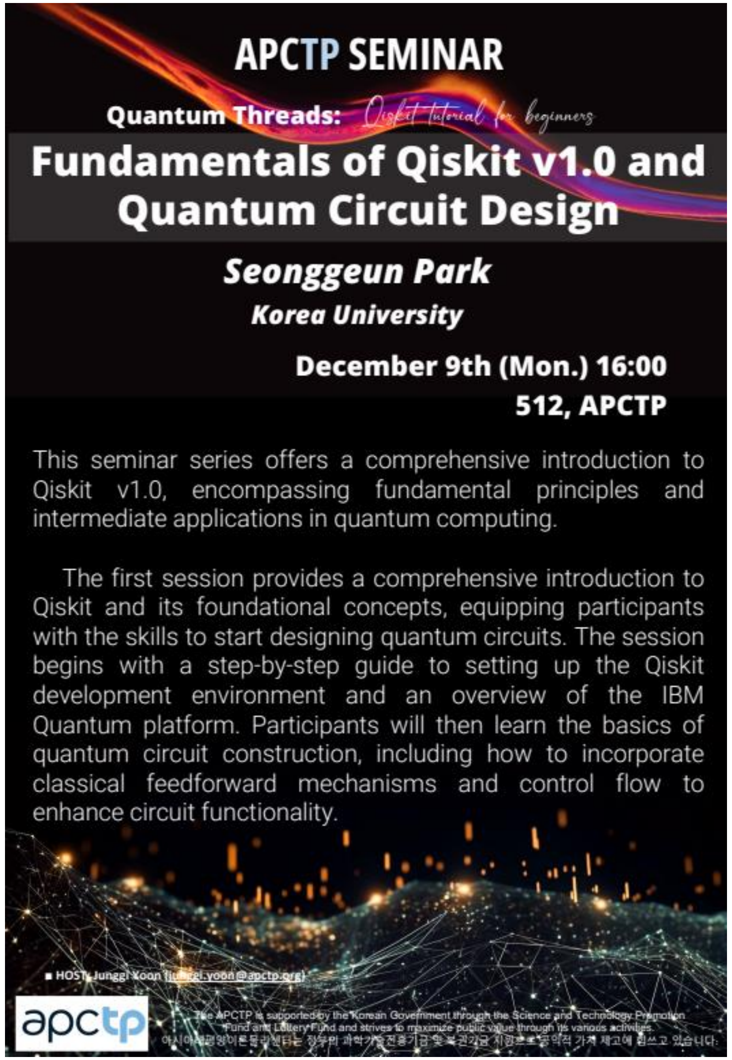
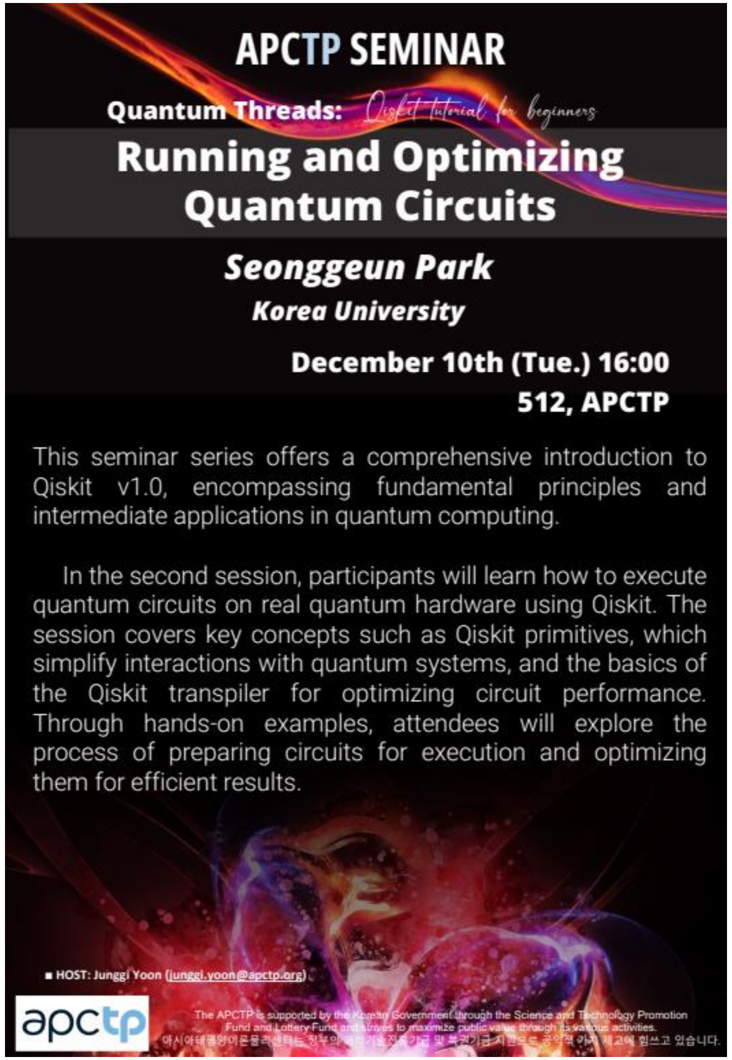
