

NONLINEAR FUNCTIONS
OF SEVERAL VARIABLES
(SYSTEMS)

$$\underline{A} \underline{x} = \underline{b} \quad \text{LINEAR}$$

$$\underline{F}_{m \times 1} = \begin{cases} f(x, y) \\ g(x, y) \end{cases} = \begin{cases} 0 \\ 0 \end{cases}$$

NONLINEAR
IN x & y

LINEARIZE

$$f(x, y) \sim f(x_0, y_0) +$$

$$\left. \frac{\partial f}{\partial x} \right|_0 \underbrace{(x - x_0)}_{\Delta x} + \left. \frac{\partial f}{\partial y} \right|_0 \underbrace{(y - y_0)}_{\Delta y}$$

$$g(x, y) \sim g(x_0, y_0) +$$

$$\left. \frac{\partial g}{\partial x} \right|_0 \Delta x + \left. \frac{\partial g}{\partial y} \right|_0 \Delta y$$

MATRIX FORM

$$\begin{Bmatrix} f(x,y) \\ g(x,y) \end{Bmatrix} = \begin{Bmatrix} f(x_0, y_0) \\ g(x_0, y_0) \end{Bmatrix} +$$

$$\begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix}_0 \begin{Bmatrix} \Delta x \\ \Delta y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

2×2 2×1

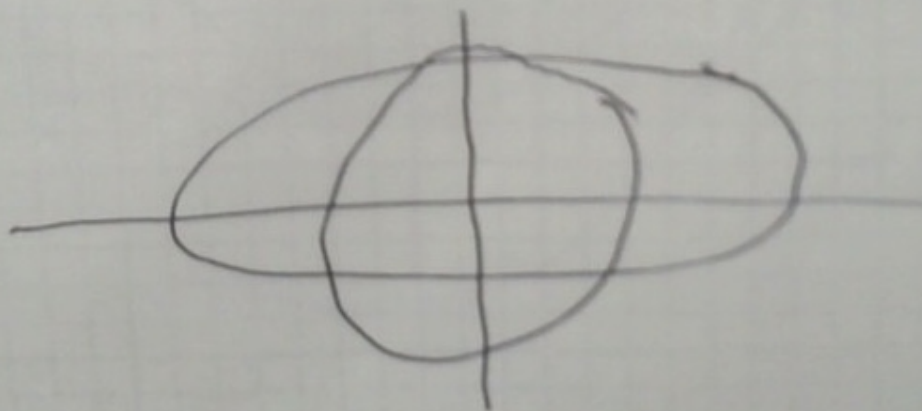
JACOBIAN

EXAMPLE

$$f(x, y) = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$$

$$g(x, y) = x^2 + y^2 - 1 = 0$$

FIND x, y



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$$

