09 Absoluter Nullpunkt

Cedric Renda, Fritz Kurz

October 14, 2021

Abstract

From the universal gas equation we know that for a given volume, filled with a given amount of gas, temperature is linear to pressure. This indicates, that there is an absolute zero for temperature, because otherwise there would have to be negative pressure. In this experiment, (after calibrating our measuring device) we want to find said absolute zero by measuring the pressure of a constant amount of gas in a constant volume at two known temperatures (in our case ice water and the steam of boiling water) and then calculation absolute zero from that. Our measurments were off by more than the calculated error suggested, so we had to try and find the reason for that. We suspect that the error could come from the limitation of the sensor used to measure the pressure.

Contents

1 Introduction

In this experiment we want to use the universal gas equation $PV = nRT$ to determine the lowest limit of the so called thermodynamic Celsius temperature scale. At this lowest point of temperature the enthalpy and entropy of a cooled ideal gas reaches its minimum values. By definition this point is the zero point of the SI base unit of temperature Kelvin, which is defined as follows**??**:

- The kelvin, symbol K, is the SI unit of thermodynamic temperature. It is defined by taking the fixed numerical value of the Boltzmann constant$k$ to be 1.380649×10^{-23} when expressed in the unit JK^{-1}, which is equal to $\mathrm{kg\ m^2\ s^{-2}\ K^{-1}}$, where the kilogram, metre and second are defined in terms of h, c and $\Delta v C_s$.

In the process to find this coldest temperature possible in the absolute temperature, we use a glass bulb filled with helium,a pressure sensor and two well known temperatures to measure and calculate the zero point. Once the pressure and temperature of the helium in the glass bulb is know, all kinds of temperatures can be measured with the change of pressure in the sealed glass bulb. To show this the temperature of liquid nitrogen is measured at the end of the experiment.

2 Experiment

Our setup consists of a glass bulb, connected to a voltmeter used to measure pressure, a pump for evacuating the bulb and a balloon, which is again connected to a bottle filled with helium (see figure 2).

Our first task is to calibrate the pressure sensor as it outputs just voltage, not pressure directly. For that we need to measure voltages of two known pressures, in our case the air pressure p_L in the lab and the one of near vacuum p_t. To evaulate the air pressure there is a mercury barometer available in the lab. First we get the signal U_L from exposing the sensor to air pressure. After that, we evacuate the bulb with the pump and get U_t, the voltage at near vacuum (the pump should be able to produce a pressure significantly below 0.2 mbar). From those two measurments we can evaluate the slope C and the offset p_0 of the so called characteristic curve of the sensor. Now we can translate voltage to pressure with the following formula.

$$p(U) = p_0 + CU \tag{1}$$

The next step is the experimental part. As said before we want to measure the pressure of a given amount of gas in a constant volume at two different known temperatures.

The first step is to fill the glass bulb with helium. We want to be sure that there is no air remaining in the bulb, so we evacuate it first, fill it with helium, and then repeat this

Figure 1: Sealed glass bulb filled with helium after being put in liquid nitrogen to measure the temperature of liquid nitrogen.

process. To fill the bulb with helium without too much pressure, we first fill the balloon to around the size of a football and fill the bulb from there, not from the helium bottle directly. We remove the hose at opening 6 before exposing the bulb to heat. Because the pressure in the bulb will increase, air will not get into the bulb.

We place a water cooker under the bulb with a container, so the bulb will be exposed to the vapor as good as possible. After the voltage has settled we close opening 6, read the signal of the sensor U_k and convert it to pressure p_k with formula (1). We can determine the boiling temperature of water with our measured air pressure and a convesion table.

Next we place the bulb in a container filled with ice and water. Again, after the voltage signal has settled, we can read U_e and calculate p_e.

In the last part of the experiment, we want to use our setup as a gas thermometer and measure the temperature of liquid nitrogen. For that we completly submerge the bulb as it is in a container of liquid nitrogen, read the voltage U_n and calculate p_n.

