Experiment 05 - Mechanic Resonance

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Abstract

As dampened oscillations are important all over physics, it is valuable to know how to determine the dampening constant of a given system. In this experiment we measure three different dampening constants of a rotating pendulum with two different methods. We first look at the dampened natural oscillation, and then add a motor and analyse the resulting forced oscillation. The results we get are close to each other, but do not match perfectly. We try to find optimizations to the methods to improve the results for further experiments.

Contents

1 General

1.1 Introduction

The dampened harmonic oscillator is omnipresent in many if not all fields of physics. Despite often looking at idealized systems without dampening, in the real world there are always some imperfections which result in dampening. The best example here probably is friction. As dampening often is not dependent on time, we can assume that there is some constant α describing it.

Our goal in this experiment is to find the dampening constant α for three different dampenings. We will do this in two different ways: In the first part, we want to use the dampened oscillation without any external force, and in the second part we look at a forced oscillation.

1.2 Setup

The general setup of the experiment consists of a rotating pendulum, equipped with an electromagnetic brake and a motor as shown in Fig. 1.

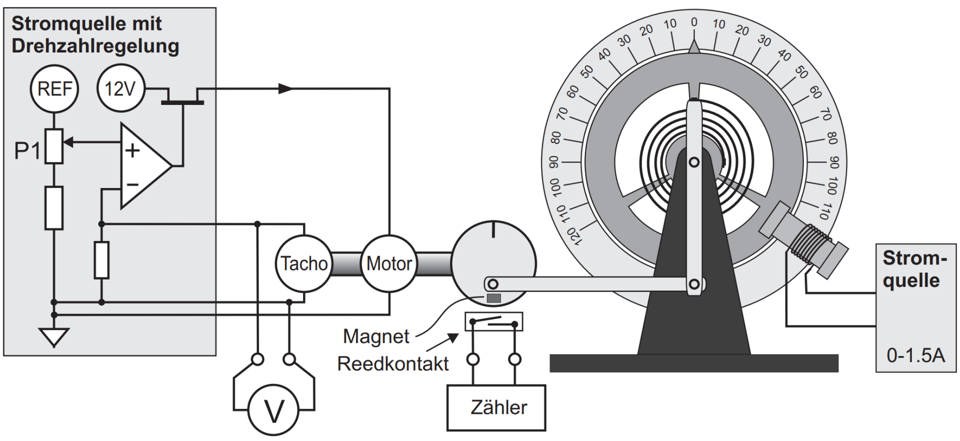

Figure 1: Experiment Setup: Rotating pendulum with motor and electromagnetic brake. [1]

The system itself already has some dampening caused by friction, but as we want to test different dampenings, we use the electromagnetic brake to create controlled dampening. We want to determine the dampening constants $\alpha_{1,2,3}$ of the system for current values $I_{1,2,3}$ of the brake of 0.64, 0.9 and 1.2 A. Because for the second part of the experiment we need an external force, there is also a motor attached to the system.

2 Dampened Natural Oscillation

2.1 Experiment

In this part of the experiment we want to use the dampened natural oscillation of our system to determine the dampening constants $\alpha_{1,2,3}$. The formula for the amplitude of this system with a given starting value A_0 is as follows.

$$A(t) = A_0 e^{-\alpha t} \tag{1}$$

To determine α, we can measure the amplitude of the system for different times t, and then fit the model (1) to our measurement points numerically.

We always start with an amplitude A_0 of $110 \pm 0.5°$. For the first dampening α_1 with a current $I_1 = 0.64 \pm 0.01\,\mathrm{A}$ on the electromagnetic brake, we measure the amplitudes A at every third period, for $n_1 = 24$ periods in total. As a higher dampening results in a faster decay time, we measure $n_2 = 14$ periods with $I_2 = 0.9\,\mathrm{A}$, measuring every second period, and $n_3 = 8$ periods for $I_3 = 1.2\,\mathrm{A}$, measuring every single period. For every dampening, we also measure the total time t_i of those n_i periods with a stopwatch. We estimate the error of every measurement for t with $\Delta t = 0.4\,\mathrm{s}$ because of the human reaction time.

