Experiment 41, Beta Decay

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Abstract

In this experiment we examine a the β^--decay of an strontium source. When a β^--decay happens an electron is emitted with a given energy. Measuring the activity with an detector close to the source and the geometric properties of the setup, allowed us to determine the activity of the source itself. Further we measured the absorption curve giving us the ability to calculate the maximal initial energy of the electrons emitted from the decay of Yttrium in three different ways. From two methods we got the very close results to each other while being a bit of to the literature value. The third method compares reasonably good with the literature.

Contents

1 General

1.1 Introduction

Decays and radiation play a big part in physics. Radiation happens at every scales of our universe, from particle physics to astrophysics it has an important role. For us humans radiation can be very dangerous, as multiple disastrous examples in our history teach us. We also use decays to calculate the age of a given substance. If a particle like a neutron or a positron decays there are some other particles emitted. This is what we call radiation.

There are three types of decay in the nucleus. We call them α-, β- and γ-radiation. They differ from each other in the process and which particles are emitted. In this experiment we will look at β-decays.

There are two different types of β-decays. In a β^+-decay, a proton becomes a neutron while emitting a positron. On the other hand, in a β^--decay, a neutron becomes a proton and emits an electron. The second type is the one we will be looking at here.

1.2 Setup

The device we are using here is called a counter tube. It consists of a capacitor filled with gas. Every time an energetic particle enters the counter tube, the particle will ionize some of the gas particles, which will do the same thing again with other gas particles. This leads to an avalanche of energetic particles that can be detected by the counter tube, resulting in one count. The time used to create this avalanche limits the counting rate of the counter tube. Our counting tube has an integrated timer, so we can enter the amount of seconds we want it to run and it automatically counts for that amount of time.

As a source we use Strontium (^{90}Sr). This first decays into Yttrium (^{90}Y), which itself is unstable and decays into the stable Zirconium(^{90}Zr). For safety reasons, the source is covered by a thin 0.1 mm layer of steel.

2 Counter Tube and Properties of the Source

2.1 Introduction

The counter tube we are using for this experiment has to be operated at a specific voltage. If the voltage is too low, either no or not all particles are detected. If on the other hand the voltage is too high, self-discharging can occur, which leads to too many counts. The range, at which the device operates as we want is called the Geiger plateau. So first we will have to find this plateau, so that we can do further measurements.

The activity, the number of decays in a given time, of a radioactive source is measured in Becquerel (Bq). One Becquerel stands for one decay per second. In order to use our source properly, we need to know its activity. Because even without a radioactive source there is some radiation in our atmosphere, so we have to take account to that. As our device only measures at a limited place, but the source decays in all directions, we have to factor that in to know the real activity of the source. The fraction of the covered directions is called the acceptance ε of the detector.

2.2 Experiment

2.2.1 Counter Tube Characteristics

To perform later measurements, we first need to know at which voltage we have to operate the counter tube. In order to do that, we need to find the so called Geiger plateau. The Geiger plateau is the plateau section of the curve where the number of counts is plotted against the voltage (Fig. 1).

In order to do that, we measure the number of counts $N(t)$ in $t = 30\,\mathrm{s}$ for different voltage settings V. We start at 200 V and increase the voltage by 100 V for every measurement. As the voltage range of our counter tube only goes up to 980 V, we make the last measurement there.

Because the scale of the voltmeter is 20 V, we have an uncertainty in V of $\Delta V = 10\,\mathrm{V}$.

2.2.2 Activity of the Source

In this part we want to determine the actual activity of our ^{90}Sr Source. In order to do that we first measure the total amount of decays N in $t = 90\,\mathrm{s}$. To determine the background radiation N_{BG}, we measure a second time, but this time with the source shielded from the counter with a 9 mm steel plate. Subtracting N_{BG} from N gives us N_{eff}.

