Physics Research Project

Cosmology with Galaxy Formation

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Abstract

Measuring the abundance of dark energy in the universe is a key field of modern cosmology. The relatively large abundance of dark energy is a major factor controlling the current evolution of the universe. Dark energy is believed to be the dominant force controlling the expansion of the universe and its apparent acceleration. Cosmological tests are useful tools used to constrain the parameters of the universe; many cosmological tests exist but they are all limited in sensitivity. We applied the dN/dz test to counts of galaxies in order to examine several of the most popular models of dark energy abundance in the universe. We confirm the suitability of using counts of galaxies as tracers for the evolving volume of the universe, while also stating its shortcomings when applied to astronomical observations. We also used our models to place constraints on some popular models of an evolving dark energy equation of state and numerically quantify how the relative abundance of dark energy affects the counts of galaxies which would be observed.

1. Introduction

Observations from astronomy can be key to studying and understanding the dynamics and geometry governing the evolution of the universe. One of the most important of these observations was the discovery that galaxies were receding away from each other, this discovery came at the beginning of the 20th century and is the basis of any modern cosmology theory. It is now widely accepted that the universe we live in is expanding. The next major revelation relating to the nature of the universe came in the 1990s, observations of Type Ia supernova suggested that the acceleration of the universe wasn't slowing down but was in fact accelerating [1] [2]. This questioned many of the ideas of the universe as these astronomical observations could not be explained by the then-accepted theory that the universe is primarily made up of matter and this matter dominates the dynamics and evolution of the universe. Additional data such as observations of the baryon to matter ratios of galaxy clusters and the baryon density predicted by nucleosynthesis suggested that matter should only make up about 30% of the total universe [3]. These observations called for a new component to be added to the models that described what the universe was made of. For this new component to explain the observations that were seen from astronomy then it needed to dominate in recent times over all other components of the universe and have a negative pressure associated with it so that it could counter the gravitational attraction of matter, it must also only be effective at cosmological distances. This new component is now commonly referred to as "dark energy".

This dark energy component of the universe is one of the key areas of interest in modern cosmology. Exactly what this dark energy is and how it has become the dominating force in the universe is one of the major unsolved problems within modern cosmology and astronomy [4]. Due to the undetectable nature of dark energy there are several candidates for what it could be. One of these is the energy of the vacuum which is represented by the cosmological constant term seen in Einstein's equations, it has a constant energy density in time and is spatially uniform. A major issue for this is that the energy associated with the vacuum that is observationally seen to drive the acceleration of the universe is many orders of magnitude below the energy level of the vacuum that is theoretically predicted by quantum field theory [5]. Many different models and theories of dark energy have been proposed in an attempt to explain how the vacuum energy could change and evolve with time to fit in with our current observations of dark energy. These models are known as quintessence models and they all suggest different ways in which dark energy could vary, as quintessence models are potentially spatially inhomogeneous, can change according to the matter density or oscillate with time as the universe evolves [6]. Other dark energy models include evolving scalar fields and modifications to general relativity. Another problem associated with dark energy is the lack of understanding of the exact physics and dynamics that governs the dark energy present in the universe, understanding and constraining the different models and theories for dark energy is key to attempting to describe the unknown physics that describes the function of dark energy.

Knowing and parametrising exactly how much dark energy is in the universe could be key to understanding how and if it evolves with time. The total constituents of the universe can be categorised by the dimensionless density parameter. The total density parameter of the universe is defined as

$$\Omega \equiv \frac{\rho}{\rho_c} \tag{1.1}$$

where Ω is the total density parameter, ρ is the observed density of the universe and ρ_c is the critical density of the universe ($\Omega = 1$ corresponds to a flat universe). The critical density is then defined as

$$\rho_c \equiv \frac{3{H_0}^2}{8\pi G} \tag{1.2}$$

where H_0 is the Hubble parameter and G is the gravitational constant. The total density parameter of the universe is linked to the separate density parameters of its constituents via the following equation,

$$\Omega = \Omega_m + \Omega_\mathrm{k} + \Omega_\Lambda \tag{1.3}$$

where Ω_m is the density parameter of matter, Ω_k is the density parameter of curvature and Ω_Λ is the density parameter of the cosmological constant, initial observations of Type IA Supernova put $\Omega_\Lambda \sim 0.7$ [7]. The matter density parameter and cosmological constant density parameter are then defined as

$$\Omega_m = \frac{8\pi G \rho_0}{3{H_0}^2} \tag{1.4}$$

$$\Omega_\Lambda = \frac{\Lambda c^2}{3{H_0}^2} \tag{1.5}$$

$$\Omega_k = -\frac{kc^2}{{H_0}^2} \tag{1.6}$$

where ρ_0 is the density of matter in the universe, Λ is the cosmological constant and c is the speed of light [8]. Assuming dark energy is spatially smooth it can be described by the following equation of state (EoS),

$$w \equiv \frac{P}{\rho} \tag{1.7}$$

where P is pressure and ρ is the energy density. For the cosmological constant w is a constant and for quintessence models w can change value in time. Current observational data constrain the equation of state to be $w \approx -1$ to within about 5% [4] strongly favouring a cosmological constant for dark energy.

Dark energy is currently causing the universe to undergo a period of accelerated expansion, the expansion of the universe is observationally seen in the recessional velocities of galaxies, which is given by Hubble's law,

$$v = H_0 d \tag{1.8}$$

where v is the velocity of the receding object and d is the distance between the observer and the receding object. Currently the Hubble parameter is believed to have a value of $H_0 = 100h\ km\ s^{-1}\ Mpc^{-1}$ where h is a dimensionless number which represents uncertainty within H_0, currently it is believed that $h \sim 0.7$ [9] [10].

Another important measurement in cosmology is redshift. Astronomical objects can be categorised by their redshift, z. The redshift of an object is the fractional shift in the wavelength of the light emitted from the receding object due to the motion of the object relative to the observer. The redshift of an object is defined as

$$z \equiv \frac{\nu_e}{\nu_o} - 1 = \frac{\lambda_o}{\lambda_e} - 1 \tag{1.9}$$

where λ_e and ν_e are the wavelength and frequency of the light emitted from the object and λ_o and ν_o are the observed wavelength and frequency.

Cosmological tests are a useful way for us to understand the universe better. These include the Tolman surface brightness (SB) test and the time dilation of supernova light curves, both of which test for the expansion of the universe. Results from the Tolman test reinforce the accepted model of an expanding universe [11]. Cosmological tests are used to outline the history of the universe including its expansion and compare it with predictions from different models. A certain model is assumed and defined by a certain set of parameters and then compared to observations. All cosmological tests fundamentally use a comparison of some measure of relative distance to redshift. Different types of cosmological tests include supernovae analysis, the Hubble diagram versus redshift, Angular diameter versus redshift, number counts as a function of redshift, indirect tests of age versus redshift, local dynamical measurements of the mass density and independent measures of H_0 and t_0. However, these methods all have their own pros and cons. For example, supernovae explosions are still not fully understood, progenitor systems are unknown, and it is not known if they evolve or not. Furthermore, supernovae are not proper standard candles, but they are good at indicating distance. Moreover, supernovae measurements alone only determine an allowed region on the Ω_M, Ω_Λ plane. To specify a value of Ω_Λ, further constraints are needed. The Hubble diagram versus redshift requires standard candles, angular diameter versus redshift requires standard rulers, source counts as a function of redshift require sources that have a constant comoving density, or standard populations, and indirect tests of age versus redshift are model dependant and require standard clocks.

In the past many different types of astronomical observations have been used to determine the density parameters of the universe. For example the merger history of dark matter halos has been used to constrain the cosmological density parameter to $\Omega_\Lambda = 0.84^{+0.16}_{-0.17}$ [12]. As there are a large number of observables that can place constraints on density parameters it is often useful to use a combination of data sets that all place different constraints on these parameters in order to be able to try to narrow down and get a clearer idea of these exact numbers. Some of the most recent studies that have attempted to place constraints on the total abundance of dark energy in the universe have used the Joint Lightcurve Analysis (JLA) from supernovae and analysis of baryonic acoustic oscillations (BAO) or weak gravitational lensing [13]. Probing the geometry of the universe is another way in which astronomical observations can place constraints on the density parameters in the universe. The idea behind this is that relative dark energy and matter densities will change the geometry of the universe. The ability to probe the geometry of the universe is therefore a useful tool for differentiating between different cosmological models. One of the classical tests for probing the constituents and nature of the universe has been the dN/dz test. This test involves studying the evolution of the cosmological volume element by measuring the redshift distribution of a tracer with a given number density, if the number density is approximately constant then the counts of the tracer will vary with the volume of the universe. Previous studies of this have involved looking at how the number of dark matter halos varies with circular velocity, applying this test and studying how the data from different surveys has allowed constraints to be applied to different cosmological parameters [14] [15].

The current state of the research in this area is minimal. The dN/dz test was first postulated as a tool to further our understanding of how dark energy evolves by Newman and Davis in 2000, and since then the idea has not seen any significant developments. No compelling research has come from this area since then, however, the idea still remains. Newman and Davis were dominant figures in this field, but still only dedicated three papers to it, the last being in 2002. Newman and Davis wrote two papers about measuring the cosmic equation of state with counts of galaxies and a paper about measuring the cosmic equation of state with galaxy clusters using the DEEP2 survey.

The essence of the Newman and Davis papers is the following; Observations of the apparent numbers of galaxies per unit redshift per steradian compared to their present day abundance directly give measurements of the cosmological volume element and consequently of fundamental cosmological parameters such as the equation of state of dark energy [16].

2. Theory

2.1 Dark Energy Parameterizations

There are many different interesting parameterizations of dark energy, here we shall look at seven of them. It is useful to define the function

$$E(z)^2 = \left(\frac{H(z)}{H_0}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_{DE})\left[\Omega_{m,0}(1+z)^3 + \Omega_{DE,0}f(z)\right] \tag{2.1}$$

where $H(z)$ Is the Hubble parameter, G is the gravitational constant, ρ_m is the energy density of matter, ρ_{DE} is the energy density of dark energy, $\Omega_{m,0}$ and $\Omega_{DE,0}$ are the matter density parameter and dark energy density parameter at present day values respectively and $f(z) = \exp\left[3\int_0^z \frac{1+w(\tilde{z})}{1+\tilde{z}}d\tilde{z}\right]$. This gives rise to the following $H(z) = H_0\,E(z)$ where H_0 is the Hubble constant and $E(z)$ is the defined function [17].

