A PRACTICAL CLOCK SYNCHRONIZATION ALGORITHM FOR UWB POSITIONING SYSTEMS

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ABSTRACT

A clock synchronization scheme is crucial for obtaining accuracy in time-based positioning systems. Existing clock synchronization schemes are mostly based on a simplified linear clock model, which unfortunately have a poor long-term synchronization accuracy. Assuming a two-way time transfer protocol, we propose a novel clock synchronization algorithm based on a precise physical clock model to realize joint clock synchronization and propagation delay estimation. The Cramer-Rao lower bound (CRLB) is derived for this clock model and shows that the estimation is asymptotically efficient and converges to the lower bound. For long time spans, this model performs better than a linear clock model.

Index Terms— Clock drift, ultra-wide band, two way time transfer, clock synchronization

1. INTRODUCTION

Ultra-wide band (UWB) signals are promising for short range positioning applications which require high accuracy. A UWB positioning system involves a number of nodes with known coordinates, and locating a target node with unknown coordinates by UWB signals can be implemented in several ways [1]. Among them, time-based techniques, including time-of-arrival (TOA) and time-difference-of-arrival (TDOA), rely on the measured travel time of the UWB signals between nodes. This can provide a high positioning accuracy due to the high time resolution of a UWB signal.

Among positioning system-level issues, clock synchronization is a crucial factor that relates directly to the accuracy of time-based positioning techniques [2]. In TOA/TDOA mode, the travel time of signals is used to deduce the absolute or relative distances between the reference nodes and the target node. As each reference node has its own free-running clock, proper clock synchronization mechanisms need to be used to ensure a common time frame [3, 4]. Clock synchronization is based on estimating clock parameters (e.g., phase and frequency offset), and for obtaining accurate results, the underlying clock model should be accurate and appropriate. The majority of existing clock synchronization protocols are based on a simplified linear clock model [5], which however lead to a low synchronization accuracy for longer time spans due to negligence of the higher-order terms such as frequency drift [6, 7]. It has been shown that a quadratic clock model is precise and can be derived from the physical clock structure [8, 9]. This model includes frequency drift and is more precise for real-world clocks than the commonly used linear model, especially in the long-term. Such a model has been widely adopted in variety of practical applications that need accurate

clock synchronization or time transfer, e.g. the computation of Coordinated Universal Time (UTC) and the clock comparison in global Navigation Satellite Systems (GNSS) [10, 11].

In this paper, we consider joint clock synchronization and propagation delay estimation, and propose a novel algorithm which is based on a quadratic clock model. We assume a pairwise Two Way Time Transfer (TWTT) communication protocol[12], and first estimate the clock drift using an extension of a traditional technique to allow for arbitrary instead of periodic message transfer times. Subsequently, the clock offset, clock skew and propagation time delay are estimated using a linear model wherein we correct for the clock drift and its estimation variance. An advantage of this new two-step algorithm is that existing protocols based on a linear clock model can still be used (including the network synchronization protocols [13]) while the drift correction extends the accuracy to longer periods. Simulation results show that the Root Mean Square Error (RMSE) of the estimator under the quadratic clock model asymptotically converges to the Cramer-Rao lower bound, which ensures a high estimation precision over long time spans.

2. SYSTEM MODEL

We consider an indoor positioning system using UWB signals, where a number of base stations with their own free-running clocks are connected by UWB communication links. One of the nodes is taken as the reference and the other nodes are synchronized to this reference via pair-wise TWTT communications.

2.1. Clock Model

Each node has its own local clock. A clock model relates the 'local time' of a clock to global time, i.e., a reference clock. A precise model based on physical characteristics of practical clocks is given in [8, 9], which describes the local clock of node i in the network as

$$t_i = \phi_i + \omega_i t + \frac{1}{2} D_i t^2 + \epsilon_i(t) \qquad (1)$$

where t is the global time, t_i is the local time for node i, ϕ_i is the clock offset, ω_i is the clock skew (difference in frequency), D_i is the frequency drift, and $\epsilon_i(t)$ is the time jitter due to random noise within the clock.