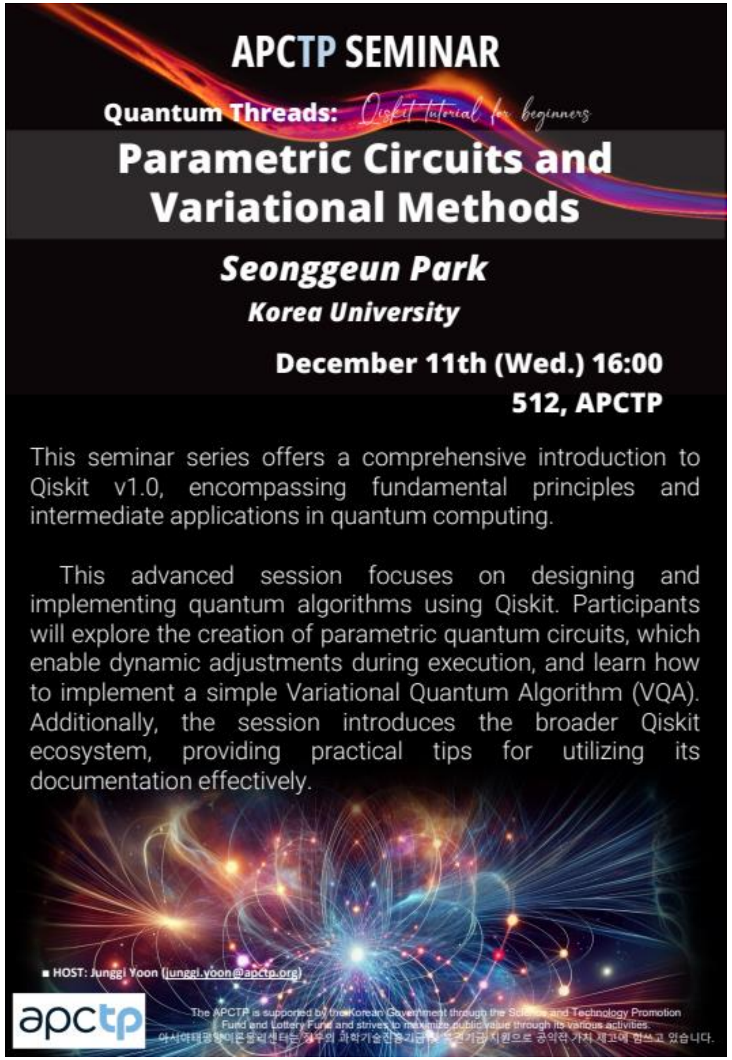
Qiskit Tutorial for Beginner Series at APCTP