Lambda Cold Dark Matter-redshift parametrization (ΛCDM) has $w = -1$. This is a well-known and standard model in cosmology, and it gives a good fit to many observations.

Linear redshift parametrisation [18], [19], in which the equation of state is given by

$$w(z) = w_0 - w_1 z \tag{2.2}$$

This is one of the simplest parametrisations of dark energy. In this model we can obtain the standard ΛCDM model with a constant equation of state, $w = -1$, by choosing $w_1 = 0$ and $w_0 = -1$. The function can then be written from equation (2.1) as

$$E(z)^2 = \Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{3(1+w_0+w_1)} \times exp(-3w_1 z) \tag{2.3}$$

where Ω_m is the matter density parameter. However, at high redshifts this model diverges from the data which places strong constraints on w_1 [20].

Chevallier-Polarski-Linder parametrization (CPL) [21], [22] has an equation of state of the following form,

$$w(z) = w_0 + \left(\frac{z}{z+1}\right) w_1 \tag{2.4}$$

The function for this parameterization can be written, from the general form given in equation (2.1), as

$$E(z)^2 = \Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{3(1+w_0+w_1)} \times exp\left[-\left(\frac{3w_1 z}{1+z}\right)\right] \tag{2.5}$$

However, this model cannot be applied to the full history of the universe.

Barboza-Alcaniz parametrisation (BA) [23] produces the following equation of state,

$$w(z) = w_0 + \frac{z(1+z)}{1+z^2} w_1 \tag{2.6}$$

which has a well behaved $w(z)$ as z approaches -1. From equation (2.1), the function for this model can be written as

$$E(z)^2 = \Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{3(1+w_0)} \times (1+z^2)^{\frac{3w_1}{2}} \tag{2.7}$$

Low Correlation parametrisation (LC) has the following equation of state,

$$w(z) = \frac{(-z+z_c)w_0 + z(1+z_c)w_c}{(1+z)z_c} \tag{2.8}$$

This model uses additional parameters to define the evolving EoS, here $w_0 = w(z=0)$ and $w_c = w(z=c)$. Most models take $z_c = 0.5$ as data suggest $z_c \sim 0.3$ [17]. The function from equation (2.1) can be rewritten to give

$$E(z)^2 = \Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{3(1-2w_0+3w_{0.5})} \times exp\left[\frac{9(w_0+w_{0.5})z}{1+z}\right] \tag{2.9}$$

where $w_{0.5} = w(z=0.5)$.

Jassal-Bagla-Padmanabhan parametrisation (JBP) has the form,

$$w(z) = w_0 + \frac{z}{(1+Z)^2} w_1 \tag{2.10}$$

This parameterization has the following function, by combining equations (2.10) and (2.1),

$$E(z)^2 = \Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{3(1-w_0)} \times exp\left[\frac{3w_1 z^2}{2(1+z)^2}\right] \tag{2.11}$$

Wetterich-redshift parametrisation (WP) [24] allows dark energy to contribute to the total energy of the universe at earlier times. It has an equation of state of the following from,

$$w(z) = \frac{w_0}{[1 + w_1 \ln(1+z)]^2} \tag{2.12}$$

where w_1 is known as the bending parameter and defines the redshift at which the equation of state goes from being constant to some other behaviour. Using equation (2.12) in equation (2.1), the following function is obtained,

$$E(z)^2 = \Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{3\left[1+\frac{w_0}{1+w_1\ln(1+z)}\right]} \tag{2.13}$$

Plotting the equations of state for the different parameterisations for dark energy gives the figure below in which their evolutions in time can be seen.

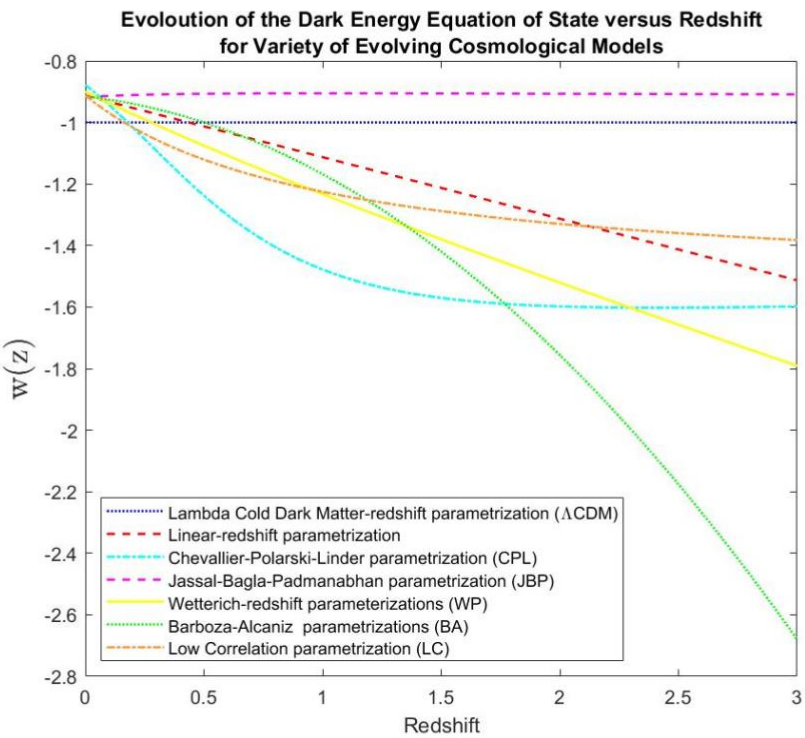

Figure 1 graph showing the change in the dark energy EoS against redshift for evolving dark energy models, the values of the parameters taken for w_0 and w_1 are given in table 2.

These evolving forms of dark energy are used to attempt to try to accurately describe how dark energy has become the dominating force in the universe since matter domination ended about 9.8 billion years ago.

An open universe has the following function,

$$E(z)^2 = \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda \tag{2.14}$$

where Ω_Λ is the cosmological constant density parameter, Ω_k is the curvature density parameter, which is defined as $\Omega_k = -\frac{kc^2}{{H_0}^2}$ with $k < 0$ for an open universe, and Ω_m is the matter density parameter.

2.2 Distance Measures

We shall now introduce the different distance measures that are often used in cosmology. One of the most fundamental measurements is the Hubble Distance, D_H, defined as $D_H \equiv \frac{c}{H_0} = 4285.7\ h^{-1}Mpc = 1.32 \times 10^{26} h^{-1}\ m$ where c is the speed of light and H_0 is the Hubble constant.

Next is the line-of-sight comoving distance, D_C. This is the constant distance between two astronomical objects in our universe that are locked into the Hubble flow. The Hubble flow is the movement of astronomical objects due to the expansion of the universe; this means that the comoving distance is a distance scale that evolves with the expanding universe. It is the proper distance multiplied by a factor of $(1+z)$. The total line-of-sight comoving distance is

$$D_C = D_H \int_0^z \frac{dz'}{E(z')} \tag{2.15}$$

where D_H is the Hubble Distance, $E(z)$ is the function and z is redshift.

Following this we can introduce the transverse comoving distance, D_M.The Transverse comoving distance between two astronomical objects at the same redshift but are some angle, $\delta\theta$, apart, it is also equivalent to the proper motion distance and is determined as the ratio of an astronomical objects transverse velocity to its proper motion [25]. The transverse comoving distance is related to the line-of-sight comoving distance by

$$D_M = \begin{cases} D_H \dfrac{1}{\sqrt{\Omega_k}} \sinh\left[\sqrt{\Omega_k}\, \dfrac{D_C}{D_H}\right] & \text{for } \Omega_k > 0 \\ D_C & \text{for } \Omega_k = 0 \\ D_H \dfrac{1}{\sqrt{|\Omega_k|}} \sin\left[\sqrt{|\Omega_k|}\, \dfrac{D_C}{D_H}\right] & \text{for } \Omega_k < 0 \end{cases} \tag{2.16}$$

where D_H is the Hubble distance, D_C is the line-of-sight comoving distance and Ω_k is the curvature density parameter.

Subsequently we may introduce the angular diameter distance, D_A. The angular diameter distance is the ratio of an astronomical objects' physical transverse size to its angular size. It is known for having a turnover at approximately redshift 1 as it does not increase as redshift tends to infinity. This makes far off astronomical objects appear larger in angular size. We can relate the angular diameter distance to the transverse comoving distance using

$$D_A = \frac{D_M}{1+z} \tag{2.17}$$

where D_M is the transverse comoving distance and z is redshift [25]–[27].

Next we shall introduce the luminosity distance, D_L. The luminosity distance is determined by the relation between the bolometric flux to the bolometric luminosity and given by the following equation

$$D_L \equiv \sqrt{\frac{L}{4\pi\, S}} \tag{2.18}$$

where L is the bolometric luminosity and S is the bolometric flux. The luminosity distance can also be related to the transverse comoving distance and angular diameter distance by

$$D_L = (1+z)D_M = (1+z)^2 D_A \tag{2.19}$$

where D_M is the transverse comoving distance, D_A is the angular diameter distance and z is redshift [25], [26]. The surface brightness of a receding object is decreased by $(1+z)^{-4}$ while the angular area is also decreased as ${D_A}^{-2}$ [8].

