

# Important Traffic Flow Characteristics

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<https://www.GoTrafficGo.com>

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# Outline

- Arrival Patterns
- Fundamental Diagrams
- Hysteresis
- Capacity Drop
- Phantom Traffic Jam
- Traffic Oscillation

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- Fundamental Diagrams
- Hysteresis
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# Arrival patterns

- Uniform arrivals
- Random arrivals - inhomogeneous Poisson distribution:  
(1) isolated intersections (2) free-flow traffic on freeway
- Platooned arrivals: more realistic urban intersections



Uniform Arrivals

(a)



Random Arrivals

(b)



Reality = Platooned Arrivals—No Theoretical Solution Available

(c)

## Arrival patterns: Poisson distribution

**Poisson distribution:** A *discrete probability distribution* that expresses **the probability of [a given number  $k$  of events that occur in a fixed time interval]** if these events occur **independently of the time since the last event.**

- $$\bullet \text{Pois}(\lambda) = P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$k = 0, 1, 2, \dots$  - the number of occurrences

$e$  - Euler number

- $\bullet \lambda = \mathbf{E}[\text{Pois}(\lambda)] = \mathbf{V}[\text{Pois}(\lambda)]$

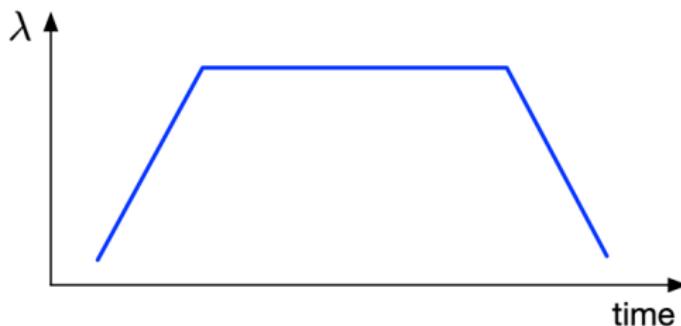
Expected Value	Variance
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# Arrival patterns: Poisson distribution

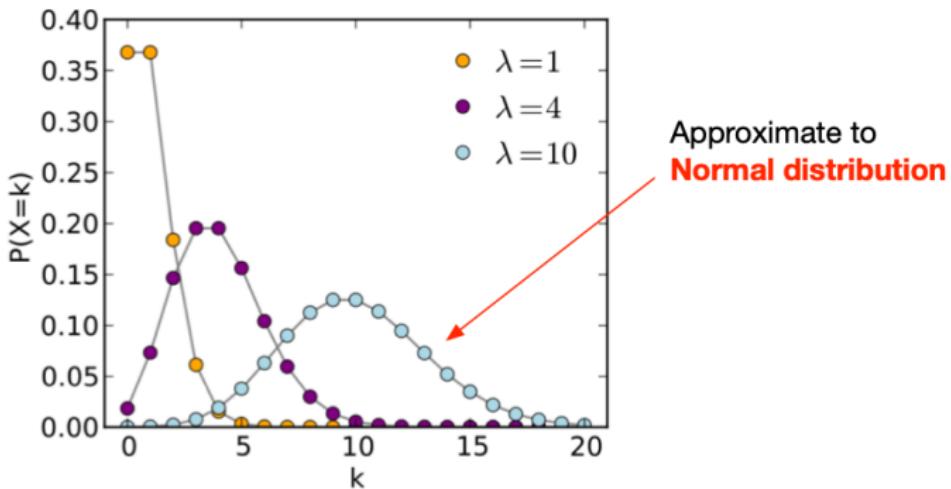
**Inhomogenous Poisson distribution:**

$$\lambda = \lambda(t)$$

Rush-hour demand:



# Arrival patterns: Poisson distribution



The horizontal axis is the index  $k$ , the number of occurrences.  $\lambda$  is the expected rate of occurrences. The vertical axis is the probability of  $k$  occurrences given  $\lambda$ . The function is defined only at integer values of  $k$ ; the connecting lines are only guides for the eye.

# Outline

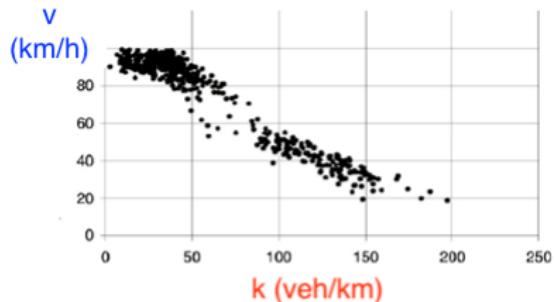
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# Observations

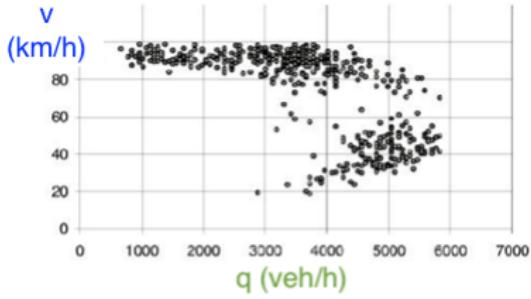
In terms of a **cross-section** of a road, the traffic is always in a specific **state** that is characterized by

- flow (rate),  $q$ , veh/h,
- density,  $k$ , veh/km,
- (space-mean) speed,  $v$ , km/h.

