

Joint Blind Source Separation and Declipping: A Geometric Approach for Time Disjoint Sources

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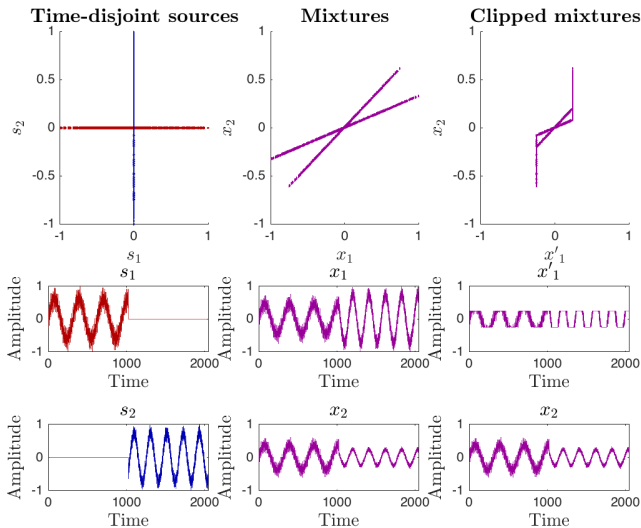
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Overview

Our problem is modelled by:

- 1 The **Independent Component Analysis** (ICA) model for source separation.
- 2 The **hard clipping** model.

Overview



Independent Component Analysis (ICA) model

- Latent variable model:

$$\mathbf{x} = \mathbf{A}\mathbf{s} \quad (1)$$

- \mathbf{s} : source signals ($D \times N$).
- \mathbf{x} : mixtures (observations) ($F \times N$).
- \mathbf{A} : mixing matrix ($F \times D$).

Determines how much of each **source** is in each **mixture**.

Clipping distortion

- What if one of the observed mixtures is **clipped**?

$$x'_{1,n} = \begin{cases} \text{sgn}(x_{1,n})\theta & \text{if } |x_{1,n}| > \theta, \\ x_{1,n} & \text{if } |x_{1,n}| \leq \theta. \end{cases} \quad (2)$$

- Existing ICA methods do not work.

Assumptions

- Assumptions are needed to make problem solvable.
- Mixtures of **time-disjoint** sources yield **source lines** (Plumbley, 2010).
- Noiseless.
- Two mixtures.

Geometric method: 5 steps

- 1 Mixing matrix estimation.
- 2 Quantisation of *trivial time points*.
- 3 Declipping mixture(s) by ℓ_1 minimisation.
- 4 Quantisation of declipped points.
- 5 Source estimation.

Example case: two sources, one clipped mixture.

1. Mixing matrix estimation

- Ignore clipped time points.
- Each line passes through the origin, so each unclipped point gives a *source line* slope:

$$\nabla_{\mathbf{M}} = (x'_{2,1}, \dots, x'_{2,M}) \odot \left(\frac{1}{x'_{1,1}}, \dots, \frac{1}{x'_{1,M}} \right) \quad (3)$$

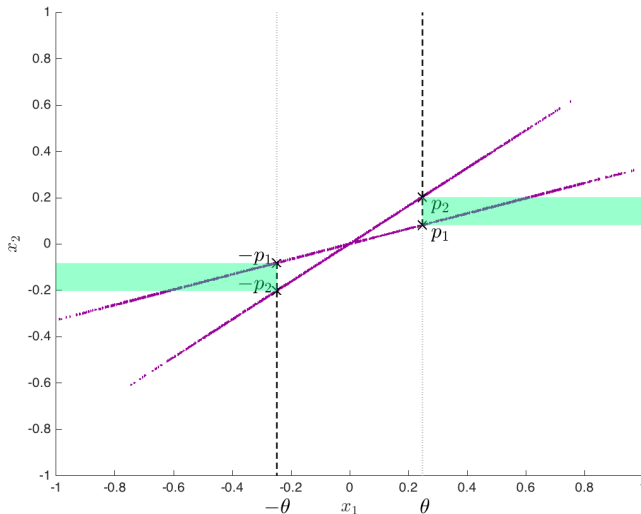
- Use the unique slopes $\nabla_{\mathbf{s}}$ to build the estimated mixing matrix $\hat{\mathbf{A}}$.

2. Quantisation of trivial time points

Points are repairable trivially if:

- Source label is known.
- Slope of the source line is known.
- Only one mixture is clipped.