The frequency drift D_i of low-cost crystal oscillators (XOs) and temperature compensated crystal oscillators (TCXOs) is at the level of 10^{-14}sec/sec^2 and $10^{-15} \sim 10^{-14}\text{sec/sec}^2$, respectively, while for expensive oven controlled crystal oscillators (OCXOs) and small atomic frequency standards (such as Rubidium clocks) it can be up to the level of $10^{-15} \sim 10^{-17}\text{sec/sec}^2$ and $10^{-17} \sim 10^{-19}\text{sec/sec}^2$ respectively [14]. Since the drift is usually quite small, a linear clock model can be used to approximate the quadratic model over short periods for high precision OCXOs and atomic frequency standards.
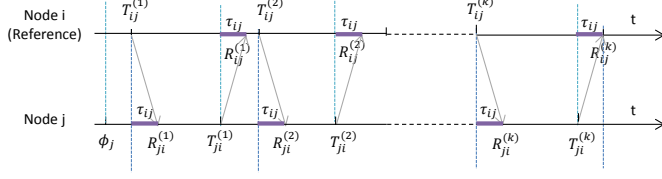
Fig. 1. Two-way time transfer (TWTT) communication between node pairs

However, the effect of the clock drift cannot be neglected over longer time spans especially for low-cost XOs and TCXOs, in which case the quadratic model is preferred.

2.2. Pair-wise TWTT model

The Two Way Time Transfer (TWTT) communication protocol [12] is a classical approach to obtain synchronization with increased accuracy over one-way protocols. Consider a TWTT mechanism between a pair of nodes, as shown in Fig. 1. In this model, we take node i as the reference and node j as the node to be synchronized. Node i initializes the communication by sending a time package with its current clock time recorded as time-stamp T_{ij} to node j. When node j receives this package, it records the time of reception R_{ji} according to its local clock. At a later time, node j responds to node i by sending its current local time as time-stamp T_{ji} back to node i, and node i records its reception time as time-stamp R_{ij}. We denote the communication link from node i to node j as down-link and the reverse link as up-link. In this paper we assume the channel propagation time τ_{ij} is constant and the same in both directions (for a linear clock model, extensions involving mobile nodes are studied in [15]). As the message sending times are accurately controlled by the local clock, the time-stamps T_{ij} and T_{ji} are exactly recorded by the local clocks. The arrival times of received messages is usually recorded by a time-interval counter, which causes measurement noise q_{ij} and q_{ji} that is added to the receiving time-stamps R_{ij} and R_{ji}, respectively. This measurement noise dominates the clock jitter $\epsilon(t)$ and is the main source of uncertainty in the model. For simplicity we assume that q_{ij} and q_{ji} are zero mean Gaussian distributed with variance σ_m^2.

Assume that there are K TWTT periods in total, $t_d^{(k)}$ and $t_u^{(k)}$ denote the global time instants of the kth down-link and up-link, respectively. For the down-link, we obtain (with node i as reference node)

$$T_{ij}^{(k)} = t_d^{(k)}$$
$$R_{ji}^{(k)} = \phi_j + \omega_j(t_d^{(k)} + \tau_{ij}) + \frac{1}{2}D_j(t_d^{(k)} + \tau_{ij})^2 + q_{ji}^{(k)} \quad (2)$$

whereas for the up-link, we obtain

$$T_{ji}^{(k)} = \phi_j + \omega_j t_u^{(k)} + \frac{1}{2}D_j t_u^{(k)2}$$
$$R_{ij}^{(k)} = t_u^{(k)} + \tau_{ij} + q_{ij}^{(k)} \quad (3)$$

2.3. Problem Formulation

The objective of the paper is, given observations on K TWTT communications, estimate the unknown parameters ϕ_j, ω_j, D_j and τ_{ij}. We propose a new algorithm which divides the estimation into two steps: first, we estimate D_j and calculate its standard deviation σ_D; second, taking D_j as a known parameter with standard deviation

σ_D, we reduce (2) and (3) to a linear model and jointly estimate the remaining clock parameters as well as the propagation delay. While such a two-step approach is sub-optimal compared to a joint (ML) estimation of all parameters, the advantage is that both steps can be carried out using straightforward linear algebra techniques, while existing approaches based on linear clock models need minimal modifications.

3. CLOCK DRIFT ESTIMATION

Clock drift estimation schemes originate from local measurements among clocks where the two clocks under comparison are at the same place and are measured periodically [16]. We extend these schemes to the general remote TWTT clock synchronization case by combining the pair-wise TWTT model proposed in (2) and (3) with the local clock drift estimation schemes used in [16].

