**Human-Oriented Robotics** Winter term 2013/2014
Prof. Dr. Kai Arras, Social Robotics Lab University of Freiburg
Lab instructors: Timm Linder, Luigi Palmieri, Billy Okal Department of Computer Science

# Exercise 3: Basics of Probabilistic Reasoning

**Submission:** Send your solution to `palmieri@informatik.uni-freiburg.de` until November 25, 2013 with subject line "`[exercises] Sheet 3`". All files (Matlab scripts, exported figures, hand-written notes in pdf/jpg format) should be put into a single zip file named `lastname_sheet3.zip`.

### Exercise 3.1: Joint Distribution

Assume $\mathbf{x} = \{x_1, x_2, ..., x_K\}$ are discrete random variables. By marginalizing out the variables in order, show that the representation

$$p(\mathbf{x}) = \prod_{k=1}^{K} p(x_k|\mathrm{pa}_k) \tag{1}$$

for the joint distribution of a direct graph is correctly normalized, i.e.

$$\sum_{x_1}\sum_{x_2}...\sum_{x_K} p(\mathbf{x}) = 1,$$

provided that all conditional distributions $p(x_k|\mathrm{pa}_k)$ are correctly normalized. For example, for the $K$-th node

$$\sum_{x_K} p(x_K|\mathrm{pa}_K) = 1$$

### Exercise 3.2: Probabilistic Graphical Models

a) Consider a malfunctioning coffee-serving robot that you have to repair. From your experience you can state the following: a loose cable is a possible cause for loss of commands over the bus that controls the robot's arm and is also an explanation for high CPU load of the robot's built-in embedded computer (because, for example, a certain task throws an exception and restarts constantly). In turn, either of these could cause the arm to malfunction and spill coffee. An increased temperature of the robot's PC can also be explained by a high CPU load.
Represent these causal links in a probabilistic graphical model. Let $a$ stand for *loose cable*, $b$ for *loss of commands*, $c$ for *high CPU load*, $d$ for *spilled coffee*, and $e$ for *increased PC temperature.*

b) From the given directed graphs (Figures 1 and 2) find the corresponding distribution. Remember that:

$$p(\mathbf{x}) = \prod_{k=1}^{K} p(x_k|\mathrm{pa}_k) \tag{2}$$

### Exercise 3.3: Markov Chains

a) Find the order of all Markov Chains shown in Figure 3

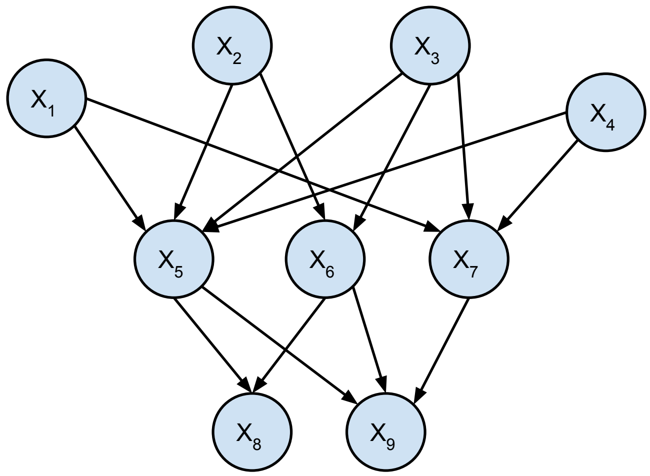


Figure 1: Probabilistic Graphical Model $a$

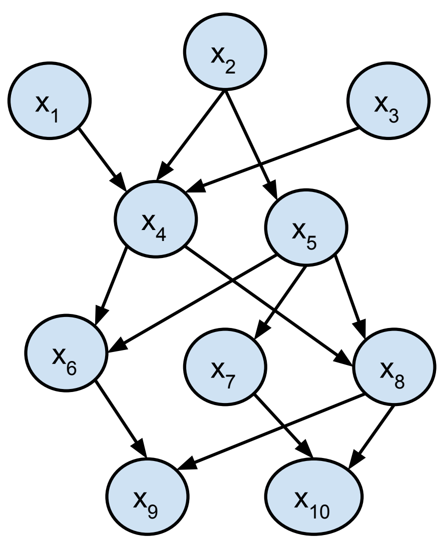


Figure 2: Probabilistic Graphical Model $b$

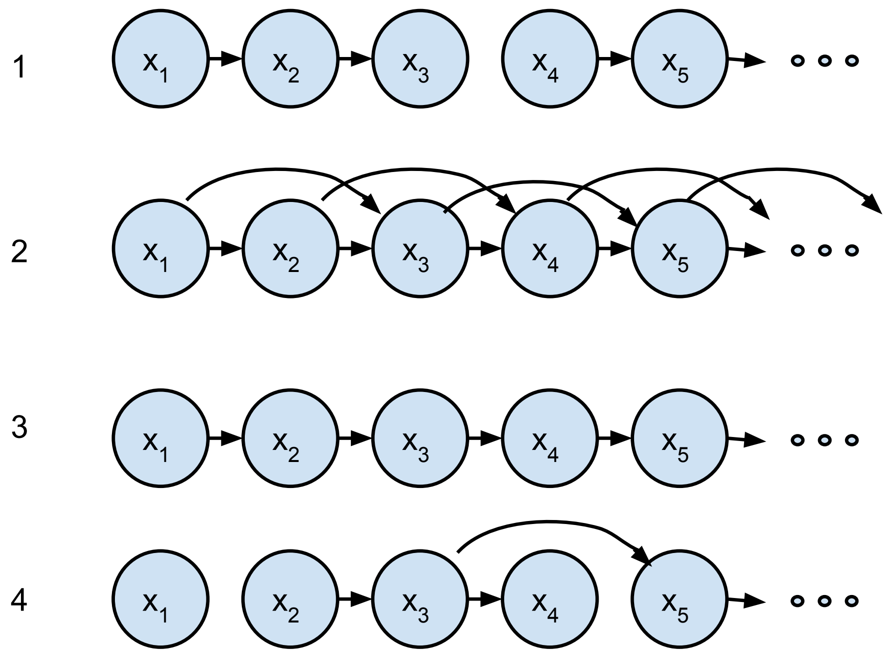


Figure 3: Markov Chains