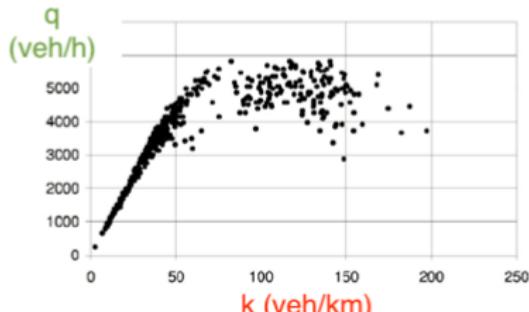
# Observations



Observation spots in a v-k diagram



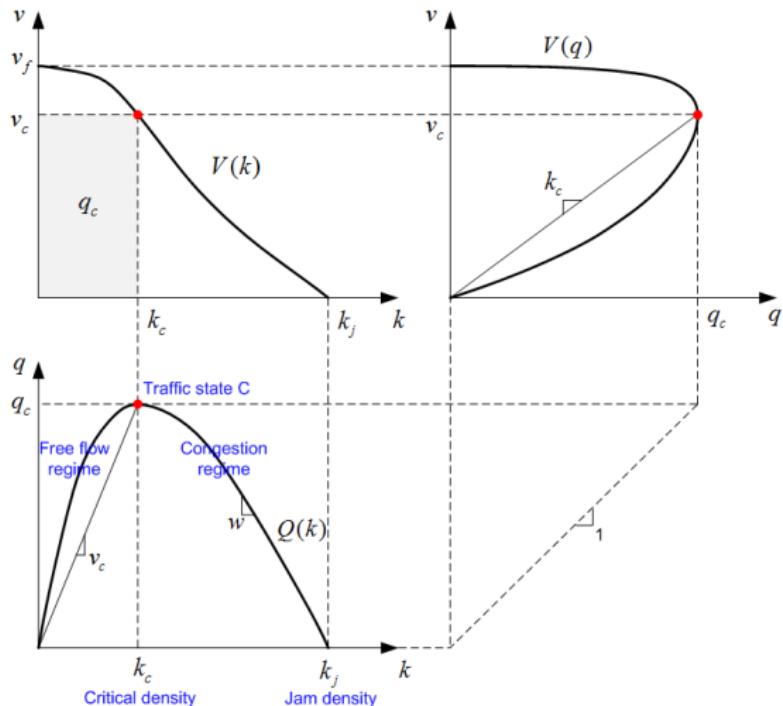
Observation spots in a v-q diagram



Observation spots in a q-k diagram

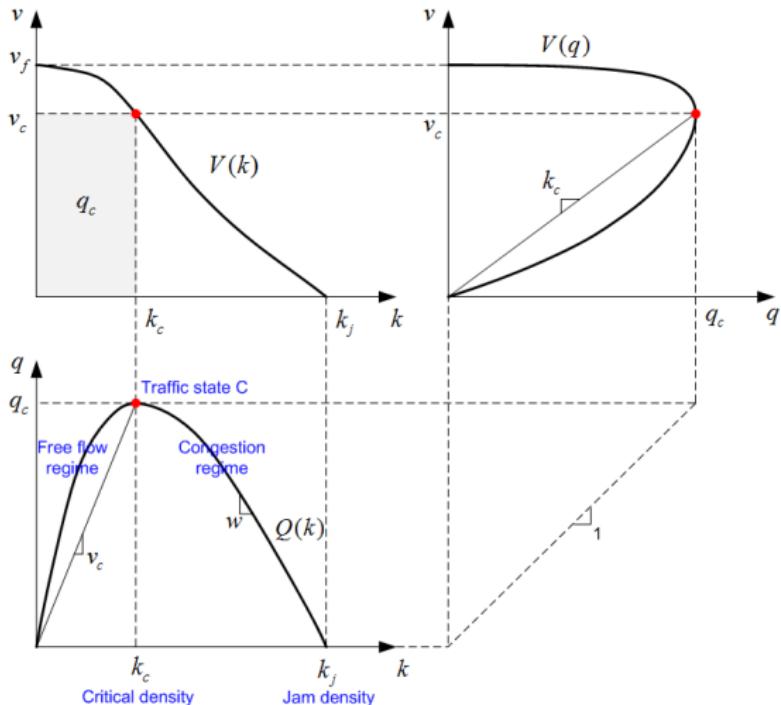
# Fundamental diagrams (FD)

