Joint Blind Source Separation and Declipping: A Geometric Approach for Time Disjoint Sources

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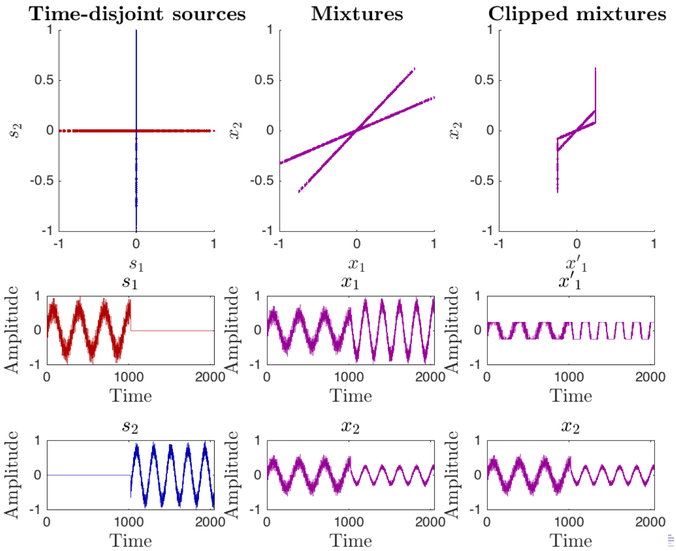
Overview

Our problem is modelled by:

1. The Independent Component Analysis (ICA) model for source separation.
2. The hard clipping model.

Overview


Time-disjoint sources
Mixtures
Clipped mixtures
s_2
s_1
x_2
x_1
x'_1
Amplitude
Time
x_2

Independent Component Analysis (ICA) model

- Latent variable model:
$$\mathbf{x} = \mathbf{A}\mathbf{s} \tag{1}$$
- $\mathbf{s}$: source signals (D×N).
- $\mathbf{x}$: mixtures (observations) (F×N).
- $\mathbf{A}$: mixing matrix (F×D).
Determines how much of each source is in each mixture.

Clipping distortion

- What if one of the observed mixtures is clipped?

$$x'_{1,n} = \begin{cases} \mathrm{sgn}(x_{1,n})\theta & \text{if } |x_{1,n}| > \theta, \\ x_{1,n} & \text{if } |x_{1,n}| \leq \theta. \end{cases} \tag{2}$$

- Existing ICA methods do not work.

Assumptions

- Assumptions are needed to make problem solvable.
- Mixtures of time-disjoint sources yield source lines (Plumbley, 2010).
- Noiseless.
- Two mixtures.

Geometric method: 5 steps

1. Mixing matrix estimation.
2. Quantisation of *trivial time points*.
3. Declipping mixture(s) by $\ell 1$ minimisation.
4. Quantisation of declipped points.
5. Source estimation.

Example case: two sources, one clipped mixture.

1. Mixing matrix estimation

- Ignore clipped time points.
- Each line passes through the origin, so each unclipped point gives a *source line* slope:

$$\nabla_{\mathbf{M}} = \left(x'_{2,1}, \ldots, x'_{2,M}\right) \odot \left(\frac{1}{x'_{1,1}}, \ldots, \frac{1}{x'_{1,M}}\right) \tag{3}$$

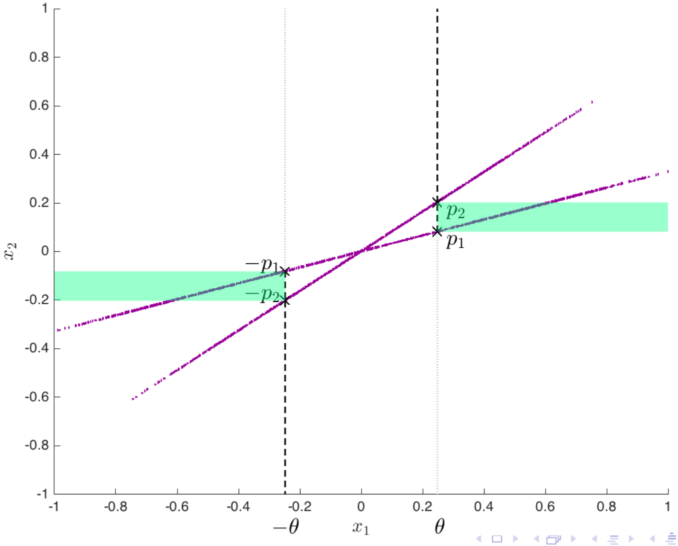
- Use the unique slopes $\nabla_{\mathbf{s}}$ to build the estimated mixing matrix $\hat{\mathbf{A}}$.

