Cumulative Count Curve and Queueing Analysis

CIVE.5490, UMass Lowell

Zhengbing He, Ph.D.
https://www.GoTrafficGo.com

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Outline

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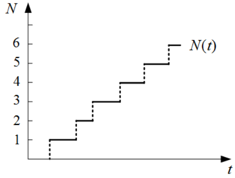
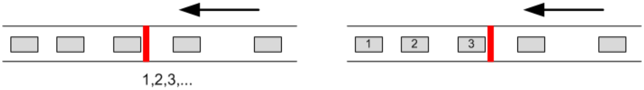
Definition

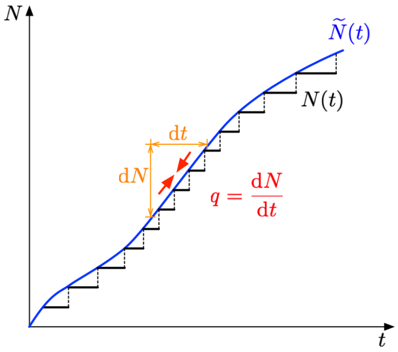
- Cumulative count curve is also called **N-curve**



- Two equivalent ways to construct N-curve at a fixed location:
  - Counting the number of arrival vehicles at a fixed location
  - Numbering the passing vehicles



Smooth approximation

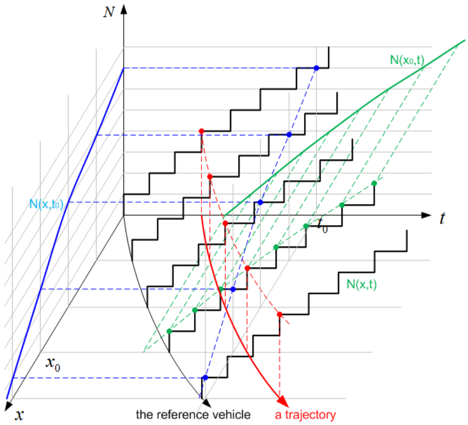


Main advantage: differential calculus can be used

$$q(t) = \frac{\mathrm{d}\tilde{N}(t)}{\mathrm{d}t}$$

Three-dimensional N-curve: $N(x, t)$

Flow at x: $q(x) = \frac{\partial N(x,t)}{\partial t}$ **density** at t: $k(t) = \frac{\partial N(x,t)}{\partial x}$



Background

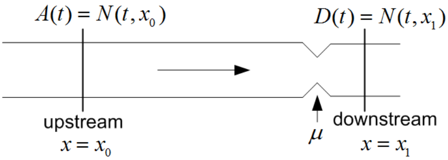
- Cumulative count curve is started from a basic technique known as 'mass curve analysis' in hydrologic synthesis
- Introduced to transportation by Moskowitz (1954)[3] and Gazis and Potts (1965)[4];
  Gordon Newell (1971,1982,1993)[2][5][6] demonstrated their full potential.

- **Suitable cases**
  - The time-space diagram: when more than one vehicle trajectory must be depicted;
  - The time-count diagram: when it displays the cumulative curves upstream and downstream of a series of **bottlenecks**, and in particular the 'arrival' and 'departure' curves at a single bottleneck.

Outline

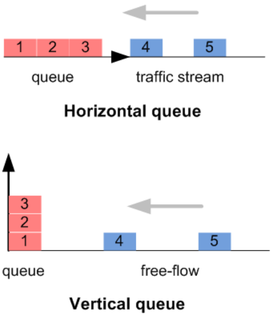
Scenario description

- A bottleneck with capacity μ;
- Two detectors located at the upstream x_0 and downstream x_1
  - A(t): Cumulative arrival count;
  - D(t): Cumulative departure count;
- Conservation: no on- or off-ramp in between;
- Vertical queue assumption.



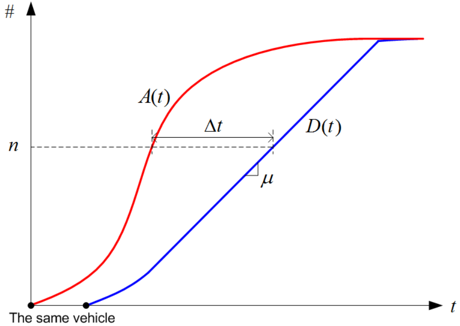
Vertical queue

- **Vertical queue**[7] presumes that vehicles stack up upon one another at the congestion point instead of backing up over the length of the road (horizontal queue);
  This simplification is widely accepted by traffic flow theorists;
- It allows vehicles *in an analysis to drive at the free flow speed until reaching the point of congestion*;
- Following FIFO rule (no overtaking).



Cumulative arrival and departure count curve

- Starting counting with a reference vehicle;
- Δt: travel time from x_0 to x_1;
- **Question**: when $D(t)$ turns to a straight line, and when it turns back to a curve?