Abstracting from the inhomogeneous and non-stationary traffic, we describe the empirical characteristics using an **equilibrium relationship**, which is called the **the fundamental relationship**, and describe graphically by three diagrams, i.e., **the fundamental diagram**



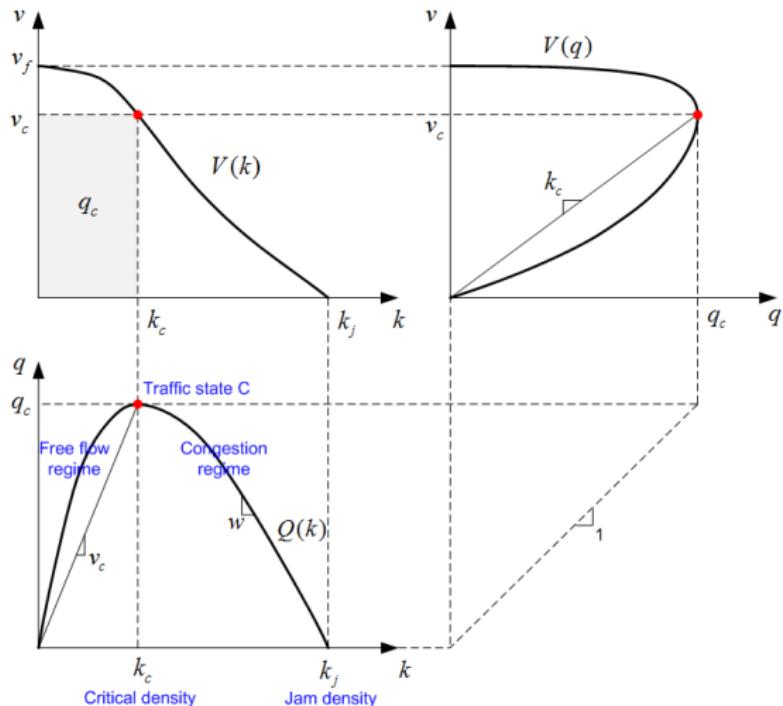
# Fundamental diagrams (FD)

- **Traffic states:**  $q, k, v$
- **Critical states:**  $q_c, k_c, v_c$
- **Free-flow branch/condition**
- **Congestion branch/condition**
- **Wave speed:**  $\omega$



# Fundamental diagrams (FD)

- **Capacity:**  $q_c = 2500$  (veh/h)
- **Critical density:**  $k_c \approx k_j/5$
- **Free flow speed:**  $v_f \approx 1.1 v_{\text{lim}}$
- **Jam density:**  $k_j \approx 150$  (veh/km)
- **Wave speed:**  $\omega \approx 15$  (km/h)



# The fundamental diagrams (FD)

Cassidy (1998) showed that carefully chosen regions in time-space where traffic conditions are stationary lead to well-defined and scatter-free empirical fundamental diagrams\*.

\* Cassidy M. Bivariate relations in nearly stationary highway traffic. *Transportation Research Part B*. 1998.



Pergamon

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## BIVARIATE RELATIONS IN NEARLY STATIONARY HIGHWAY TRAFFIC

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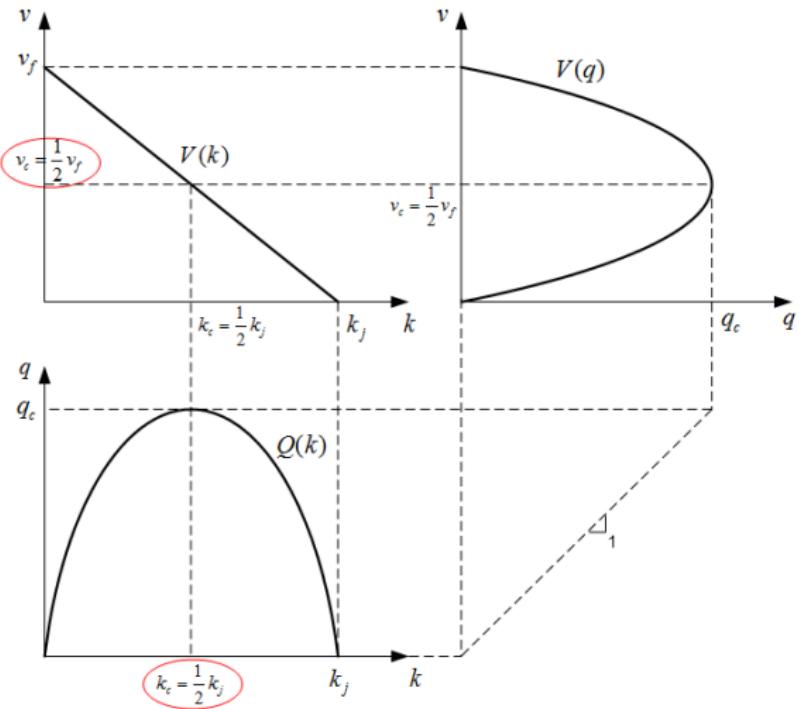
(Received 18 November 1996; in revised form 20 April 1997)