2. Quantisation of trivial time points



3. Declipping mixture(s) by ℓ_1 minimisation

$$\begin{aligned}
 &\text{minimize} && \sum_{i=1}^D \|\mathbf{r}_i\|_1 \\
 &\text{subject to} && \hat{\mathbf{A}}_1 \mathbf{r}^T \boldsymbol{\Psi}^T \mathbf{C}_u^T = x_1'' \mathbf{C}_u^T, \\
 & && \hat{\mathbf{A}}_2 \mathbf{r}^T \boldsymbol{\Psi}^T = x_2'', \\
 & && \hat{\mathbf{A}}_1 \mathbf{r}^T \boldsymbol{\Psi}^T \mathbf{C}_-^T \leq -\theta, \\
 & && \hat{\mathbf{A}}_1 \mathbf{r}^T \boldsymbol{\Psi}^T \mathbf{C}_+^T \geq \theta.
 \end{aligned} \tag{4}$$

4. Quantisation of declipped points

- Optimisation solution not guaranteed to lie on source lines.
- Each point is projected to the nearest source line (nearest slope to the slope of the point).

$$l_n = \arg \min_i \left| \hat{x}_{1,n} - \frac{x_{2,n}}{\nabla_{s_i}} \right| \quad (5)$$

$$\hat{x}_{1,n} = \frac{x_{2,n}}{\nabla_{s_{l_n}}} \quad (6)$$

5. Source estimation

- For two sources, two mixtures, source estimation is trivial.
- Mixing matrix is square, so we can use a matrix inverse:

$$\hat{\mathbf{s}} = \hat{\mathbf{A}}^{-1} \hat{\mathbf{x}} \quad (7)$$

Comparative evaluation

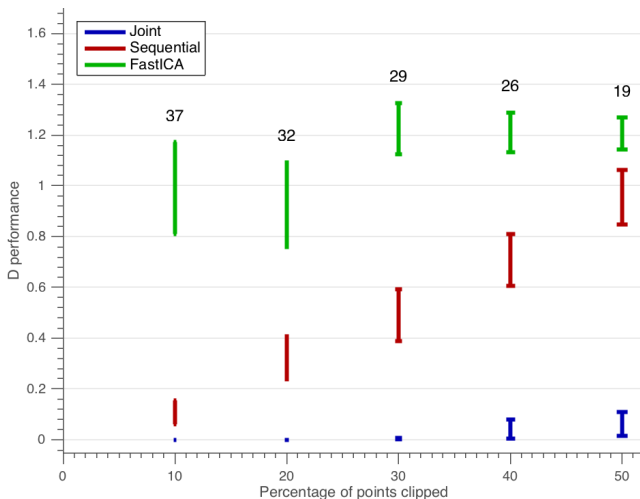
Three methods:

- 1 The proposed joint method.
- 2 A sequential version (declip mixtures, estimate sources).
- 3 FastICA on declipped mixtures.

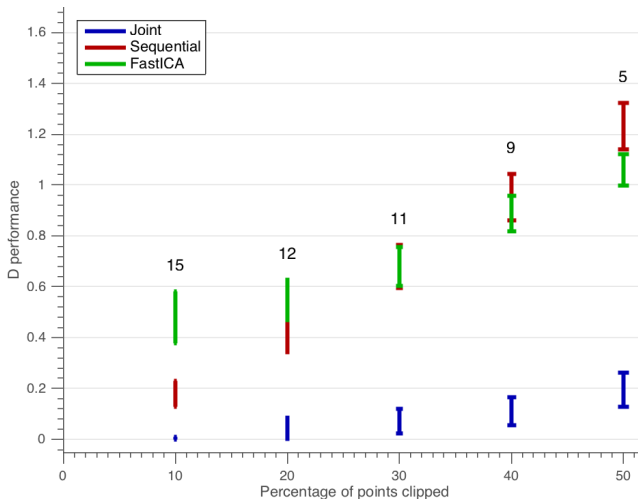
Performance measure:

$$\mathbf{D} := \min_{\epsilon = \pm 1} \left\| \frac{\hat{s}_i}{\|\hat{s}_i\|} - \epsilon \frac{s_i}{\|s_i\|} \right\|_2^2 \quad (8)$$

Results (Sine wave sources)



Results (Gaussian sources)



Results (Speech sources)