Next we want to measure the acceptance ε of the device. The formula for determining that is as follows.

$$\varepsilon = \sin^2\left(\frac{\theta}{2}\right) \text{ [1]} \tag{1}$$

Here, θ is the half angle of the triangle given by the point of our source and the width of the aperture. In order to know θ, we measure the radius r of the aperture and the distance d between the source and the detector. As $d = 101\,\mathrm{mm}$ is given by the manual [1], we assume there is an uncertainty of $\Delta d = 0.5\,\mathrm{mm}$. The scale of the vernier we use to measure the radius is to 0.05 mm, so we assume an error of $\Delta r = 0.025\,\mathrm{mm}$.

2.3 Results

The measurements of the characteristic curve of the detector are shown in Fig. 1, together with the fit to the Geiger plateau, which will be explained later. The middle of the plateau is at around 740 V, so we will operate the system at this voltage for the rest of the experiment.

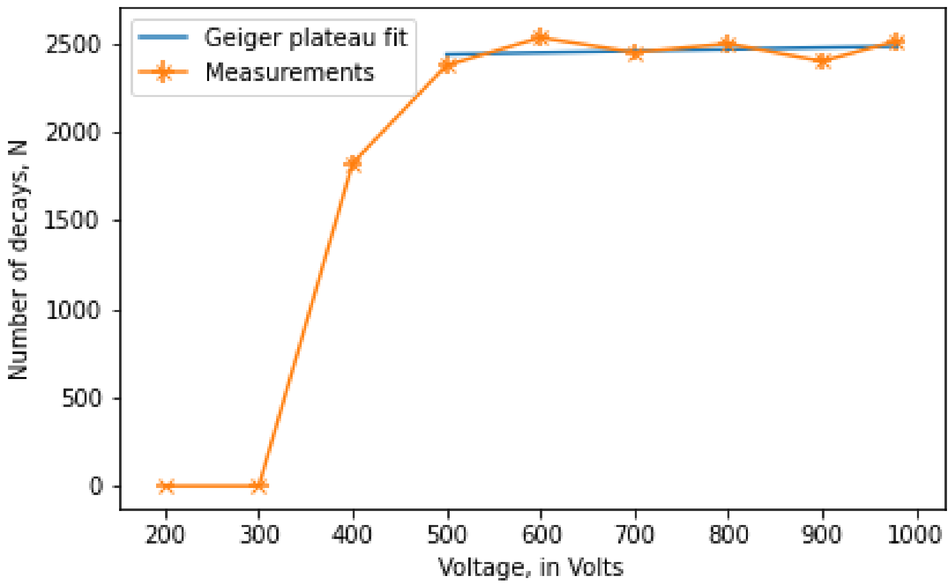

Figure 1: Number of counts plotted against voltage for duration of 30 s. Uncertainty only on voltage.

The uncertainty of the number of counts is described by the Poisson-distribution, therefore the uncertainty on any count is $\Delta N = \sqrt{N}$ [1]. We measured an activity of $N = 7453 \pm 86$ decays, together with $N_{BG} = 50 \pm 7$ decays without the source.

As said, the distance is given as $d = 101 \pm 0.5\,\mathrm{mm}$, and the diameter of the aperture we measured as $2 * r = 18.05 \pm 0.025\,\mathrm{mm}$.

2.4 Data analysis

2.4.1 Counter Tube Characteristics

As shown in Fig. 1, the Geiger plateau of the characteristic curve starts at around 500 V and goes up to our last measurement point at 980 V. This gives us a value of $V_{Ideal} = 740\,\mathrm{V}$ as middle and by that operating voltage of our detector. To know the quality of the detector, we look at its voltage dependency, so the slope of the plateau at V_{Ideal}. We fit a linear approximation with the method of least squares to the measured points, resulting the blue line in Fig. 1, and a slope of $9.5 \cdot 10^{-2}\mathrm{V}^{-1}$. So for every variation of the voltage by 100 V, the number of counts differs by 9.5. As for high number counts this is way smaller than the uncertainty we get from the Poisson-distribution, we can ignore that additional error in further calculations. In general, the smaller the gradient of the plateau, the better the detector.