**Abstract**—This paper demonstrates that reproducible bivariate relations exist among traffic variables, such as flow and occupancy, when traffic conditions are approximately stationary. The inspection of cumulative curves of vehicle arrival number and vehicle occupancy has revealed that sustained periods of nearly stationary conditions do arise in the traffic stream. By plotting average values of the data corresponding to each nearly stationary condition, well-defined relations are observed. These scatterplots of near-stationary data are contrasted with plots of data that were measured over consecutive time intervals of fixed duration and this reveals that data from certain conditions do not necessarily fall on a curve describing near-stationary traffic.

Note: **stationary** and **homogeneous** traffic is always in a state located on the **bold black** lines.

# Greenshield fundamental diagram

- $v = V(k) = \frac{v_f}{k_j}(k_j - k)$
- $q = V^{-1}(q) = k_j v(1 - \frac{v}{v_f})$
- $q = Q(k) = \frac{v_f}{k_j} k(k_j - k)$
- **Pros:** One equation

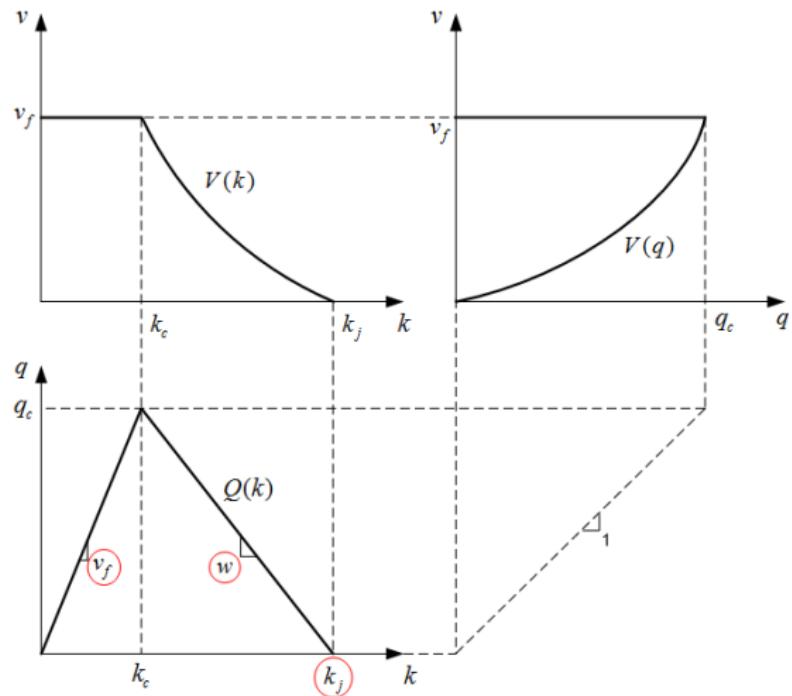


# Triangular fundamental diagrams (TFD)

- Determining the fundamental relationship only need **three parameters**:  $v_f$ ,  $\omega$ ,  $k_j$ , which is very convenient in traffic modeling.

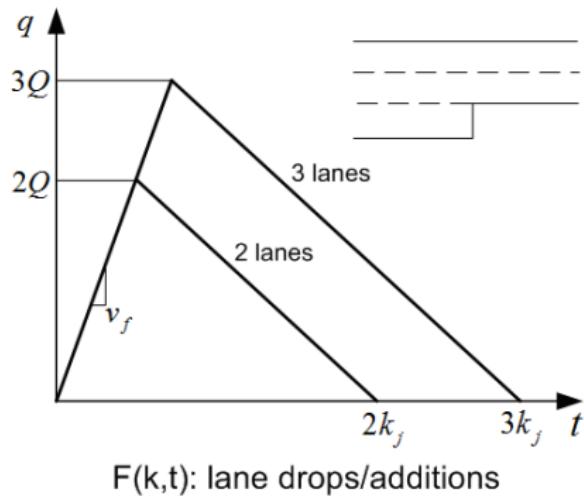
- $$q = \min\{v_f k, -\omega(k - k_j)\}$$

- Question:  
Critical states:  $k_c$ ,  $q_c$ ?



# Practice: Lane drop

When an accident happens, a lane of a three-lane road is blocked.  
Try to draw and compare the (T)FDs of the **two-lane** and **three-lane** roadway.

