Human-Oriented Robotics
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Exercise 3: Basics of Probabilistic Reasoning – Solutions –

Exercise 3.1: Joint Distribution

We want to show that:

$$\sum_{x_1} ... \sum_{x_K} p(x) = 1$$

$$\sum_{x_1} ... \sum_{x_K} p(x) = \sum_{x_1} ... \sum_{x_K} \prod_{k=1}^{K} p(x_k|\mathrm{pa}_k) = 1$$

We assume that the nodes in the graph have been numbered such that x_1 is the root node and no arrows lead from a higher numbered node to a lower numbered node. We can then marginalize over the nodes in reverse order, starting with x_K. We can write:

$$\prod_{k=1}^{K} p(x_k|pa_k) = p(x_K|\mathrm{pa}_K) \prod_{k=1}^{K-1} p(x_k|\mathrm{pa}_k)$$

then

$$\sum_{x_1} ... \sum_{x_K} \prod_{k=1}^{K} p(x_k|pa_k) = \sum_{x_1} ... \sum_{x_K} p(x_K|\mathrm{pa}_K) \prod_{k=1}^{K-1} p(x_k|\mathrm{pa}_k)$$

since each of the conditional distributions is assumed to be correctly normalized and none of the other variables depend on x_K, so :

$$\sum_{x_K} p(x_K|\mathrm{pa}_K) = 1$$

We can write:

$$\sum_{x_1} ... \sum_{x_K} p(x_K|\mathrm{pa}_K) \prod_{k=1}^{K-1} p(x_k|\mathrm{pa}_k) = \sum_{x_1} ... \sum_{x_{K-1}} \prod_{k=1}^{K-1} p(x_k|\mathrm{pa}_k)$$

Repeating this process K 2 times we are left with

$$\sum_{x_1} p(x_1|\emptyset) = 1$$

Exercise 3.2: Probabilistic Graphical Models

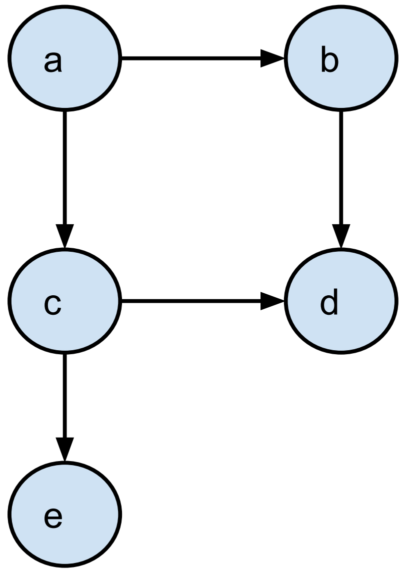

Figure 1: Probabilistic Graphical Model a

a) The resulting probabilistic graphical model is the one in Figure **??**

b) Figure a :

$$p(\mathbf{x}) = p(x_1)p(x_2)p(x_3)p(x_4)p(x_5|x_1, x_2, x_3, x_4)p(x_6|x_2, x_3)p(x_7|x_1, x_3, x_4)p(x_8|x_5, x_6)p(x_9|x_5, x_6, x_7)$$

Figure b :

$$p(\mathbf{x}) = p(x_1)p(x_2)p(x_3)p(x_4|x_1, x_2, x_3)p(x_5|x_2)p(x_6|x_4, x_5)p(x_7|x_5)p(x_8|x_4, x_5)p(x_9|x_6, x_8)p(x_{10}|x_7, x_8)$$

Exercise 3.3: Markov Chains

a) The Model 1 and 4 are not Markov Chains. Number 2 is a second order Markov Chain. Number 3 is a first order Markov Chain.