2.4.2 Activity of the Source

To calculate the effective number of particles N_{eff} detected by the counter tube, we have to subtract N_{BG} from N. The error propagation method we will use most in this report is the Gaussian one. Therefore we introduce it here in a general form and reference to it later: For a function $R(A, B, \dots)$, where $A \pm \Delta A, B \pm \Delta B, \dots$ are measured values, it is as follows.

$$\Delta R = \sqrt{\left(\frac{\partial R}{\partial A}\Delta A\right)^2 + \left(\frac{\partial R}{\partial B}\Delta B\right)^2 + \dots} \tag{2}$$

Using this method gives us an effective decay of $N_{eff} = N - N_{BG} = 7403 \pm 87$ decays in 90 s.

As we measured the diameter but not the radius of the aperture, we have to divide the value in two. Using Gauss again, we get a value of $r = 9.025 \pm 0.0125$ mm. To determine the acceptance ε of the detector, we need to know the half angle θ. Given our measurements, and using basic trigonometry, θ is given as $\theta(r, d) = \arctan(r/d)$. Once again we use Gauss to determine the error propagation, leaving us with $\theta = 5.11 \pm 0.03$ deg. Now, equation (1) finally gives us $\varepsilon = 1.98 \pm 0.02 \cdot 10^{-3}$. The total activity of the source is now given as $A_{tot} = \frac{N_{eff}}{t \cdot \varepsilon} = 41454 \pm 632$ Bq, using Gauss.

As the source is surrounded by a thin layer of steel which will prevent some radiation from getting to the source. For ^{90}Sr decay, that is around 55%, for ^{90}Y decay, that is around 10%. Using the fact that in this case around half of the decays are ^{90}Sr and the other half ^{90}Y, we can calculate the activity of the source without the steel layer as $\tilde{A}_{tot} = \frac{A_{tot}(45+90)}{2*(100)} = 69090 \pm 1011$ Bq.

2.5 Activity and Dose Calculations

Here we want to calculate the lifetime τ of Sr and Y. If n_0 is the amount of atoms at the start, there are n_0/e atoms are left after τ. We can calculate τ using the half-live of Sr and Y. The half-live of Sr is $h_{Sr} = 28.8$y, while the half-live of Y is $h_Y = 64$h [1]. The number of atoms remaining is given by $n(t) = n_0 e^{-t/\tau}$. The formula for getting the lifetime is then $\tau = h/\log(2)$, which leaves us with $\tau_{Sr} = 42$y and $\tau_Y = 92$h.

As the activity of the source is also described as $A(t) = \tilde{A}_{tot} e^{-t/\tau}$, we can calculate, when the activity is down to $A = 10000$Bq, which leads us to $t = -\tau \log(A/\tilde{A}_{tot}) = 80$y. So in 80 years, the activity of our source will be around 10 000 Bq.

The average radiation people in Switzerland are exposed to in a year is $S = 5.5\,\mathrm{mSv}$. Knowing the activity of the source we can calculate how long it would take to reach that dose if someone incorporated it. We can describe the dose with $D = tA(d_{Sr} + dY)$, where $d_{Sr} = 2.80 \cdot 10^{-8}\mathrm{SvBq}^{-1}$ and $d_Y = 2.70 \cdot 10^{-9}\mathrm{SvBq}^{-1}$ are the dose coefficients for Sr and Y [1]. To know the time t_{Dose} it would take, we can rearrange this formula to

$$t_{Dose} = \frac{D}{A(d_{Sr} + d_Y)} = 2.6\,\mathrm{s}.$$

After that time, the average annual dose would be reached, so it is better to not incorporate the source!

3 Determining E_{max}

3.1 Experiment

The goal of this task is to find the maximum energy E_{max} of the beta particles leafing the source. We know that higher energy particles can penetrate thicker layers of material. With this knowledge, we measure the amount of particles which successfully penetrate our testing material to get an absorption curve of the material. From this curve we use three methods to get our E_{max}.

To get this absorption curve, we lay aluminium plates of twelve different sizes in x-axis in the steel slider of our testing apparatus and measure the count rate N over a time of 90 seconds. The aluminium plates used are in a range from 0mm to 4mm in thickness. After placing each plate in the measuring apparatus we let the automatic counter count all electrons reaching the sensor for 90 seconds. The counted electrons then get logarithmically plotted against the thickness of the plates.

From the plotted absorption curve we then can make our calculations of E_{max}.