Qiskit Tutorial for Beginner Series at APCTP

- A Quick Overview of Basic Quantum Information
- Introduction to Qiskit & IBM Quantum Platform
- Setting Up the Qiskit Development Environment
- Designing Quantum Circuits
- Implementing Classical Feedforward and Control Flow

- Executing Quantum Circuits using Qiskit Primitives
- Transpiling Quantum Circuits

- Implementing Parametric Quantum Circuits & VQA
- Browsing the Qiskit Ecosystem

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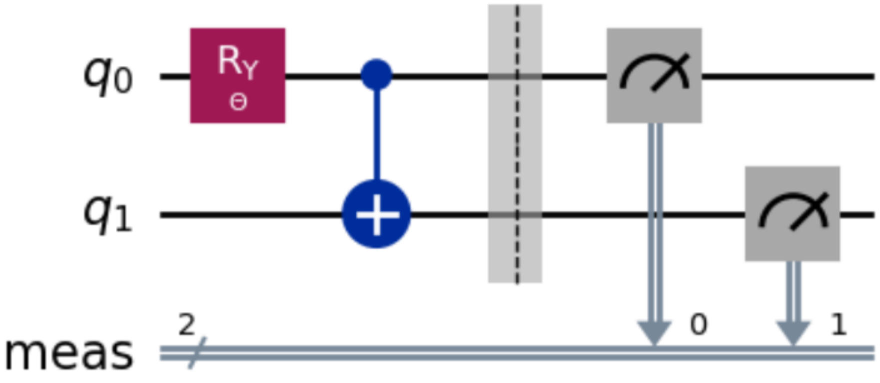
Contents

I.
Implementing Parametric Quantum Circuits & VQA

Parametric Quantum Circuits(PQC)

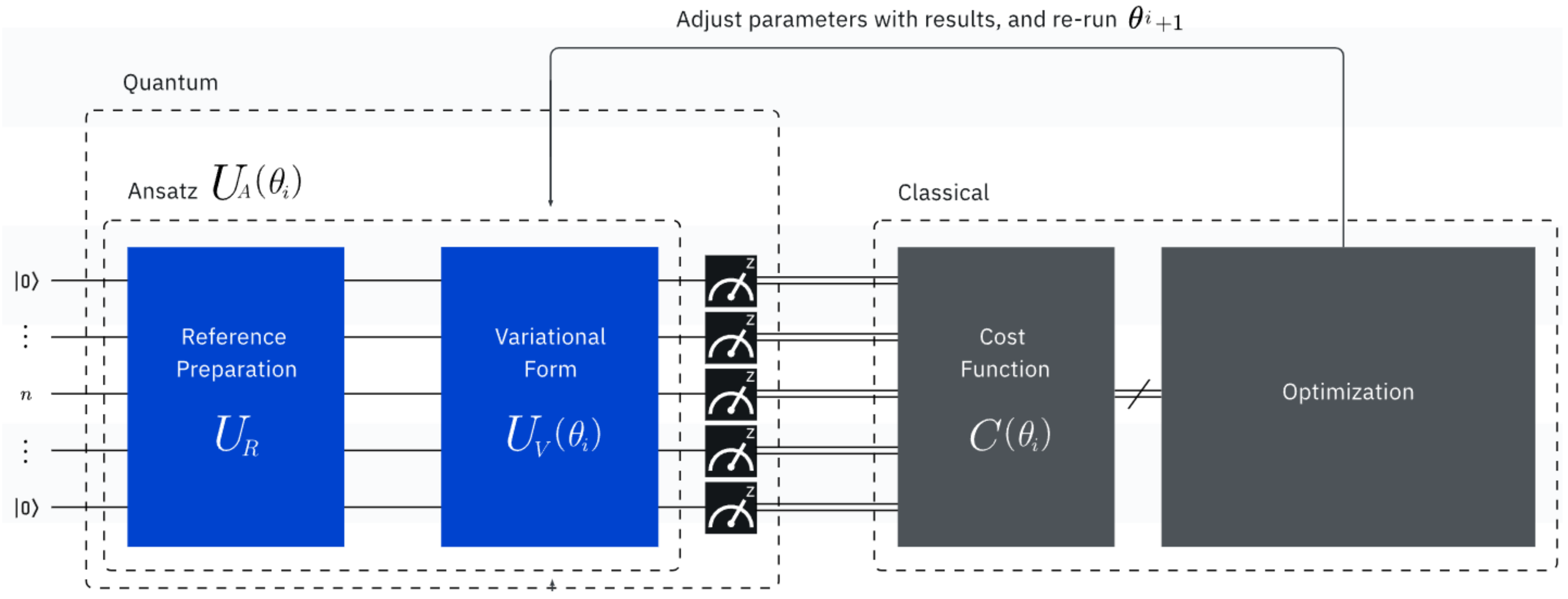
- Quantum circuit that includes gates with parameters that are not yet fixed
- PQC are commonly used in Variational Quantum Algorithm (VQA) and Quantum Machine Learning (QML)

```
from qiskit.circuit import Parameter

# create the parameter
theta = Parameter('θ')
par_bell_meas = QuantumCircuit(2)

# parameterize the rotation
par_bell_meas.ry(theta, 0)
par_bell_meas.cx(0, 1)

par_bell_meas.measure_all()

par_bell_meas.draw('mpl')
```



