

NONLINEAR FUNCTIONS  
OF SEVERAL VARIABLES  
(systems)

$\tilde{A} \tilde{x} = \tilde{b}$  LINEAR

$$\underbrace{\tilde{F}_{m \times 1} = \begin{cases} f(x, y) \\ g(x, y) \end{cases}}_{\text{NONLINEAR IN } x \text{ & } y} = \begin{cases} 0 \\ 0 \end{cases}$$

LINEARIZE

$$f(x, y) \sim f(x_0, y_0) +$$

$$\frac{\partial f}{\partial x} \bigg|_0 \underbrace{(x - x_0)}_{\Delta x} + \frac{\partial f}{\partial y} \bigg|_0 \underbrace{(y - y_0)}_{\Delta y}$$

$$g(x, y) \sim g(x_0, y_0) +$$

$$\frac{\partial g}{\partial x} \bigg|_0 \Delta x + \frac{\partial g}{\partial y} \bigg|_0 \Delta y$$

## MATRIX FORM

$$\begin{Bmatrix} f(x,y) \\ g(x,y) \end{Bmatrix} = \begin{Bmatrix} f(x_0, y_0) \\ g(x_0, y_0) \end{Bmatrix} +$$

$$\begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix}_0 \begin{Bmatrix} \Delta x \\ \Delta y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$2 \times 2 \qquad \qquad \qquad 2 \times 1$

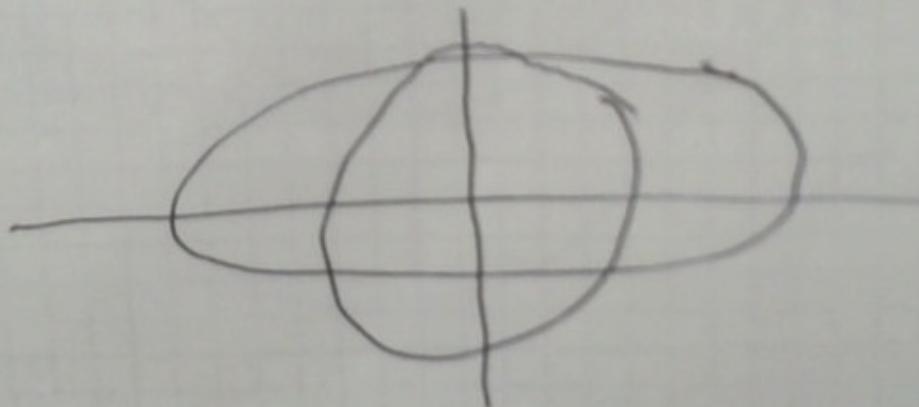
JACOBIAN

EXAMPLE

$$f(x, y) = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$$

$$g(x, y) = x^2 + y^2 - 1 = 0$$

FIND  $x, y$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$$

