
Fast Computer Vision based Geometry Estimation

Bachelor Thesis

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Subject

Image Processing

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Abstract

Introduction

Approach

Conclusion

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Abbreviations

ASK Amplitude Shift Keying

Chapter 1

Introduction

Chapter 2

Theory

This chapter takes a closer look at the theory and technology applied in this thesis.

2.1 Camera Calibration in OpenCV

This section describes the theory behind the calibration process in OpenCV 4.3.0, which is the latest version at the time of writing this thesis.

2.1.1 The pinhole camera model

The functions OpenCV provides to calibrate the camera use the so-called pinhole camera model [1]. This model describes, how a 3D-point specified in world-coordinates (P_w) is transformed to a 3D-point in camera-coordinates (P_c) and then further projected onto the image plane (p). After this step, the point is described as a 2D-point in pixel coordinates. Figure 2.1 illustrates this setup.

Figure 2.1: Pinhole-model (not up to date).

The transition from the world-coordinates to the camera-coordinates can be described as

$$\underbrace{\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix}}_{P_c} = \underbrace{\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}}_R \underbrace{\begin{pmatrix} X_w \\ Y_w \\ Z_w \end{pmatrix}}_{P_w} + \underbrace{\begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}}_t.$$

The vector P_w is first rotated by R and then translated by t . This can be written in one single matrix:

$$\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{pmatrix} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix} \Leftrightarrow P_c = (R \mid t) \begin{pmatrix} P_w \\ 1 \end{pmatrix}. \quad (2.1)$$

As a result of the theorem of intersecting lines, the projection from P_c to p is described as

$$\underbrace{\begin{pmatrix} u \\ v \end{pmatrix}}_p = \begin{pmatrix} f_x \cdot X_c / Z_c \\ f_y \cdot Y_c / Z_c \end{pmatrix} + \begin{pmatrix} c_x \\ c_y \end{pmatrix}.$$

where f_x and f_y are the focal length f (in world units) normalized by their respective pixel size (in world units). Thus f_x and f_y are the same, if the pixels are quadratic.

By adding the principal point $(c_x \ c_y)^T$, which is usually close to the image center, it is taken into account, that pixel-coordinates are specified with respect to the upper left corner of the image plane. . It is now simpler to write this in homogeneous coordinates:

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \sim s \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \underbrace{\begin{pmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix}}_K \begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} \Leftrightarrow s \begin{pmatrix} p \\ 1 \end{pmatrix} = K \cdot P_c, \quad (2.2)$$

where s is an arbitrary scaling factor and K is called the camera matrix. The overall transition from world- to pixel-coordinates is the result of combining 2.1 and 2.2:

$$s \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{pmatrix} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix} \Leftrightarrow s \begin{pmatrix} p \\ 1 \end{pmatrix} = K (R \mid t) \begin{pmatrix} P_w \\ 1 \end{pmatrix} \quad (2.3)$$

The rotation and translation in $(R \mid t)$ are called the extrinsic parameters. The camera matrix K contains analogously the linear intrinsic parameters.

2.1.2 The distortion model in OpenCV

Non linear distortions, which appear before the projection in 2.2 should be considered too. OpenCV takes the effects of radial, tangential and thin prism distortion into account [1].

Radial Distortion

Radial distortion is caused by the flawed curvature of the lens [2]. It can be modeled with

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} x' \frac{1+k_1 r^2+k_2 r^4+k_3 r^6}{1+k_4 r^2+k_5 r^4+k_6 r^6} \\ y' \frac{1+k_1 r^2+k_2 r^4+k_3 r^6}{1+k_4 r^2+k_5 r^4+k_6 r^6} \end{pmatrix}, \quad (2.4)$$

where x' and y' are coordinates, described in camera-coordinates, normalized with Z_c

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} X_c/Z_c \\ Y_c/Z_c \end{pmatrix}$$

from and r is the radius as taken with respect to the principal point $(c_x \ c_y)^T$

$$r^2 = x'^2 + y'^2.$$

This type of distortion is symmetrical about the optical axis. Figure 2.2 illustrates the effect on a rectangle.

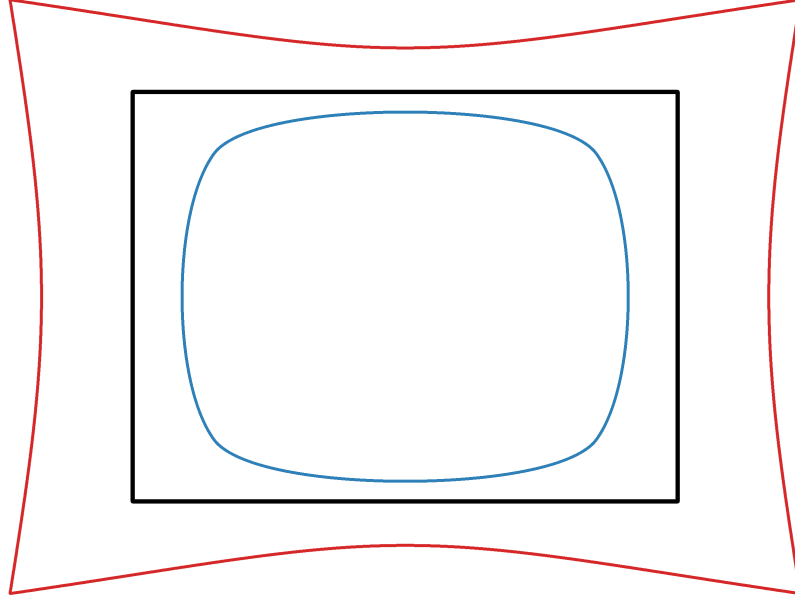


Figure 2.2: Different types of radial distortion. Black: no distortion, red: $k_1 = 10, k_2 = 11, k_3 = 12, k_4 = 5, k_5 = 6, k_6 = 7$, blue: $k_1 = -2.2, k_2 = -1.2, k_3 = -0.8, k_4 = -0.4, k_5 = -0.3, k_6 = -0.2$

