Human-Oriented Robotics
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University of Freiburg
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Exercise 2: Introduction to Probability

Submission: Send your solution to `palmieri@informatik.uni-freiburg.de` until November 18, 2013 with subject line "`[exercises] Sheet 2`". All files (Matlab scripts, exported figures, handwritten notes in pdf/jpg format) should be put into a single zip file named `lastname_sheet2.zip`.

Exercise 2.1: Conditional Probability

a) Suppose that we have three colored boxes r (red), b (blue), and g (green). Box r contains 3 apples, 4 oranges, and 3 limes, box b contains 1 apple, 1 orange and 0 limes, and box g contains 3 apples, 3 oranges and 4 limes. If a box is chosen at random with probabilities $p(r) = 0.2$, $p(b) = 0.2$, $p(g) = 0.6$, and a piece of fruit is removed from the box (with equal probability of selecting any of the items in the box), then what is the probability of selecting an apple?

b) If we observe that the selected fruit is in fact an orange, what is the probability that it came from the green box?

Exercise 2.2: Bayes' Rule

a) Suppose we live in a world with only three possible robot positions $X = (x_1, x_2, x_3)$. Let Z be a Boolean sensor variable characterized by the following probabilities:

$$\begin{array}{ll} p(z|x_1) = 0.8 & p(\neg z|x_1) = 0.2 \\ p(z|x_2) = 0.4 & p(\neg z|x_2) = 0.6 \\ p(z|x_3) = 0.1 & p(\neg z|x_3) = 0.9 \end{array}$$

Suppose that the marginal distribution of the robot position is uniform, $p(x_i) = \frac{1}{3}$. Calculate the posterior $p(x_i|z)$ for each of the locations $X = (x_1, x_2, x_3)$.

Exercise 2.3: Expectation, Variance and Covariance

a) Suppose that the two variables x and z are statistically independent. Show that the mean and variance of their sum satisfies

$$\mathrm{E}[x + z] = \mathrm{E}[x] + \mathrm{E}[z] \tag{1}$$

$$\mathrm{Var}[x + z] = \mathrm{Var}[x] + \mathrm{Var}[z] \tag{2}$$

b) Show that if two variables x and y are independent, then their *covariance* is zero. The covariance is defined as

$$\mathrm{Cov}[x, y] = \mathrm{E}[(x - \mu_x)(y - \mu_y)] \tag{3}$$

c) John computes the *covariance matrix* of a random vector as follows. Bob tells him that he is wrong. Explain why.

$$C = \begin{bmatrix} 9 & 4 \\ 4 & 1 \end{bmatrix} \tag{4}$$

Exercise 2.4: Probability distributions and moments

This exercise requires you to import the provided Matlab file `distributions.mat` into your Matlab workspace. It will then contain a 10000×5 matrix called `data`.

a) Each of the five columns of *data* contains 10.000 values that were randomly sampled from unknown probability distributions. Without using Matlab's built-in functions, calculate the

- mean
- variance
- standard deviation
- 3rd central moment
- 4th central moment

for each column. Instead of using for-loops, use vectorized functions such as `sum` and `repmat`.

b) Now verify your results using the built-in functions `mean`, `var`, `std` and `moment`.

c) Plot each of the five distributions into the same figure using the `plot` command. Use $[-5, 20]$ as the `x`-axis limits and `'.-'` as the line style. You can use the `histc` command with e. g. 100 bins to compute a histogram of the relative frequencies of the values in each column of `data`. Normalize the frequencies such that for each distribution, they sum up to 1.0.

d) By looking at the plot, which data set do you think was sampled from which type of probability distribution explained in the lecture?

e) If you suspect any of the distributions to be Gaussian, what were the parameters originally used to build the distribution?