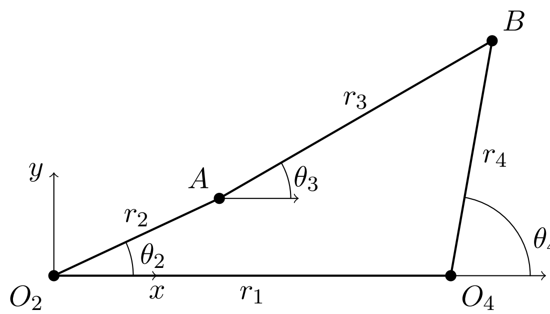
Notebook Example: 4-Bar Linkage Position Analysis (Nonlinear System)

A planar four-bar linkage consists of four rigid links connected by ideal pin joints. The ground link has length r_1 and connects two fixed pivots O_2 and O_4. The input crank has length r_2 and rotates about O_2 with a prescribed input angle θ_2. The coupler has length r_3 and connects joint A to joint B. The output rocker has length r_4 and rotates about O_4 with angle θ_4.



Given: $r_1, r_2, r_3, r_4,\ \theta_2$ Solve for: θ_3, θ_4

Figure 1: Planar four-bar linkage.

For a given set of link lengths (r_1, r_2, r_3, r_4) and a prescribed input angle θ_2, determine the configuration of the linkage by solving for the unknown angles

$$\theta_3 \quad \text{and} \quad \theta_4.$$

This is a nonlinear system of equations because the geometry involves $\sin(\cdot)$ and $\cos(\cdot)$ terms.

Coordinate setup

Place the mechanism in the xy-plane with fixed pivots

$$O_2 = (0, 0), \qquad O_4 = (r_1, 0).$$

Let the joint locations be:

$$A = O_2 + (r_2 \cos\theta_2,\ r_2 \sin\theta_2), \qquad B = O_4 + (r_4 \cos\theta_4,\ r_4 \sin\theta_4).$$

The coupler constraint is that the distance between A and B equals r_3.

Loop-closure (nonlinear system)

The vector loop-closure equation is

$$\mathbf{r}_2 + \mathbf{r}_3 = \mathbf{r}_1 + \mathbf{r}_4,$$

which, in components, yields the nonlinear system

$$f_1(\theta_3, \theta_4) = r_2 \cos\theta_2 + r_3 \cos\theta_3 - r_1 - r_4 \cos\theta_4 = 0,$$
$$f_2(\theta_3, \theta_4) = r_2 \sin\theta_2 + r_3 \sin\theta_3 - r_4 \sin\theta_4 = 0.$$

Define

$$F(\mathbf{x}) = \begin{bmatrix} f_1(\theta_3, \theta_4) \\ f_2(\theta_3, \theta_4) \end{bmatrix}, \qquad \mathbf{x} = \begin{bmatrix} \theta_3 \\ \theta_4 \end{bmatrix}.$$

We seek $\mathbf{x}$ such that $F(\mathbf{x}) = \mathbf{0}$.