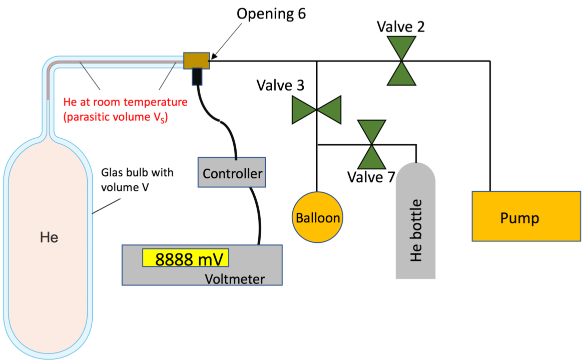

Figure 2: Schematic taken from the experiment manual

3 Results

Our measurments and calculations as seen below lead us to the following result for absolute zero t_0:

$$t_0 = -269.59 \pm 5.1\,°\mathrm{C} \tag{2}$$

The exact result is $-273.150 \pm 0.005\,°\mathrm{C}$.

Our result for the temperature t_n of liquid nitrogen is:

$$t_n = -194.6 \pm 3.8\,°\mathrm{C} \tag{3}$$

with the boiling temperature of nitrogen at $-195.8\,°\mathrm{C}$ (Wikipedia).

4 Data analysis

For the calibration of the pressure sensor we need to calculate the slope and offset between the pressure given by the pressure sensor in mV and the reference values. For this we used the ambient pressure and vacuum as the two reference points. To measure the ambient pressure, the mercury barometer in the lab was used. From the barometer we could read the uncorrected $p_L = 720,9$ m $\cdot\, 10^{-3}$ Hg (tL) with the corresponding roomtemperature of $23,1$ °C.

In the data analysis section you describe the post processing of the data. How did you obtain the data that you plot in the figures? The raw data does not necessarily need to be presented in the report. An important part of this paragraph are the measurement uncertainties. You should provide the uncertainties of all experimental results, i.e. in the form of error bars. Further, you should explain the origin of these uncertainties.

There are many way how you can include equations and mathematical terms in your report. The easiest is to write them inside the text like this: $\Gamma = 1.5\,\mu\mathrm{m}\,\mathrm{s}^{-2}$. If you need to write