Method 1 we look at the absorption curve and find the maximum thickness x_{max} of the aluminium, where the radiation is starting to fail to penetrate the aluminium. This thickness is then used to read out the maximal energy from a given figure [1] witch shows the range of the electrons in aluminium as a function of the distance-density product ρx. On top of this value we add the energy lost E_{loss} which the electrons lose to penetrate the thin stainless steel foil placed over the source. These two values added describe the initial energy of source $E_{maxsource}$. Like above the we get this value from a plot given in the manual [1], which shows the energy loss of electrons for 0.1mm stainless steel, as a function of electron energy.

Method 2 Here we use all the measured counts with a activity above our background noise N_{BG} ($x < x_{max}$), to make a fit on a giving model from the manual [1]. The model used is an exponential which when plotted in a logarithmic scale gives us an straight line. Using the slope μ of the straight line we can calculate the E_{max} from a given equation.

Method 3 in the last method we first correct our background noise. Then we plot the corrected counts N_{eff} against $E_{max} - E(x)$ in a double logarithmic scale. From this plot we calculate the slope n to draw a new plot were we plot the n-th root of the corrected counts N_{eff} against the energy E. From this we get an extrapolated intersecting line were the counts are zero giving us a new E_{max} value.

3.2 Results

Results method 1 From the measured absorption curve displayed in figure 2, we can read x_{max} of 3.5 ± 0.25mm. This leads us to an E_{max} of 1.9 ± 0.5MeV and an $E_{maxsource}$ of 2.0 ± 0.5MeV.

Results method 2 The slope μ calculated from values above the background noise N_{BG} is $1.59 \pm 0,07$. The corresponding slope is plotted in figure 2. Resulting in a E_{max} of $2.1 \pm 0,3$MeV and an $E_{maxsource}$ of 2.2 ± 0.3MeV.

Results method 3 With the last method an E_{max} of $1.9 \pm 0,3$MeV is found. Leading to an $E_{maxsource}$ of 2.0 ± 0.3MeV.

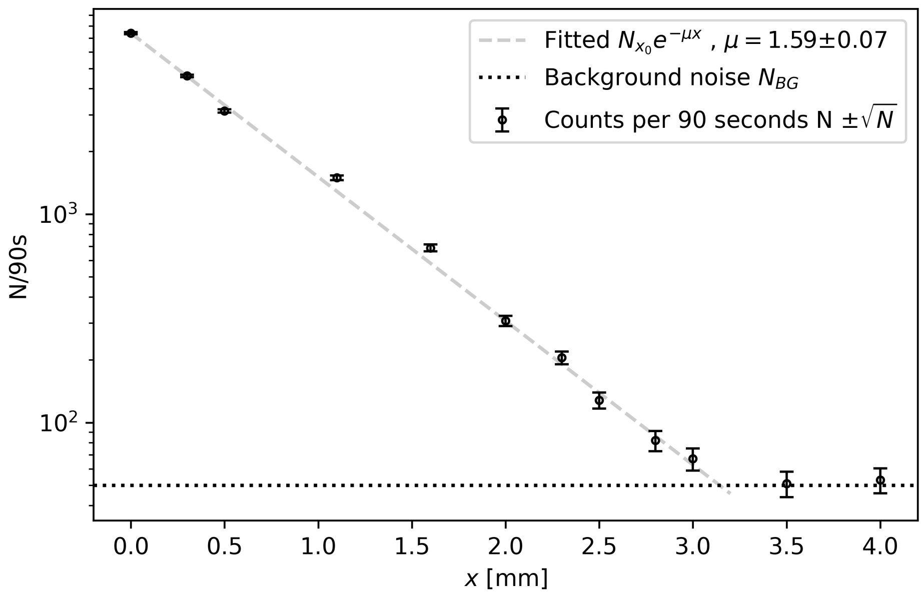

Figure 2: Number of counts per 90 s is logarithmically plotted against the thickness of the aluminium plates [mm]. The uncertainty of the counted numbers is displayed as error bars on the measured counts.

3.3 Analysis

For the absorption curve we measured all the given aluminium plates with a caliper. The measured thickness of the different plates is found in the lab sheet in the appendix. An error of the scale of the caliper is $\pm 0,05$mm. Since this error is relatively small to the errors further in the experiment we neglected it.

