CIVL 2120
Mechanics of Materials

Formula Sheet

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1 Stress

1.1 Normal Stress

$$\sigma = \frac{P}{A} \tag{1}$$

Positive sign indicates tensile stress, negative sign indicates compressive stress.

$$P = \int_A \sigma \, dA \tag{2}$$

1.2 Shearing Stress

$$\tau = \frac{P}{A} \tag{3}$$

1.3 Bearing Stress

$$\sigma_b = \frac{P}{A} = \frac{P}{td} \tag{4}$$

where t is the thickness of the plate and d is the diameter of the bolt. We use the **projected area** of the bolt on the plate, instead of the curved surface directly.

1.4 Stresses on Oblique Plane

Normal Stress

$$\sigma_\theta = \frac{F}{A_\theta} = \frac{P}{A_0} \cos^2\theta \tag{5}$$

Shearing Stress

$$\tau_\theta = \frac{V}{A_\theta} = \frac{P}{A_0} \sin\theta\cos\theta \tag{6}$$

where $F = P\cos\theta$ is the force **normal** to the plane, $V = P\sin\theta$ is the force **tangential** to the plane, and θ is the angle between the force P and the normal of the plane.

1.5 Factor of Safety

$$FS = \frac{P_{\text{ultimate}}}{P_{\text{allow}}} = \frac{\sigma_{\text{ultimate}}}{\sigma_{\text{allow}}} \tag{7}$$

2 Axial Loading

2.1 Normal Strain

$$\epsilon = \frac{\delta}{L} \tag{8}$$

2.2 Hooke's Law

$$\sigma = E\epsilon \tag{9}$$

where E is the Young's modulus.

2.3 Deformation

$$\delta = \frac{PL}{EA} \tag{10}$$

For multiple rods in series,

$$\delta = \sum_i \frac{P_i L_i}{E_i A_i} \tag{11}$$

For multiple rods in parallel,

$$\delta = \frac{PL}{\sum_i E_i A_i} \tag{12}$$

2.4 Poisson's Ratio

$$\nu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\epsilon_y}{\epsilon_x} = -\frac{\epsilon_z}{\epsilon_x} \tag{13}$$

$$\begin{cases} \epsilon_x & = \frac{\sigma_x}{E} \\ \epsilon y & = \epsilon_z = -\frac{\nu\sigma_x}{E} \end{cases} \tag{14}$$

Note that $0 < \nu < \frac{1}{2}$.

2.5 Generalised Hooke's Law

$$\begin{cases} \epsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E} \\ \epsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E} \\ \epsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E} \end{cases} \tag{15}$$

Or in a more compact matrix form

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{bmatrix} \tag{16}$$

2.6 Dilatation

$$\begin{aligned} e &= \epsilon_x + \epsilon_y + \epsilon_z \\ &= \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma + z) \end{aligned} \tag{17}$$

Under uniform pressure

$$e = -\frac{2(1-2\nu)}{E}p \tag{18}$$

2.7 Bulk Modulus

$$k = \frac{E}{3(1-2\nu)} \tag{19}$$

$$e = -\frac{p}{k} \tag{20}$$

3 Torsion

3.1 Stress and Torque

$$\int \rho \, dF = \int \rho\tau \, dA = T \tag{21}$$

3.2 Shearing Strain

$$\gamma = \frac{\rho\phi}{L} \tag{22}$$

where ϕ is the angle of twist.

$$\gamma_{\text{max}} = \frac{c\phi}{L} \tag{23}$$

where c is the outer radius.

3.3 Hooke's Law for Twisting

$$\tau = G\gamma \tag{24}$$

c.f. Equation 9

$$\sigma = E\epsilon$$

3.4 Shearing Stress

$$\tau = \frac{T\rho}{J} \tag{25}$$

where J is the polar moment of inertia.

$$\tau_{\text{max}} = \frac{Tc}{J} \tag{26}$$

3.5 Polar Moment of Inertia

$$J = \int_A r^2 \, dA \tag{27}$$

For hollow cylinders,

$$J = \frac{1}{2}\pi \left(c_2^2 - c_1^2\right) \tag{28}$$

3.6 Angle of Twist

$$\phi = \frac{TL}{GJ} \tag{29}$$

where G is the modulus of rigidity.

c.f. Equation 10

$$\delta = \frac{PL}{EA}$$

For multiple components in series,

$$\phi = \sum_i \frac{T_i L_i}{G_i J_i} \tag{30}$$

4 Pure Bending

4.1 Strain Due to Bending

$$\epsilon_x = -\frac{y}{\rho} \tag{31}$$

where y is the signed distance measured from the neutral axis.

4.2 Stress Due to Bending

$$\sigma_x = -\frac{My}{I} \tag{32}$$

$$\sigma_{x_{\max}} = -\frac{Mc}{I} \tag{33}$$

4.3 Section Modulus

$$S = \frac{I}{c} \tag{34}$$

4.4 Radius of Curvature

$$\rho = \frac{EI}{M} \tag{35}$$

The reciprocal $\frac{1}{\rho}$ is the curvature of the neutral surface.

4.5 Eccentric Loading

$$\sigma_x = \frac{P}{A} - \frac{My}{I} \tag{36}$$

For couples acting not on the plane of symmetry,

$$\sigma_x = \frac{P}{A} + \frac{M_z y}{I_z} - \frac{M_y z}{I_y} \tag{37}$$

The negative sign in the M_z term is due to the compression above the xz plane (in the positive y axis) and tension below (in the negative y axis). The M_y term is positive because of the tension on the left of the xy plane (in the positive z axis) and compression on the right (in the negative z axis).

4.6 Neutral Axis

$$\tan\phi = \frac{I_z}{I_y}\tan\theta \tag{38}$$