MMAE 450 — Homework #3

FTCS for Transient Heat Conduction: Verification and Sphere Cooling

Due: Tuesday, February 3, 2026 (11:59 pm)

Learning goals. By completing this assignment, you will:

- implement the **Forward Time, Centered Space (FTCS)** method for diffusion,
- investigate the distinction between *stability* and *accuracy* for explicit time integration,
- enforce **Dirichlet**, **Neumann (zero flux)**, and **Robin (convection)** boundary conditions,
- apply FTCS to a physically meaningful cooling problem.

Homework 3: FTCS for the 1D Heat Equation

Problem 1: 1D Transient Heat Conduction with Zero-Flux Boundary

Consider a one-dimensional rod of length L governed by the transient heat equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}, \qquad 0 < x < L, \quad t > 0.$$

The rod is initially at a uniform temperature T_0. The boundary and initial conditions are:

- **Left end (symmetry / zero flux):**

$$-k \left. \frac{\partial T}{\partial x} \right|_{x=0} = 0.$$

- **Right end (Dirichlet):**

$$T(L, t) = T_L.$$

- **Initial condition:**

$$T(x, 0) = T_0.$$

Problem Parameters

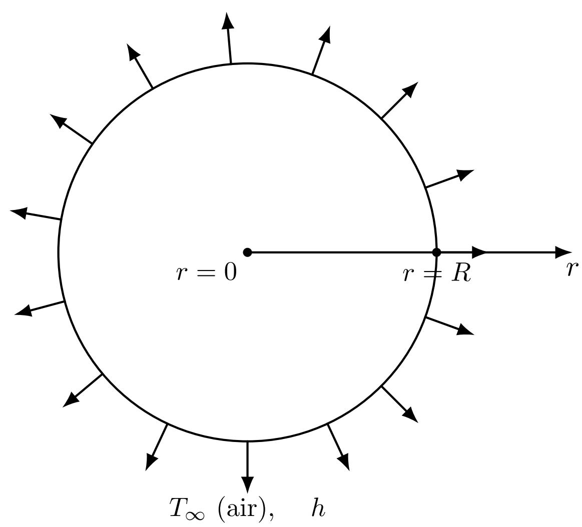
Quantity	**Symbol**	**Value**
Rod length	L	1.0 m
Initial temperature	T_0	100 °C
Right-end temperature	T_L	20 °C
Thermal conductivity	k	401 $\mathrm{W/(m \cdot K)}$
Density	ρ	8960 $\mathrm{kg/m^3}$
Specific heat	c_p	385 $\mathrm{J/(kg \cdot K)}$
Thermal diffusivity	α	$\dfrac{k}{\rho c_p}$

Tasks:

1. Discretize the governing equation using the Forward Time, Centered Space (FTCS) method on a uniform spatial grid.
2. Derive the finite-difference update equation for the interior nodes.
3. Derive the FTCS update equation at the left boundary using the zero-flux condition.
4. Implement the method in Python and simulate the transient temperature evolution.
5. Plot the midpoint temperature $T(L/2, t)$ versus time for $0 < t < 5000\,\mathrm{s}$ and comment on the transient behavior.

Problem 2: Cooling of a Copper Sphere (Regularity at Center, Convection at Surface)

A solid copper sphere of radius $R = 0.10\,\mathrm{m}$ is initially at a uniform temperature $T_0 = 100\,^\circ\mathrm{C}$. At time $t = 0$ the sphere is removed from an oven and exposed to still air at $T_\infty = 20\,^\circ\mathrm{C}$ with a constant convective heat transfer coefficient h.



Assume:

- radial symmetry (temperature depends only on radius r and time t),
- constant material properties,
- no internal heat generation.

Material properties and convection data

The thermal diffusivity is

$$\alpha = \frac{k}{\rho c_p}.$$

Property	Symbol	Value
Thermal conductivity	k	$401\,\mathrm{W/(m\cdot K)}$
Density	ρ	$8960\,\mathrm{kg/m^3}$
Specific heat capacity	c_p	$385\,\mathrm{J/(kg\cdot K)}$
Convective heat transfer coefficient (air)	h	$10\,\mathrm{W/(m^2{\cdot}K)}$

Governing equation and conditions (given)

With radial symmetry, the transient heat equation in spherical coordinates is

$$\frac{\partial T}{\partial t} = \alpha \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) \right], \qquad 0 < r < R,\ t > 0.$$

Boundary condition at the surface:

$$-k \left. \frac{\partial T}{\partial r} \right|_{r=R} = h\,(T(R,t) - T_\infty) \qquad \text{(convection at the surface).}$$

Regularity condition at the center:

$$T(r,t) \text{ remains bounded as } r \to 0 \qquad \text{(finite temperature at the origin).}$$

Tasks

1. Discretize the radial domain using $N+1$ nodes:

$$r_i = i\Delta r, \qquad i = 0, 1, \ldots, N, \qquad \Delta r = \frac{R}{N}.$$

2. Implement FTCS time stepping for the sphere. Your implementation must:
   - enforce the **regularity condition** at $r = 0$ (bounded temperature at the origin) using the *center control-volume balance law* from lecture,
   - enforce the **Robin (convection)** condition at $r = R$ (e.g. via a ghost-node relation or an equivalent one-sided discretization),
   - update only the interior nodes with the spherical FTCS stencil.

3. Choose at least three time steps (or λ values) and demonstrate:
   - a stable run,
   - a run near the stability limit,
   - an unstable run (nonphysical oscillations or blow-up).

4. For your *best stable* run, plot the center temperature $T(0,t)$ as a function of time over a *reasonable* time interval (long enough to observe a noticeable decrease toward T_∞). State the final simulation time used.