

# 09 Absoluter Nullpunkt

Cedric Renda, Fritz Kurz

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## Abstract

From the universal gas equation we know that for a given volume, filled with a given amount of gas, temperature is linear to pressure. This indicates, that there is an absolute zero for temperature, because otherwise there would have to be negative pressure. In this experiment, (after calibrating our measuring device) we want to find said absolute zero by measuring the pressure of a constant amount of gas in a constant volume at two known temperatures (in our case ice water and the steam of boiling water) and then calculation absolute zero from that.

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## 1 Introduction

In this experiment we want to use the universal gas equation [1]

$$PV = nRT \tag{1}$$

where  $P$  is the pressure,  $V$  the volume,  $n$  the number of particles,  $R$  universal gas constant and  $T$  the temperature, to determine the lowest limit of the so called thermodynamic

Celsius temperature scale. At this lowest point of temperature the enthalpy and entropy of a cooled ideal gas reaches its minimum values. By definition this point is the zero point of the SI base unit of temperature Kelvin, which is defined as follows [2]:

- The kelvin, symbol K, is the SI unit of thermodynamic temperature. It is defined by taking the fixed numerical value of the Boltzmann constant  $k$  to be  $1.380649 \times 10^{-23}$  when expressed in the unit  $\text{JK}^{-1}$ , which is equal to  $\text{kg m}^2 \text{s}^{-2} \text{K}^{-1}$ , where the kilogram, metre and second are defined in terms of  $h$ ,  $c$  and  $\Delta\nu C_s$ .

In the process to find this coldest temperature possible in the absolute temperature, we use a glass bulb filled with a known gas, a pressure sensor and two well known temperatures to measure and calculate the zero point. Once the pressure and temperature of the gas in the glass bulb is know, all kinds of temperatures can be measured with the change of pressure in the sealed glass bulb. To show this the temperature of liquid nitrogen is measured at the end of the experiment.



Figure 1: Sealed glass bulb filled with helium after being put in liquid nitrogen to measure the temperature of liquid nitrogen.

## 2 Experiment

Our setup consists of a glass bulb with a pre-installed pressure sensor, connected to a voltmeter, a pump for evacuating the bulb and a balloon, which is again connected to a bottle filled with helium as shown in figure 2.

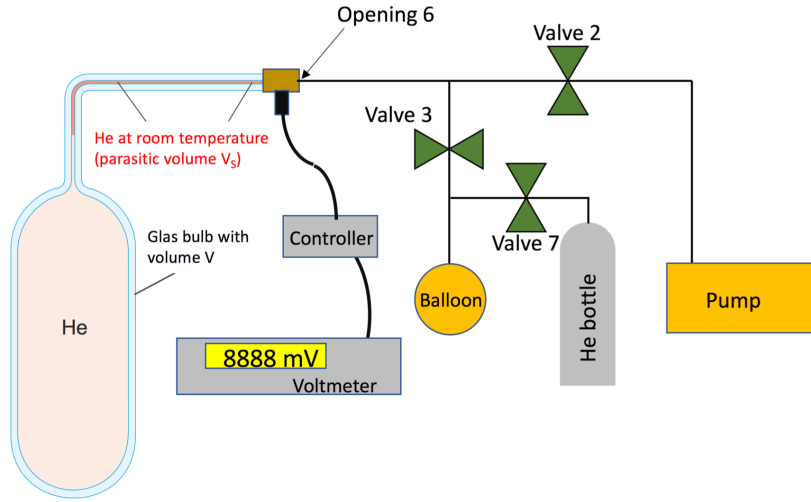


Figure 2: Schematic taken from the experiment manual

**1. Step** Our first task is to calibrate the pressure sensor as it outputs just voltage, not pressure directly. For that we need to measure voltages of two known pressures, in our case the air pressure  $p_L$  in the lab and the one of near vacuum  $p_t$ . To evaluate the air pressure there is a mercury barometer available in the lab. First we get the signal  $U_L$  from exposing the sensor to air pressure. After that, we evacuate the bulb with the pump and get  $U_t$ , the voltage at near vacuum (the pump should be able to produce a pressure significantly below 0.2 mbar). From those two measurements we can evaluate the slope  $C$  and the offset  $p_0$  of the so called characteristic curve of the sensor. Now we can convert the measured voltage to a pressure value with the following formula.

$$p(U) = p_0 + CU \quad (2)$$

**2. Step** The next step is the experimental part. As said before we want to measure the pressure of a given amount of gas in a constant volume at two different known temperatures.