Variational Quantum Algorithm

Quantum computing approach that combines quantum circuits with classical optimization techniques to find approximate solutions to complex problems by interatively adjusting quantum parameters to minimize a cost function.



Reference Preparation

A reference state is an initial quantum state that is chosen as a starting point for the optimization process

We initialize our system with a reference state to help our variational algorithm converge faster

The reference state can be set in
1. fixed form: $U_R|0\rangle$
2. Parameterized quantum circuit: $U_R(\vec{x})|0\rangle$

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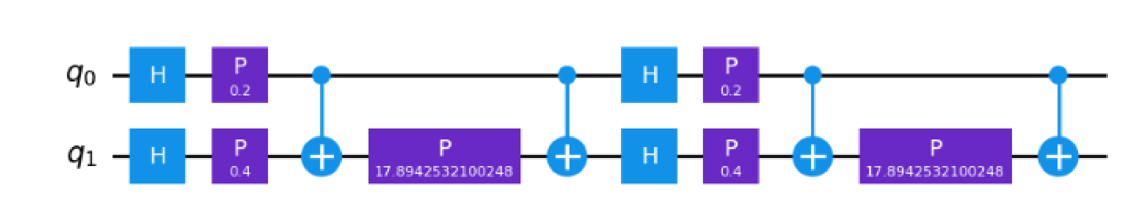
```python
from qiskit.circuit.library import ZZFeatureMap

data = [0.1, 0.2]

zz_feature_map_reference = ZZFeatureMap(feature_dimension=2, reps=2)
zz_feature_map_reference = zz_feature_map_reference.assign_parameters(data)
zz_feature_map_reference.decompose().draw("mpl")
```