Each plate got measured in the apparatus for 90 seconds giving us twelve counts N_i with an corresponding error of $\pm\sqrt{N_i}$ for each different plate used. This values are shown in Figure 2 with error bars.

Analysis method 1 We determined from eye that after 3.5mm the counted electrons are in the same range as the background noise picked up by the detector. So we set our x_{max} to 3.5mm with an estimated error of ± 0.25mm.

If we follow the last section described steps, we multiply the x_{max} value with the given ρ_{Al} ($2.69 \mathrm{g\,cm^{-3}}$) from the manual [1]. This resulting value is then used to find the corresponding energy from the diagram [1]. Having a double logarithmic plot the error to read out the diagram is estimated half of the smallest grid drawn. In the range of $\rho_{Al} x_{max} = 941 \mathrm{mg\,cm^{-2}}$ the result for E_{max} is 1.9 ± 0.5MeV.

Then we use this E_{max} value to read the E_{loss} from another table in the manual [1] giving us a loss energy of 108keV. This table used shows the energy loss of the electrons for 0.1mm stainless steel and has an scale of 4keV. Resulting in an reading error of ± 2keV. Ending in a $E_{maxsource} = E_{max} + E_{loss} = 2.0 \pm 0.5$MeV.

Analysis method 2 In the second method we had to fit the from the manual [1] given model

$$N(x) = N_0 e^{\mu x}$$

to our values. This was done with the numpy polyfit method, retuning $\mu = 1.59$ with an given error of 0.07. Using the given equation [1] to calculate

$$E_max = \left(\frac{17\rho_{Al}}{\mu}\right)^{1/1.43} = 2.093,$$

we only have to use the Gauss method on this equation to get the uncertainty on the maximum energy $\Delta E_{max} = 0.31$. For the loss energy the same procedure as in method 1 is used. Combined this results in E_{max} of $2.1 \pm 0,3$MeV and an $E_{maxsource}$ of 2.2 ± 0.3MeV.

Analysis method 3 For the last method we first had to subtract the background noise

$$N_{eff} = N - N_{BG}.$$

This points are plotted as a function of $E_{max} - E(x)$ on a double logarithm scale as seen in figure 3.