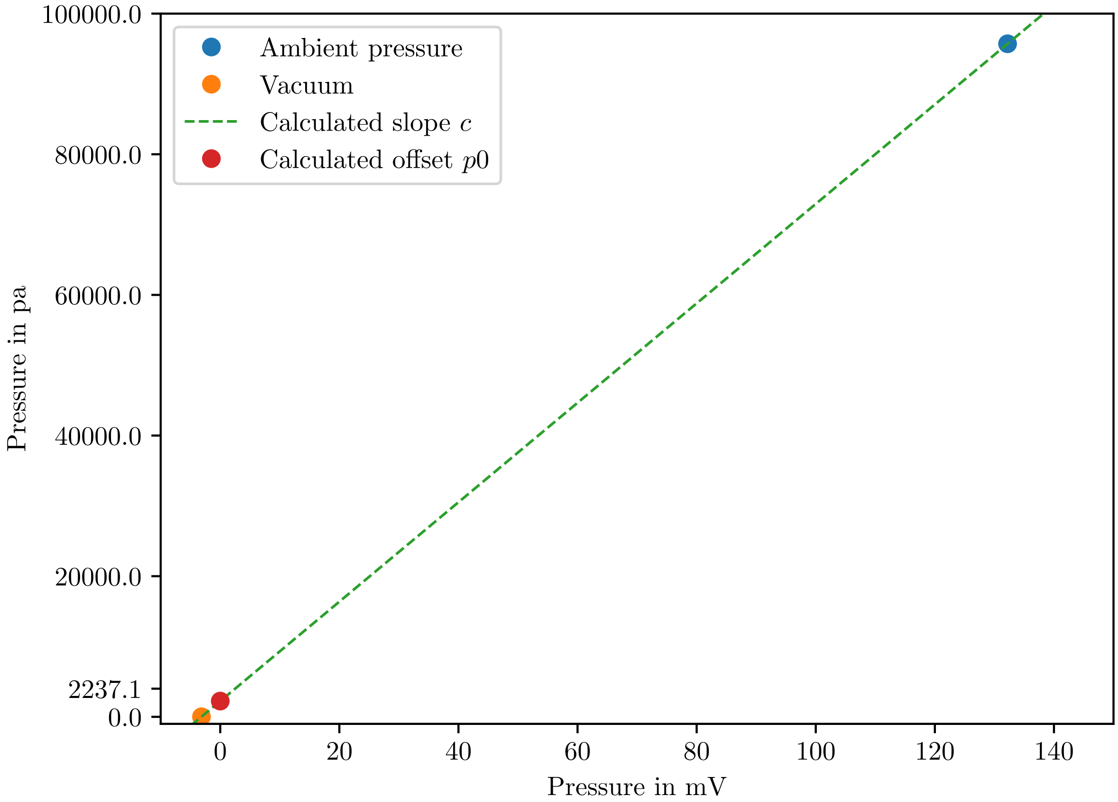

Figure 3: Calibration of pressure sensor.

a long equation, it is recommended to use e.g. the align environment.

$$\Gamma = \frac{a}{4\kappa} \times ... \tag{4}$$

All figures or tables that are part of your report have to be referenced somewhere in the text, ideally in order of their appearance ("as shown in Fig. **??**"). Figures have to have axis labels with units and a sensible scale. If more than on data set is plotted you need to provide a legend. This may be a sentence in the caption ("red dots denote data measured with 1 mV, blue crosses were measured with 10 mV").

After you have presented the data you need to interpret it. To this end you want to discuss the theoretical model that describes your data and you will derive model parameters from your measurement data (i.e. by fitting it to the data). Here, you will again elaborate on the confidence interval of the derived values (error propagation). This is an important part of the report and it will be the basis for the next paragraph.

5 Error Calculation

First we have to take account of the possible error of the pressure sensor. From the experiment manual we get that there is an absolute error of $\pm 0.075\,\mathrm{mV}$ Due to different temperatures when the sensor was originally calibrated and the time we do the experiment, there is also an error of $\pm 0.08\,\mathrm{mV}$, also according to the manual. In addition to that, the slope of the curve can vary by $\pm 0.1\,\%$. We can now calculate our error for voltage measurments $\Delta U = 0.075 + 0.08 + 0.001 \times U$. With our measurments we get $U_L = 132.21 \pm 0.29\,\mathrm{mV}$ and $U_t = -3.72 \pm 0.16\,\mathrm{mV}$

We then need to know the errors of p_L and p_t. From the manual we know that $p_t = 10 \pm 10\,\mathrm{Pa}$. The measurment of the barometer is very exact, so we assume that there is no relevant error in that.

We want to know the error of the pressure measurments we make during the experiment. According to formula (1), we need to know the errors Δp_0, ΔC, and ΔU. The last one we know already. First we determine ΔC, the error of the slope of the characteristic curve of the sensor. The formula for that is $C = \frac{p_T - p_l}{U_T - u_l}$. With Gaussian error propagation we can evaluate $\Delta C = 1.7$, so $C = 704.1 \pm 1.7$ Second we want to know Δp_0. The formula for p_0 is $p_0 = p_t - CU_t$ We use Gaussian error propagation again and find that $\Delta p_0 = 109.8\,\mathrm{Pa}$

Now we have everything to determine the error of a pressure measurment. The formula for that is given as $p = p_0 + CU$. So again with the same method we find the function

$$\Delta p(U) = \sqrt{\Delta p_0^2 + (U\Delta C)^2 + (C\Delta U)^2} \tag{5}$$

which we must use for every pressure measurment we make. Our measurments give us $p_e = 70345 \pm 264\,\mathrm{Pa}$ and $p_k = 95240 \pm 320\,\mathrm{Pa}$

Now we are finally able to calculate Δt_0, again by the Gaussian method. Unlike befor, we don't want to take derivatives by hand, so for approximating $\frac{\partial t_0}{\partial x_i}(x_1, ..., x_n)$, we use the differential quotient $\frac{t_0(x_1,...,x_i+h,...,x_n) - t_0(x_1,...,x_i,...,x_n)}{h}$ and use a small $h = 10^{-5}$. By doing that for all parameters $x_i, i = 1, ..., n$

(in formula xxx) and then using the Gaussian method, we get to $t_0 = -267.67 \pm 5.02\,°\mathrm{C}$.