Based on (2), take the first-order discrete derivative of $R_{ji}^{(k)}$,

$$
\begin{aligned}
y_{ji}^{(k)} &\triangleq \frac{R_{ji}^{(k+1)} - R_{ji}^{(k)}}{T_{ij}^{(k+1)} - T_{ij}^{(k)}} \\
&= \omega_j + \frac{1}{2}D_j\left[\left(T_{ij}^{(k+1)} + T_{ij}^{(k)}\right) + 2\tau_{ij}\right] + \frac{q_{ji}^{(k+1)} - q_{ji}^{(k)}}{T_{ij}^{(k+1)} - T_{ij}^{(k)}} \\
&\quad (k \leq K-1)
\end{aligned} \quad (4)
$$

where $y_{ji}^{(k)}$ can be regarded as the 'average frequency' of the local clock at node j (taking node i as the reference) during the kth time interval. Similarly, the second-order discrete derivative of $R_{ji}^{(k)}$ relates to the linear frequency drift, as

$$
\begin{aligned}
y_{ji}^{(k)} - y_{ji}^{(r)} &= \frac{1}{2}D_j\left[\left(T_{ij}^{(k+1)} + T_{ij}^{(k)}\right) - \left(T_{ij}^{(r+1)} + T_{ij}^{(r)}\right)\right] + \\
&\quad \left(\frac{q_{ji}^{(k+1)} - q_{ji}^{(k)}}{T_{ij}^{(k+1)} - T_{ij}^{(k)}} - \frac{q_{ji}^{(r+1)} - q_{ji}^{(r)}}{T_{ij}^{(r+1)} - T_{ij}^{(r)}}\right) \\
&\quad (k \leq K-1, \quad r \leq K-1)
\end{aligned} \quad (5)
$$

where k and r are the kth and rth down-link communication.

Let $\boldsymbol{y}_{ji} = \left[y_{ji}^{(1)}, y_{ji}^{(2)}, \cdots, y_{ji}^{(k-1)}\right]^T$, which is a vector containing all the average frequencies during every down-link period. According to (4), $\boldsymbol{y}_{ji}$ is defined by

$$\boldsymbol{y}_{ji} = (\mathrm{diag}\,(\boldsymbol{H}\boldsymbol{t}_{ij}))^{-1}\,\boldsymbol{H}\boldsymbol{r}_{ji} \quad (6)$$

where

$$
\boldsymbol{H} = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \cdots & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix}_{(K-1)\times K},
$$

$$\boldsymbol{t}_{ij} = \left[T_{ij}^{(1)}, T_{ij}^{(2)}, \cdots, T_{ij}^{(K)}\right]^T, \boldsymbol{r}_{ji} = \left[R_{ji}^{(1)}, R_{ji}^{(2)}, \cdots, R_{ji}^{(K)}\right]^T.$$

We can also write equation (5) as

$$\boldsymbol{S}\boldsymbol{y}_{ji} = \boldsymbol{Z}\boldsymbol{r}_{ji} = \boldsymbol{A}\boldsymbol{t}_{ij}D_j + \boldsymbol{Z}\boldsymbol{q}_{ji} \quad (7)$$

where

$$\boldsymbol{S} = \begin{bmatrix} 0 & \cdots & 0 & -1 & 1 & 0 & \cdots & 0 \\ \vdots & & -1 & 0 & 0 & 1 & & \vdots \\ & \ddots & & \vdots & \vdots & & \ddots & \\ 0 & \iddots & \iddots & \vdots & \vdots & & \ddots & 0 \\ -1 & 0 & \cdots & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}_{\frac{K-1}{2} \times (K-1)}$$

$$\boldsymbol{A} = \frac{1}{2} \begin{bmatrix} 0 & \cdots & 0 & -1 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & & -1 & -1 & 0 & 1 & 1 & & \vdots \\ & \ddots & & \vdots & \vdots & & \ddots & \\ 0 & \iddots & \iddots & \vdots & \vdots & & \ddots & 0 \\ -1 & -1 & \cdots & 0 & 0 & 0 & \cdots & 1 & 1 \end{bmatrix}_{\frac{K-1}{2} \times K}$$

$$\boldsymbol{Z} = \boldsymbol{S} \left(\operatorname{diag} \left(\boldsymbol{H} \boldsymbol{t}_{ij} \right) \right)^{-1} \boldsymbol{H}, \; \boldsymbol{q}_{ji} = \begin{bmatrix} q_{ji}^{(1)} & q_{ji}^{(2)} & \cdots & q_{ji}^{(K)} \end{bmatrix}^T.$$