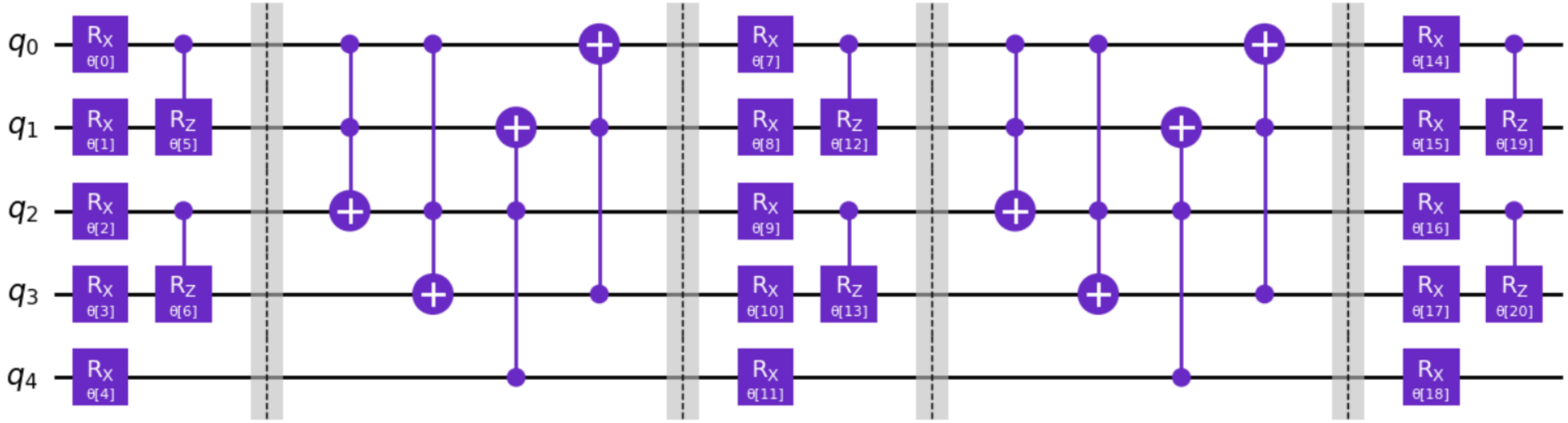
Output:



Variational Form and Ansatz

Variational algorithms operate by exploring and comparing a range of quantum states $|\psi(\vec{\theta})\rangle$, which depend on a finite set of k parameters $\vec{\theta} = (\theta^0, \ldots, \theta^{k-1})$

These states can be prepared using a PQC



Variational Form and Ansatz

To iteratively optimize from a reference state $U_R|0\rangle = |\rho\rangle$ to a target state $|\psi(\vec{\theta})\rangle$, we need to define a variational form $U_V(\vec{\theta})$ that representa a collection of parameterized states for our variational algorithm to explore:

$$\begin{aligned} |0\rangle \xrightarrow{U_R} U_R|0\rangle = |\rho\rangle &\xrightarrow{U_V(\vec{\theta})} U_A(\vec{\theta})|0\rangle \\ &= U_V(\vec{\theta})U_R|0\rangle \\ &= U_V(\vec{\theta})|\rho\rangle \\ &= |\psi(\vec{\theta})\rangle \end{aligned}$$

Cost Function

In general, cost functions are used to describe the goal of a problem and how well a trial state is performing with respect to that goal.

This definition can be applied to various examples in chemistry, machine learning, finance, optimization, and so on.

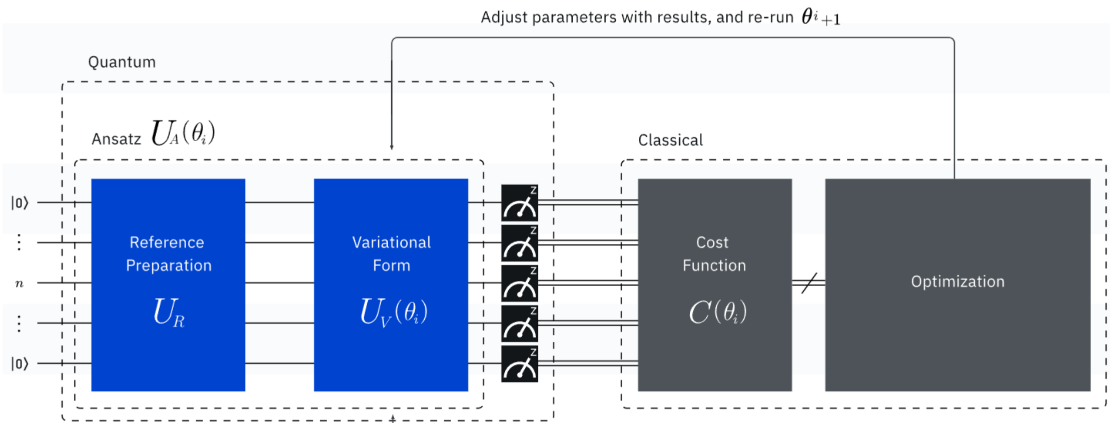
Our objective is to minimize the expectation value of the observable representing energy (Hamiltonian $\widehat{\mathcal{H}}$):