6 Discussion

Absolute zero is known with pretty good precision. The exact result is inside our uncertainty interval, which means that our measurements are possible. In the case of the temperature of liquid nitrogen, we know that the boiling point at room pressure is around $-195.8\,°\mathrm{C}$. We do not know the exact temperature of the liquid we measured, but it has to be below the boiling point. The nitrogen is exposed to room temperature, which is much warmer, so it will not be a lot below said boiling point. As our range covers temperatures right below the boiling point, this measurement also seems possible.

The results we have got with our experiment have pretty big uncertainties. So if we would want to redo the experiment, we would have to improve on that. We see the biggest problem in the sensor used. The device is meant to operate at voltages from 0 mV upwards. When calibrating, we had to measure voltages below that, which obviously is not optimal. So we would try to find a device more suited to its purpose.

7 Conclusion

In the concluding paragraph you summarize the result, with the emphasis on what you have discovered in this work. You can end this with an outlook on future research, i.e. how could the results be improved or what would be a logical follow up experiment.

8 Dos and Don'ts

- Be honest with yourself and with the reader. Try to find possible loopholes in your conclusion and explicitly mention them.
- Be aware that **scientific fraud** is an important topic that we (and the entire ETH) take very seriously. There are many forms of fraud, from copying text without referencing it to forging data. If unsure, ask your assistants about specific issues.
- Good writing is largely a question of practice and of experience. Why not read some scientific papers to study how professionals write? We are happy to recommend some literature.
- A good practice is to begin each paragraph with a 'topic sentence' that conveys the main message of the paragraph. As an example, the experimental section might start with "We performed experiments with a mechanical resonator inside a vacuum chamber." From this short sentence, the reader gathers immediately that the paragraph is about the experimental setup.
- Avoid using passive voice for extended paragraphs. The use of active language can make the text more interesting to read and is by default preferred by many English writers. For instance, instead of writing "The data points were measured over the course of 405 seconds", you can write "We measured data points over the course of 405 seconds" or simply "The data acquisition lasted 405 seconds". Of course, sometimes it may be better to use passive voice in order to describe basic processes, e.g. "Samples were cleaned for 3 minutes in acetone". The choice is yours - try out what fits better in specific cases.
- Write in short sentences. Always put clarity before artistic form. Whenever possible, avoid interrupting your sentences with brackets, formulas, or complex mathematical signs. Your text is much easier to read when you group such additional information at the end of a sentence, in a table, or in the caption of a figure. Remember that your readers might need their full attention for the physics involved (and do not want to decipher complex sentences).
- Avoid slang and terms that might not be known to the reader. One of the most difficult tasks is to explain something very complicated in simple terms that newspaper readers might understand. If you have to use specialized terms, try to explain them when they first appear.
- When you use abbreviations like 'AFM', make sure you use the full term once. "Nanoscale surfaces can be characterized with an atomic force microscope (AFM)."
- As a rule, use "cannot" instead of "can't", "will not" instead of "won't", "do not" instead of "don't" and so on (the title of this section is an exception).
- Graphs should not be overloaded with information. Make the essential features stand

out. Presenting scientific data is an art!

- Graphs should be drawn with the help of a software. In any case, graphs have to fulfill all relevant criteria of good scientific practice, such as well-scaled and labeled axes (including units), and the data points must be clearly visible and contain error bars where applicable.
- Fitting parameters only need to be provided for actual physical models, not for a "guide to the eye". Measured and derived values should be given with error bars (confidence intervals) and an appropriate number of significant digits.
- Figure captions are an important part of a figure. Ideally, a reader that is familiar with the field should understand your results by merely looking at your figures and reading the captions. Figure captions are an ideal place to give specific numbers that are not absolutely required in the main text (such as 'applied voltage' or 'laser power').
- The reports should be written with a text processing software (e.g. Latex).

References

[1] BIPM, The International System of Units (SI Brochure) [9th edition, 2019], https://www.bipm.org/en/publications/si-brochure/

[2] M. C. Cross, A. Zumdieck, R. Lifshitz, and J. L. Rogers, Phys. Rev. Lett. **93**, 224101 (2004).