

09 Absoluter Nullpunkt

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Abstract

From the universal gas equation we know that for a given volume, filled with a given amount of gas, temperature is linear to pressure. This indicates, that there is an absolute zero for temperature, because otherwise there would have to be negative pressure. In this experiment, (after calibrating our measuring device) we want to find said absolute zero by measuring the pressure of a constant amount of gas in a constant volume at two known temperatures (in our case ice water and the steam of boiling water) and then calculation absolute zero from that.

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1 Introduction

In this experiment we want to use the universal gas equation [1]

$$PV = nRT \quad (1)$$

were P is the pressure, V the volume, n the number of particles, R universal gas constant and T the temperature, to determine the lowest limit of the so called thermodynamic Celsius temperature scale. At this lowest point of temperature the enthalpy and entropy of a cooled ideal gas reaches its minimum values. By definition this point is the zero point of the SI base unit of temperature Kelvin, which is defined as follows [2]:

- The kelvin, symbol K, is the SI unit of thermodynamic temperature. It is defined by taking the fixed numerical value of the Boltzmann constant k to be 1.380649×10^{-23} when expressed in the unit JK^{-1} , which is equal to $\text{kg m}^2 \text{s}^{-2} \text{K}^{-1}$, where the kilogram, metre and second are defined in terms of h , c and $\Delta\nu C_s$.

In the process to find this coldest temperature possible in the absolute temperature, we use a glass bulb filled with a known gas, a pressure sensor and two well known temperatures to measure and calculate the zero point. Once the pressure and temperature of the gas in the glass bulb is known, all kinds of temperatures can be measured with the change of pressure in the sealed glass bulb. To show this the temperature of liquid nitrogen is measured at the end of the experiment.

2 Experiment

Our setup consists of a glass bulb with a pre-installed pressure sensor, connected to a voltmeter, a pump for evacuating the bulb and a balloon, which is again connected to a bottle filled with helium as shown in figure 2.

1. Step Our first task is to calibrate the pressure sensor as it outputs just voltage, not pressure directly. For that we need to measure voltages of two known pressures, in our case the air pressure p_L in the lab and the one of near vacuum p_t . To evaluate the air pressure there is a mercury barometer available in the lab. First we get the signal U_L from exposing the sensor to air pressure. After that, we evacuate the bulb with the pump and get U_t , the voltage at near vacuum (the pump should be able to produce a pressure significantly below 0.2 mbar). From those two measurements we can evaluate the slope C and the offset p_0 of the so called characteristic curve of the sensor. Now we can convert the measured voltage to a pressure value with the following formula.

$$p(U) = p_0 + CU \quad (2)$$



Figure 1: Sealed glass bulb filled with helium after being put in liquid nitrogen to measure the temperature of liquid nitrogen.

2. Step The next step is the experimental part. As said before we want to measure the pressure of a given amount of gas in a constant volume at two different known temperatures.

To begin we need to fill the glass bulb with helium. Since we want no air remaining in the bulb, we evacuate it first, fill it with helium, and then repeat this process once. Filling the bulb with too much pressure should be avoided. Thus we first fill a balloon with helium and later fill the glass bulb from this balloon. To have enough helium in the balloon, the size should be around the size of a football. From there, we fill the glass bulb with helium and not from the helium bottle directly. We remove the hose at opening 6 in figure 2 before exposing the bulb to heat. Because the pressure in the bulb will increase, air will not get into the bulb.

We place a water cooker under the bulb with a container, so the bulb will be exposed to the vapour as good as possible. After the voltage has settled we close opening 6 from figure 2, read the signal of the sensor which is our U_k and convert it to pressure value p_k with the equation 2. We can determine the boiling temperature of water with our measured air pressure and a conversion table supplied in the lab.