Tangential distortion

Real optical systems are also subject to tangential distortion. This type of distortion has a decentering effect and occurs, if the line through the optical center of the lens and the principal point is not col-linear with the optical axis [2]. The model

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} x' + 2p_1x'y' + p_2(r^2 + 2x'^2) \\ y' + 2p_2x'y' + p_1(r^2 + 2y'^2) \end{pmatrix} \quad (2.5)$$

takes this into account. Figure 2.3 shows the distortion this model introduces.

Thin prism distortion

Thin prism distortion is partially caused by lens imperfections and camera assembly [2]. It introduces additional radial and tangential distortion, modeled with

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} x' + s_1r^2 + s_2r^4 \\ y' + s_3r^2 + s_4r^4 \end{pmatrix}. \quad (2.6)$$

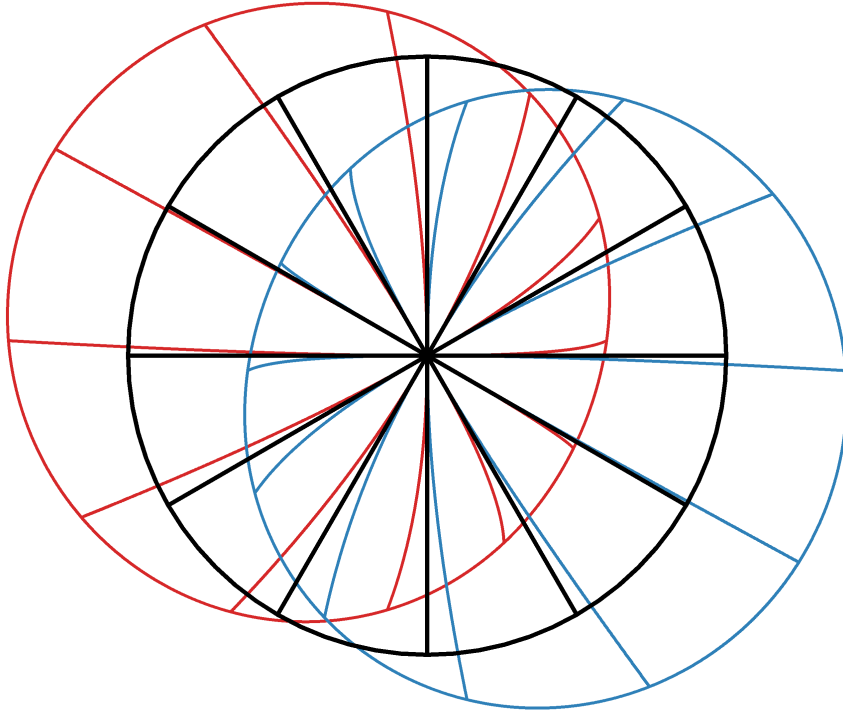


Figure 2.3: Different types of tangential distortion. Black: no distortion, red: $p_1 = -0.15$, $p_2 = -0.4$, blue: $p_1 = 0.15$, $p_2 = -0.4$

The combined model in OpenCV

The models in 2.4, 2.5 and 2.6 combined result in

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} x' \frac{1+k_1r^2+k_2r^4+k_3r^6}{1+k_4r^2+k_5r^4+k_6r^6} + 2p_1x'y' + p_2(r^2 + 2x'^2) + s_1r^2 + s_2r^4 \\ y' \frac{1+k_1r^2+k_2r^4+k_3r^6}{1+k_4r^2+k_5r^4+k_6r^6} + 2p_2x'y' + p_1(r^2 + 2y'^2) + s_3r^2 + s_4r^4 \end{pmatrix}. \quad (2.7)$$

In summary, a point in normalized camera-coordinates $(x' \ y')^T$ is distorted as modeled in 2.7, which leads to the point $(x'' \ y'')^T$. To get the distorted pixel-coordinates (subscript d), the projection has to be applied

$$\begin{pmatrix} u_d \\ v_d \end{pmatrix} = \begin{pmatrix} x'' f_x + c_x \\ y'' f_y + c_y \end{pmatrix} \Leftrightarrow \begin{pmatrix} u_d \\ v_d \\ 1 \end{pmatrix} = K \begin{pmatrix} x'' \\ y'' \\ 1 \end{pmatrix},$$

which completes the model so far. Keep in mind, that when correcting the distortion, equation 2.7 has to be inverted.

Chapter 3

Evaluation

Chapter 4

Development

This chapter covers the developing process in more detail.

Chapter 5

Results

This chapter covers the most important results.

Chapter 6

Conclusion

References

- [1] (). Camera Calibration and 3D Reconstruction, [Online]. Available: https://docs.opencv.org/4.3.0/d9/d0c/group__calib3d.html.
- [2] J. Weng, P. Cohen, and M. Herniou, “Camera calibration with distortion models and accuracy evaluation,” *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 14, no. 10, pp. 965–980, 1992.

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Statement of Plagiarism

We declare that, apart from properly referenced quotations, this report is our own work and contains no plagiarism; it has not been submitted previously for any other assessed unit on this or other degree courses.

Place	Date
Rapperswil	April 22, 2020

Signatures

Cedric Renda

Manuel Tischhauser

Appendix A

Requirements

A.1 Assignment

A.2 Requirement Specification