The covariance matrix of the measurement noise on (7) is

$$\boldsymbol{Q}_r = E \left(\left(\boldsymbol{Z} \boldsymbol{q}_{ji} \right) \left(\boldsymbol{Z} \boldsymbol{q}_{ji} \right)^T \right) = \sigma_m^2 \boldsymbol{Z} \boldsymbol{Z}^T$$

This model generalizes the model proposed in [16] by not relying on constant fixed intervals between the transmissions. Based on this model, the BLUE (Best Linear Unbiased Estimator) [17] for D_j and its variance $\sigma_{\hat{D}_j}$ are given by

$$\hat{D}_j = \left(\boldsymbol{t}_{ij}^T \boldsymbol{A}^T \boldsymbol{Q}_r^{-1} \boldsymbol{A} \boldsymbol{t}_{ij} \right)^{-1} \boldsymbol{t}_{ij}^T \boldsymbol{A}^T \boldsymbol{Q}_r^{-1} \boldsymbol{Z} \boldsymbol{r}_{ji}$$
$$\sigma_{\hat{D}_j}^2 = \left(\boldsymbol{t}_{ij}^T \boldsymbol{A}^T \boldsymbol{Q}_r^{-1} \boldsymbol{A} \boldsymbol{t}_{ij} \right)^{-1} \tag{8}$$

4. CLOCK OFFSET, CLOCK SKEW AND DISTANCE ESTIMATION

4.1. Model Establishment

Suppose $\hat{D}_j = D_j + q_D$, where q_D is the noise in the estimator. Inserting this into (2) and (3), we obtain

$$R_{ji}^{(k)} = \phi_j + \omega_j \left(T_{ij}^{(k)} + \tau_{ij} \right) + \frac{1}{2} \left(\hat{D}_j - q_D \right) \left(T_{ij}^{(k)} + \tau_{ij} \right)^2 + q_{ji}^{(k)}$$
$$T_{ji}^{(k)} = \phi_j + \omega_j \left(R_{ij}^{(k)} - \tau_{ij} - q_{ij}^{(k)} \right) + \frac{1}{2} \left(\hat{D}_j - q_D \right) \left(R_{ij}^{(k)} - \tau_{ij} - q_{ij}^{(k)} \right)^2 \tag{9}$$

Let $\alpha_j = \frac{1}{\omega_j}$, $\beta_j = -\frac{\phi_j}{\omega_j}$, then (9) can be re-written as

$$\begin{bmatrix} R_{ji}^{(k)} - \frac{1}{2} \hat{D}_j \left(T_{ij}^{(k)} \right)^2 & 1 & -1 & -\hat{D}_j T_{ij}^{(k)} \\ T_{ji}^{(k)} - \frac{1}{2} \hat{D}_j \left(R_{ij}^{(k)} \right)^2 & 1 & 1 & \hat{D}_j R_{ij}^{(k)} \end{bmatrix} \begin{bmatrix} \alpha_j \\ \beta_j - \frac{1}{2} \hat{D}_j \alpha_j \tau_{ij}^2 \\ \tau_{ij} \\ \alpha_j \tau_{ij} \end{bmatrix}$$
$$= \begin{bmatrix} T_{ij}^{(k)} \\ R_{ij}^{(k)} \end{bmatrix} + \begin{bmatrix} q_{ji}^{(k)} - \frac{1}{2} q_D \alpha_j \left(T_{ij}^{(k)} + \tau_{ij} \right)^2 \\ -q_{ij}^{(k)} - \frac{1}{2} q_D \alpha_j \left(R_{ij}^{(k)} - \tau_{ij} \right)^2 \end{bmatrix}, \; k = 1, 2, \ldots, K \tag{10}$$