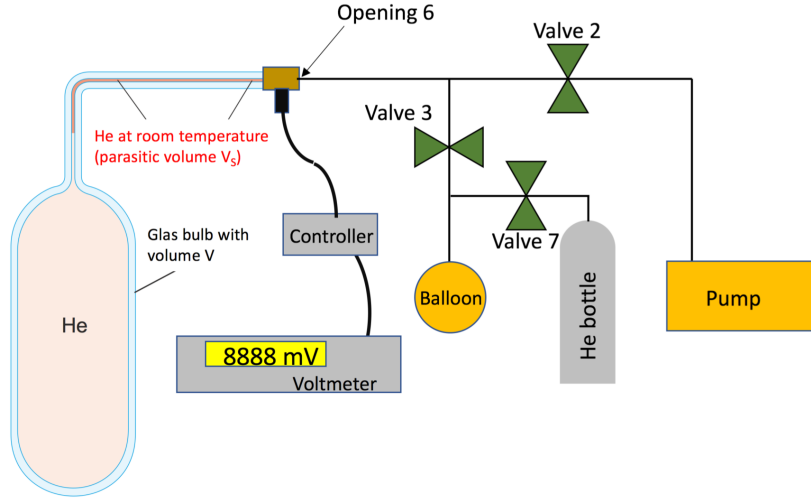


Figure 2: Schematic taken from the experiment manual

Next we place the bulb in a container filled with ice and water to have a temperature as close to zero degrees Celsius as possible. Again, after the voltage signal has settled, we can read our pressure value U_e and calculate p_e .

3. Step In the last part of the experiment, we want to use our setup as a thermometer and measure the temperature of liquid nitrogen. For that we completely submerge the sealed glass bulb as it is, in a container of liquid nitrogen, read the voltage U_n corresponding to the pressure value and calculate p_n .

3 Data analysis

In this chapter the origin of the data is explained and the formulas used to calculate the results are discussed. Table 1 shows the measured voltages from the pressure sensor. Since some measurements were done a second time the measurement results are split into first and second attempt. Therefore a mean of the two attempts is also provided in the last column of the table 1.

Calibration For the calibration of the pressure sensor we need to calculate the slope C and offset p_0 between the pressure given by the pressure sensor in mV and the reference values as visualized in figure 3. For this we used the ambient pressure and vacuum as the two reference points.

To measure the ambient pressure, the mercury barometer in the lab was used. From the barometer we could read the uncorrected $p_L = 720,9 \text{ m} \cdot 10^{-3} \text{ Hg (tL)}$ with the corresponding room temperature of $23,1 \text{ }^\circ\text{C}$. With this temperature we got a corrective value of 2,99

Table 1: Measured pressure values in mV at certain temperatures or in a vacuum.

vacuum / temperatures	U_L 23.1 °C	U_t vacuum	U_k 98.412°C	U_e 0°C	U_N < −196°C
first attempt	132,21	-3,15	131,28	96,18	-
sec. attempt	-	-3,72	131,54	96,86	24,63
Mean	132,21	-3,435	131,41	96.52	24,63