To begin we need to fill the glass bulb with helium. Since we want no air remaining in the bulb, we evacuate it first, fill it with helium, and then repeat this process once. Filling the bulb with too much pressure should be avoided. Thus we first fill a balloon with helium and later fill the glass bulb from this balloon. To have enough helium in the balloon, the

size should be around the size of a football. From there, we fill the glass bulb with helium and not from the helium bottle directly. We remove the hose at opening 6 in figure 2 before exposing the bulb to heat. Because the pressure in the bulb will increase, air will not get into the bulb.

We place a water cooker under the bulb with a container, so the bulb will be exposed to the vapour as good as possible. After the voltage has settled we close opening 6 from figure 2, read the signal of the sensor witch is our  $U_k$  and convert it to pressure value  $p_k$  with the equation 2. We can determine the boiling temperature of the water with our measured air pressure and a conversion table supplied in the lab.

Next we place the bulb in a container filled with ice and water to have a temperature as close to zero degrees Celsius as possible. Again, after the voltage signal has settled, we can read our pressure value  $U_e$  and calculate  $p_e$ .

**3. Step** In the last part of the experiment, we want to use our setup as a thermometer and measure the temperature of liquid nitrogen. For that we completely submerge the sealed glass bulb as it is, in a container of liquid nitrogen, read the voltage corresponding to the pressure value  $U_n$  and calculate  $p_n$ .

### 3 Results

Our measurements and calculations as seen below lead us to the following result for absolute zero  $t_0$ :

$$t_0 = -269.59 \pm 5.1 ^\circ\text{C} \quad (3)$$

The exact result is  $-273.150 \pm 0.005 ^\circ\text{C}$ .

Our result for the temperature  $t_n$  of liquid nitrogen is:

$$t_n = -194.6 \pm 3.8 ^\circ\text{C} \quad (4)$$

with the boiling temperature of nitrogen [3] at  $-195.8 ^\circ\text{C}$  .

### 4 Data analysis

In this chapter the origin of the data is explained and the formulas used to calculate the results are discussed. The table 1 shows the measured voltages from the pressure sensor. Since some measurements were done a second time the measurement results are split into first and second attempt. Therefore a mean of the two attempts is also provided in the last column of the table 1.

Table 1: Measured pressure values in mV at certain temperatures or in a vacuum.

vacuum / temperatures	$U_L$ 23.1 °C	$U_t$ vacuum	$U_k$ 98.412°C	$U_e$ 0°C	$U_N$ < −196°C
first attempt	132,21	-3,15	131,28	96,18	-
sec. attempt	-	-3,72	131,54	96,86	24,63
Mean	132,21	-3,435	131,41	96.52	24,63

**Calibration** For the calibration of the pressure sensor we need to calculate the slope  $C$  and offset  $p_0$  between the pressure given by the pressure sensor in mV and the reference values as visualized in figure 3. For this we used the ambient pressure and vacuum as the two reference points.

