Python Setup and Introduction to Matrix Algebra

Numerical Structure and Stability

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Lecture Objectives

By the end of this lecture, you should be able to:

- Recognize what a correct Python environment looks like
- Understand why matrix algebra is central to computational mechanics
- Appreciate that numerical issues can arise even in simple systems

Why We Start with the Python Environment

- All computational examples rely on a working Python setup
- Errors early in the semester tend to compound later
- A correct setup enables reproducibility and confidence

Key Message

If your environment works today, it will work all semester.

What a Successful Setup Looks Like

A working environment allows you to:

- Activate a virtual environment
- Launch Jupyter Notebook
- Select the correct kernel
- Import core libraries without errors

Minimal Verification

```
import numpy, sympy, matplotlib
```

Why Matrix Algebra Matters

Almost every computational mechanics problem reduces to:

$$\mathbf{K}\mathbf{x} = \mathbf{f}$$

- Unknowns $\rightarrow$ vector
- Physical laws $\rightarrow$ matrix
- Loads and sources $\rightarrow$ right-hand side

Big Picture

Matrix algebra is the language of computational mechanics.

Matrices Encode Structure

A matrix represents more than numbers:

- Coupling between variables
- Physical constraints
- Geometry and material behavior

Matrix–vector multiplication describes how information propagates through a system.

Numerical Sensitivity

- Computers use finite-precision arithmetic
- Small numerical errors are unavoidable
- Some systems amplify these errors dramatically

Terminology

This sensitivity is called *conditioning*.

Well-Conditioned vs Ill-Conditioned Systems

- **Well conditioned**: small input errors produce small output errors
- **Ill conditioned**: small input errors produce large output errors

Important

Numerical instability is often a property of the problem, not the code.

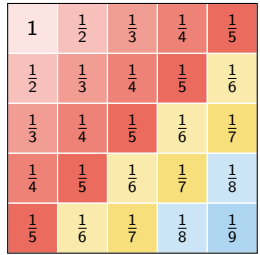
The Hilbert Matrix

The Hilbert matrix is defined by:

$$H_{ij} = \frac{1}{i + j - 1}$$

- Simple definition
- Smooth entries
- Extremely ill conditioned

$H_{5\times 5}$



Larger entries cluster near the top-left.

Why the Hilbert Matrix Matters

- Appears harmless
- Defeats standard numerical solvers
- Demonstrates limits of floating-point arithmetic

Notebook Demonstration

In the accompanying notebook, we:

- Construct Hilbert matrices of increasing size
- Solve linear systems numerically
- Observe loss of accuracy

Key Observation

As matrix size increases, numerical reliability collapses.

What Went Wrong?

- The mathematical problem is well defined
- The algorithm is correct
- Floating-point arithmetic limits accuracy

Critical Lesson

The computer did not fail — the mathematics did.

Implications for This Course

- Matrix structure matters
- Solver choice matters
- Blind trust in numerical output is dangerous

These themes will reappear throughout the semester.

Looking Ahead

In upcoming lectures, we will:

- Define condition numbers formally
- Study direct and iterative solvers
- Connect conditioning to stiffness matrices
- Develop strategies for numerical robustness

Summary

- A correct Python setup is essential
- Matrix algebra underpins computational mechanics
- Numerical instability is unavoidable but manageable

Next Step

Work through the Hilbert matrix notebook carefully.