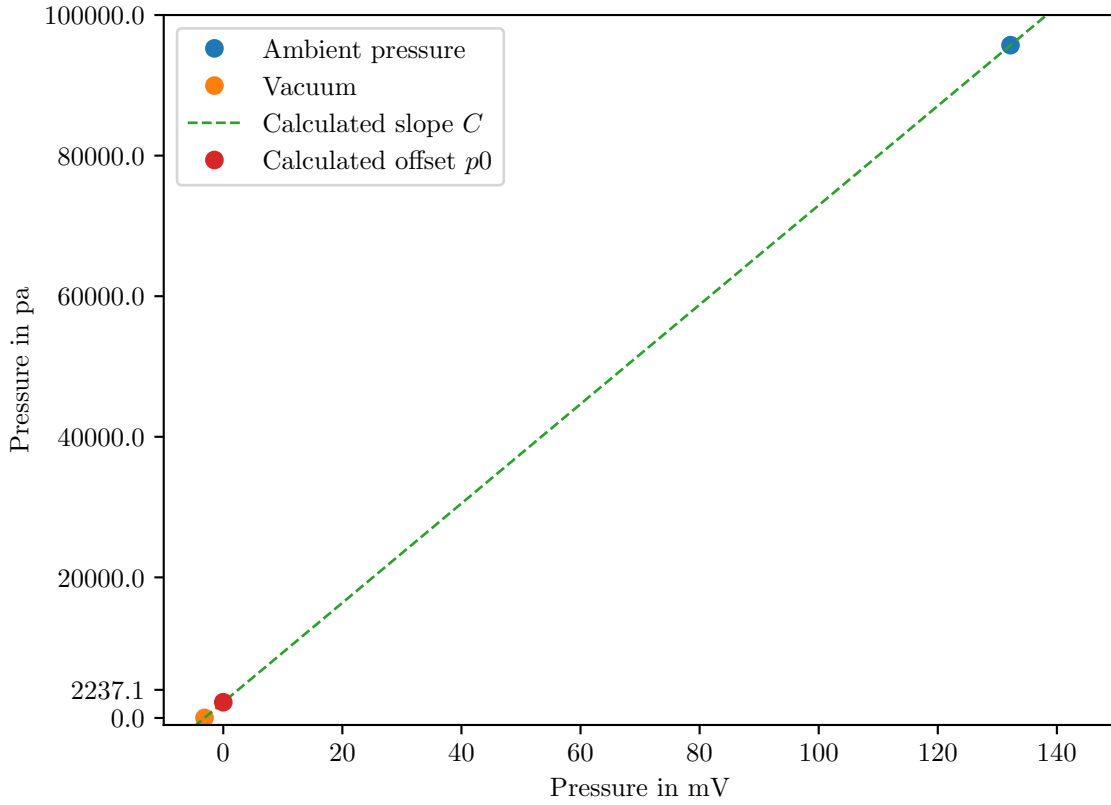


Figure 3: Calibration of pressure sensor.

which we had to subtract from the measured pressure to get 717,91 Torr. One Torr corresponds to 133,32 Pa which resulted in an ambient pressure of 95711 Pa. With the pressure sensor we got for the ambient room pressure with the same temperature $U_L = 132, 21$ mV.

For the second reference point the vacuum we assumed that it should be possible to achieve

a vacuum of $p_t = 0.1 \pm 0.1$ mbar [4]. The corresponding pressure value in the glass bulb measured is $U_t = -3.15$ mV.

To get the slope we calculate

$$C = \frac{p_L - p_t}{U_L - U_t}$$

and for the offset we calculate

$$p_0 = p_L - C \cdot U_L = p_t - C \cdot U_t.$$

With these two values, the pressure measured in mV and the equation 2 we get the converted value in Pascal.

Absolute Zero To calculate the absolute zero temperature, we take two reference points, where we can measure the temperature and the corresponding pressure very exactly. From this and the knowledge of the ideal gas equation 1, we can extrapolate to the absolute zero point t_0 . The first point we get with the temperature source of boiling water as described in the paragraph 2 and the second point from ice water from the same paragraph. From the manual[4] we use the equations 3 to calculate our absolute zero temperature t_0 .

$$\begin{aligned} a &= (1 + \varepsilon)p_E - (1 + \varepsilon + \gamma t_K)p_K \\ b &= \varepsilon(p_K - p_E)t_K + (1 + \gamma t_K)p_K t_L - p_E(t_L + t_K) \\ c &= p_E t_L t_K \\ t_0 &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{aligned} \tag{3}$$

where $\gamma = 1,0 \cdot 10^{-5} \text{C}^{-1}$ is the cubic expansion coefficient of the glass bulb and $\varepsilon = 0,001$ is the volume of the harmful space divided by the volume of the glass bulb itself.