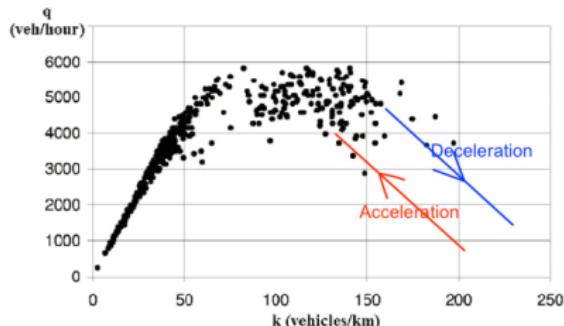


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- Fundamental Diagrams
- **Hysteresis**
- Capacity Drop
- Phantom Traffic Jam
- Traffic Oscillation

# Hysteresis

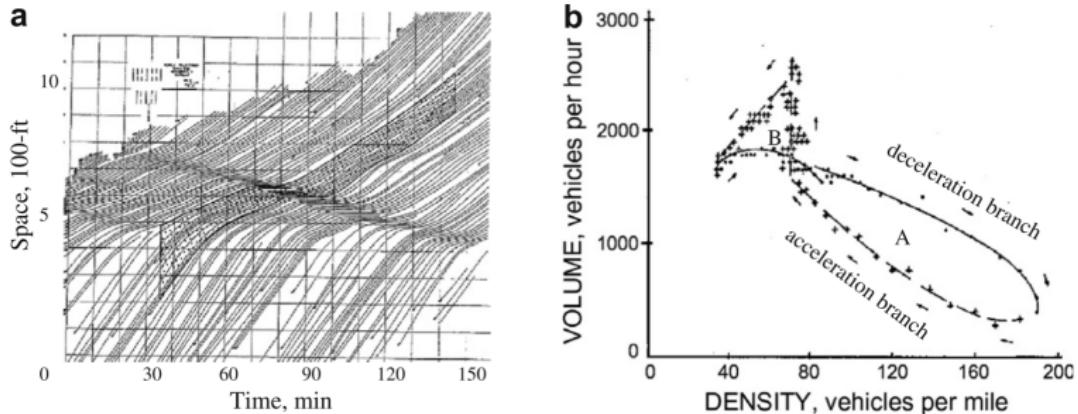
- Dictionary: the **lag** in response exhibited by a body in reacting to changes in the forces.
- The first theory for explaining hysteresis is due to Newell (1962). He conjectured the existence of **two different congested branches** in the fundamental diagram. He argued that in the flow-density diagram, the deceleration branch should be **above** the acceleration branch.



Observation spots in a  $q$ - $k$  diagram

# Hysteresis

- Zhang (1999) postulated that these branches may **intersect and reverse** the relative position of acceleration and deceleration branches, possibly describing **multiple loops**.

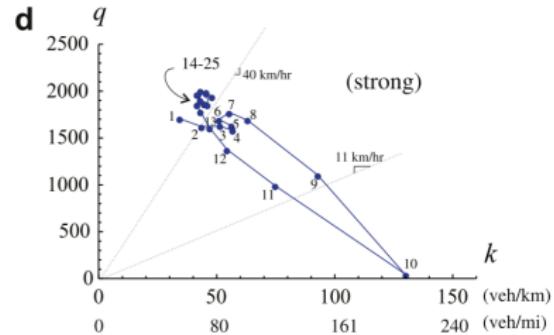
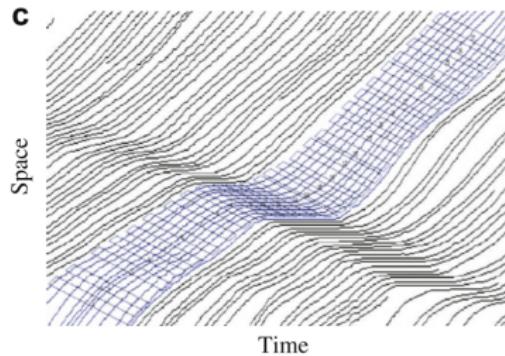


Hysteresis shown in Treiterer and Myers (1974)

# Hysteresis

- Hysteresis shown in Laval (2010) plotted by using Edie's definition.

Imagine: quick deceleration and slow acceleration



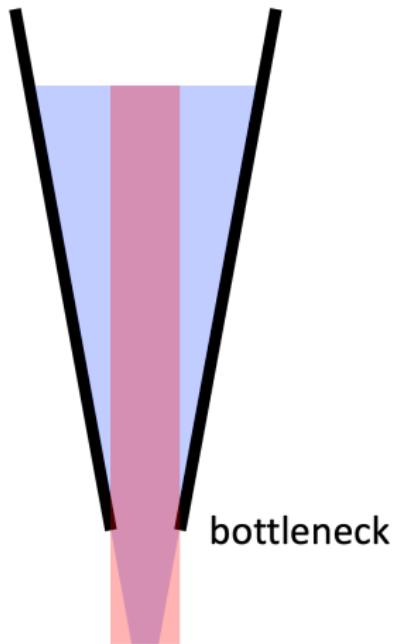
- Note the difference caused by different measurements.

