

Exploring infinities in Vienna and Kobe

Tatsuya Goto & Takehiko Gappo

A version with hyperlinks is available from the QR code on the right!



Set Theory: Mathematical Study of Infinity

Set theory is the branch of mathematical logic. One of the simplest, but deepest questions in set theory is:

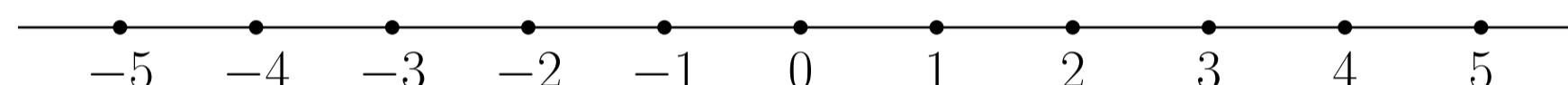
How many points are on the real line?

Even modern research of set theory often stems from this question.

Set theory started with Cantor's realization that there are different sizes of infinities.

Theorem (Cantor, 1873).

There are strictly more points on the real line than there are integers.



Cantor's conjecture, called the Continuum Hypothesis, is that there are no infinities whose size is strictly between the number of all integers and the number of all points on the real line.

Theorem (Gödel, 1938; Cohen, 1963).

The Continuum Hypothesis cannot be neither proved nor refuted (within the standard framework of mathematics, namely ZFC).

Vienna's Central Role in Set Theory and Logic

For set theorists, Vienna is

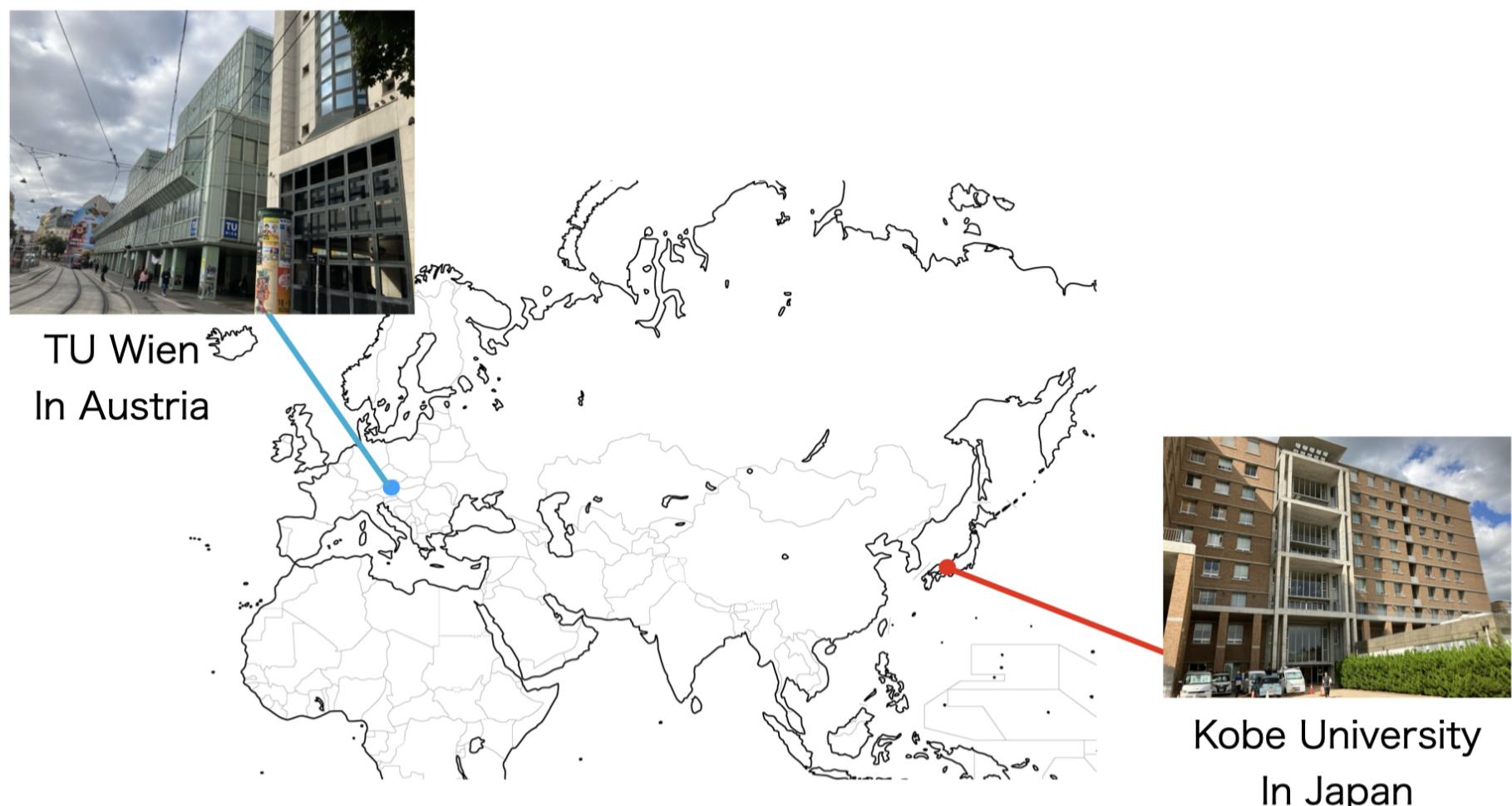
1. a city with **world-class research institutes** and leading experts,
2. a place offering **abundant funding opportunities** for fundamental scientific research,
3. a vibrant **hub of international scientific exchange**, with numerous seminars and conferences.

