Réponses TEG_MBT_A 2017-2018

Ex1 a) $\lambda_1 = 3$ $\lambda_2 = -3$ $\lambda_3 = 6$

b) $\lambda_1 = 3$ Vect propre $\vec{u} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$

$\lambda_2 = -3$ - - - - $\vec{v} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

$\lambda_3 = 6$ ——— $\vec{w} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

c) A inversible car ses val propres sont non nulles.

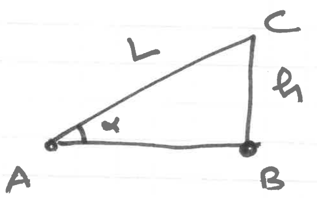
Val propres de A^{-1} sont $\mu_1 = \frac{1}{\lambda_1} = \frac{1}{3}$ $\mu_2 = \frac{1}{\lambda_2} = \frac{-1}{3}$

et $\mu_3 = \frac{1}{\lambda_3} = \frac{1}{6}$

Vect propre de A^{-1} :

μ_1 Vect propre $\vec{u}$; μ_2 Vect propre $\vec{V}$; μ_3 Vect propre $\vec{W}$

Ex2



Constantes $V = \dot{x}'$, h.

Variables : L, $x = AB$, α

$\tan(\alpha) = \frac{h}{x}$

$(1 + \tan^2(\alpha))\,\alpha' = -\frac{h}{x^2}\,x'$

$\alpha' = -\frac{h\,x'}{x^2 + h^2} = -\frac{h\,x'}{L^2} \simeq -0,02$ rad/s

Ex3 $x_0 = 0$ $|\Delta f - df| = \left(\frac{1}{10}\right)^3$

$x_0 = 100$ $|\Delta f - df| = 3 + \left(\frac{1}{10}\right)^3$

Ex4 $f(x) = \frac{\ln(x)^2}{x}$ $x > 0$

$D(f) =]0, +\infty[$

$f'(x) = \frac{\ln(x)\,(2 - \ln(x))}{x^2}$

T.V

x	0		1		e^2		$+\infty$
$f'(x)$							
$f(x)$	$+\infty$	$\searrow$	0	$\nearrow$	$\frac{4}{e^2}$	$\searrow$	0

$$f''(x) = \frac{2}{x^3}\left(\ln(x)^2 - 3\ln(x) + 1\right)$$

$f''(x) = 0$ $\qquad x_1 = e^{\frac{3-\sqrt{5}}{2}}$ $\qquad x_2 = e^{\frac{3+\sqrt{5}}{2}}$

x	0		x_1		x_2		$+\infty$
$f''(x)$		$+$	0	$-$	0	$+$	
		Convexe		Concave		Convexe