This generalizes the linear model proposed in [13]. For K TWTT communications, the equations can be stacked as

$$\begin{bmatrix} \boldsymbol{c}_{ji} & \boldsymbol{1}_{2K} & \boldsymbol{e} & \boldsymbol{u}_{ij} \end{bmatrix} \begin{bmatrix} \alpha_j \\ \beta_j - \frac{1}{2} \hat{D}_j \alpha_j \tau_{ij}^2 \\ \tau_{ij} \\ \alpha_j \tau_{ij} \end{bmatrix} = \boldsymbol{m}_{ij} + \boldsymbol{q}_j \tag{11}$$

where

$$\boldsymbol{c}_{ji} = \left[R_{ji}^{(1)} - \frac{1}{2} \hat{D}_j \left(T_{ij}^{(1)} \right)^2, \; T_{ji}^{(1)} - \frac{1}{2} \hat{D}_j \left(R_{ij}^{(1)} \right)^2, \right.$$
$$\left. R_{ji}^{(2)} - \frac{1}{2} \hat{D}_j \left(T_{ij}^{(2)} \right)^2, \cdots, T_{ji}^{(K)} - \frac{1}{2} \hat{D}_j \left(R_{ij}^{(K)} \right)^2 \right]^T,$$

$$\boldsymbol{1}_{2K} = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T, \; \boldsymbol{e} = \begin{bmatrix} -1 & +1 & \cdots & -1 & +1 \end{bmatrix}^T,$$
$$\boldsymbol{u}_{ij} = \begin{bmatrix} -\hat{D}_j T_{ij}^{(1)} & \hat{D}_j R_{ij}^{(1)} & -\hat{D}_j T_{ij}^{(2)} & \cdots & \hat{D}_j R_{ij}^{(K)} \end{bmatrix}^T,$$
$$\boldsymbol{m}_{ij} = \begin{bmatrix} T_{ij}^{(1)} & R_{ij}^{(1)} & T_{ij}^{(2)} & \cdots & R_{ij}^{(K)} \end{bmatrix}^T$$

and $\boldsymbol{q}_j$ is the noise vector. When $\alpha_j \approx 1$, $\boldsymbol{q}_j$ can be approximated by

$$\boldsymbol{q}_j = \boldsymbol{q}_{1j} - \boldsymbol{q}_{2j}$$

where $\boldsymbol{q}_{1j} = \begin{bmatrix} q_{ji}^{(1)} & -q_{ij}^{(1)} & q_{ji}^{(2)} & \cdots & -q_{ij}^{(K)} \end{bmatrix}^T$,

$$\boldsymbol{q}_{2j} = \frac{1}{2} q_D \left[\left(T_{ij}^{(1)} + \tau_{ij} \right)^2, \; \left(R_{ij}^{(1)} - \tau_{ij} \right)^2, \cdots, \left(R_{ij}^{(K)} - \tau_{ij} \right)^2 \right]^T.$$

Using the independence of $q_{ij}^{(k)}$ and $q_{ji}^{(r)}$, and of $q_{ij}^{(k)}$ and $q_{ji}^{(r)}$ (for $k \neq r$), considering q_D not correlated with $q_{ij}^{(k)}$ and $q_{ji}^{(k)}$ for K large enough, and assuming τ_{ij} to be small, the noise covariance matrix of $\boldsymbol{q}_j$ can be obtained as

$$\boldsymbol{\Sigma}_j = E \left(\boldsymbol{q}_j \boldsymbol{q}_j^T \right) = E \left(\boldsymbol{q}_{1j} \boldsymbol{q}_{1j}^T \right) + E \left(\boldsymbol{q}_{2j} \boldsymbol{q}_{2j}^T \right)$$
$$\cong \sigma_m^2 \boldsymbol{I}_{2K \times 2K} + \frac{1}{4} \sigma_{\hat{D}}^2 \left(\boldsymbol{m}_{ij} \odot \boldsymbol{m}_{ij} \right) \left(\boldsymbol{m}_{ij} \odot \boldsymbol{m}_{ij} \right)^T \tag{12}$$

where $\odot$ denotes element-wise multiplication. The resulting model is summarized as

$$\boldsymbol{A}_{ij} \boldsymbol{\theta}_j = \boldsymbol{m}_{ij} + \boldsymbol{q}_j$$

where

$$\boldsymbol{A}_{ij} = \begin{bmatrix} \boldsymbol{c}_{ji} & \boldsymbol{1}_{2K} & \boldsymbol{e} & \boldsymbol{u}_{ij} \end{bmatrix}, \boldsymbol{\theta}_j = \begin{bmatrix} \alpha_j \\ \beta_j - \frac{1}{2} \hat{D}_j \alpha_j \tau_{ij}^2 \\ \tau_{ij} \\ \alpha_j \tau_{ij} \end{bmatrix}.$$