2. Quantisation of trivial time points

Points are repairable trivially if:

- Source label is known.
- Slope of the source line is known.
- Only one mixture is clipped.

2. Quantisation of trivial time points



3. Declipping mixture(s) by $\ell 1$ minimisation

$$
\begin{aligned}
\text{minimize} \quad & \sum_{i=1}^{D} \|\mathbf{r}_i\|_1 \\
\text{subject to} \quad & \hat{\mathbf{A}}_1 \mathbf{r}^T \boldsymbol{\Psi}^T \mathbf{C_u}^T = x_1'' \mathbf{C_u}^T, \\
& \hat{\mathbf{A}}_2 \mathbf{r}^T \boldsymbol{\Psi}^T = x_2'', \\
& \hat{\mathbf{A}}_1 \mathbf{r}^T \boldsymbol{\Psi}^T \mathbf{C}_-^T \leq -\theta, \\
& \hat{\mathbf{A}}_1 \mathbf{r}^T \boldsymbol{\Psi}^T \mathbf{C}_+^T \geq \theta.
\end{aligned} \tag{4}
$$

4. Quantisation of declipped points

- Optimisation solution not guaranteed to lie on source lines.
- Each point is projected to the nearest source line (nearest slope to the slope of the point).

$$l_n = \arg\min_i \left| \hat{x}_{1,n} - \frac{x_{2,n}}{\nabla_{s_i}} \right| \tag{5}$$

$$\hat{x}_{1,n} = \frac{x_{2,n}}{\nabla_{s_{l_n}}} \tag{6}$$

5. Source estimation

- For two sources, two mixtures, source estimation is trivial.
- Mixing matrix is square, so we can use a matrix inverse:

$$\widehat{\mathbf{s}} = \widehat{\mathbf{A}}^{-1}\widehat{\mathbf{x}} \tag{7}$$

Comparative evaluation

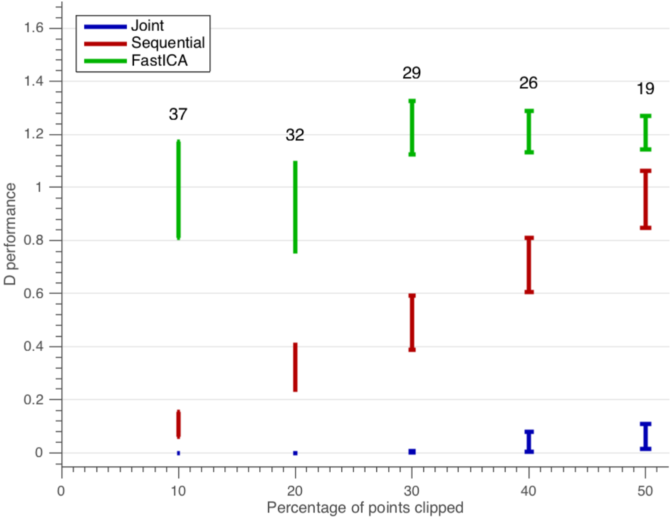
Three methods:

1. The proposed joint method.
2. A sequential version (declip mixtures, estimate sources).
3. FastICA on declipped mixtures.

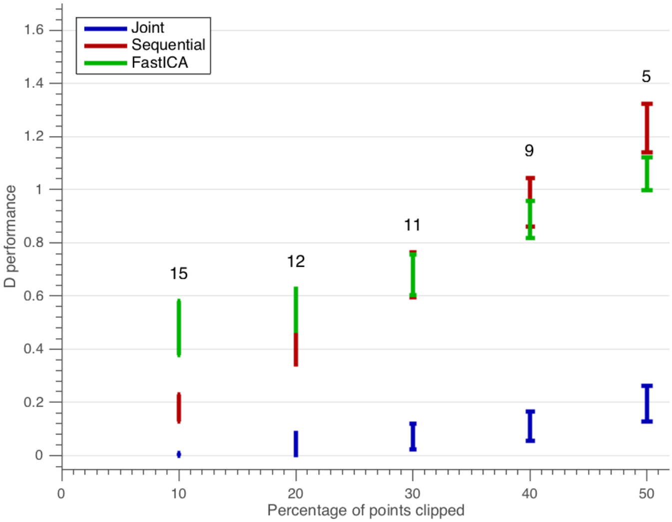
Performance measure:

$$\mathbf{D} := \min_{\epsilon=\pm 1} \left\| \frac{\widehat{s_i}}{\|\widehat{s_i}\|} - \epsilon \frac{s_i}{\|s_i\|} \right\|_2^2 \tag{8}$$

Results (Sine wave sources)



Results (Gaussian sources)



Results (Speech sources)