Further to this, we will introduce the comoving volume, V_C. The comoving volume is the volume measure in which the number density of non-evolving objects that are locked into the Hubble flow is constant with redshift. It can also be defined as the proper volume multiplied by $(1+z)^3$. The comoving volume element per unit redshift per steradian is

$$dV_c = D_H\, \frac{(1+z)^2 {D_A}^2}{E(z)}\, d\Omega\, dz \tag{2.20}$$

where the derivative of the line-of-sight comoving distance with respect to redshift is the reciprocal of the function defined in equation (2.1), the angular diameter distance, D_A, transforms a solid angle, $d\Omega$, into a proper area, the $(1+z)^2$ transforms a proper area into a comoving area [25], [27]. It is important to note that galaxies themselves don't grow with the expansion of the universe as they are gravitationally bound. Therefore, as the universe expands galaxies appear smaller. If we integrate the comoving volume element we obtain the total comoving volume, over all of the sky, out to redshift z which gives us

$$V_C = \begin{cases} \left(\frac{4\pi {D_H}^3}{2\Omega_k}\right)\left[\frac{D_M}{D_H}\sqrt{1+\Omega_k \frac{{D_M}^2}{{D_H}^2}} - \frac{1}{\sqrt{\Omega_k}}\sinh^{-1}\left(\sqrt{\Omega_k}\,\frac{D_M}{D_H}\right)\right] & \text{for } \Omega_k > 0 \\ \frac{4\pi}{3}{D_M}^3 & \text{for } \Omega_k = 0 \\ \left(\frac{4\pi {D_H}^3}{2\Omega_k}\right)\left[\frac{D_M}{D_H}\sqrt{1+\Omega_k \frac{{D_M}^2}{{D_H}^2}} - \frac{1}{\sqrt{|\Omega_k|}}\sin^{-1}\left(\sqrt{|\Omega_k|}\,\frac{D_M}{D_H}\right)\right] & \text{for } \Omega_k < 0 \end{cases} \quad (2.21)$$

where ${D_H}^3$ is known as the Hubble volume, D_H is the Hubble distance, Ω_k is the curvature density parameter and D_M is the transverse comoving distance. The comoving volume element and its integral are both commonly used in predicting number counts of galaxies.

Another useful measure is magnitude, m. This defines the unitless brightness of an astronomical object. The apparent bolometric magnitude is defined by

$$m = -2.5\log_{10}\left(\frac{L}{4\pi\, {D_L}^2}\right) + constant \quad (2.22)$$

where L is the bolometric luminosity of the astronomical object, D_L is the luminosity distance to the astronomical object and $constant$ is some defined zero point (magnitude systems are usually defined such that the star Vega has an apparent magnitude equal to zero). Magnitude is a logarithmic scale where objects with larger magnitudes are fainter.

2.3 Schechter Function

Schechter functions are used to describe the relative densities and abundance of galaxies as a function of their mass or luminosity. A Schechter function $\phi(M)$ will give the number of galaxies per unit mass, this means that $\phi(M)dM$ will describe the number of galaxies with a mass between M and $M + dM$. A Schechter function can be used to find the number density of galaxies within a mass or luminosity range by integrating it with respect to either mass or luminosity.

Schechter functions are useful in astronomy in order to be able to fit astronomical data to a general mathematical form so that galactic mass or luminosity distribution can be evaluated. The typical form of the Schechter function is as follows,

$$\phi(M) = \left(\frac{\phi^*}{M^*}\right)\left(\frac{M}{M^*}\right)^{\alpha} \exp\left(-\frac{M}{M^*}\right) \quad (2.23)$$

where ϕ^* is the normalisation of the Schechter function, $M^*$is the turnover mass, α is the slope of the low mass end of the Schechter function and M is the stellar mass.

The Schechter function can be changed to a form where galaxies can be characterised by their apparent bolometric magnitude instead of luminosity or mass. This can be a lot more useful within astronomy as apparent bolometric magnitude can be measured a lot more easily than mass or luminosity (measuring the luminosity of a galaxy requires knowing how far away it is but measuring the apparent magnitude does not). The relationship between apparent magnitude and luminosity is given in equation (2.22).

Using equation (2.22) we can see that we can shift a Schechter function defined in terms of luminosity to one defined in terms of apparent magnitude using

$$m - m^* = -2.5log\left(\frac{L}{L^*}\right) \tag{2.24}$$

where m^* is the apparent magnitude of an astronomical object with luminosity L^*, and m is the apparent magnitude of an astronomical object with luminosity L. This gives a Schechter function written in terms of magnitude as

$$\phi(m) = \frac{\ln(10)}{2.5}\phi^* 10^{0.4(\alpha+1)(m-m^*)}\exp\left(-10^{0.4(m-m^*)}\right) \tag{2.25}$$

where m is the apparent magnitude and m^* is the turnover apparent magnitude.

This form of a Schechter function can be used to count the number of observed galaxies within a given magnitude range, i.e. the number density of galaxies we would expect to observe when counted down to a given magnitude limit.

3. Method

For this we looked at several different types of universes. The table below shows the different parameters for the models we will be looking at.

Name	Ω_m	Ω_Λ	Ω_k	w
ΛCDM	0.3	0.7	0	-0.8, -0.9, -0.95, -1, -1.05, -1.1, -1.2
Einstein-de Sitter	1	0	0	0
Open CDM	0.3	0	0.7	0

Table 1: A table of the different values of the density parameters and the equation of state for different cosmologies.

We chose to look at redshift range of zero to three. Setting $D_H = 4285.7\ Mpc$ and using a linear redshift parameterization as defined by equation (2.3) for ΛCDM and Einstein-de Sitter universes and equation (2.14) is used for an open universe, unless otherwise stated.

3.1 Angular Diameter Distance

Plotted the angular diameter distance, D_A, given in equation (2.17) against redshift for the various cosmological models in Table 1.

3.2 Luminosity Distance

Plotted the luminosity distance, D_L, given in equation (2.19) against redshift for the various cosmological models previously stated in Table 1.

3.3 Comoving Volume Element

Plotted the comoving volume element, dV_C, given in equation (2.20) against redshift for the various cosmological models previously stated in Table 1. Used numerical integration to integrate the reciprocal of the function stated in equation (2.3) with respect to redshift.

3.4 Change in Magnitude from a ΛCDM Universe

Plotted the change in apparent bolometric magnitude of galaxies for different cosmologies given in Table 1 compared to a ΛCDM cosmology against redshift. The apparent bolometric magnitude for astronomical objects is given in equation (2.22) and the change in apparent bolometric magnitude for different cosmologies is given by

$$\Delta m = m_{cosm} - m_{\Lambda CDM} = -2.5 \log_{10}\left(\frac{{D_{L_{\Lambda CDM}}}^2}{{D_{L_{\mathrm{cosm}}}}^2}\right) \tag{3.1}$$

where m_{cosm} is the magnitude of galaxies in the cosmology we are interested in and previously stated, $m_{\Lambda CDM}$ is the magnitude of galaxies in a ΛCDM cosmology, $D_{L_{\Lambda CDM}}$ is the luminosity distance in a ΛCDM cosmology and $D_{L_{\mathrm{cosm}}}$ is the luminosity distance in the cosmology we are interested in.

3.5 Change in Volume from a ΛCDM Universe

Plotted the ratio of the change in volume of galaxies for different cosmologies given in Table 1 compared to a ΛCDM cosmology to the volume of a ΛCDM cosmology against redshift. This is given by the following equation

$$\Delta\, vol = \frac{vol_{cosm} - vol_{\Lambda CDM}}{vol_{\Lambda CDM}} \tag{3.2}$$

where vol_{cosm} is the volume of galaxies in the cosmology we are interested in and $vol_{\Lambda CDM}$ is the volume of galaxies in a ΛCDM cosmology. The comoving volume is given by equation (2.21).

3.6 Values of w_0 and w_1

This table shows the values of w_0 and w_1 that we have chosen to use for the cosmologies we have modelled that have an evolving dark energy component. The values taken all lie within the constraints placed on these limits from the SNE Ia JLA and BAO data [28] [29].

Model	Parameters	
	w_0	w_1
Lambda Cold Dark Matter-redshift parametrization (ΛCDM)	-1	0
Linear-redshift parametrization	-0.913	0.200
Chevallier-Polarski-Linder parametrization (CPL)	-0.878	-0.600
Barboza-Alcaniz parameterization (BA)	-0.916	-0.084
Low Correlation parameterization (LC)	-0.912	-1.121
Jassal-Bagla-Padmanabhan parametrization (JBP)	-0.918	0.050

Table 2: A table of the different values for w_0 and w_1 used for the different dark energy parameterizations looked at here.

3.7 Schechter Function

The plotted Schechter function was taken from a study by Mortlock et al. [30]. The paper was based off the combined photometry of several astronomy surveys including the Ultra Deep Survey, Cosmic Assembly Near-Infrared Deep, Extragalactic Legacy Survey and the Great Observatories Origins. The Schechter function was constructed from both a low and high probe of the galaxy mass range in the redshift range $0.3 < z < 3$. The Schechter function used was the form of the Schechter function for the redshift range $0.3 < z < 0.5$. The single Schechter function used takes the following form

$$\phi(M) = \phi^* \ln(10)(10^{M-M^*})^{1+\alpha} \exp(-10^{M-M^*}) \tag{3.3}$$

The values of the Schechter function at given redshifts are given below, note that the stellar mass and turnover mass have units of dex

Redshift Range	$\boldsymbol{M^*}$	$\mathbf{log}(\boldsymbol{\phi}^*)$	$\boldsymbol{\alpha}$
$0.3 < z < 0.5$	11.32 ± 0.07	-3.2 ± 0.08	-1.41 ± 0.02
$0.5 < z < 1.0$	11.16 ± 0.04	-3.12 ± 0.05	-1.34 ± 0.02
$1.0 < z < 1.5$	11.04 ± 0.04	-3.21 ± 0.06	-1.31 ± 0.03
$2.0 < z < 2.5$	11.15 ± 0.06	-3.74 ± 0.09	-1.51 ± 0.03
$2.0 < z < 2.5$	11.02 ± 0.10	-3.78 ± 0.14	-1.56 ± 0.06
$2.5 < 3.0$	11.04 ± 0.11	-4.03 ± 0.16	-1.69 ± 0.06

Table 3: Table showing the different values and uncertainties for the variables in the single Schechter function across various redshift ranges.

The Schechter function was plotted in the mass range $6 < M < 12$ in units of dex.