4.2. Weighted least-squares solutions

If we regard the entries of $\boldsymbol{\theta}_j$ as independent parameters, then we can estimate $\boldsymbol{\theta}_j$ using Weighted Least Squares (WLS) [17], where we take into account that $\boldsymbol{\Sigma}_j$ is not a diagonal matrix, resulting in

$$\hat{\boldsymbol{\theta}}_j = \left(\boldsymbol{A}_{ij}^T \boldsymbol{\Sigma}_j^{-1} \boldsymbol{A}_{ij} \right)^{-1} \boldsymbol{A}_{ij}^T \boldsymbol{\Sigma}_j^{-1} \boldsymbol{m}_{ij}$$

Alternatively, we can introduce a constraint on $\boldsymbol{\theta}_j$ to express the multiplication relation between α_j, τ_{ij} and $\alpha_j \tau_{ij}$ as

$$\boldsymbol{\theta}_j^T \boldsymbol{D} \boldsymbol{\theta}_j + \boldsymbol{b}^T \boldsymbol{\theta}_j = 0$$

where

$$\boldsymbol{D} = \begin{bmatrix} 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{b}^T = \begin{bmatrix} 0 & 0 & 0 & -1 \end{bmatrix}.$$

The Lagrange function associated with the resulting constrained WLS problem can be formulated as [17]

$$L \left(\boldsymbol{\theta}_j, \lambda \right) = \| \boldsymbol{A}_{ij} \boldsymbol{\theta}_j - \boldsymbol{m}_{ij} \|_{\boldsymbol{\Sigma}_j}^2 + \lambda \left(\boldsymbol{\theta}_j^T \boldsymbol{D} \boldsymbol{\theta}_j + \boldsymbol{b}^T \boldsymbol{\theta}_j \right) \tag{13}$$

where λ is the Lagrange multiplier. Solving this by setting $\partial L(\boldsymbol{\theta}_j, \lambda)/\partial \boldsymbol{\theta}_j$ to zero gives

$$\hat{\boldsymbol{\theta}}_j(\lambda) = \left(\boldsymbol{A}_{ij}^T \boldsymbol{\Sigma}_j^{-1} \boldsymbol{A}_{ij} + \lambda \boldsymbol{D} \right)^{-1} \left(\boldsymbol{A}_{ij}^T \boldsymbol{\Sigma}_j^{-1} \boldsymbol{m}_{ij} - \frac{1}{2} \lambda \boldsymbol{b} \right) \tag{14}$$

where λ is obtained by solving the constraint equation

$$\hat{\boldsymbol{\theta}}_j^T(\lambda) \boldsymbol{D} \hat{\boldsymbol{\theta}}_j(\lambda) + \boldsymbol{b}^T \hat{\boldsymbol{\theta}}_j(\lambda) = 0 \tag{15}$$

By solving the nonlinear equation (15) in an iterative way (e.g. Newton's method [18]), the proper λ can be found, after which $\hat{\boldsymbol{\theta}}_j$ follows.
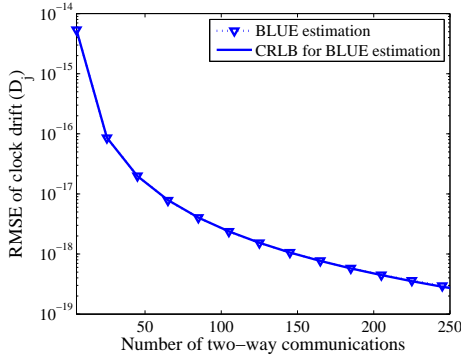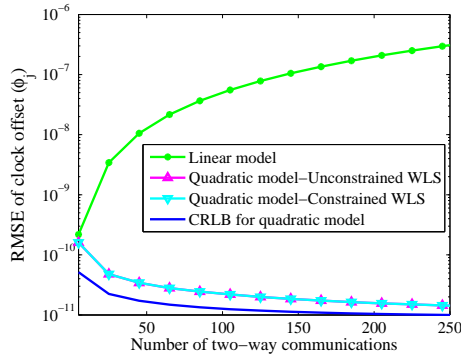
Fig. 2. RMSE plot of estimated clock drift
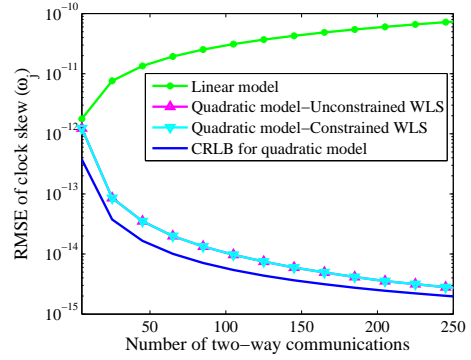