The Gaussian error propagation for a function $R(A, B, \dots)$, where $A \pm \Delta A, B \pm \Delta B, \dots$ is as follows.

$$\Delta R = \sqrt{\left(\frac{\partial R}{\partial A}\Delta A\right)^2 + \left(\frac{\partial R}{\partial B}\Delta B\right)^2 + \dots} \tag{2}$$

2.2 Results

The amplitudes A measured in the first experiment can be seen in figure 2. There is an estimated error bar of $\pm 2°$ for all values other than the first one. The first value was fixed at an amplitude of $A_0 = 110°$ with an $1°$ scale, resulting in a $\pm 0.5°$ error. In figure 2 we see an linear behaviour of the measured values if plotted in a logarithmic scale getting close to the expected exponential decay.

The uncertainty Δt of the measured time is in the magnitude of $0,4$s with the stop watch. Since this is barly visible in the plot it was removed.

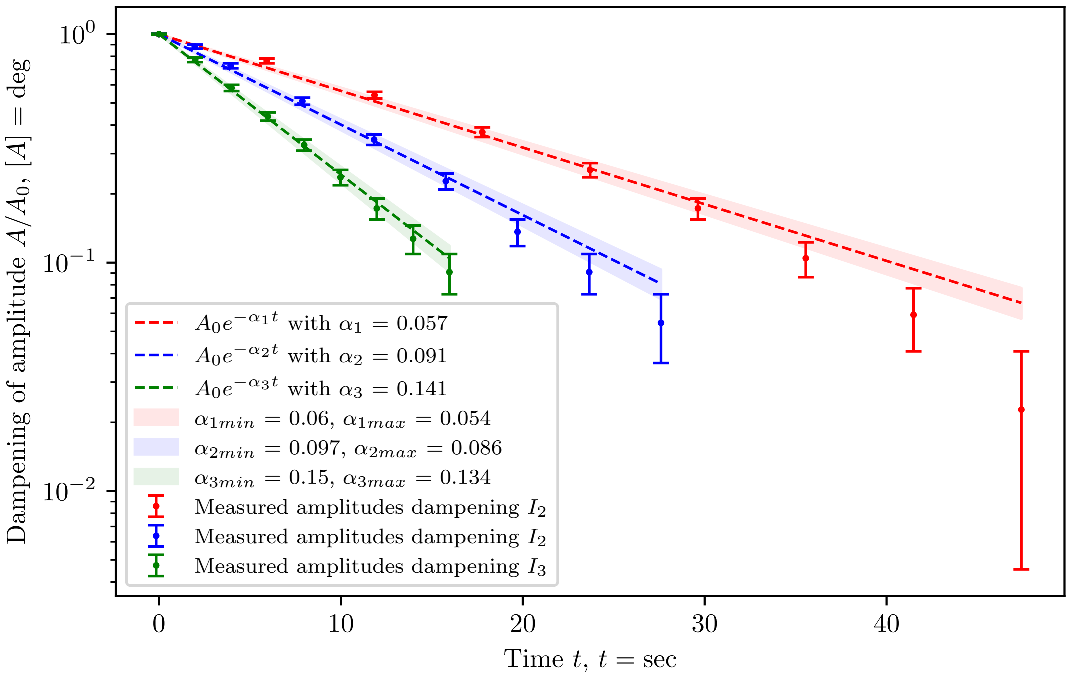

Figure 2: Measured amplitudes with an estimated error of $\Delta A = \pm 2^{\circ}$ plotted on a logarithmic scale. Shown next to the exponential decay with $\alpha_{1,2,3}$ as decay rates and $A_0 = 110^{\circ}$ as starting value of the measurement. To every α value a calculated maximum and minimum error band is shown.

2.3 Data Analysis

For the first experiment we followed the steps described in the subsection 2.1. To determine the value α_1, we taken all the measured amplitudes with the corresponding times and performed a least square fit on the expected model 1. The least square fit for α_1 on the measured values is shown in figure 3 as an red dashed line. Since this curve gets not very close to the measured values the measuring technique is probably not the most exact method used. We expected an error of $\pm 2^{\circ}$ while reading the scale in motion. But since the values changed very fast and the reading was done by eye an even grater uncertainty can be expected.

