Designing a particle physics experiment

Data analysis 2023 – Group project III

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Motivation

In this project you perform a simulation study to optimize the layout of a simple particle physics experiment. The experiment should measure decays of charged kaons (K^+) into a charged pion (π^+) and a neutral pion (π^0):

$$K^+ \to \pi^+\pi^0 \, .$$

The K^+ is a particle that consist of an up quark and a strange antiquark. It is unstable and decays to a π^+ and a π^0 with a probability of about 21%. The π^+ consists of an up quark and a down antiquark, the π^0 is a mixture of up and down quark/antiquark pairs. In reality, the π^+ and π^0 are also unstable, but for the sake of simplicity we will neglect this in our simulation study.

1 Setup

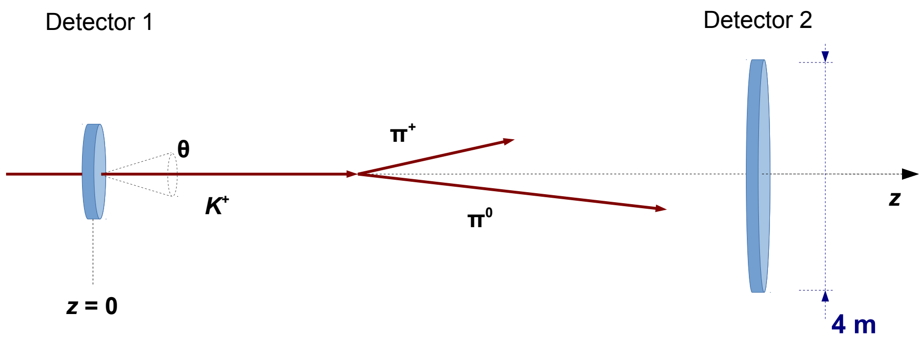

Figure 1: Sketch of the setup.

A sketch of the planned setup is shown in Fig. 1. The experiment uses a beam of K^+ and consists of two detectors. The first detector is used to detect the incoming K^+ and the second detector is used to detect the π^+ and the π^0 that are emitted in the decay of the kaon. This second detector has a circular cross section with a diameter of 4 m and is centred on the beam axis. The purpose of your study is to find the optimal distance between the two detectors such that the number of K^+ decays with both pions detected in the downstream detector is as large as possible. You have to take into account two effects: by decreasing the distance between the two detectors, you lose events because less and less K^+ decay before the second detector; by increasing the distance between the two detectors, you lose events because more and more π^+ and π^0 miss the acceptance of the second detector.

2 Determination of the average decay length of the K^+

You do not know the momentum of the beam at which you conduct your experiment, but you do know that the beam contains 84% of π^+ and only 16% of K^+. Since you do not know the momentum, you need to determine the average decay length of the K^+ from data recorded in a previous experiment. That experiment used the same beam, but measured a different decay mode and could not distinguish between kaon and pion decays. The file `dec_lengths.dat` contains 100'000 decay lengths measured in that previous experiment. You also know that the average decay length of π^+ in that beam was measured to be $\beta\gamma c\tau_\pi = 4.188$ km.

Using your knowledge of the composition of the beam and of the average decay length of the π^+, perform a fit to the data in order to extract the average decay length of the K^+. Compute the uncertainty on your result, and check if your result is compatible with the the known mean lifetime of the K^+, which you can look up on the web page of the Particle Data Group (`pdg.lbl.gov`).

3 Infinitely narrow beam along the z axis

First, assume that the K^+ travel exactly along the z axis. Generate a sample of $K^+ \to \pi^+\pi^0$ decays as follows:

(a) Generate the position of the K^+ decay vertex according to an exponential distribution with the mean decay length that you have determined above;

(b) Generate the decay angles of the π^+ and π^0 isotropically distributed in the K^+ rest frame;

(c) Boost the four-vectors of the two pions into the laboratory frame as described in the Appendix below.

Finally, determine for how many of the generated decays both pions fly into the acceptance of the downstream detector. Do this for different z positions of the downstream detector and determine the z position that maximizes the number of such events.

4 Divergent beam

Repeat the study assuming now that the K^+ beam has a finite angular divergence. Simulate this by generating K^+ flight directions according to a Gaussian profile with a standard deviation of $\sigma_\theta = 1$ mrad, where θ is the angle between the z axis and the K^+ flight direction.

Appendix: K^+ decay and Lorentz transformation

It is convenient to describe the particles in terms of four-vectors:

$$P = (E, p_x, p_y, p_z) \ ,$$

where $E = \sqrt{m^2 + |p|^2}$, $|p| = \sqrt{p_x^2 + p_y^2 + p_z^2}$ and m is the rest mass of the particle (you can look up the rest masses of the K^+, π^+ and π^0 on the web page of the Particle Data Group (pdg.lbl.gov). The total four-vector is conserved in the decay:

$$P_K = P_{\pi^+} + P_{\pi^0} \ .$$

From momentum conservation it follows that the momenta of the two pions are produced back-to-back in the rest frame of the K^+, i.e. that their momenta in this frame have equal magnitude and point in opposite directions. The angular distribution of the two pions in the K^+ rest frame is isotropic.

If the K^+ travels parallel to the z axis, the four-momenta of the pions in the laboratory frame are related to those in the K^+ rest frame, denoted with a *, as

$$\begin{bmatrix} E \\ p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 & \beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta\gamma & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} E^* \\ p_x^* \\ p_y^* \\ p_z^* \end{bmatrix} ,$$

where

$$\beta = |p_K|/E_K$$

and

$$\gamma = E_K/m_K$$

If the K^+ does not travel parallel to the z axis, you have to modify the directions of the two pions accordingly by performing a rotation.