3.8 Number Density

The number density was obtained from the form of the Schechter function, the function was numerically integrated in Matlab with respect to stellar mass between given mass ranges to obtain the number density of galaxies between that mass range and the lower mass limit of 6.

3.9 Number of Galaxies per Square Degree on the Sky

From the integrated Schechter function and the comoving volume we were able to find the number of galaxies per square degree in the sky for different cosmologies. This was done by first finding the number density from the Schechter function in a given mass range. The number density was then multiplied by a comoving volume shell of a ΛCDM cosmology. This gave the number of galaxies within various redshift ranges which, to make into an astronomical observation, was divided by the total number of square degrees in the sky (41 253). This gave the number of galaxies per square degree in the sky for a ΛCDM universe as a function of redshift. This number was plotted against redshift in the corresponding intervals of comoving volume shells for ΛCDM. A more detailed description of the equations used to find the number of galaxies is given below

$$\int_{M_1}^{M_2} \phi(M)\, dM = n \tag{3.4}$$

$$\frac{n \times V_{\Lambda CDM}}{41\ 257} = N \tag{3.5}$$

$$V_{CDM} = \frac{4\pi}{3}(\Delta D_m)^3 \tag{3.6}$$

where M_1 and M_2 are the lower and upper limit of the mass respectively (the plot shown uses $M_1 = 10$ and $M_2 = 12$), n is the number density calculated by the Schechter function, $V_{\Lambda CDM}$ is the volume of the shells in a ΛCDM cosmology and ΔD_m is the comoving distance corresponding to that particular volume shell and redshift interval. The redshift intervals used ran from 0 to 3 with a width of 0.25, i.e. $z = 0.00 \rightarrow 0.25, 0.25 \rightarrow 0.5, 0.5 \rightarrow 0.75 \ldots\ 2.75 \rightarrow 3.00$.

For modelling a new cosmology, we first found the ratio of the luminosity distance of the new cosmology to the luminosity distance of ΛCDM in that given redshift interval. This was used to calculate a shift in the mass range used to find the number density of the galaxies. This new number density is then used to find a new number of galaxies per square degree in the sky using the comoving volume shell of the corresponding cosmology. The detailed equations used to find this number density are given below

$$r = \frac{{D_{L_{\Lambda CDM}}}^2}{{D_L}^2} \tag{3.7}$$

$$M_1' = log_{10}\left(\frac{10^{10}}{r}\right) \tag{3.8}$$

$$\int_{M_1'}^{M_2} \phi(M)\, dM = n' \tag{3.9}$$

$$\frac{n' \times V}{41\ 257} = N' \tag{3.10}$$

where $D_{L_{\Lambda CDM}}$ and D_L is the luminosity distance of a ΛCDM and given cosmology (such as Einstein-de Sitter) respectively in the redshift range and v is the volume of the shells in the redshift range of the new cosmology we are finding the number of galaxies in the sky for.

We decided to not shift the upper limit (M_2) of the integral used to find the number density of galaxies. This is because the Schechter function has a turnover mass ~ 11 so any shift of the upper limit $M_2 = 12$ will not significantly affect the number density calculated.

3.10 Change in the Number of Galaxies per Square Degree on the Sky

This was obtained by simply taking the number of galaxies per square degree in the sky away from the number of galaxies per square degree in the sky for a ΛCDM universe for each of the different cosmological models. We did this for both non-evolving models and models with an evolving dark energy equation of state.

4. Results

The following contains all results obtained using the methods that were discussed in section 3.

4.1 Angular Diameter Distance

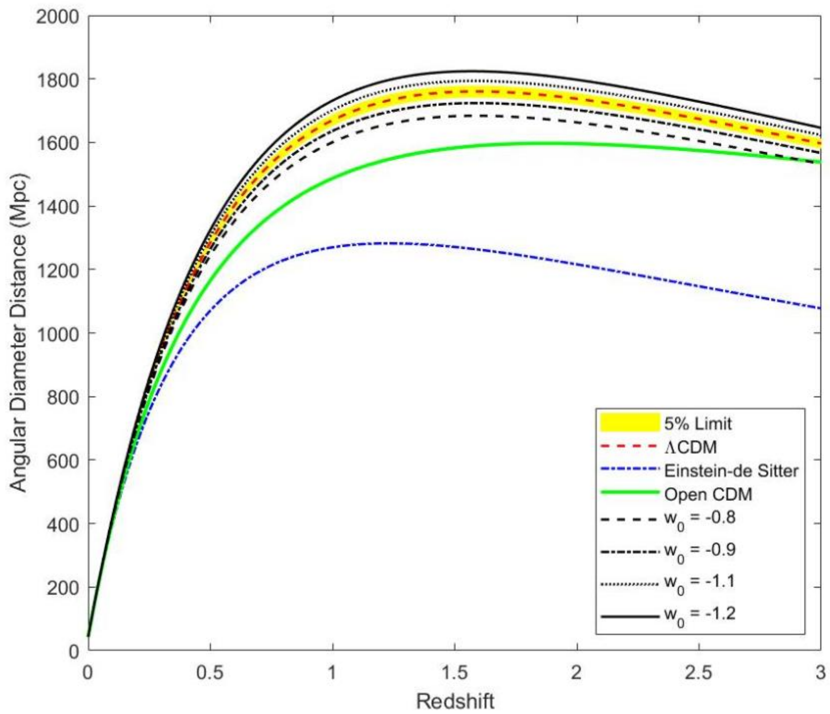

Figure 2: Plot of the angular diameter distance versus redshift for a variety of cosmoogical models.

4.2 Luminosity Distance

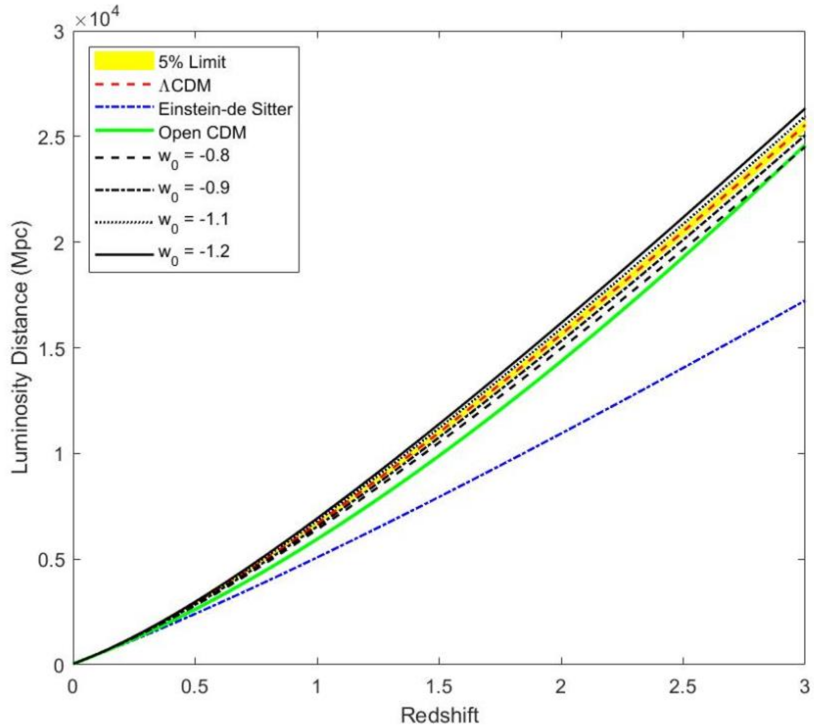

Figure 3: Plot of the luminosity distance versus redshift for a variety of cosmological models.

4.3 Comoving Volume Element

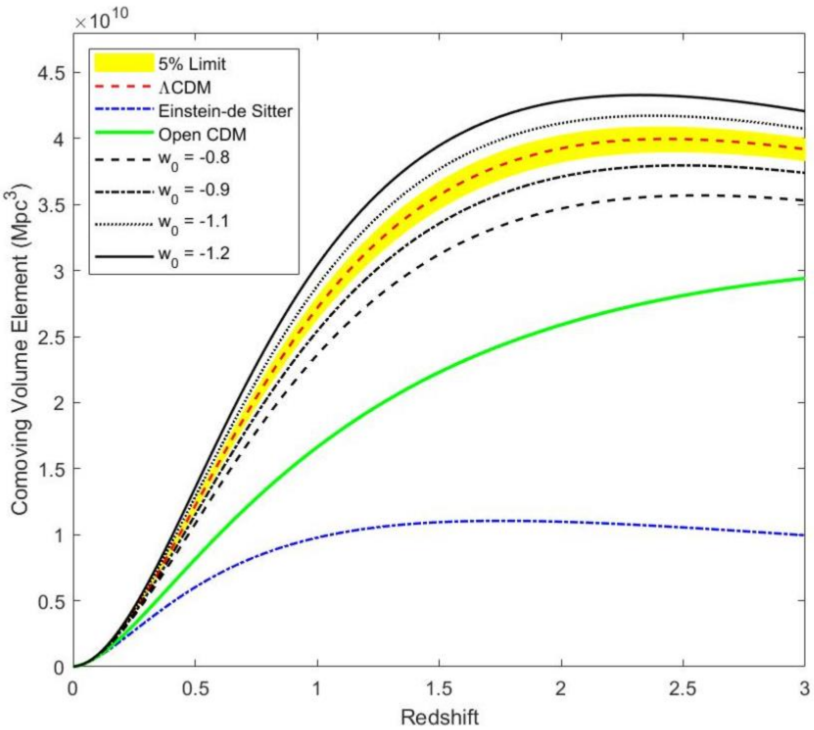

Figure 4: Plot of the comoving volume per unit redshift per steradian versus redshift for a variety of cosmological models