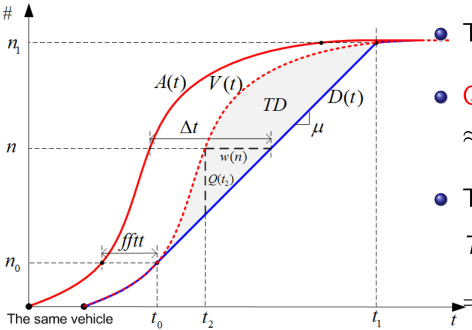


Analysis

To better analyze the queueing system, we shift A(t) right by free flow travel time ($fftt = (x_1 - x_0)/v_f$), and obtain

$$\text{Virtual Arrival Count Curve: } V(t) = A(t - fftt).$$

The information we can get is:



- The delay of vehicle n: $w(n)$;
- Queue at t_2
  $\approx$ excess veh accum.: $Q(t_2)$;
- The total delay:
  $TD = \int_{t_0}^{t_1} [V(t) - D(t)]dt$
  $= \int_{t_0}^{t_1} Q(t)dt;$

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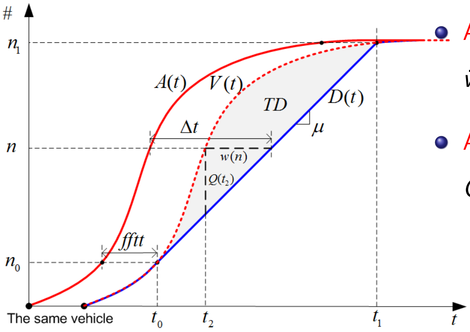


- Average delay per vehicle:

  $\bar{w} = \frac{TD}{n_1 - n_0}$;

- Average queue:

  $\bar{Q} = \frac{TD}{t_1 - t_0}$;

Little law

Since $\bar{w} = \frac{TD}{n_1 - n_0}$ and $\bar{Q} = \frac{TD}{t_1 - t_0}$, the so-called LITTLE LAW is

$$\bar{Q} = \lambda \bar{w}$$

where $\lambda = \frac{n_1 - n_0}{t_1 - t_0}$ is the average arrival flow to the system (in this bottleneck case, $\lambda = \mu$); i.e.,

average customers in storage ($\bar{Q}$)
$=$ average arrival rate (λ) $\times$ average time spent in the system ($\bar{w}$)

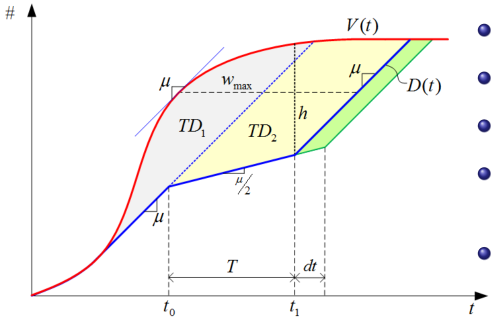
Example- *size of cafeterias in meal time*
'Easy' to estimate λ and $\bar{w}$: $\lambda = 10$ (customer/min) and $\bar{w} = 30$ (min/meal). So $\bar{Q} = 300$ chairs

Examples: incident analysis

Problem description: During a rush hour, a roadway is congested by the increasing demand. After the rush hour, the congestion dissipates due to decease of the demand.

Suppose that at time $\boxed{t_0}$ during the rush hour, an incident happens, and lasts for period $\boxed{T}$. During the period, the capacity drops from $\boxed{\mu}$ to $\boxed{\frac{\mu}{2}}$. Try to draw relevant N-curves and analyze traffic delays by using the N-curves.

Examples: incident analysis



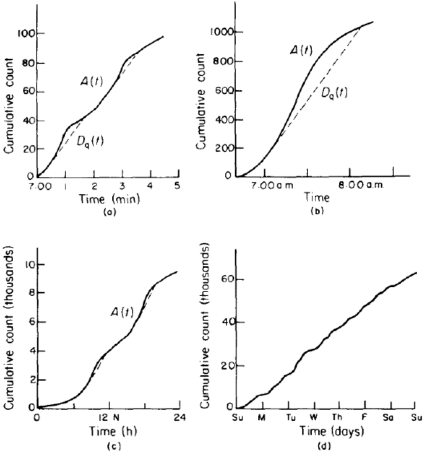
- TD w/o incident: TD_1;
- TD due to incident: TD_2;
- Max waiting time: w_{max};
- Vehicle number in queue at the clearance time: h;
- **Marginal delay** due to incident duration: T:

$$\frac{\partial TD(T)}{\partial T} = \lim_{dt \to 0} \frac{TD(T+dt) - TD(T)}{dt} = \frac{Area\ of\ \ Green}{dt} \simeq h$$

Outline

Scales of counts and time

- **Seconds or minutes**: queue (a);
- **Hours**: peak demand or rush hours (b,c);
- **Days**: different patterns on different days (d);