Fig. 3. RMSE plot of estimated clock skew

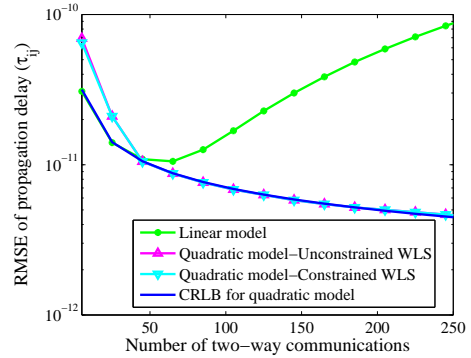**Fig. 4**. RMSE plot of estimated clock offset

Fig. 5. RMSE plot of estimated propagation delay

5. SIMULATIONS

The performance of the algorithm is assessed using simulations. We take the clock drift uniformly distributed in the range $[-1, 1] \times 10^{-14}$ sec/sec^2, the clock skew in $[-1, 1] \times 10^{-3}$ sec/sec, the clock offsets in $[-1, 1]$ sec, and the propagation delays in $[1 \times 10^{-7}, 1 \times 10^{-6}]$ sec, which are typical values in real-world applications. The TWTT transmission times are uniformly distributed in $[0, 1] \times 100$ sec. The standard deviation of q_m is 10^{-10} sec. Applying the linear model given by [13] and the quadratic model given by (10), 10,000 independent Monte Carlo simulations are run. In the following, the averaged simulation results are presented.

From Fig.2 we observe that the proposed clock drift estimation scheme is convergent and reaches the CRLB. From Fig.3, Fig.4 and Fig.5 we can see that the linear model will fail in the long-term due to the neglect of the clock drift, because the RMSE of the clock offset and the clock skew under the linear model are absolutely divergent. In contrast, the proposed WLS estimation algorithm under the quadratic clock model is convergent and the estimation asymptotically reaches the CRLB. Thanks to this precise clock model, the estimation error decreases when the number of samples increases, which leads to a better synchronization accuracy for longer time spans. These results indicate that, compared to the linear clock model, the estimation under the quadratic clock model has several advantages. The time synchronization parameters under the quadratic clock model are accurate over a much longer time, while the estimation under the linear clock model deviates already over short time-spans, in particular for low-cost clocks which usually have a larger clock drift term. Accumulating more sampling data usually helps estimators to become more accurate by averaging out the measurement noise. But for the linear clock model, longer sampling times lead to a bias due to the clock drift which limits the estimation accuracy. The estimation error under the quadratic clock model asymptotically approaches zero because the BLUE is consistent.

6. CONCLUSIONS

In this paper, a novel algorithm is proposed to estimate clock offset, clock skew, clock drift, and propagation delay via TWTT communications between pairs of nodes, while assuming a quadratic clock model which is more accurate than the usually used linear model. The proposed algorithm is divided into two steps. First, the clock drift and its variance is estimated using a straightforward double differencing scheme. In the second step, by inserting the estimated clock drift coefficient, the remaining parameters are estimated using modifications of existing techniques. In particular, the equations become linear and can be efficiently solved using WLS. Simulation results show that the estimator under the quadratic clock model is asymptotically efficient while the estimator under the linear clock model diverges. Thus, we can conclude that the estimator using a linear clock model may work for short time spans with limited synchronization accuracy while the proposed estimator for a quadratic clock model can be adopted to both short and long time span estimation with high synchronization accuracy. The proposed scheme is readily generalized to synchronization in a mobile network, cf. [13, 15].

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