To calculate an error on the calculated decaying rate α_1 another two fits were done. For the first one α_{1max} we used the same set of data but added the estimated errors to the original values before performing the fit. On the second one α_{1min} we subtracted the estimated error. Having the maximum and minimum α_1 we calculate the error

$$\Delta\alpha_1 = \frac{1}{2}(\alpha_{1max} - \alpha_{1min})$$

This error is shown in figure 3 and figure 2 as the error band.

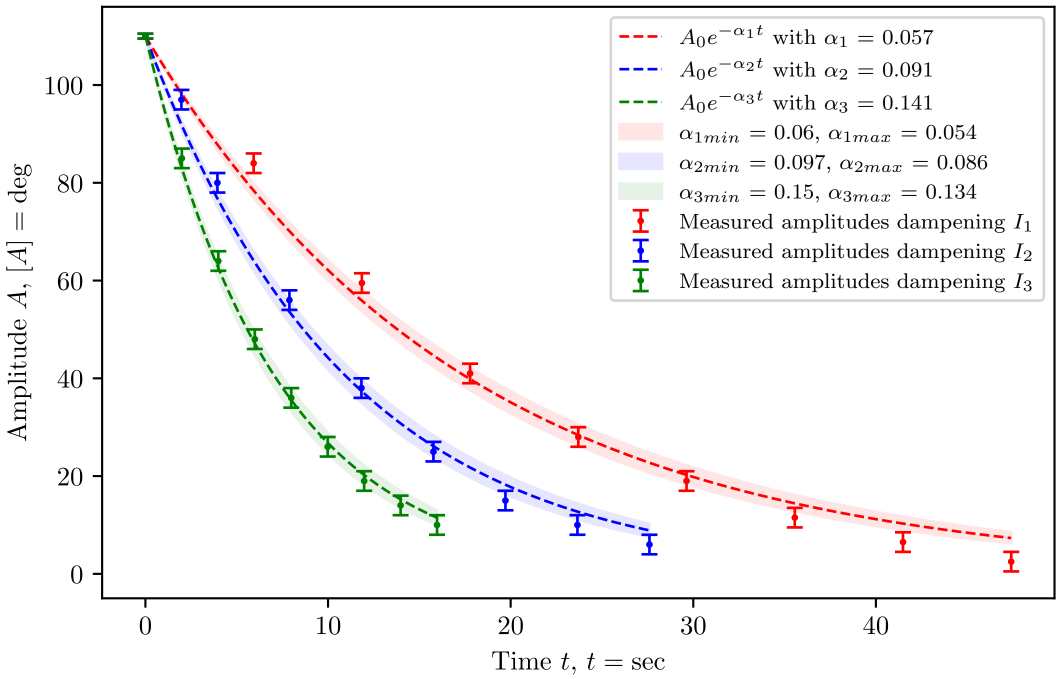

Figure 3: Measured amplitudes with an estimated error of $\Delta A = \pm 2°$. Shown next to the exponential decay with $\alpha_{1,2,3}$ as decay rates and $A_0 = 110°$ as starting value of the measurement. To every α value a calculated maximum and minimum error band is shown.

The same procedure as for the value α_1 was done for $\alpha_{2,3}$. resulting in the following values for $\alpha_{1,2,3}$:

$$\alpha_1 = 0.057 \pm 0.003\,\mathrm{s}^{-1}$$
$$\alpha_2 = 0.091 \pm 0.006\,\mathrm{s}^{-1}$$
$$\alpha_3 = 0.141 \pm 0.008\,\mathrm{s}^{-1}$$

3 Forced Oscillation

3.1 Calibration

To perform our experiment we need to know the exact angular frequency ω of our motor. As we are only able to read the output voltage of the motor, we have to calibrate it before actually performing the experiment. In order to do that, we measure the time t that it takes the motor to do N rotations at a voltage V_{Tacho}. This gives us the period $T = t/N$.

From that we know the angular frequency $\omega = 2\pi/T$ of the motor at V_{Tacho}. The relation between V_{Tacho} and ω is given by $C_T = \omega/V_{Tacho}$.