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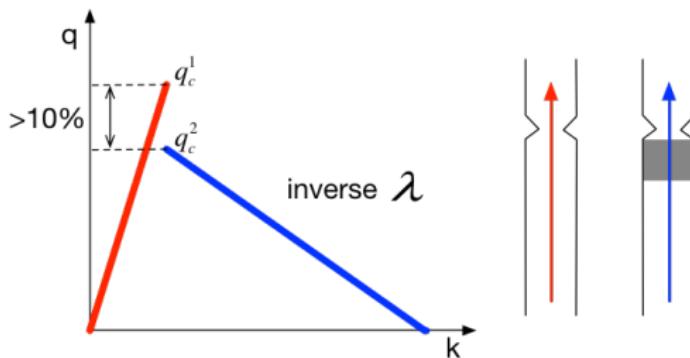
# Capacity drop

See Video: *Rice Experiment*



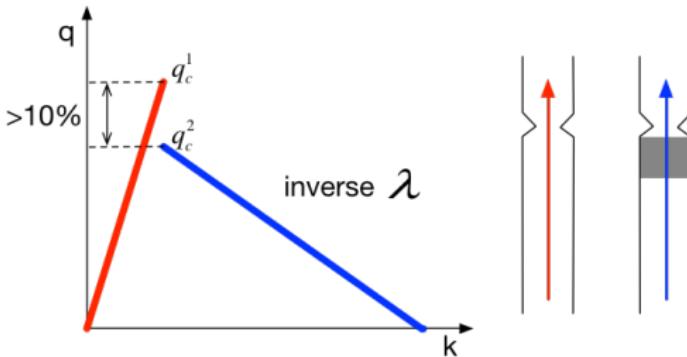
# Capacity drop

- Observation:
  - No congestion: **higher** capacity;
  - After being congested: **lower** capacity;
- Once being congested, the traffic **cannot completely** restore the capacity.



# Capacity drop

- Capacity drop is the **major hazard** of an active bottleneck
- For example, the maximum flow (capacity) of the lane is 1500 veh/h, but only 1000 veh/h can pass through **when the congestion forms**. The efficiency of traffic is thus **reduced drastically**.



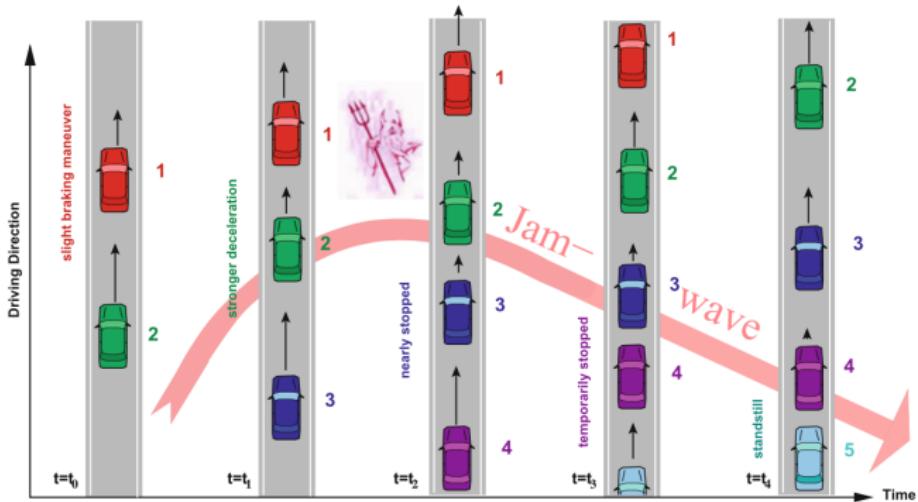
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# Phantom traffic jam

- Traffic is congested **without clear causes**, i.e., we cannot see clear bottlenecks, such as ramps, accidents.
- **Small disturbances** (a driver hitting the brake too hard, or getting too close to another car) in heavy traffic can become amplified into a self-sustaining traffic jam.
- In mathematics, it is related to **traffic instability**.
- Resulting in **stop-and-go waves** (introduced in the following section).
- See video.

