Lecture 8 - Non-standard filters

In the previous lectures we have looked at a series of *linear* filters which have constant coefficients. In this lecture we will finish the course by considering some interesting, applicable and widely used filters which can be realised easily in the digital domain. I avoid the interesting topic of *adaptive filters*, as these require a full course on their own.

8.1 Sub-space filters

Consider a digital signal $x[n]$ for index $n = N : M$. Construct an *embedding matrix* from this sequence from a set of p *lagged versions* of x.

$$\mathbf{X} = \begin{pmatrix} x[N-1] & x[N] & \ldots & x[M-1] \\ x[N-2] & x[N-1] & \ldots & x[M-2] \\ x[N-3] & x[N-2] & \ldots & x[M-3] \\ \vdots & & & \vdots \\ x[N-p] & x[N-p+1] & \ldots & x[M-p] \end{pmatrix}$$

Sub-space filters act by operating on this matrix, decomposing it into its *singular values*. The *singular value decomposition* of $\mathbf{X}$ is written as:

$$\mathbf{X} = \mathbf{U}\mathbf{S}\mathbf{V}^{\mathsf{T}}$$

where $\mathbf{U}$ is an orthogonal set of projections of $\mathbf{X}$ onto the vectors contained in (the orthonormal matrix) $\mathbf{V}$. The matrix $\mathbf{S}$ is diagonal and contains the singular values. You do not need worry about this method - although it is worth noting that the singular values are the positive square roots of the eigenvalues of the matrix $\mathbf{X}\mathbf{X}^{\mathsf{T}}$. The main point is that, given $\mathbf{X}$ the matrices $\mathbf{U}, \mathbf{S}, \mathbf{V}$ are *uniquely determined*.

So what use is all this? Taking the above equation we can re-write it such that

$$\mathbf{U} = \mathbf{W}\mathbf{X}$$

Let's consider just the first component of $\mathbf{U}$, a set of samples $u_1[n]$ say.

$$u_1[n] = \sum_{k=1}^{p} w_1[k]x[n-k]$$

which is just a *general moving average FIR filter*. This means that each column of $\mathbf{U}$ is a different FIR filter, whose coefficients are determined from the observed data, rather than being 'designed'.

Fig. 8.1 shows 1000 samples ($f_s = 11.3$ kHz) from a Bessie Smith blues Ballad. Fig 8.2 shows the resultant components from the matrix $\mathbf{U}$ when $p = 10$. Note that they are increasingly higher frequency.

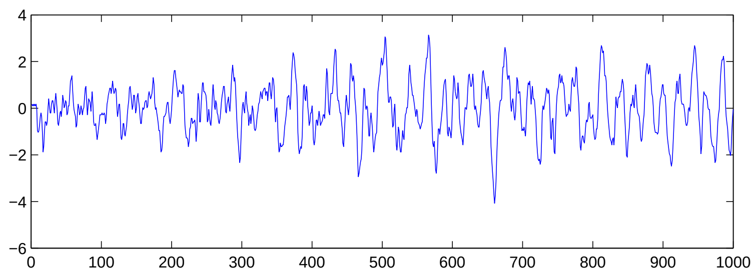

Figure 8.1: *Music example.*

8.2 Order statistic filters – the median filter

Consider a set of digital samples from $x[n-p] : x[n]$. Order the set from smallest to largest, and let this sample set be $z[1] : z[p]$. Order statistic filters thence perform linear filtering on this ordered set, rather than the original.

What use might this be? Consider a signal, corrupted by spiking noise, as in Fig 8.3. We see that the signal (a sine wave) is corrupted by *impulsive noise*. The effect of this noise is that the maximal and minimal values we observe in any small window on the data are likely to be caused by the noise process, not from the signal.

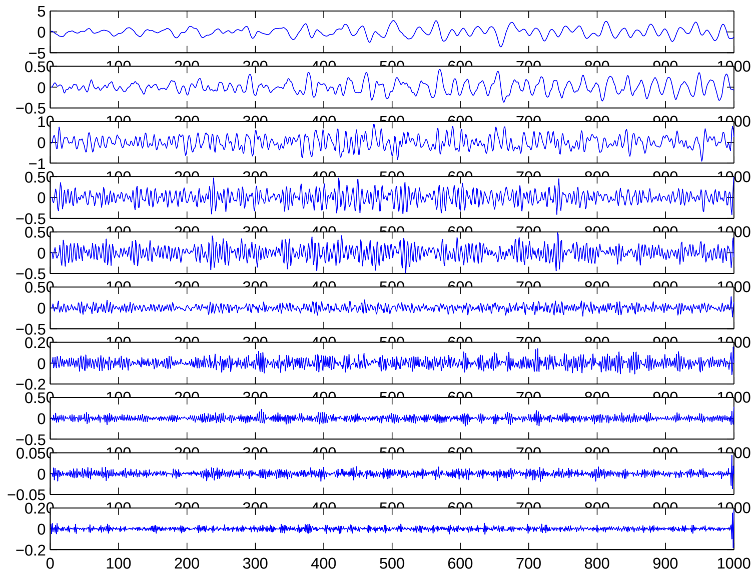

Figure 8.2: *Music – subspace filtered.*

If we apply a filter mask to the ordered set $z[k]$ such that the output of the filter is just the middle value in the set (I assume that p is odd). This value is the *median* of the values and the filter is refered to as the *median filter*. How well it does at removing impulsive noise is shown in the lower plot of the figure.

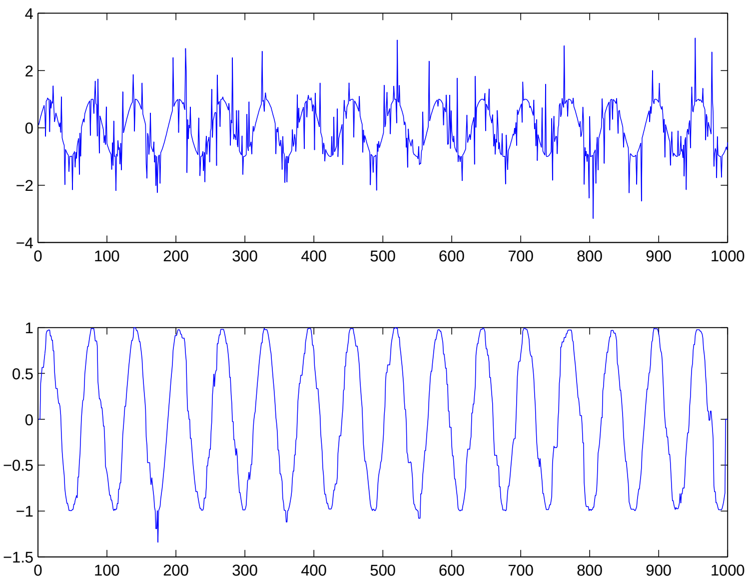

Figure 8.3: *(top) Spiky signal. (bottom) median filtered output,* $p = 7$