In our case, we do this measurement three times for three different voltages. Again we use $\Delta t = 0.4\,\mathrm{s}$. As our voltmeter reads to an accuracy of $0.001\,\mathrm{V}$, we use an uncertainty for the voltage of $\Delta V_{Tacho} = 0.0005\,\mathrm{V}$. Using Gaussian error propagation (2) we get a final value of $C_T = 2.45 \pm 0.01\,\mathrm{V}^{-1}\mathrm{s}^{-1}$. This lets us translate a voltage measurement V into the angular frequency ω with the formula

$$\omega = C_T V. \tag{3}$$

3.2 Experiment

To perform this experiment we measure the amplitude A of the system for different angular frequencies ω of the motor around the eigenfrequency ω_0 of the system. We do our measurements for ω in a range of around $\omega_0 \pm 0.5\,\mathrm{s}^{-1}$. Because we will use the maximal frequency A_{max} later, we do one measurement as close as possible to the Eigenfrequency ω_0 of the system. It is very important to let the system get to its stationary state after changing ω, because before that, the amplitude oscillates quite a bit. For each dampening, we measure between 12 and 14 different points.

3.3 Results

The resonance curves we measured can be seen in Fig.4. The uncertainties on the amplitudes A come from two different places. First, we can not be entirely sure that the system has reached its stationary state. Second there is an uncertainty of $\pm 0.5°$ because of the scale of the device itself. We estimate the resulting uncertainty to $\Delta A = 2°$.

The uncertainties of the angular frequencies ω come directly from the already calculated error on C_T and the accuracy of the voltmeter. Using Gaussian error propagation (2) on formula 3 lets us calculate $\Delta\omega$ for every measurement. Those uncertainties are in the magnitude of $\Delta\omega \approx 0.005 s^{-1}$, so they can be ignored (that is also why they are not shown in Fig. 4).

3.4 Data Analysis

We now want to estimate the dampening constants $\alpha_{1,2,3}$ using the resonance curves. We will show the process of doing that for α_1, for the other two it is exactly the same.

The dampening constant α is given as half of the width of the curve at height $A_\alpha = A_{max}/\sqrt{2}$. As we did one measurement per dampening at resonance frequency, this will be our value for A_{max}. To estimate the values of the resonance curve at this height, we make a linear approximation of the curve between the measurement points closest to A_α. Then we calculate, where the two resulting linear functions are equal to A_α. This gives us the