4.4 Change in Magnitude from a ΛCDM Universe

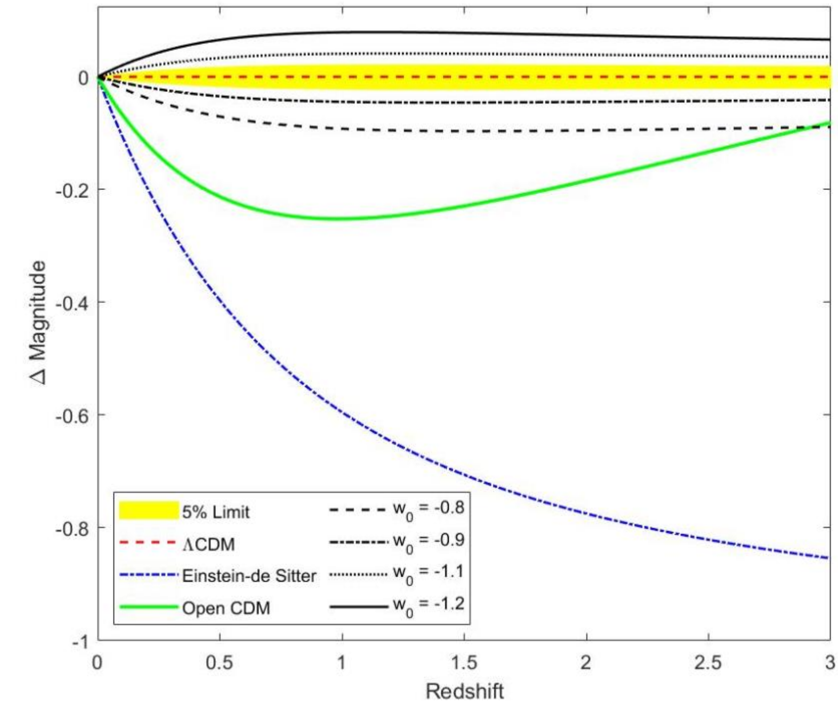

Figure 5: Plot of the change in magnitude from a ΛCDM universe versus redhsift for a variety of cosmological models.

4.5 Change in Volume from a ΛCDM Universe

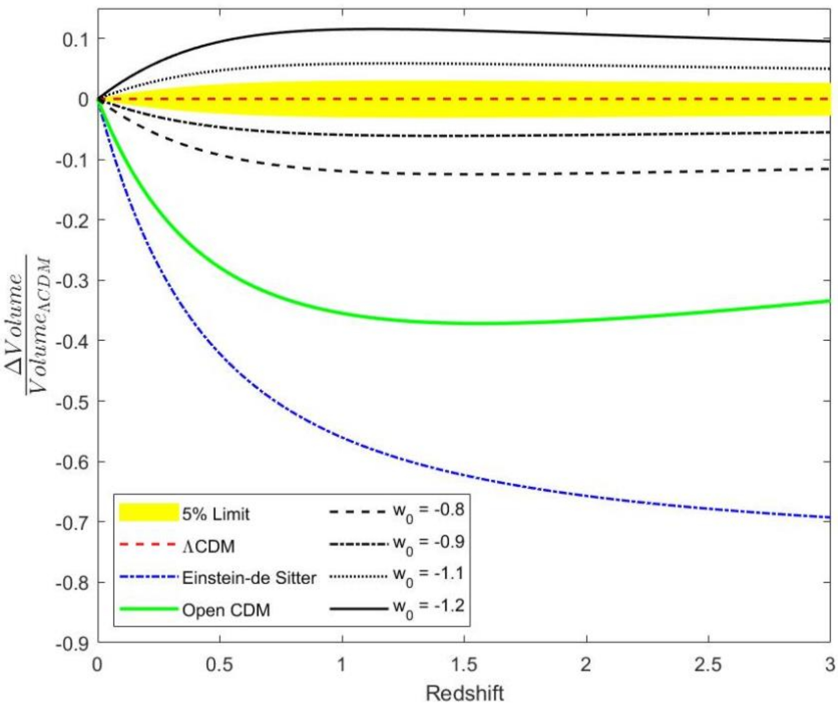

Figure 6: Plot of the change in volume from a ΛCDM universe versus redshift for a variety of cosmological models.

4.6 Parameterization for w_1

The table below shows the constrained values placed on w_1 for the corresponding w_0 values for the different dark energy parameterizations. These constraints were based on the 5% limit of a ΛCDM universe with $w = -1$ for the comoving volume element.

Parameterization	Values of w_0	Value range for w_1
Linear-redshift Parameterization	-1	$-0.08 \leq w_1 \leq 0.08$
	-0.913	$-0.06 \leq w_1 \leq 0.29$
Chevallier-Polarski-Linder parametrization (CPL)	-1	$-0.18 \leq w_1 \leq 0.18$
	-0.878	$-0.64 \leq w_1 \leq -0.26$
Barboza-Alcaniz parameterization (BA)	-1	$-0.10 \leq w_1 \leq 0.10$
	-0.916	$-0.28 \leq w_1 \leq -0.07$
Low Correlation parameterization (LC)	-1	$-1.06 \leq w_1 \leq -0.95$
	-0.912	$-1.08 \leq w_1 \leq -0.96$
Jassal-Bagla-Padmanabhan parametrization (JBP)	-1	$-0.09 \leq w_1 \leq 0.10$
	-0.918	$-0.25 \leq w_1 \leq -0.07$
Wetterich-redshift parameterizations (WP)	-1	$-0.06 \leq w_1 \leq 0.06$
	-0.900	$-0.19 \leq w_1 \leq -0.08$

Table 4: This table shows the different values w_1 have been constrained to using the aforementioned method.

4.7 Schechter Function for a ΛCDM Universe

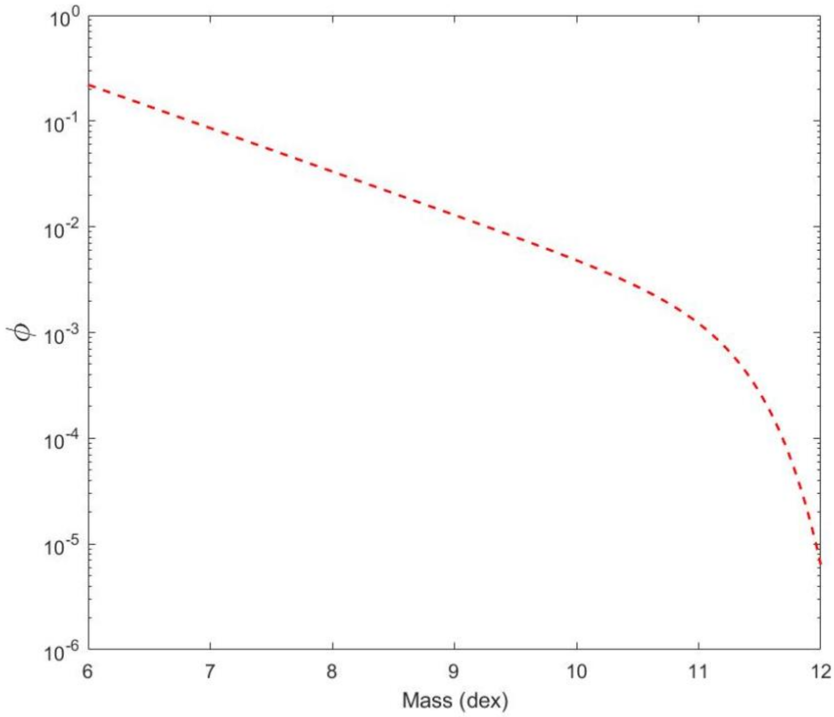

Figure 7: Plot of the Schechter function versus mass for a ΛCDM universe for the redshift range $0.3 < z < 0.5$.

4.8 Number Density for a ΛCDM Universe

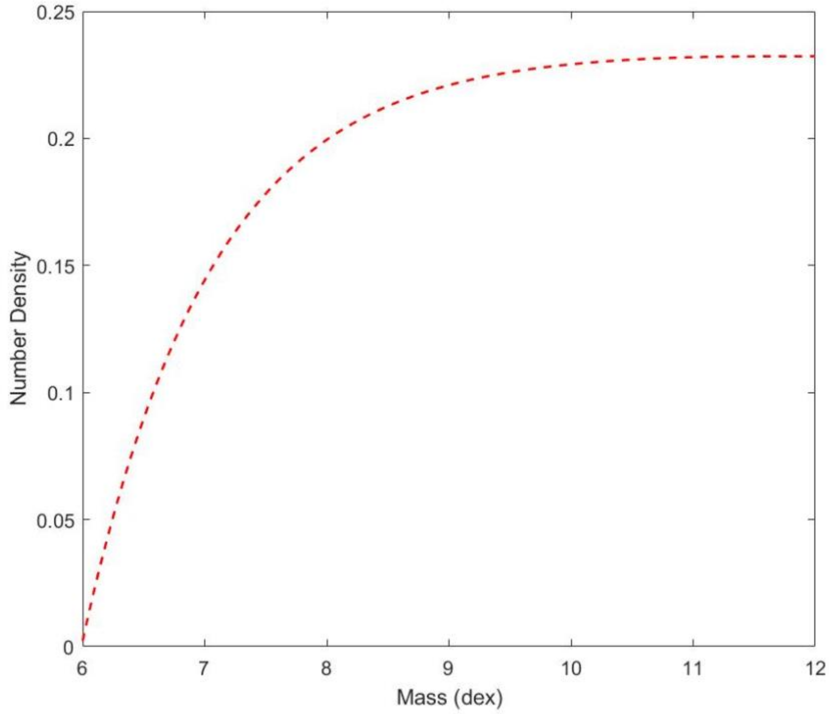

Figure 8: Plot of the number density versus mass of a ΛCDM universe fro the redshift range $0.3 < z < 0.5$.