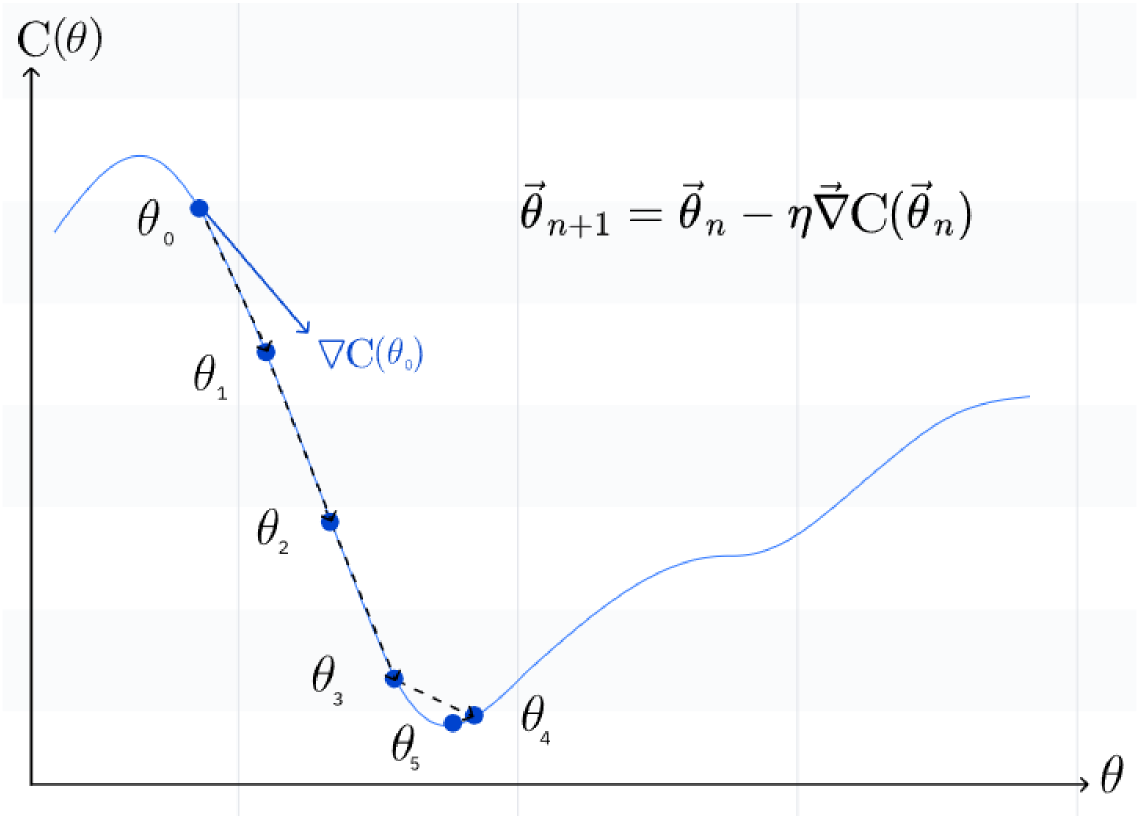
$$\min_{\vec{\theta}} \langle \psi(\vec{\theta}) | \hat{\mathcal{H}} | \psi(\vec{\theta}) \rangle$$

We can use the “Estimator” primitive to evaluate the expectation value and pass this value to an optimizer to minimize.