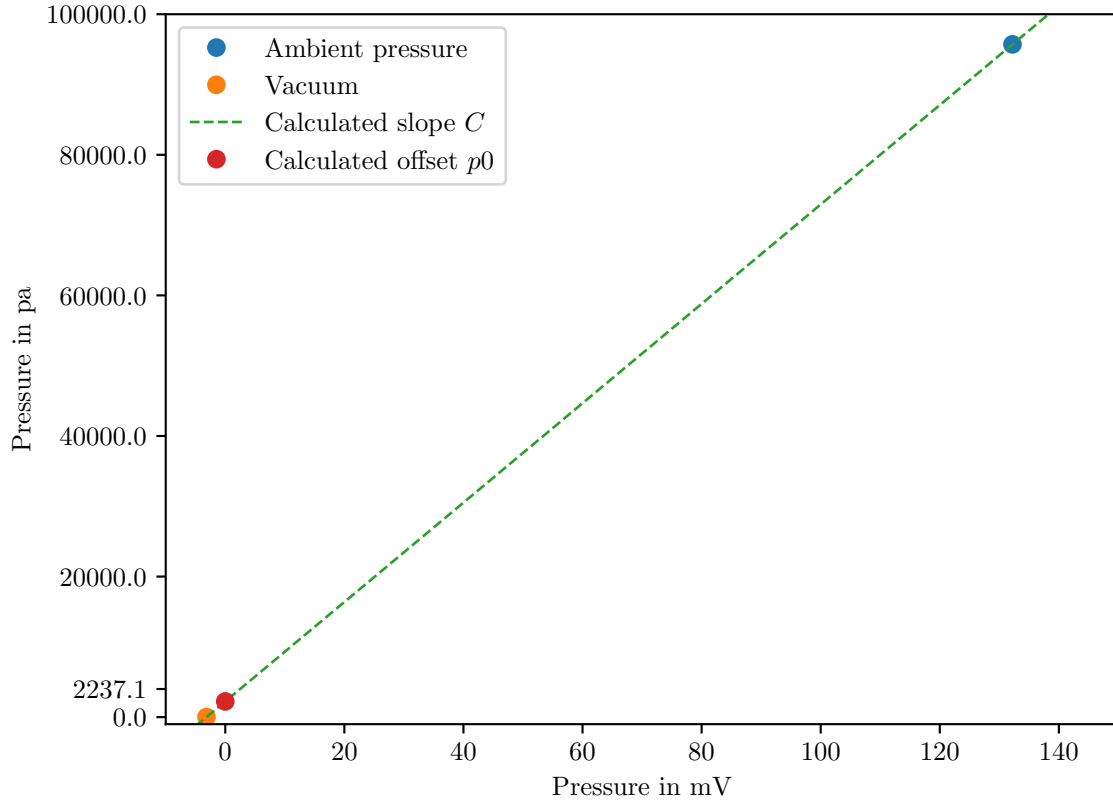


Figure 3: Calibration of pressure sensor.

To measure the ambient pressure, the mercury barometer in the lab was used. From the barometer we could read the uncorrected  $p_L = 720,9 \text{ m} \cdot 10^{-3} \text{ Hg (tL)}$  with the corresponding room temperature of  $23,1^\circ\text{C}$ . With this temperature we got a corrective value of 2,99 which we had to subtract from the measured pressure to get 717,91 Torr. One Torr corresponds to 133,32 Pa which resulted in an ambient pressure of 95711,6712 Pa. With the pressure sensor we got for the ambient room pressure with the same temperature  $U_L = 132,21 \text{ mV}$ .

For the second reference point the vacuum we assumed that it should be possible to achieve a vacuum of  $p_t = 0,1 \pm 0,1 \text{ mbar}$ . The corresponding pressure value in the glass bulb measured is  $U_t = -3,15 \text{ mV}$ .

To get the slope we calculate

$$C = \frac{p_L - p_t}{U_L - U_t}$$

and for the offset we calculate

$$p_0 = p_L - C \cdot U_L = p_t - C \cdot U_t.$$

With these two values, the pressure measured in mV and the equation 2 we get the converted value in Pascal.

**Absolute Zero** To calculate the absolute zero temperature, we take two reference points, where we can measure the temperature and the corresponding pressure very exactly. From this and the knowledge of the ideal gas equation 1, we can extrapolate to the absolute zero point  $t_0$ . The first point we get with the temperature source of boiling water as described in the paragraph 2 and the second point from iced water from the same paragraph. From the manual[4] provided with the lab we use the equations 5 to calculate our absolute zero temperature  $t_0$ .

$$\begin{aligned} a &= (1 + \varepsilon)p_E - (1 + \varepsilon + \gamma t_K)p_K \\ b &= \varepsilon(p_K - p_E)t_K + (1 + \gamma t_K)p_K t_L - p_E(t_L + t_K) \\ c &= p_E t_L t_K \\ t_0 &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{aligned} \tag{5}$$

where  $\gamma = 1,0 \cdot 10^{-5}^\circ\text{C}^{-1}$  is the cubic expansion coefficient of the glass bulb,  $\varepsilon = 0,001$  is the volume of the harmful space divided by the volume of the glass bulb itself.