4.9 Number of Galaxies per Square Degree on the Sky

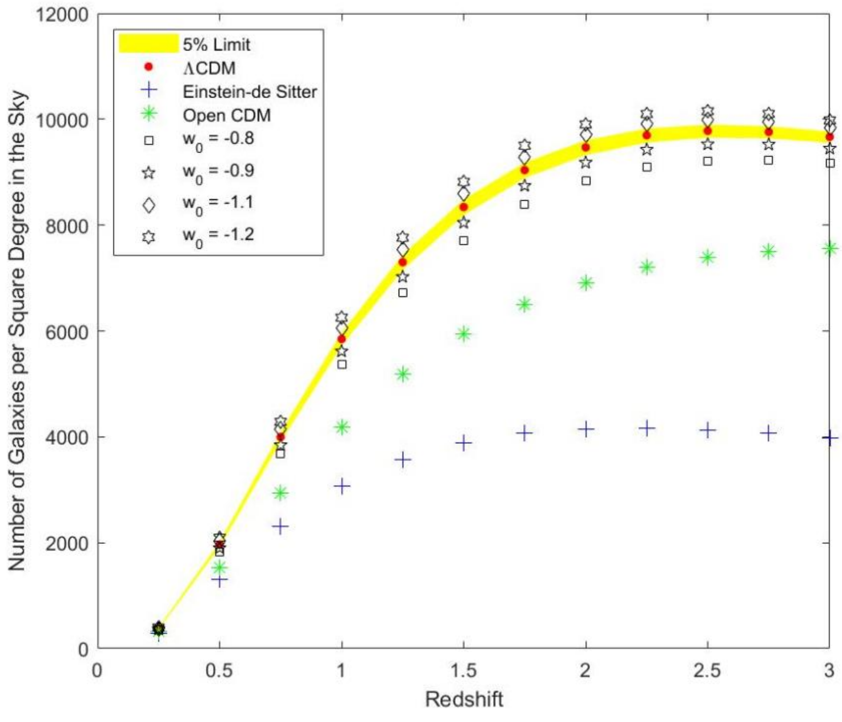

Figure 9: Plot of the number of galaxies per square degree in the sky versus redshift fro a variety of cosmological models.

The resulting table shows the number of galaxies per square degree on the sky for ΛCDM with $w = -1$ with 5% limit for a certain redshift.

Redshift	**Number of galaxies per square degree**
0.25	377^{+5}_{-4}
0.50	1965^{+32}_{-42}
0.75	3996^{+75}_{-76}
1.00	5844^{+108}_{-113}
1.25	7294^{+127}_{-134}
1.50	8336^{+133}_{-142}
1.75	9033^{+131}_{-141}
2.00	9462^{+125}_{-136}
2.25	9691^{+117}_{-128}
2.50	9775^{+107}_{-120}
2.75	9754^{+99}_{-111}
3.00	9660^{+92}_{-100}

Table 5: A table showing the number of galaxies per square degree in the sky for a given redshift for a ΛCDM universe. The limits on the number are representative of the 5% limit placed on the dark energy equation of state.

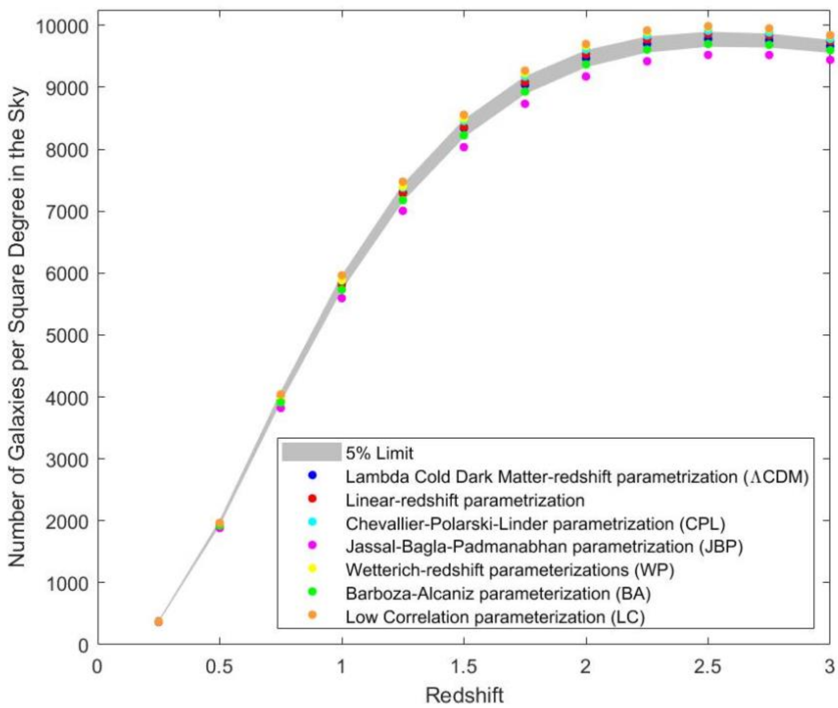

Figure 10: Plot of the number of galaxies per square degree in the sky versus redshift fro a variety of evolving cosmological models.

4.10 Difference in the Number of Galaxies per Square Degree on the Sky from a ΛCDM Universe

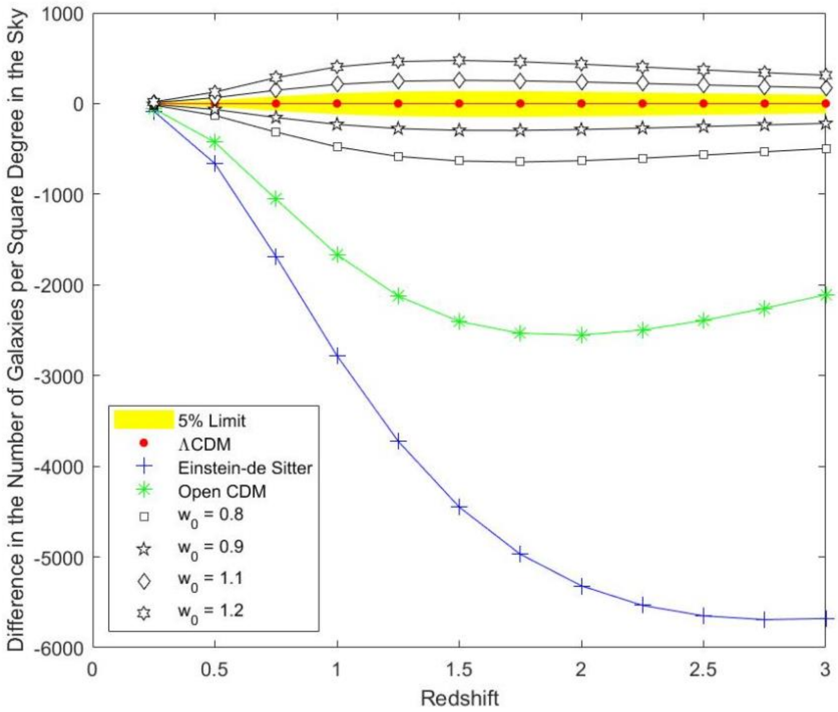

Figure 11: Plot of the difference in the number of galaxies per square degree in the sky from a ΛCDM universe versus redshift fro a variety of cosmological models.

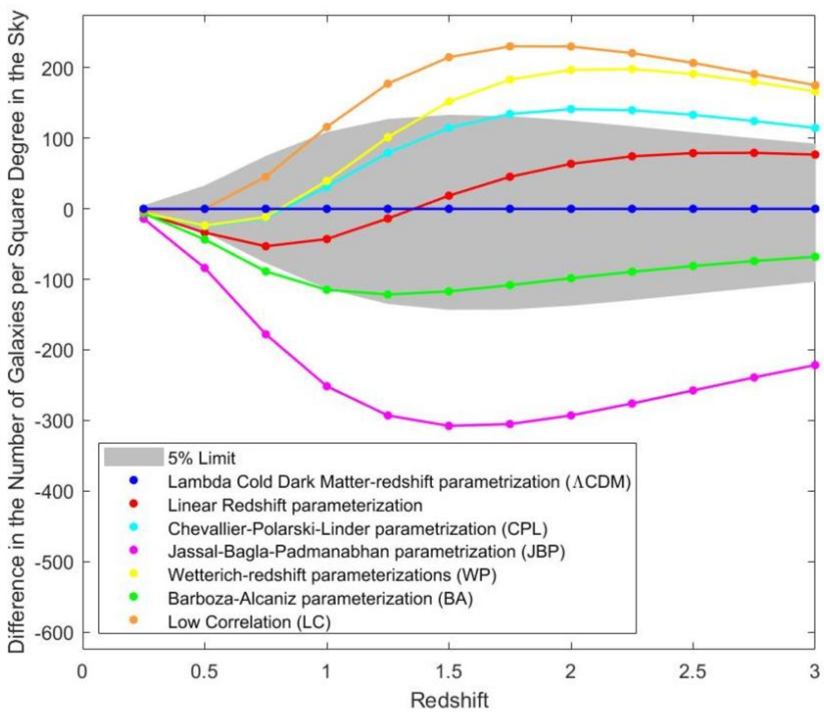

Figure 12: Plot of the difference in the number of galaxies per square degree in the sky from a ΛCDM universe versus redshift fro a variety of evolving cosmological models.

5. Discussion

5.1 Angular Diameter Distance

The angular diameter distance plot shows all cosmologies behaving in a similar manner. They all increase rapidly as redshift increases, until the turnover at approximately redshift 1, except for an open CDM universe in which the turnover is at approximately redshift 2, they then begin to slowly decrease. At redshifts less than 0.5, the different cosmologies are virtually indistinguishable from each other. While a ΛCDM universe with $w \sim -1$ and an open CDM universe are all relatively close together by redshift 3. An Einstein-de Sitter universe never reaches the same height and at redshift 3 has a much lower angular diameter distance. Furthermore, at redshift 3 the angular diameter distance of an open CDM universe and a ΛCDM universe with $w = -0.8$ are almost identical. Physically this means that in all universes we look, the angular diameter distance increases to a point and then decreases. Meaning at redshifts approximately greater than 1, astronomical objects start to become closer to us than what they ought to be. This also leads to them appearing larger in the sky than they ought to be. This magnifying effect would be due to the fact that the proper distance from us to the astronomical object would have been smaller when the light was emitted, and the universe has since expanded. So, light has travelled less distance than what it would have travelled if it was emitted today A universe with a greater abundance of dark energy will have a higher angular diameter peak, this means more distant astronomical objects will be seen before the turnover effect arises . If we take current observations of $w \approx -1$ to within about 5%, then at redshift 3 the galaxies in our universe have an angular diameter distance of $D_A(z = 3) = 1\ 597^{+13}_{-15}\ Mpc$. A 5% limit on ΛCDM with $w = -1$ equals a 1.8% change in the angular diameter distance at redshift 3. At redshift 3 our models have angular diameter distances that vary between 1 077 Mpc and 1 646 Mpc.