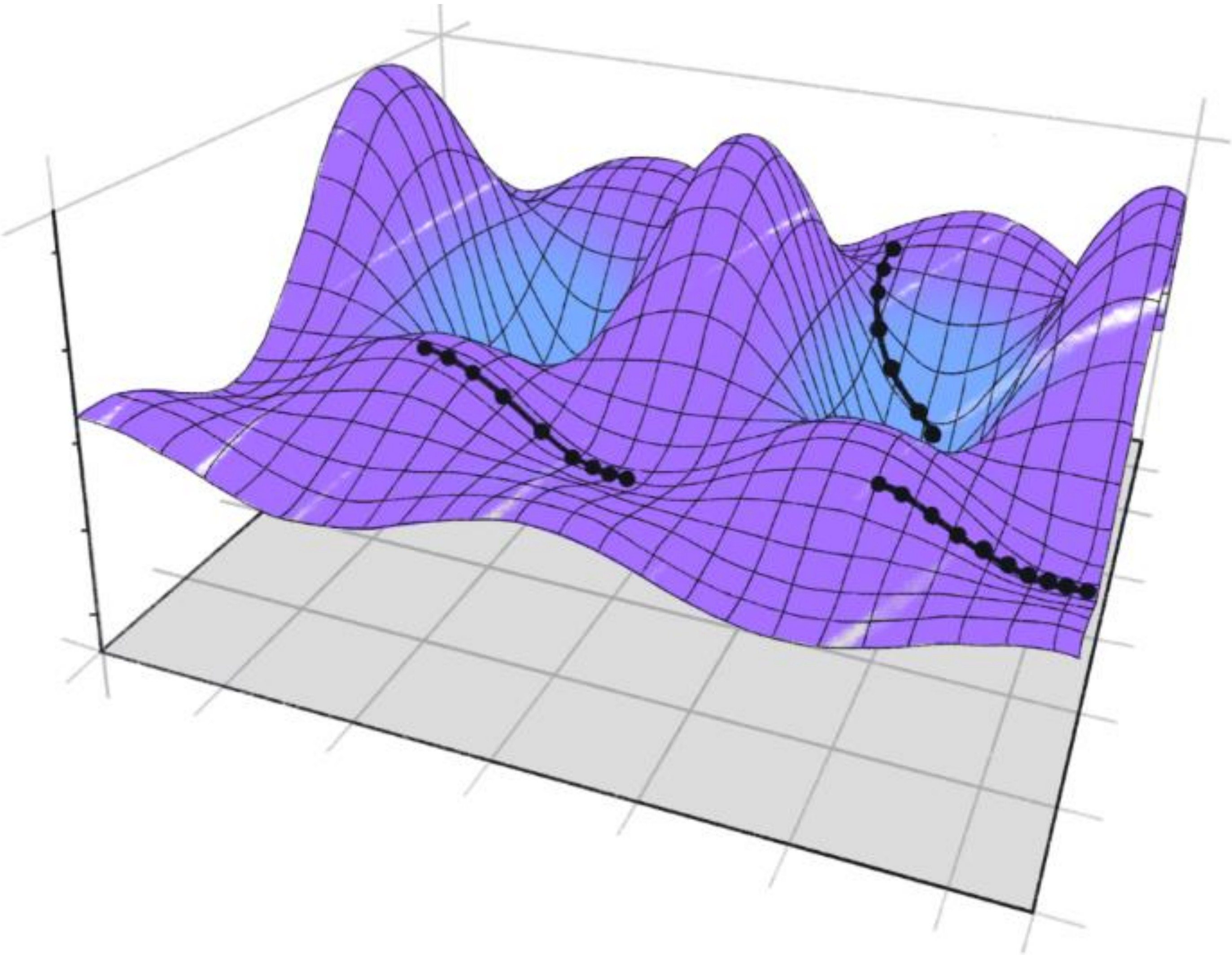
We will get a set of optimal parameter values $\overrightarrow{\theta^*}$