## 5 Error Calculation

First we have to take account of the possible error of the pressure sensor. From the experiment manual we get that there is an absolute error of  $\pm 0,075 \text{ mV}$ . Due to different

temperatures when the sensor was originally calibrated and the time we do the experiment, there is also an error of  $\pm 0.08 \text{ mV}$ , also according to the manual. In addition to that, the slope of the curve can vary by  $\pm 0.1 \%$ . We can now calculate our error for voltage measurements  $\Delta U = 0.075 + 0.08 + 0.001 \times U$ . With our measurements we get  $U_L = 132.21 \pm 0.29 \text{ mV}$  and  $U_t = -3.72 \pm 0.16 \text{ mV}$

We then need to know the errors of  $p_L$  and  $p_t$ . From the manual we know that  $p_t = 10 \pm 10 \text{ Pa}$ . The measurement of the barometer is very exact, so we assume that there is no relevant error in that.

We want to know the error of the pressure measurements we make during the experiment. According to formula (1), we need to know the errors  $\Delta p_0$ ,  $\Delta C$ , and  $\Delta U$ . The last one we know already. First we determine  $\Delta C$ , the error of the slope of the characteristic curve of the sensor. The formula for that is  $C = \frac{p_T - p_L}{U_T - U_L}$ . With Gaussian error propagation we can evaluate  $\Delta C = 1.7$ , so  $C = 704.1 \pm 1.7$ . Second we want to know  $\Delta p_0$ . The formula for  $p_0$  is  $p_0 = p_t - CU_t$ . We use Gaussian error propagation again and find that  $\Delta p_0 = 109.8 \text{ Pa}$ .

Now we have everything to determine the error of a pressure measurement. The formula for that is given as  $p = p_0 + CU$ . So again with the same method we find the function

$$\Delta p(U) = \sqrt{\Delta p_0^2 + (U\Delta C)^2 + (C\Delta U)^2} \quad (6)$$

which we must use for every pressure measurement we make. Our measurements give us  $p_e = 70345 \pm 264 \text{ Pa}$  and  $p_k = 95240 \pm 320 \text{ Pa}$ .

Now we are finally able to calculate  $\Delta t_0$ , again by the Gaussian method. Unlike before, we don't want to take derivatives by hand, so for approximating  $\frac{\partial t_0}{\partial x_i}(x_1, \dots, x_n)$ , we use the differential quotient

$$\frac{t_0(x_1, \dots, x_i + h, \dots, x_n) - t_0(x_1, \dots, x_i, \dots, x_n)}{h}$$

and use a small  $h = 10^{-5}$ . By doing that for all parameters  $x_i, i = 1, \dots, n$  in equation 5 and then using the Gaussian method, we get to  $t_0 = -267.67 \pm 5.02^\circ \text{C}$ .

## 6 Discussion

Absolute zero is known with pretty good precision. The exact result is inside our uncertainty interval, which means that our measurements are possible. In the case of the temperature of liquid nitrogen, we know that the boiling point at room pressure is around  $-195.8^\circ \text{C}$ . We do not know the exact temperature of the liquid we measured, but it has to be below the boiling point. The nitrogen is exposed to room temperature, which is much warmer, so it will not be a lot below said boiling point. As our range covers temperatures right below the boiling point, this measurement also seems possible.

The results we have got with our experiment have pretty big uncertainties. So if we would want to redo the experiment, we would have to improve on that. We see the biggest problem in the sensor used. The device is meant to operate at voltages from 0 mV upwards. When calibrating, we had to measure voltages below that, which obviously is not optimal. So we would try to find a device more suited to its purpose.



## References

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## 7 Appendix