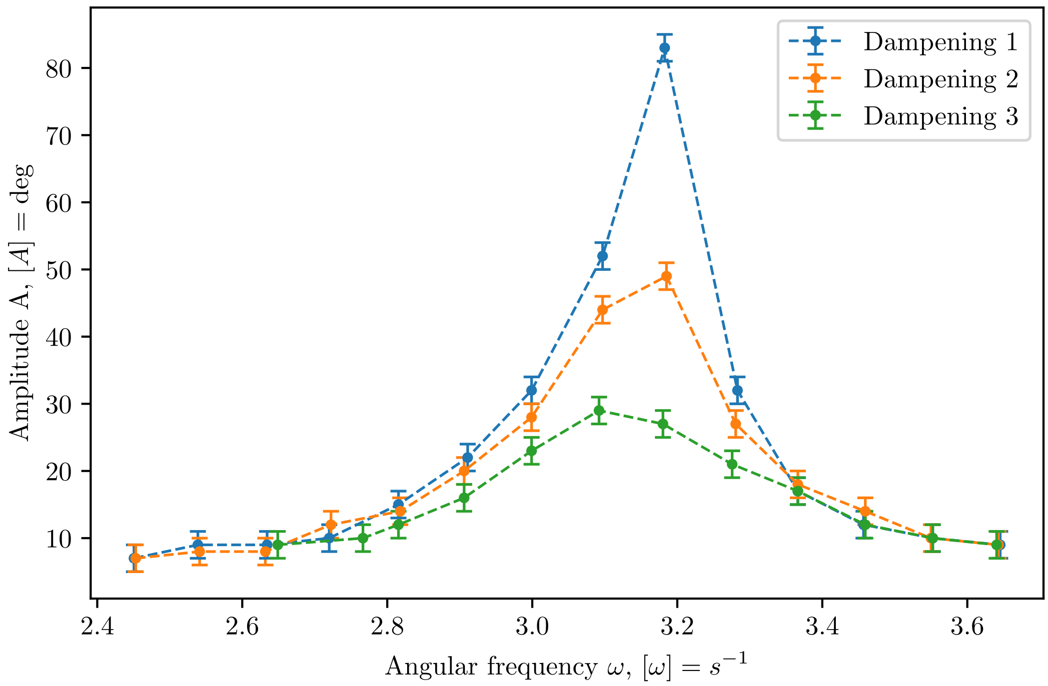

Figure 4: Measured resonance curves with errors. Errors are only shown on amplitude as they are too small to see on angular frequencies.

values ω_1 and ω_2. We now get $\sigma_1 = \omega_0 - \omega_1$ and $\sigma_2 = \omega_0 + \omega_2$. The dampening constant is then approximated with

$$\alpha = \frac{\sigma_1 + \sigma_2}{2}.$$

To calculate the error of this value, we perform basically the same calculations as before, once for the upper limit of our error range for the resonance curve and once for the lower limit, as indicated in Fig.5.

As said before, we do the exact same thing for the other two dampenings. In the end, this leaves us with the following values for $\alpha_{1,2,3}$:

$$\alpha_1 = 0.058 \pm 0.001\,\mathrm{s}^{-1}$$
$$\alpha_2 = 0.104 \pm 0.003\,\mathrm{s}^{-1}$$
$$\alpha_3 = 0.16 \pm 0.01\,\mathrm{s}^{-1}$$

4 Discussion and Conclusion

The values we get for α using the two different methods are pretty close to each other. This indicates that they are also not too far off the real values. As the methods and setups

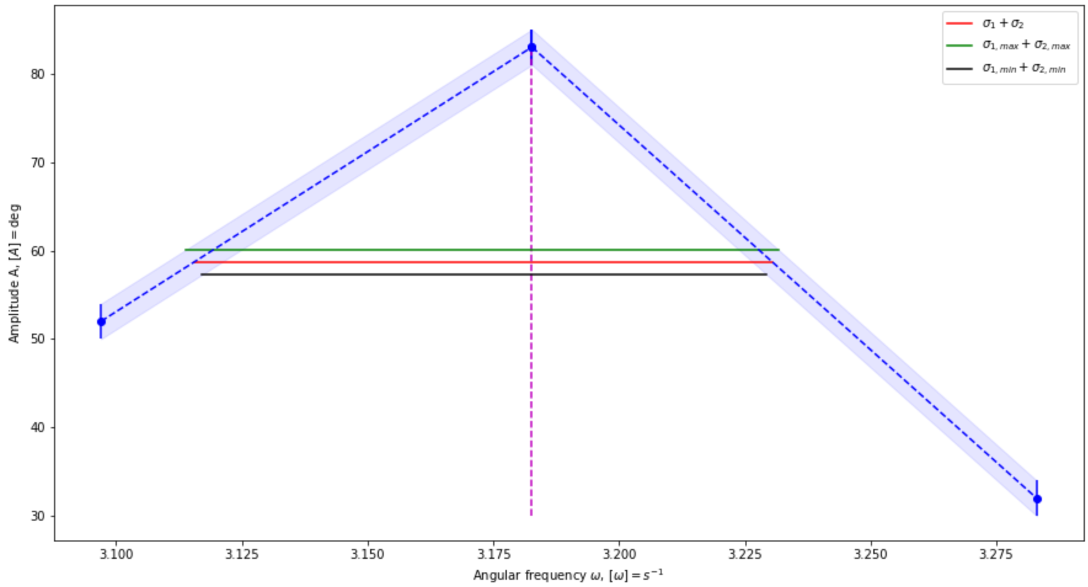

Figure 5: Important part of resonance curve for dampening 1. The horizontal lines are the geometrical representation of how α and $\Delta\alpha$ are calculated.

differ from each other, it is hard to know if the real dampening constants are really exactly the same, or if for example the motor has an influence on it. Nonetheless, we could improve the results of both methods quite a bit:

In general, it was quite hard to see the exact values of the amplitude on the scale. We could improve this by filming the experiment and then analyse the video in slow-motion. Another, better way to improve it would be to let a computer measure the amplitudes.