Liquid nitrogen In the last part of the experiment we used the glass bulb as an thermometer to measure the temperature of liquid nitrogen. The measured voltage $U_N = 24,63$ mV leads us to $p_N = 20118$ Pa. With the given equation 4 from the manual [4] we got an temperature $t_n = -194.6$.

$$\begin{aligned} A &\equiv \frac{p_E}{t_E - t_0} + \frac{\varepsilon(p_E - p_N)}{t_L - t_0} \\ t_N &= \frac{A t_0 + p_N}{A - \gamma p_N} \end{aligned} \tag{4}$$

4 Error Calculation

First we have to take account of the possible error of the pressure sensor. From the experiment manual we get that there is an absolute error of ± 0.075 mV. Due to different

temperatures when the sensor was originally calibrated and the time we do the experiment, there is also an error of $\pm 0.08 \text{ mV}$, also according to the manual. In addition to that, the slope of the curve can vary by $\pm 0.1 \%$. We can now calculate our error for voltage measurements $\Delta U = 0.075 + 0.08 + 0.001 \times U$. With our measurements we get $U_L = 132.21 \pm 0.29 \text{ mV}$ and $U_t = -3.72 \pm 0.16 \text{ mV}$

We then need to know the errors of p_L and p_t . From the manual we know that $p_t = 10 \pm 10 \text{ Pa}$. The measurement of the barometer is very exact, so we assume that there is no significant error in that.

We want to know the error of the pressure measurements we make during the experiment. According to formula (1), we need to know the errors Δp_0 , ΔC , and ΔU . The last one we know already. First we determine ΔC , the error of the slope of the characteristic curve of the sensor. The formula for that is $C = \frac{p_T - p_t}{U_T - U_t}$. With Gaussian error propagation we can evaluate $\Delta C = 1.7$, so $C = 704.1 \pm 1.7$. Second we want to know Δp_0 . The formula for p_0 is $p_0 = p_t - CU_t$. We use Gaussian error propagation again and find that $\Delta p_0 = 109.8 \text{ Pa}$.

Now we have everything to determine the error of a pressure measurement. The formula for that is given as $p = p_0 + CU$. So again with the same method we find the function

$$\Delta p(U) = \sqrt{\Delta p_0^2 + (U\Delta C)^2 + (C\Delta U)^2} \quad (5)$$

which we must use for every pressure measurement we make. Our measurements give us $p_e = 70345 \pm 264 \text{ Pa}$ and $p_k = 95240 \pm 320 \text{ Pa}$.

Now we are finally able to calculate Δt_0 , again by the Gaussian method. Unlike before, we don't want to take derivatives by hand, so for approximating $\frac{\partial t_0}{\partial x_i}(x_1, \dots, x_n)$, we use the differential quotient

$$\frac{t_0(x_1, \dots, x_i + h, \dots, x_n) - t_0(x_1, \dots, x_i, \dots, x_n)}{h}$$

and use a small $h = 10^{-5}$. By doing that for all parameters $x_i, i = 1, \dots, n$ in equation 3 and then using the Gaussian method, we get to $t_0 = -269.6 \pm 5.1 \text{ }^\circ\text{C}$.

5 Results

Our measurements and calculations as seen below lead us to the following result for absolute zero t_0 :

$$t_0 = -269.6 \pm 5.1 \text{ }^\circ\text{C} \quad (6)$$

The exact result is $-273.150 \pm 0.005 \text{ }^\circ\text{C}$.

Our result for the temperature t_n of liquid nitrogen is:

$$t_n = -194.6 \pm 3.8 \text{ }^\circ\text{C} \quad (7)$$

with the boiling temperature of nitrogen [3] at $-195.8 \text{ }^\circ\text{C}$.

6 Discussion

Absolute zero is known with pretty good precision. The exact result is inside our uncertainty interval, which means that our measurements are possible. In the case of the temperature of liquid nitrogen, we know that the boiling point at room pressure is around -195.8°C . We do not know the exact temperature of the liquid we measured, but it has to be below the boiling point. The nitrogen is exposed to room temperature, which is much warmer, so it will not be a lot below said boiling point. As our range covers temperatures right below the boiling point, this measurement also seems possible.

The results we have got with our experiment have pretty big uncertainties. So if we would want to redo the experiment, we would have to improve on that. We see the biggest problem in the sensor used. The device is meant to operate at voltages from 0 mV upwards. When calibrating, we had to measure voltages below that, which obviously is not optimal. So we would try to find a device more suited to its purpose.

