Illinois Institute of Technology
MMAE 350 – Computational Mechanics
Exam 1

Time: 75 minutes
Total: 50 points

Name: ____________________________________

Instructions:

- This exam is closed book and closed notes.
- A calculator is permitted.
- Show all work clearly.
- Answers without supporting work may receive limited credit.

Part I – Gaussian Elimination (6 points)

Consider the system

$$\begin{aligned} 2x + y - z &= 3 \\ 4x + 3y + z &= 7 \\ -2x + y + 2z &= 1 \end{aligned}$$

(a) (3 pts) Compute the multiplier m_{21} used to eliminate the first column (no pivoting).

(b) (3 pts) After eliminating the first column, what is the updated coefficient in row 2, column 2?

Part II – Thomas Algorithm (6 points)

Consider the tridiagonal system

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

(a) (3 pts) Perform the first forward elimination step and compute the modified diagonal entry d_2'.

(b) (3 pts) In 2–3 sentences, explain why the Thomas algorithm is more efficient than standard Gaussian elimination for this type of matrix.

Part III – Gauss–Seidel Method (6 points)

Consider the linear system

$$\begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 2 \\ 6 \\ 2 \end{bmatrix}.$$

(a) (4 pts) Perform one Gauss–Seidel sweep starting from

$$\boldsymbol{x}^{(0)} = (0, 0, 0),$$

and compute

$$x_1^{(1)}, \quad x_2^{(1)}, \quad x_3^{(1)}.$$

(b) (2 pts) In 2–3 sentences, explain why this system is expected to converge under Gauss–Seidel iteration.

Part IV – Newton's Method (1D) (6 points)

Let

$$f(x) = x^3 - 2x - 5.$$

(a) (2 pts) Compute $f'(x)$.

(b) (2 pts) Perform one Newton iteration starting from $x_0 = 2$. Compute x_1.

(c) (2 pts) At iteration x_k, Newton's method replaces $f(x)$ with a linear approximation. In 2–3 sentences, explain how x_{k+1} is obtained from this linearization.

Part V – Newton's Method for a System (8 points)

Consider

$$f(x, y) = x^2 + y^2 - 4,$$
$$g(x, y) = x - y - 1.$$

with initial guess $(1, 1)$.

(a) (2 pts) Compute the residual vector at $(1, 1)$.

(b) (4 pts) Compute the Jacobian matrix at $(1, 1)$.

(c) (2 pts) In 2–3 sentences, explain why solving the linear system inside Newton's method corresponds to a first-order Taylor approximation.

Part VI – Big Picture (6 points)

In 4–6 complete sentences, compare the following three methods:

- Gaussian elimination
- Gauss–Seidel
- Newton's method

In your comparison, clearly address:

- Which types of problems each method is used to solve (linear or nonlinear),
- Whether the method is direct or iterative,
- One situation where each method would be appropriate.

Part VII – Code Interpretation (4 points)

Consider the following Python code:

```
x = 0
for k in range(3):
    x = 2*x + 1
print(x)
```

(a) (3 pts) What value is printed?

(b) (1 pt) In one sentence, describe what this loop is doing.

Part VIII – Pseudocode (4 points)

Write clear pseudocode for Newton's method to solve

$$f(x) = 0.$$

Part IX – Matrix Algebra (4 points)

Consider

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & -2 \\ 0 & -1 & 4 \end{bmatrix}, \quad \boldsymbol{x} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}.$$

(a) (2 pts) Compute $\boldsymbol{b} = A\boldsymbol{x}$.

(b) (2 pts) Write a short block of Python code (using basic loops, not built-in matrix multiplication) that computes $\boldsymbol{b} = A\boldsymbol{x}$ for a general 3×3 matrix A and vector $\boldsymbol{x}$.