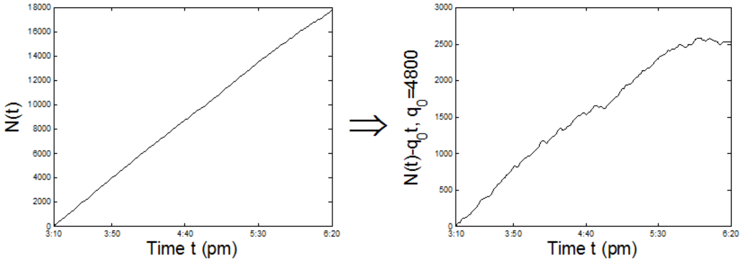


Oblique N-curve

In freeways accumulation of arrival counts is very fast since the flow is large. The detail is hard to be observed for the regular N-curve. Oblique coordinate system is used to show arrival changes, which keeps the relative quantity (See, for example, [8][9]):

$$(N\ ,\ \ t) \Longrightarrow (N - q_0 t\ ,\ \ t)$$

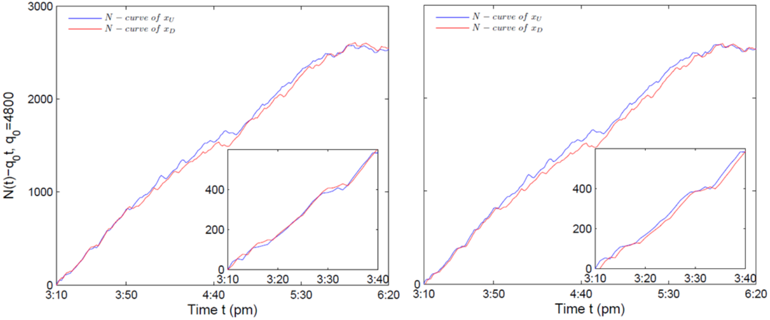
where q_0 is a coefficient.



N-curves in practice

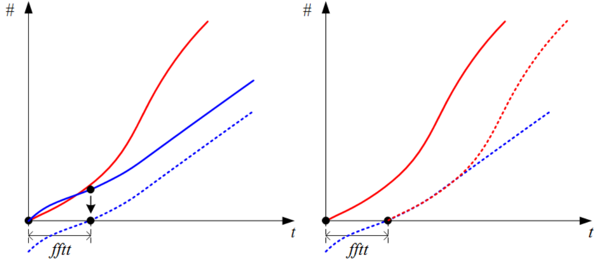
A four-lane section of the North-bound M42 freeway near Birmingham int'l airport, England. The data were aggregated every one minute from 3:10 pm to 6:20 pm in Nov. 28th, 2008. The distance of two detectors were 1.279 km and in between no on- or off-ramp

Left: raw data; *right*: shifted for the same reference vehicle[10]



Modification for the same reference vehicle

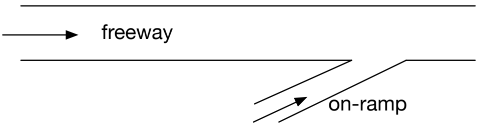
- Raw data of two detectors: the same start points in time
- Our goal: N-curves starting from the same vehicle
- Therefore, we take the free flow part as a reference and shift the downstream N-curve as follow:



Shifting down until *fftt*   Testing: overlapping in free flow parts

Homework

- Suppose a freeway with a merge
- Make rational assumptions for the network topology and the demands in a rush hour
- Describe the interaction between freeway and on-ramp using "Newell-Daganzo Merge Model"
- Install your detectors at proper locations
- Simulate the traffic in the rush hour by coding in Matlab based on N-curves and the vertical queue assumption
- Draw N-curve figures, and analyze the process of congestion formation and dissipation



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[1] Daganzo CF. Fundamentals of Transportation and Traffic Operations. 1997:133-135,259.

[2] Newell GF. Applications of queueing theory (2nd edition). Chapman and Hall London, 1982.

[3] Moskowitz, K. Waiting for a gap in a traffic stream. Proc. Highway Res. Board, 33, 385-395, 1954.

[4] Gazis, D.C. and R.B. Potts (1965). The over-saturated intersection. Proc. 2nd Int. Symp. on the Theory of Road Traffic Flow, (J. Almond, editor), pp. 221-237, OECD, Paris, France.

[5] Newell, G.F. Applications of queueing theory, Chapman Hall, London, 1971.

[6] Newell, G.F. A simplified theory of kinematic waves in highway traffic, I general theory, II queuing at freeway bottlenecks, III multi-destination flows. Transportation Research Part B, 281-313, 1993.

[7] Wikipedia, `http://en.wikipedia.org/wiki/Vertical_Queue`, accessible on March 6th, 2011.

[8] Cassidy M. Bivariate relations in nearly stationary highway traffic. Transportation Research Part B. 1998.

[9] Cassidy M. Some traffic features at freeway bottlenecks. Transportation Research Part B. 1999;33:25-42.

[10] He Z, Laval J, Ma S. Traffic state interpolation using a stochastic extension of Newell's "three-detector method" (under review)

Thank you!