Optimization





Barren Plateuas



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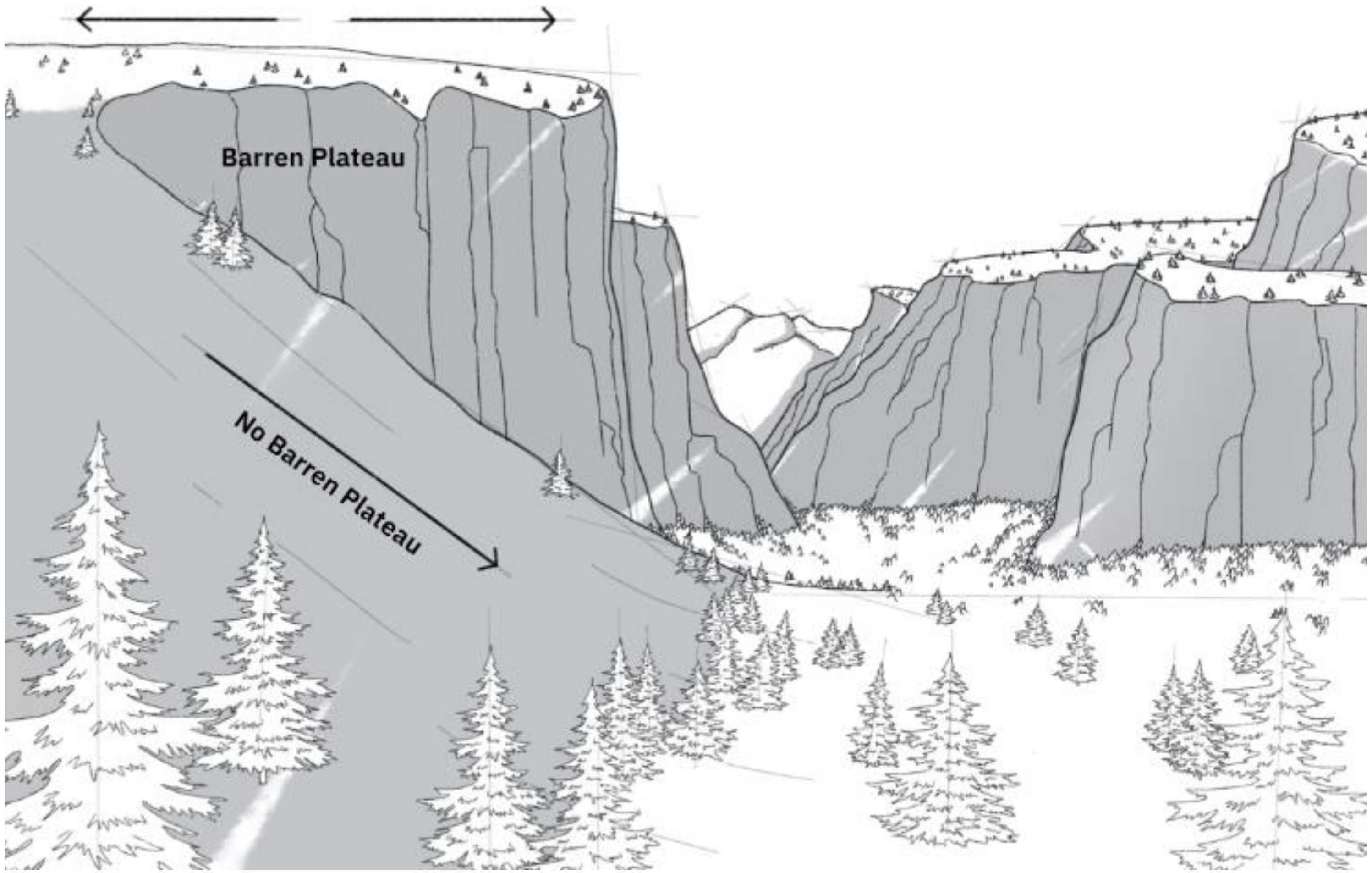

Example: Variational Quantum Eigensolver(VQE)

VQE (Variational Quantum Eigensolver) is an algorithm designed to solve eigenvalue problems on a quantum computer, primarily used in quantum chemistry and quantum physics to find the minimum energy state of a molecule.

https://learning.quantum.ibm.com/course/variational-algorithm-design/instances-and-extensions

II.
Qiskit ecosystem

Qiskit Ecosystem

https://www.ibm.com/quantum/ecosystem