# Phantom traffic jam



## Phantom traffic jam

The phantom forms as follows (Note: one equilibrium state corresponds to a speed and a gap):

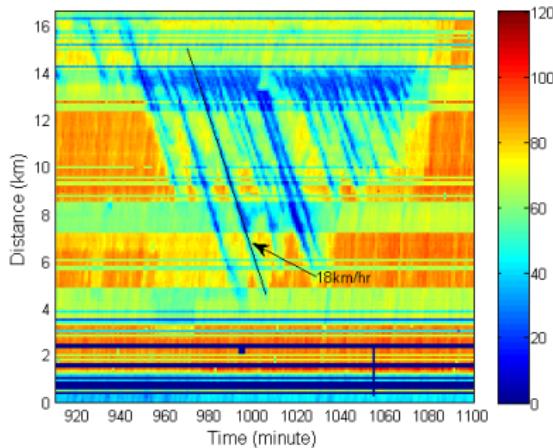
- All vehicles move with a equilibrium speed  $v_0$  and gap  $g(v_0)$ ;
- Vehicle 1 suddenly slightly decelerates from  $v_0$  to  $v_1$ ;
- When vehicle 2 **realizes it (reaction time)** and decelerates to  $v_1$  from  $v_0$ , the gap has been smaller than  $g(v_1)$
- To adapt the smaller gap, vehicle 2 has to decelerate to  $v_2 < v_1$ , and the gap becomes  $g(v_2) < g(v_1)$
- Vehicle 3 has to decelerate to  $v_3 < v_2$
- ...., until one vehicle has to stop

# Outline

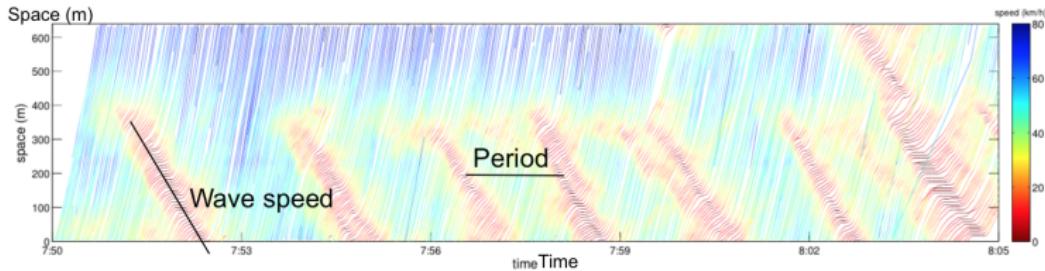
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# Oscillation or stop-and-go wave

- From the perspective of a driver, it is:  
slowly move → decelerate → stop → accelerate →  
slowly move → ... (see video)
- From the macroscopic perspective, it is:



# Oscillation or stop-and-go wave

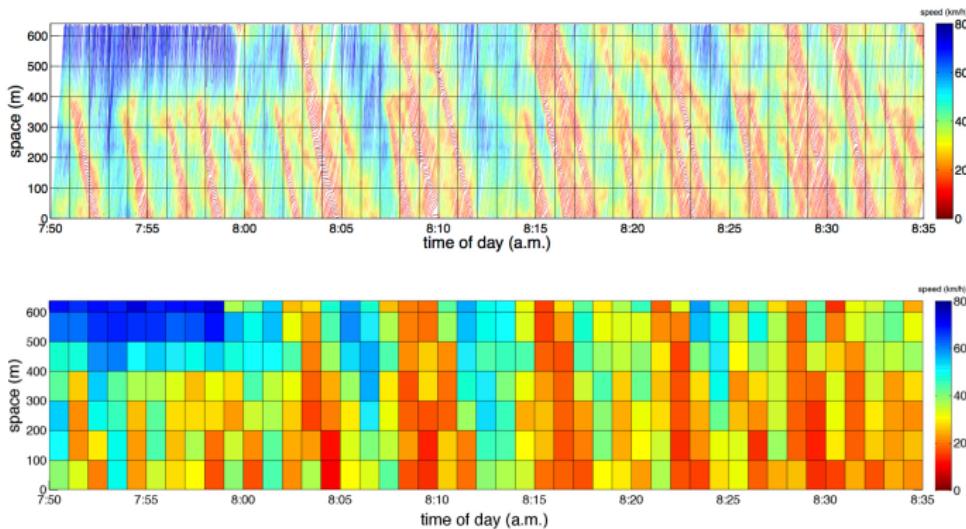


- **Main properties:**
  - Wave speed:  $-10 \text{ km/h} \sim -20 \text{ km/h}$
  - Period:  $2 \text{ min} \sim 15 \text{ min}$
  - Amplitude (max speed-min speed):  $20 \sim 60 \text{ km/h}$
- **Side effects:** more fuels and emissions, make drivers tired and thus more danger.

# Oscillation or stop-and-go wave

## Time-space diagram

in which x-axis is time, y-axis is space, and the color inside (or z-axis) represents speed, is a foundational tool of various transportation research and applications, such as identifying bottlenecks and understanding traffic characteristics.