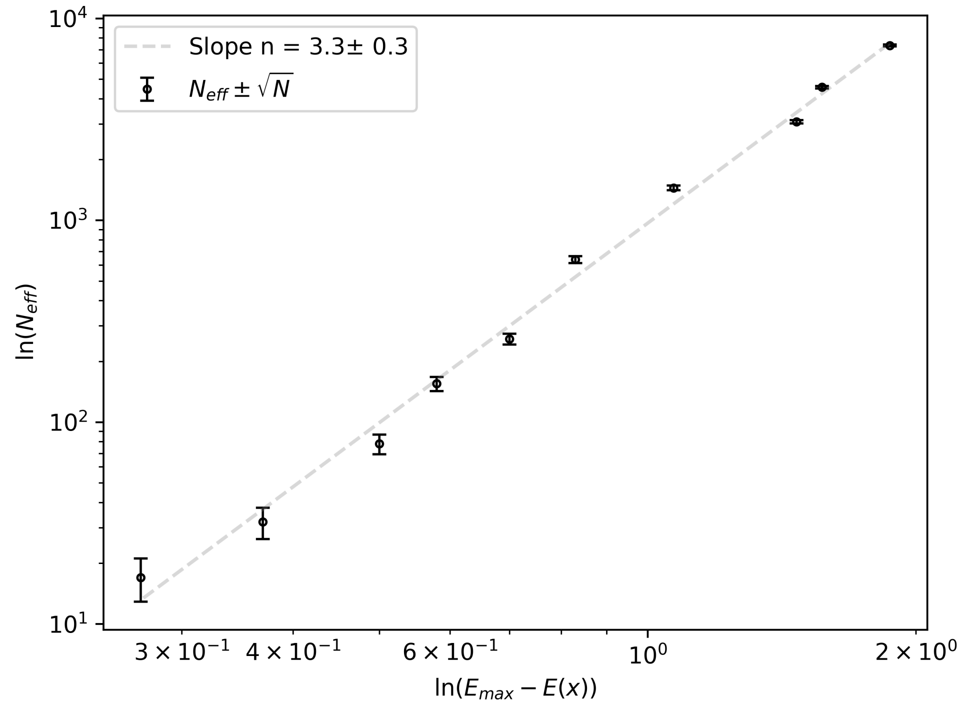

Figure 3: Number without background noise of counts per 90 s is double logarithmically plotted against the maximum energy from method 1 subtracted from the energies at the different sizes of the aluminium plates $E_{max} - E(x)$. Where is is the thickness of the plates in millimetre. Trough this point a linear fit is done with an slope value $n = 3.3 \pm 0.3$.

From this point we determine the slope trough a polyfit in numpy. This gives us an slope $n = 3.28$ and an error of 0.258 which are used to generate the next plot.

Using this slope to calculate the values $N_{eff}^{1/n}$ and plotting it against the energy $E(x)$ as seen in figure 4. The same procedure to find a slope is done again resulting in a linear model $f(E) = a + bE + \epsilon_E$. The from the slope b and offset a we can calculate the function for $f(E) = 0$. Giving us the equation

$$E_{max} = \frac{-a}{b}$$

with an already known error of 0,85, $a = 15,21$ and $b = -8,14$ resulting in $E_{max} = 1.9$ and the Gaussian error propagation we get an error of $\pm 0,3$MeV.

With the same approach like above we get $E_{maxsource} = 2.0 \pm 0.3$MeV.

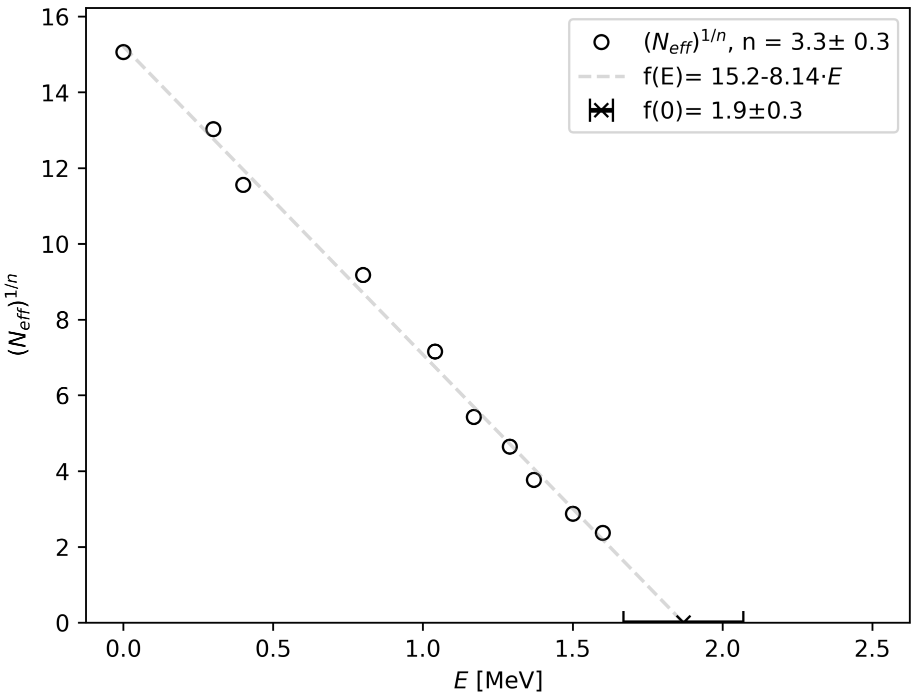

Figure 4: Number of counts per 90 s is logarithmically plotted against the thickness of the aluminium plates [mm]. The uncertainty of the counted numbers is displayed as error bars on the measured counts.

4 Conclusion and Discussion

In the first part of the experiments we looked at the detector an how well its performance is. Having an gradient which is negligible small on the Geiger plateau is a good sign that we can trust the values measured by the detector.

