Human-Oriented Robotics Winter term 2013/2014
Prof. Dr. Kai Arras, Social Robotics Lab University of Freiburg
Lab instructors: Timm Linder, Luigi Palmieri, Billy Okal Department of Computer Science

Exercise 2: Introduction to Probability – Solutions –

Exercise 2.1: Conditional Probability

a) Probability of drawing an apple:

$$
\begin{aligned}
p(a) &= \textstyle\sum_{box} p(a, box) \\
&= \textstyle\sum_{box} p(a|box)p(box) \\
&= p(a|r)p(r) + p(a|b)p(b) + p(a|g)p(g) \\
&= 0.3 * 0.2 + 0.5 * 0.2 + 0.3 * 0.6 = 0.34
\end{aligned}
$$

b) Probability of green box given orange

$$
\begin{aligned}
p(g|o) = & \tfrac{p(g,o)}{p(o)} \\
&= \tfrac{p(o|g)p(g)}{\sum_{box} p(o|box)p(box)} \\
&= \tfrac{0.18}{0.36} = 0.5
\end{aligned}
$$

Exercise 2.2: Bayes' Rule

a) Applying the Bayes' theorem:

$$p(x_i|z) = \tfrac{p(z|x_i)p(x_i)}{\sum_{i=1}^{3} p(z|x_i)p(x_i)} \tag{1}$$

$$p(x_1|z) = \tfrac{0.8*1/3}{0.8*1/3+0.4*1/3+0.1*1/3)} = 0.616 \tag{2}$$

$$p(x_2|z) = \tfrac{0.4*1/3}{0.8*1/3+0.4*1/3+0.1*1/3)} = 0.308 \tag{3}$$

$$p(x_3|z) = \tfrac{0.1*1/3}{0.8*1/3+0.4*1/3+0.1*1/3)} = 0.077 \tag{4}$$

Exercise 2.3: Expectation, Variance and Covariance

a) Since x and z are independent, their joint distribution factorizes $p(x,z) = p(x)p(z)$:

$$\begin{aligned} \mathrm{E}[x+z] &= \int\int (x+z)p(x)p(z)dxdz \\ &= \int xp(x)dx + \int zp(z)dz \\ &= \mathrm{E}[x] + \mathrm{E}[z] \end{aligned} \tag{5}$$

Similarly for the variances, we first note that

$$(x+z-\mathrm{E}[x+z])^2 = (x-\mathrm{E}[x])^2 + (z-\mathrm{E}[z])^2 + 2(x-\mathrm{E}[x])(z-\mathrm{E}[z]) \tag{6}$$

where the final term will integrate to zero with respect to the factorized distribution $p(x)p(z)$ Hence:

$$\begin{aligned} \mathrm{var}[x+z] &= \int\int (x+z-\mathrm{E}[x+z])^2 p(x)p(z)dxdz \\ &= \int (x-\mathrm{E}[x])^2 p(x)dx + \int (z-\mathrm{E}[z])^2 p(z)dz \\ &= \mathrm{var}[x] + \mathrm{var}[z] \end{aligned} \tag{7}$$

b)

$$\mathrm{cov}[xz] = \mathrm{E}[xz] - \mathrm{E}[x]\mathrm{E}[z]$$

If x and y are independent:

$$\begin{aligned} \mathrm{E}[xz] &= \textstyle\sum_x \sum_y xyp(x,y) \\ &= \textstyle\sum_x \sum_y xyp(x)p(y) \\ &= \textstyle\sum_x xp(x) \sum_y yp(y) \\ &= \mathrm{E}[x]\mathrm{E}[y] \end{aligned} \tag{8}$$

c) The covariance matrix is not valid because of the following reasons;

- The matrix is not positive semi-definite, easily checked by
- The eigenvectors are $\begin{bmatrix} -0.6569 \\ 10.6569 \end{bmatrix}$; should be all real and positive. Use **eig** in Matlab to check this.
- Determinant is -4; should be positive as a consequence of the eigenvalues. Use **det** in Matlab

Exercise 2.4: Probability distributions and moments

See provided Matlab files! First load the data, then run the scripts `CalcStats` followed by `PlotData`.