5.2 Luminosity Distance

The luminosity distance increases with redshift for all cosmologies, however the relationship is not linear as the lines become steeper at higher redshift. At redshifts less than 0.5, the different cosmologies are virtually indistinguishable from each other. After redshift 0.5, the line for Einstein-de Sitter starts to diverge from the rest but still follows a similar overall shape. The line for an open CDM universe is more curved than the rest and deviates slightly between redshifts 1 and 2.5. A universe with more dark energy has a larger luminosity distance. In an expanding universe the $(1 + z)^2$ factor in D_L makes more distant sources fainter, physically this is due to relativistic effects such as the energy loss of photons and time dilation. The ΛCDM universe, its variations and open CDM curves are very closely packed together. This would make luminosity distance a bad cosmological test as it would be extremely difficult to distinguish between these cosmologies. Furthermore, at redshift 3 a ΛCDM universe with $w = -0.8$ and an open CDM universe are practically identical. Therefore, luminosity distance may only be able to determine between an Einstein-de Sitter universe and all the other cosmologies looked at here. If we take current observations of $w \approx -1$ to within about 5%, then at redshift 3 the galaxies in our universe have a luminosity distance of $D_L(z = 3) = 25\ 550^{+210}_{-240}\ Mpc$. A 5% limit on ΛCDM with $w = -1$ equals a 1.8% change in the luminosity distance at redshift 3. At redshift 3 our models have luminosity distances that vary between 17 240 Mpc and 26 330 Mpc.

5.3 Comoving Volume Element

The comoving volume element for a ΛCDM cosmology and its variations of w rapidly increases until a turnover at approximately redshift 2 and thereafter the comoving volume element begins to slowly decrease. The comoving volume element for an Einstein-de Sitter cosmology follows a similar pattern whereby it increases and then decreases. However, the slope is much less steep, the changes are more gradual, and the curve never reaches the same height as ΛCDM and its variations. Einstein-de Sitter also has its turnover earlier on at approximately redshift 1.5. The curve for open CDM grows less rapidly than any variation of ΛCDM but more rapidly than Einstein-de Sitter. Nonetheless, the open CDM curve continues to grow and has no turnover point up to redshift 3, although the steepness does decrease as redshift increases. It is likely that the turnover point for open CDM is at a redshift greater than 3. The turnover is due to the comoving volume element being given by the angular diameter distance multiplied by other factors, see equation (2.20). the angular diameter distance has a turnover which is then seen in the comoving volume element. Different cosmologies mean the comoving volume element is stretched by the angular diameter distance by different amounts and the turnover for comoving volume element is at a higher redshift than for angular diameter distance. The more dark energy a certain type of cosmology has, the higher its peak is, giving rise to a larger comoving volume element. Logically this makes sense, as we can see that an Einstein-de sitter universe with no dark energy and no accelerated expansion, has a much lower maximum comoving volume element than other cosmologies. The different cosmologies diverge at approximately redshift 0.25, which is earlier on than with other cosmological tests, and they also differ more wildly at higher redshifts, making the comoving volume element an interesting cosmological test as different cosmological models would be easier to distinguish between. If we take current observations of $w \approx -1$ to within about 5%, then at redshift 3 the galaxies in our universe have a comoving volume element of $dV_C(z=3) = 3.918^{+0.080}_{-0.086} \times 10^{10}\ \ Mpc^3$. A 5% limit on ΛCDM with $w = -1$ equals a 4.2% change in the comoving volume element at redshift 3. At redshift 3 our models have comoving volume elements that vary between $9.951 \times 10^9\ Mpc^3$ and $4.206 \times 10^{10}\ Mpc^3$.

5.4 Change in Magnitude from a ΛCDM Universe

For the plot of the change in magnitude versus redshift, the line of ΛCDM with $w = -1$ is horizontal and flat as this is the cosmology we are comparing other cosmologies to. The variations of ΛCDM with $w < -1$ have a positive change in magnitude were as variations of ΛCDM with $w > -1$ have a negative change. Both Einstein-de Sitter and open CDM cosmologies also have a negative change in magnitude. The curve for an Einstein-de Sitter universe deviates drastically from a ΛCDM universe with the change increasing rapidly with redshift, it also shows no sign of plateauing or flattening out. For an open CDM universe, the curve diverges from a ΛCDM universe up until a minimum at approximately redshift 1, then the curve starts to move closer to a ΛCDM universe and crossing the line for a ΛCDM universe with $w = -0.8$. The variations of w in a ΛCDM universe all follow similar patterns. They diverge from a ΛCDM universe with $w = -1$ until approximately redshift 1 and then begin to flatten out and the change in magnitude decrease so slightly it becomes almost constant. At redshift 3 the open CDM and ΛCDM with $w = -0.8$ curves have a virtually identical change in magnitude. If the change in magnitude is negative this means that galaxies in the cosmology we are looking at have a lower magnitude than in a ΛCDM universe and so have brighter galaxies. If the change in magnitude is positive this means that galaxies in the cosmology we are looking at have a higher magnitude than in a ΛCDM universe and so have dimmer galaxies. This can be seen by comparing an Einstein-de Sitter universe with a ΛCDM universe, an Einstein-de Sitter universe has no dark energy and so no accelerated expansion, therefore astronomical objects are closer to us and so appear to be brighter relative to a ΛCDM universe. If we take current observations of $w \approx -1$ to within about 5%, then at redshift 3 the galaxies in our universe have a change in magnitude of between -0.01976 and 0.01833. At redshift 3 the greatest change in magnitude from ΛCDM is -0.8539 for an Einstein-de Sitter universe.

5.5 Change in Volume from a ΛCDM Universe

For the plot of the change in volume of different cosmologies compared to a ΛCDM cosmology with $w = -1$ versus redshift, the line of ΛCDM cosmology with $w = -1$ is flat and horizontal as there would be no change in volume as this is the cosmology we are using as a base to measure the change of other cosmologies. For variations of w for a ΛCDM cosmology, all curves diverge from ΛCDM until approximately redshift 1, then the curves become almost flat with the change in volume decreasing minimally and so the change in volume becomes close to constant for higher redshift. For ΛCDM cosmologies with $w > -1$, the change in volume is negative giving these cosmologies a smaller volume than ΛCDM as there would be less dark energy and so a slower rate of accelerated expansion. For ΛCDM cosmologies with $w < -1$, the change in volume is positive giving these cosmologies a larger volume than ΛCDM as there would be more dark energy and so a higher rate of accelerated expansion. The curve for an Einstein-de Sitter universe greatly deviates from ΛCDM for lower redshifts, however at approximately redshift 1, the curve starts to flatten and become less steep. Einstein-de Sitter has a negative change in volume meaning an Einstein-de Sitter universe has less volume than a ΛCDM universe with $w = -1$. The change in volume becomes more negative with redshift meaning the volume in an Einstein-de Sitter universe become increasing smaller than it would be in a ΛCDM universe at higher redshifts. This makes sense as an Einstein de-sitter universe has no dark energy and so no accelerated expansion. An open CDM universe also has a negative change in volume. It diverges away from ΛCDM until it reaches a minimum at approximately redshift 1.5, then slowly the change in volume decreases becoming less negative at higher redshifts. Open CDM does not have as great a change in volume as an Einstein-de Sitter universe. If we take current observations of $w \approx -1$ to within about 5%, then at redshift 3 the change in volume is between -0.02693 and 0.02564. At redshift 3 the greatest change in volume from ΛCDM is -0.6926 for an Einstein-de Sitter universe.

5.6 Dark Energy Parameterizations

Table 4 shows further constraints we have placed on w_1 for the evolving dark energy parametrisations. These values were obtained by choosing a constant value of w_0 and then varying w_1 so that we could determine the values of w_1 that lie within the 5% limit to which the dark energy equation of state is currently constrained to. By doing this we have been able to use the comoving volume test to place further constraints on the possible values of the redshift dependent term in these dark energy models. It has been shown that the values for w_1 are small. This implies that if dark energy does evolve in time then its changes are subtle and gradual, and the dark energy equation of state will be approximately constant for recent times with only significant deviations at larger redshifts. This can also bee seen in figure 1.

5.7 Schechter Function

The Schechter function we have used was taken from combined results of many astronomical surveys, most of which were in the near infrared range but also included optical observations, such as GOODS (The Great Observatories Origins Deep Survey). The data was used to probe galactic evolution and was chosen due to the large mass range covered by combined photometry across all of the surveys. The form of the Schechter function was taken from the redshift range $0.3 < z < 0.5$, while this doesn't completely describe the galaxy mass function, we have taken it to a reasonable approximation as being able to describe the mass function across the entire redshift range of $0 < z < 3$. Examining table 3 which gives the values of the turnover mass, normalisation and slope of the low mass end of the function for the single Schechter function it can be seen that the values of these variables do not drastically change across the entire redshift range and show this approximation of only using the initial redshift range is appropriate. As previously mentioned the goal of the Schechter function was to probe both high and low mass ranges of galaxy stellar mass function across a wide variety of photometry and redshifts, the paper this was based on successfully creates a stellar mass function for galaxies across a wide range of masses so basing our observations on this Schechter function provides an accurate description to be used to find the number of galaxies in the sky per square degree for different cosmologies.

5.8 Number Density

The number density plot is derived from the single Schechter function (equation 2.25), it shows how total galaxy density changes with an increasing upper limit on the integral of the Schechter function. Due to the nature of the Schechter function the number density has a sudden turn off around a mass of 8 in log units, at this point the number density no longer continues to grow and is fairly constant with increasing mass. This means that at the higher end of the mass scale (~ 12) we would not expect any shift in the upper limit to drastically affect the number densities of the galaxies, this supports our approximation to only shift the lower limit of the Schechter function integral when looking at different cosmology models.