The found activity of around 69kBq with an error of 1.5% the value lays in between the reference values given in the manual. Also the calculated time needed to reduce the activity of the source to 10kBq is 80 years with a half-life time of 28,8 years in Strontium, this results seem plausible. We can also see, that even with a small source like this, it would only need 2,6 seconds to get the same radiation as annually reached in Switzerland. This shows how bad it can be if somebody swallows radio active materials like this.

In the second part we looked at the energies. If we compare our literature value for Yttrium 2.282MeV with our results, we can see that the reference value lays in the confidence interval of all the results. While the expected value of method 1 and 3 are a little further away method 2 got close for the expected value.

For all methods we had to use some logarithmic tables to read values of it. This lead to some big errors on the results. Method 1 is the worst performing method in our experiments. It produced a very big uncertainty value, bigger than in the other two methods while having an expected value which is not better than method 2 or 3.

There a many ways to improve the results if the experiment was redone. On way would be to measure for longer time periods, which would result in a smaller error on the counted activity. Measuring the same part of the experiment would increase the confidence in the numbers and improve the expected values.

Especially in the absorption curve, it would be a good thing to have more data in the interesting parts. With more different strengths of aluminium we could improve the x_{max} value. Improving overall the energy measurement results.

The last part which could drastically improve the error, would be to use the function or dataset used to generate the logarithmic reference plots used in the manual. This way a lot of values can be calculated instead of read out of an plot wich can lead to high errors.

5 Questions for Physics Students

- **Describe the various ranges of operation of a gas detector. Which voltage range should be chosen to operate the detector as ionisation chamber, proportional chamber or Geiger-Mueller counter tube?**

  If the voltage of the detector is very low, most of the ions do not reach the electrodes of the tube. When we want to use the detector as an ionisation chamber, we have to increase the voltage, so that the ions ar able to reach the electrodes, but not so high that the ion current can be detected.

  As we increase the voltage further, the electric field gets bigger. Then, the gas atoms in the detector get ionized by the incoming electrons, which then inoize further gas atoms and so on. This leads to the so-called gas amplification [1]. In the right range, the charge is then proportional to the amount of incoming ions in the beginning. This is called the proportionality range.

  If we want to use it as a Geiger-Mueller counter tube, as we do in this experiment, we have to operate it at said Geiger-Plateau. This is described in the report above.

- **What determines the energy resolution of the detector? A particularly good energy resolution can be achieved with semiconductor detectors. Why? How do scintillation detector work?**

  A detector with a higher energy resolution detects decays with lower energy. A detector with a low energy resolution does not detect all decays, while a better one detects more. While in a gas detector ions are measured, the semiconductor detector measures the amount of electrons. The energy required to produce electrons is far lower than for ions, so the number of undetected decays is much smaller in a semiconductor detector than in a gas detector.

A scintillating material reacts to passing radiation by emitting light. This light gets amplified by a photomultiplier, so that it can be measured and analysed.

- **What other important aspects of detectors can you think of?** The detector should not produce any radiation itself, it should not react to outer influences (for example light in case of a scintillator).

- **How can spatial resolution be achieved with counting detectors?** We could install multiple detectors spread across the area to get spatial resolution.

- **Beta particles are either negatively charged (=electrons) or positively charged (=positrons). How are positrons used nowadays to investigate and examine materials?**

  Theβ^+-decay is used in a technique called positron emission tomography (PET). It is mostly used in (nuclear) medicine.

- **Are neutrons also subject to β-decays?** Yes, a neutron can decay into a proton emitting an electron and an antineutrino, which is called β-decay.

- **What is the difference between γ-rays and X-rays?** The difference between γ- and X-rays comes from the way they originate. X-rays happen, when the velocity of charged particles changes, while γ-rays are photons emitted by radioactive decays or nuclear reactions.

References

[1] Lab manual 41 - Beta Decay, Physikpraktikum der ETH Zurich, https://ap.phys.ethz.ch/Anleitungen/Bilingual/41_Manual.pdf.

6 Appendix

Name 1: Fritz Kurz Datum: ....

Name 2: Cédric Renda Platz Nr: ....