Kurt Gödel Research Center in UW and a **set theory research group** of TU Wien form the largest set theory research community in the world.



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In 2025, TU Wien hosted the **Logic Colloquium**, the annual European meeting of the Association for Symbolic Logic.



Interaction between TU Wien and Kobe in Set Theory

The collaboration between TU Wien and Kobe University reflects

1. a **long history of cooperation** led by senior researchers,
2. the **continuous exchange** of PhD students and junior postdocs,
3. a **shared focus** on closely related research themes.

Set theorists at both institutes have spent decades studying various kinds of infinity that appear in the real line. Their joint efforts to understand how these infinities are related have led to a major milestone known as "**Cichoń's maximum**," proved by Goldstern, Kellner, Mejía, and Shelah. They showed that all infinities in Cichoń's diagram can be different.

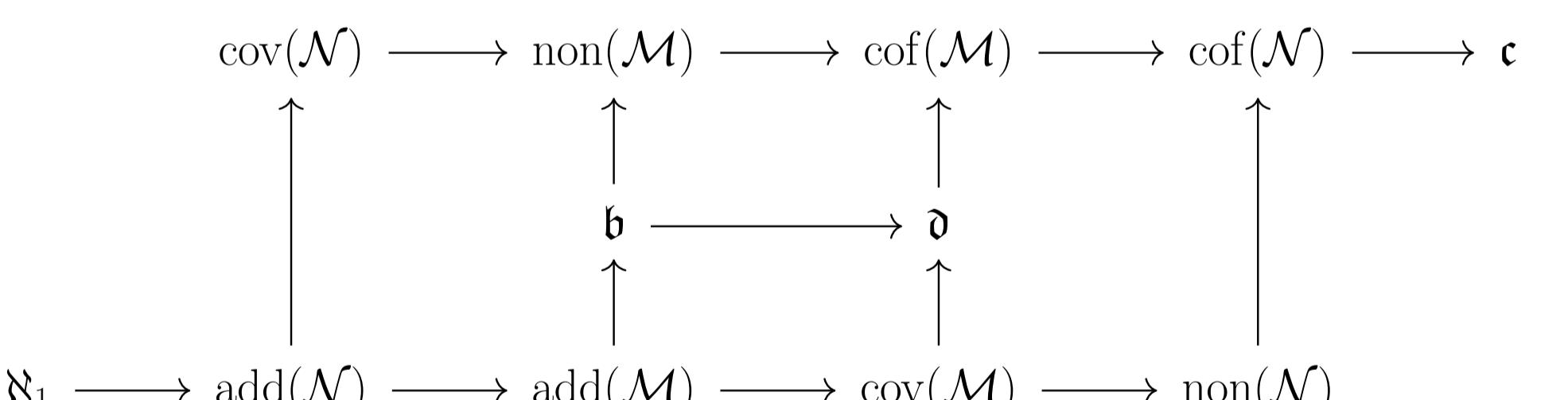


Figure 1. Cichoń's diagram. $A \rightarrow B$ means that $A \leq B$ is provable.



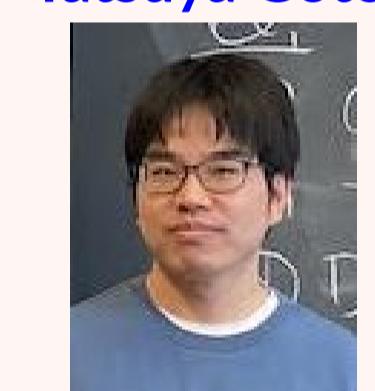
This September a research conference was held at Kobe University to celebrate Professor Brendle's 60th birthday.

Ongoing Collaborations by Young Researchers

- "Cardinal invariants of idealized Miller null sets," by Cieślak, [Gappo](#), Martínez-Celis, [Yamazoe](#).
 - "Left preservation" by [Goto](#) & [Mejía](#).
 - "The Meagre ideal for singular higher Baire spaces," by [Hayashi](#) & [van der Vlugt](#).
 - "Cardinal Invariants Associated with Probability Trees," by [Mejía](#) and [Uribe-Zapata](#).
 - "Covering of the null ideal may be singular in Cichoń's maximum," by [Mejía](#) and [Uribe-Zapata](#).
 - "Higher evasion and prediction," by [van der Vlugt](#) & [Yamazoe](#).
- and more...

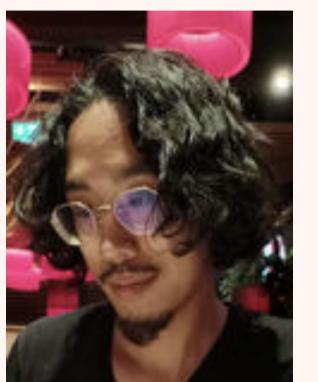
Authors' Research Profiles

Tatsuya Goto (TU Wien, JSPS postdoc)



He was born in Hyogo. He received his PhD at Kobe University in March 2025. He is currently a postdoctoral researcher at TU Wien under the JSPS Overseas Research Fellowship. He studies set-theoretic nature of infinite-dimensional spaces.

Takehiko Gappo (TU Wien, FWF postdoc)



He was born in Hokkaido. He received his PhD at Rutgers University in October 2022 and since then, he has been a postdoctoral researcher at TU Wien. Currently, he is a principal investigator of the FWF ESPRIT project, studying games of infinite length.

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