5.9 Number of Galaxies per Square Degree on the Sky

Our plot for the total number of galaxies an observer would expect to see per square degree in the sky as a function of redshift shows a very similar appearance to the dependence of the comoving volume element plotted against redshift (Figure 9), this suggests that the number of galaxies per square degree in the sky make a good tracer for the comoving volume. At a redshift of $z \leq 1.25$ we see a linear increase in the number of galaxies for a ΛCDM universe. As we increase the redshift we see a gradual drop off in the number of galaxies observed between the redshift ranges $1.25 < z < 2.25$, then at $z \sim 2.5$ the number of galaxies begins to flatten out to $\sim 9\,800$ and slowly start to decrease with increasing redshift. Included in this plot is the 5% confidence interval in the equation of state to which the dark energy is currently constrained. At a redshift of $z = 1$ our model shows that the confidence interval is separated with an upper limit of 7421 and a lower limit of 7160, at redshift $z = 3$ this interval has an upper limit of 9752 and a lower limit of 9560. This shows a relatively small separation of 192 galaxies within the total confidence interval current models constrain the dark energy equation of state to. One interesting point to note that can be seen in our plot is that in a universe with less dark energy we would expect to see less galaxies per square degree of the sky than a universe with more. This can appear to be somewhat counter intuitive as with more dark energy we would expect to count less galaxies in a given magnitude due to the accelerated expansion causing galaxies to be further away and therefore fainter, moving out of the magnitude limit we are counting in and cause an overall decrease in the number of galaxies we observe. This is seen physically in Figure 3 which shows that a cosmology with less dark energy has a lower luminosity distance. Another effect that the dark energy can cause on the number of observed galaxies is that the shells of comoving volume we are using to find the total number of galaxies are larger in a universe with increased dark energy (Figure 4), this means that the number of galaxies we will see will be larger in a cosmology with more dark energy. It is clear from our model that the effect of increased volume from the dark energy accelerating the expansion of the universe dominates over the effect of increased luminosity distance when observing the number of galaxies in the sky.

The plot shows the same flattening off for an Einstein de-Sitter universe that is seen in the comoving volume plot at a redshift of $z \sim 1$. At low values of redshift ($z < 1$) all of the cosmologies appear to contain a similar number of galaxies per square degree, at larger redshifts however the models drastically diverge from each other. At a redshift of $z = 3$ an ΛCDM universe with $w = -1$ contains roughly 6000 more galaxies per square degree than an Einstein de-Sitter universe. This is a substantial difference when compared to the difference at lower redshifts, at $z = 0.25$ a ΛCDM with $w = -1$ universe contains only 83 more galaxies, at $z = 0.5$ this difference has grown to 661 and by $z = 0.75$ the two models start to significantly diverge with a difference of 1686 galaxies. An open universe, while containing more galaxies per square degree when compared to an Einstein de-Sitter universe still does not come close to the small range that we would expect to see for a universe that contains dark energy given the current constraints placed on the equation of state. This indicates the strong contribution the dark energy creates on the increased volume of the universe which cannot be replicated by any increased volume caused by a negative curvature term seen in an open universe.

While our plots do indicate that the number of galaxies per square degree in the sky could serve as a good tracer for assessing the comoving volume element for different constraints on the dark energy equation of state. One potential issue we face with using the number of galaxies per square degree in the sky is that galactic evolution is not fully understood. This means that the number of galaxies we observe may not evolve at the same rate of how the comoving volume of the universe changes. Events such as galaxy mergers would change the number of galaxies in a given volume of the universe which would lead to false indications of a change in the comoving volume of the universe. A possible way in which this uncertainty could be reduced would be to take observations from multiple points in the sky and use many counts of galaxies in a square degree of the sky in order to be able to reduce the overall error from galactic evolution. A more complete understanding and theory of galaxy evolution would be able to solve these uncertainties as the number of galaxies we would expect to see would be fixed with a theory of how galaxy counts change with redshift.

The method used to obtain the number of galaxies for different cosmologies does come with some related issues. We use the ratio of the luminosity distances to find a new lower limit on the integral, but this does not fully describe the form of the Schechter function in a new cosmology. The actual form of the Schechter function should change in a different cosmology, the Schechter function used is designed to fit to a ΛCDM cosmology so will not fully describe cosmologies that diverge significantly from ΛCDM cosmology, such as an open or Einstein-de Sitter universe. Considering that the main areas of interest are looking at a ΛCDM cosmologies with an equation of state parameter close to $w = -1$ we have taken it to be a reasonable approximation that the form of the Schechter function we have used should work well for most of the significant cosmologies we have modelled.

Figure 10 shows the number of galaxies per square degree in the sky for cosmologies involving an evolving equation of state terms as described in [17], the numerical values for the variables involved in the evolving terms for the equation of state are given in Table 2. We have chosen these set of parameters in order to be able to try to fit the evolving models as closely as possible to the constraints placed on ΛCDM. From this we can see that the constraints placed on these parameters are able to give counts of the number of galaxies per area of the sky that mostly lie within the 5% confidence interval that dark energy is constrained to. This indicates that, with some additional restraints placed on the w_0 and w_1 terms, then these evolving dark energy models could accurately describe the change in volume element we expect to see from dark energy permeating the universe and causing an accelerated expansion. An evolving dark energy term can be appear to be more suitable to describe the dark energy present in the universe as we know that dark energy domination has only recently come into action (~ 3.9 billion years ago) to dominate the expansion of the universe and so an evolving dark energy could explain why dark energy has only recently come into effect and cause an accelerated expansion of the universe.

5.10 Difference in the Number of Galaxies per Square Degree in the Sky from a ΛCDM universe

The change in the number of galaxies seen in the sky relative to a ΛCDM universe shows more clearly the differences we would expect from astronomical data to differentiate between different cosmologies. It also more clearly shows the effect that the accelerated expansion caused by dark energy has on the expected number of galaxies in the sky. In Figure 11 we can see more explicitly how the 5% limit changes the observed number of galaxies, we can see that the 5% limit on the dark energy equation of state produces a difference of about ± 130 galaxies at $z = 2$. Again, we can clearly see a large deviation of an Einstein de-Sitter universe which contains ~ 5000 less galaxies than a ΛCDM universe at $z = 2$, this confirms the redundancy of a matter dominated cosmology with no dark energy.

The evolving models of dark energy show that with the difference in the number of galaxies goes outside of the expected 5% limit at various redshift intervals, for example the JBP, CPL, LC and WP models all diverge from this confidence interval and only linear redshift parametrisation and BA models are well behaved following the expected difference in the number of galaxies. This is expected as the parametrisations used in these models were fitted from SNe Ia JLA and BAO constraints and not from the comoving volume constraints we found in table 4. This validates the need to constrain these evolving parameters further and suggest that the constraints we have placed on them are more fitting for a comoving volume cosmology test.

6. Conclusion

There are currently many theoretical models of our universe, each model contains different numerical values for the various density parameters that make up the total density of the universe. In order to be able to differentiate between these models and get a clearer idea on which models accurately predict observations, cosmologists have developed various cosmological tests. These cosmological tests have taken many different forms, one of these has been the dN/dz test which looks at how the comoving volume element of the universe changes with redshift. The test is done by seeing how the counts of a tracer with a known number density varies with redshift. By seeing how the counts of the tracer changes with redshift the evolution of the comoving volume can be inferred. The test has been done with other tracers before, but we have developed a model for using galaxies as the tracer to measure the comoving cosmological volume. Our model used a Schechter function to find a number density of galaxies in different cosmological models and at different redshifts, we then modelled the number of galaxies per square degree against redshift using shells of comoving volume at various redshift intervals in the given cosmological model. This showed us how the number of galaxies could potentially be used to trace the comoving volume element and also gave numerical data for how the number of galaxies per area of the sky in different redshift intervals would change for different cosmologies. We modelled several different cosmological models to see how the number of galaxies would change within them, the most significant models we looked at were ΛCDM models with a constant dark energy equation of state around $w \sim -1$ as well as evolving time-dependent dark energy models.

We found that the evolution of the number of galaxies per square degree of the sky followed the evolution of the comoving volume element very closely. The number of galaxies seen in the sky for a given magnitude limit had the same redshift dependence as the comoving volume, this indicates galaxies as being good tracers for the comoving volume of the universe. We found that for a lower mass limit of $10^{10} M_{\odot}$ and upper mass limit of $10^{12} M_{\odot}$ at redshift $z = 2$ we would see a difference of about 200 galaxies separating the upper and lower limit of the 5% interval that the dark energy equation of state is currently constrained to. We looked at six different models for an evolving dark energy equation of state term, we found that by looking at these models with parameters that have been constrained with data from SNe Ia JLA and BAO data we were able to model different evolving dark energy cosmologies with a number of galaxies that agrees with the number expected from the 5% limit that the dark energy equation of state is currently believed to fall in. By modelling comoving volume against redshift we placed further constraints on the w_1 parameter that controls the evolving dark energy factor, these constraints were obtained by fitting the models to the 5% limit of the dark energy EoS that it is currently constrained to.

Galaxies do not make perfect tracers for the comoving volume of the universe, since a complete theory for galaxy formation and evolution does not currently exists. This means that the number of galaxies we observe is subject to poorly constrained evolutionary effects. Events such as galaxy mergers would have an effect in the number of observed galaxies. While this is a fault with using galaxies as tracers, the effect of galactic evolution on counts of galaxies could be reduced by counting galaxies from different parts of the sky, this would reduce the noise in tracer counts caused by galactic evolution. In addition to this, the method could be used with other cosmological tests in order to be able to more accurately determine the abundance of dark energy in the universe as well as how the dark energy in the universe varies with time. By examining multiple different points of the sky and by combining data from other cosmological tests we believe the model we have developed could be used to help to constrain and differentiate between the different cosmological models of dark energy that exist in the current literature. Constraining different models for how dark energy has evolved over the cosmological timescale would allow us to further pinpoint the underlying physics and mechanics that governs dark energy, much of which is still unknown within the field of cosmology.

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