In the first part the problem mostly was finding a good fit of the model to our measurements. We expect, that the system does not follow the model exactly, and because of that we were not able to fit it without getting quite big errors either in the beginning or in the end. The second problem was determining the error when using pre-written python functions. Here, we could probably use more sophisticated methods to do the error propagation.

In the second part we did too many measurements we later did not use at all. To get better results, we would try to find the important values A_{max}, σ_1 and σ_2 as perfect as possible, so we would perform all the measurements around the expected values for those. This should definitely help us get closer to the correct value. We also experienced problems calculating the correct error, specially when doing (linear) approximations. If we had more measurements in the regions we look at, we could probably get a more sophisticated approximation then a linear one, as the real resonance curve is by no means linear. As proposed in the beginning, as soon as we had a setup where a computer reads the amplitude of the system, it would be pretty easy to write a program that changes ω of the motor by a given margin as soon as the amplitude has reached its stationary state. We could then leave the experiment running for a longer time, which would then give us a very detailed resonance curve.

5 References

References

[1] Lab manual 05 - Mechanical Resonance, Physikpraktikum der ETH Zurich, https://ap.phys.ethz.ch/Anleitungen/Bilingual/05_Manual.pdf.

6 Questions for students

- **What is a harmonic oscillator, what is its Eigenfrequency?** A system which experiences a restoring force proportional to the displacement it experiences. Usually this is done by converting one form of energy into another,like kinetic energy into potential energy as it happens with a pendulum. The Eigenfrequency is the oscillation frequency of the system once activated.

- **How does an electromagnetic brake work?** If a conductive material is moving past a magnet a current gets induced. This current is called Eddicurrent and is closed in itself in small loops. The current loops builds up a magnetic field which opposes the inducing field. Causing a force in the opposite direction of the moving motion of the conductor. Trough this force the conductor experience a de acceleration. The faster the object moves trough the magnetic field to bigger is the breaking force. And if the magnetic force from outside is bigger also the breaking force gets bigger. Making this break bad to stop very slow movements but good at high velocity movements.

- **What is a moment of inertia?** The moment of inertia Θ is the torque M needed for a desired angular acceleration $\ddot{\varphi}$ around itself in a given axis.

$$\Theta = \frac{M}{\ddot{\varphi}}$$

- **What is the differential equation of a damped harmonic oscillator? How does one differentiate the three cases of damped harmonic motion?** The equation of an damped harmonic oscillator is

$$\frac{\partial^2 x}{\partial t^2} + 2K\omega_0\frac{\partial x}{\partial t} + \omega_0^2 x = 0.$$

With K as the damping ratio and ω_0 the undamped angular frequency. For the three cases only the damping ratio K is important.

$K > 1$ is called over damped. The system decays exponential to steady state. $K = 1$ is called critically damped. Resulting in the system returning to steady state as quickly as possible. $K < 1$ is called under damped. The system oscillates with an exponential decaying rate

- **Which frequency dominates in the stationary state of a forced oscillation (driven oscillator)?** Until the natural oscillation is decayed an superposition of two different oscillations is present. After the decay only the forced oscillation remains.

- **How does the phase shift between driving moment and forced oscillation behave, if the driving frequency is much larger than the Eigenfrequency? (This case arises for the scattering of X-ray radiation)**

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- **Resonance curves are characterized by their quality (or Q factor). What is the Q factor in the case of a forced oscillation?** The Q factor is defined as

  $$Q = \frac{E_{stored}}{E_{lost}}.$$

  In a forced oscillation the Q factor is seen in the resonance curve in the height of the peak. The bigger Q is the bigger the peak in the curve gets around de resonance frequency. It also can be seen if the phase shift is plotted against the frequency. The bigger Q is the steeper and quicker the phase shift gets.