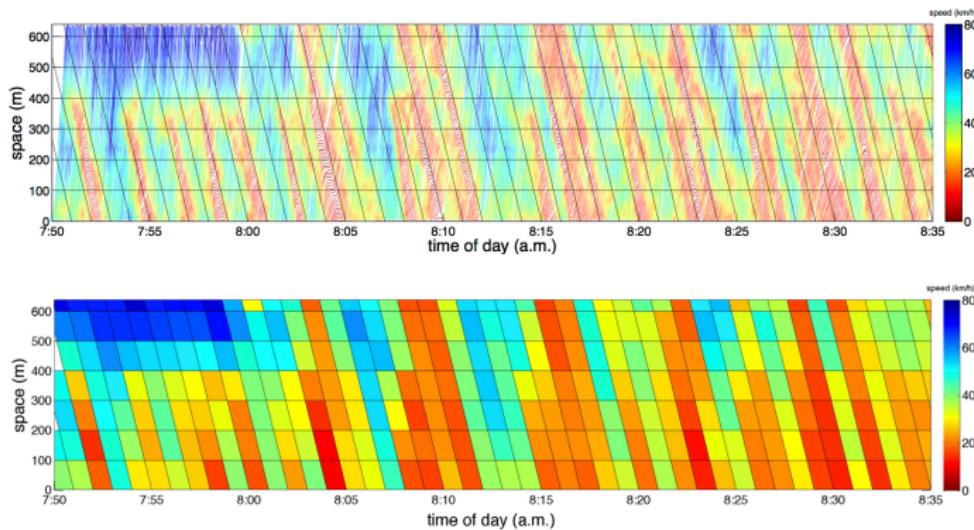
# Oscillation or stop-and-go wave

## Time-space diagram

Zhengbing He, Ying Lv, Lili Lu, Wei Guan,

*Constructing spatiotemporal speed contour diagrams: using rectangular or non-rectangular parallelogram cells,*

Transportmetrica B, 7(1):44-60, 2019

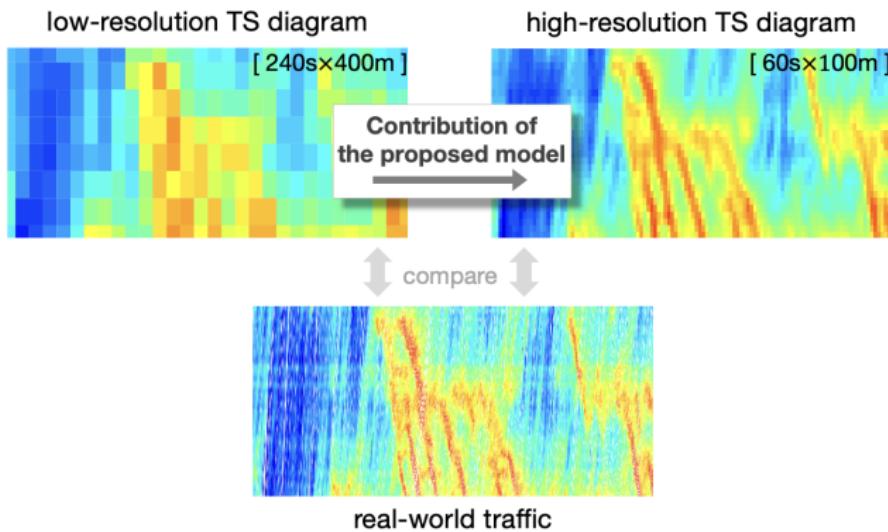


# Oscillation or stop-and-go wave

## Time-space diagram

Zhengbing He,

*Refining time-space traffic diagrams: A simple multiple linear regression model, IEEE Transactions on Intelligent Transportation Systems*

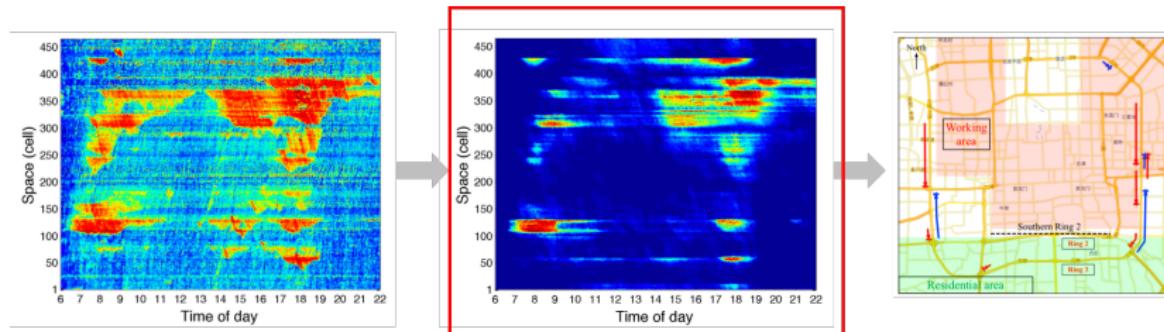


# Oscillation or stop-and-go wave

## Stochastic congestion map

Represent the likelihood of the occurrence of congestion based on the observations for many days.

Count the number of the congested cells (like, speed < 20 km/h) in the t-x diagram during  $n$  workdays, and obtain the stochastic congestion maps through dividing the counters by the total number  $n$  of workdays.



# Thank you!