41	Aktivität und Energie des ß-Strahlers

1. Zählrohrcharakteristik

V	200	300	400	500	600	700	800	900	980			
t	30	"	"	"	"	"	"	"	"			
N(V)	0	0	1826	2379	2535	2452	2501	2402	2516			

V_{ideal} = 740 V;

Spannungsabhängigkeit der Zählrate N bei V_{ideal}: $\frac{\Delta N}{\Delta V} = 9{,}5 \cdot 10^{-2}\ V^{-1}$

Je kleiner $\frac{\Delta N}{\Delta V}$, umso besser (besser / schlechter) ist der Geigerzähler.

2. Aktivität der Quelle

t = 90s .... Messdauer

N = 7453 .... Einfallende Teilchen in der Zeit t

N_{BG} = 50 .... Hintergrund in der Zeit t (mit 8 mm Messing-Absorber)

$N_{eff} = N - N_{BG}$ = 7403 .... Effektive Anzahl einfallender Teilchen in der Zeit t

Raumwinkelberechnung:

Durchmesser Stahlblende d = 18,05 mm

Halbwinkel Kegel θ = 5,11 deg

somit ist der Raumwinkelbruchteil bzw. die Akzeptanz ε = 1,98

Was ist der Raumwinkel? Beachte, dass die Quelle in alle Richtungen näherungsweise uniform strahlt. Der Detektor „sieht" aber nur einen Bruchteil davon, den sog. Raumwinkelbruchteil (genannt „Akzeptanz"). Die Berechnung des Raumwinkels soll vorgängig erlernt werden.

Aktivität der hier verwendeten Probe = 41454 Bq

Aktivität der hier verwendeten Probe, wenn man die schützende Stahlfolie der Quelle entfernen würde: 69090 Bq

3. Strahlentechnisch relevante Berechnungen

a. 1/e Zeit von Sr: 42 Jahre. 1/e Zeit von Y: 92 Stunden
   - Angenommen, die Quelle wurde am 1.1.1950 hergestellt: Was war die Aktivität zum Herstellungszeitpunkt? Die Aktivität war 385'891 Bq

b. Aktivität der Quelle sinkt auf 10 kBq in 80 Jahren

c. Wie lange dauert es, bis die mittlere Jahresdosis in der Schweiz (= 5.5 mSv) bei vollständiger Inkorporation dieser Quelle erreicht wird?

4. Absorption und β-Energie

Hinweis: Dichte $\rho(Al) = 2{,}69\ g\ cm^{-3}$

(nur für Studierende des D-PHYS)

x [mm]	$\rho(Al)\cdot x$ [mg·cm⁻²] ~~[gcm⁻²]~~	t [sec]	N	50 N_{eff}	gemessene Aktivität [Bq]	E(x) [MeV]	E_{max} -E(x) [MeV]	$(N_{eff})^{1/n}$
0	0	90	7390	7340		–	1,87	
0,3	80,7	〃	4606	4556		0,3	1,57	
0,5	134,5	〃	3124	3074		0,4	1,47	
1,1	295,9	〃	1496	1446		0,8	1,07	
1,6	430,4	〃	689	639		1,04	0,83	
2	538	〃	308	258		1,17	0,7	
2,3	618,7	〃	205	155		1,29	0,58	
2,5	672,5	〃	128	78		1,37	0,5	
2,8	753,2	〃	82	32		1,50	0,37	
3	807	〃	67	17		1,60	0,27	
3,5	941,5	〃	51	1		1,87	0	
4	1076	〃	53	3		2,18	–	

a) x_{max} = ...3,5............... mm, und somit E_{max} = ...1,87......... MeV

b) Energieverlust Stahlfolie = ...108.... keV

c) $E_{max\ Quelle}$ = ...1,978.... MeV

d) μ = ...1,63.......... m~~c~~m⁻¹, und somit E_{max}(für μ=...1,63... m ~~c~~m⁻¹) 2,06.... MeV

e) $E_{max\ Quelle}$ (μ) = 2,17....... MeV

5. E_{max} für Studierende des Departements Physik

Koeffizient n = ...3,28........... und somit E_{max} = ...1,87....... MeV

$E_{max\ Quelle}$ = ...1,978... MeV