References

- [1] E. Clapeyron, Mémoire sur la puissance motrice de la chaleur. In: Journal de l'École Polytechnique. XIV, 1834, S. 153-90.
- [2] BIPM, The International System of Units (SI Brochure) [9th edition, 2019], <https://www.bipm.org/en/publications/si-brochure/>
- [3] Yiming Zhang, Julian R. G. Evans, Shoufeng Yang: Corrected Values for Boiling Points and Enthalpies of Vaporization of Elements in Handbooks. In: Journal of Chemical & Engineering Data. 56, 2011, S.328-337,<https://doi.org/10.1021/je1011086>.
- [4] Lab manual 9 - Absoluter Nullpunkt, Physikpraktikum der ETH Zürich, https://ap.phys.ethz.ch/Anleitungen/Bilingual/09_Manual.pdf.

7 Appendix

Name 1: Cédric Renda Datum: 08.10.21Name 2: Fritz Kurz Platz Nr.: 2

9

Absoluter Nullpunkt

Eichung des Drucksensors

$$1 \text{ Torr} = 133,32 \text{ Pa}$$

Eichpunkt beim Luftdruck:

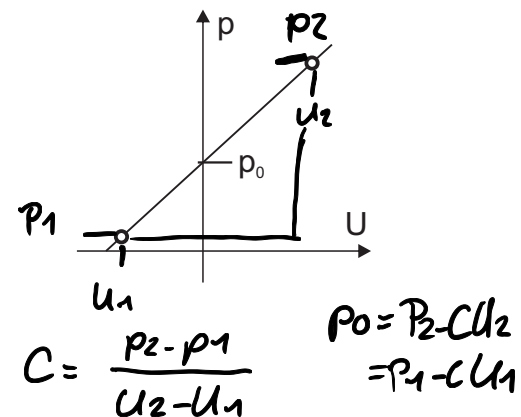
Barometerablesung (unkorrigiert)		<i>korrigiert</i>
$p_L = 720,9 \text{ mm Hg } (t_L)$	Lufttemp. $t_L = 23,1$	$p_L = 717,91 \text{ Torr}$
$p_L = 95711,7612 \text{ Pa}$	$U_L = 132,21 \text{ mV}$	

Eichpunkt bei sehr tiefem Druck: *2. Messung*

$p_t = 0,1 \pm 0,1 \text{ mbar}$	<i>-3,72</i>
$p_t = 10 \pm 10 \text{ Pa}$	$U_t = -3,15 \text{ mV}$

Sensorcharakteristik $p = p_0 + C U$:

$p_0 = 2237,1019$	$C = 707,01655$
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Bestimmung des absoluten Nullpunktes

Bei der Temperatur von kochendem Wasser:

$t_k = 99,412$	$U_k = 131,28 \text{ mV}$	$p_k = 92817,132$
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Bei der Temperatur von Eiswasser:

$t_E = 0^\circ$	$U_E = 96,18 \text{ mV}$	$p_E =$
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Näherungswert für die Temperatur des absoluten Nullpunktes

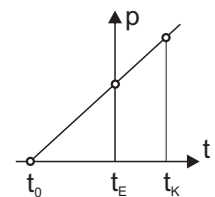
 $t_{00} =$

Vol. des schädlichen Raums / Vol. des Glaskolbens:

Platz 1 & 2: $V_s/V = \varepsilon = 0,001$ Kubischer Ausdehnungskoeffizient vom Glaskolben $= \gamma = 1,0 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$

$a =$	$b =$	$c =$
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Temperatur des absoluten Nullpunktes

 $t_0 = 269,6$ 

Bestimmung der Temperatur von flüssigem Stickstoff

$U_N = 24,63$	$p_N = 17973,48$
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Näherungswert für die Temperatur des flüssigen Stickstoffs

 $t_{N0} =$ $A =$

Temperatur des flüssigen Stickstoffs

 $t_N = -194,6 \pm 3,8$