MEASURING THE COSMIC EQUATION OF STATE WITH COUNTS OF GALAXIES

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ABSTRACT

The classical *dN/dz* test allows one to determine fundamental cosmological parameters from the evolution of the cosmic volume element. This test is applied by measuring the redshift distribution of a tracer whose evolution in number density is known. In the past, ordinary galaxies have been used for this; however, in the absence of a complete theory of galaxy formation, that method is fraught with difficulties. In this Letter, we propose studying instead the evolution of the apparent numbers of dark matter halos as a function of their circular velocity, observable via the line widths or rotation speeds of visible galaxies. Upcoming redshift surveys will allow the line width distribution of galaxies to be determined at both $z \sim 1$ and the present day. In the course of studying this test, we have devised a rapid, improved semianalytic method for calculating the circular velocity distribution of dark halos based on the analytic mass function of Sheth, Mo, & Tormen and the formation time distribution of Lacey & Cole. We find that if selection effects are well controlled and minimal external constraints are applied, the planned DEEP Redshift Survey could allow us to measure the cosmic equation-of-state parameter w to $\pm 10\%$ (as little as 3% if Ω_m has been well determined from other observations). This type of test also has the potential to provide a constraint on any evolution of w, such as that predicted by "tracker" models.

Subject headings: cosmology: miscellaneous — cosmology: observations — cosmology: theory — dark matter — galaxies: fundamental parameters

1. INTRODUCTION

For almost as long as they have been known to be located outside the Milky Way, galaxies have been used to probe the geometry of the universe (Hubble 1926). Fairly early in the history of relativistic cosmology, it was realized that if the number density of galaxies is known, their apparent numbers per solid angle and redshift interval depend directly on basic cosmological parameters (Tolman 1934). Applying this "*dN/dz* test" requires a substantial sample of distant galaxies with measured redshifts; it was first applied by Loh & Spillar (1986) using a set of 406 galaxies with estimated photometric redshifts z_p between 0.15 and 0.85. Assuming that the total comoving number density of the galaxies is constant and that the luminosity function of the galaxies retains a similar shape over that redshift range, they measured $\Omega = 0.9^{+0.6}_{-0.5}$.

However, it remains unclear whether or not the assumptions of Loh & Spillar hold; no complete theory or simulation of galaxy formation and evolution as yet exists (Benson et al. 2000). It would thus be preferable to apply such a test to objects whose abundance can be calculated semianalytically or via computer models, eliminating such ambiguities. In this Letter, we suggest that the observed numbers of galaxies (and thus indirectly dark halos) as a function of line width and redshift provide a candidate for improving on the analysis of Loh & Spillar, particularly because such quantities will be measured by surveys occurring in the next decade.

In the process of studying this possibility, we have developed a semianalytic method for determining the abundance of dark halos as a function of their circular velocity ($v_c^2 \equiv GM/r$) and redshift that can provide results similar to those of *N*-body simulations in considerably less computer time and without the effects of limited resolution. In essence, we begin with the abundances of galaxies as a function of mass predicted by the modified Press-Schechter formalism of Sheth, Mo, & Tormen

(2000), which provides an excellent fit to dark matter simulations. We then determine the probability that a halo of given mass has a formation time such that it will virialize with the desired circular velocity using the semianalytic method of Lacey & Cole (1994), which again is very well reproduced by simulations. By integrating over the distribution in mass, we may then determine the total abundance of objects with a given circular velocity at the epoch of interest. We describe this method in more detail in § 2 and the prospects for our proposed test in § 3.

2. DERIVING THE VELOCITY FUNCTION OF DARK HALOS

A number of semianalytic methods for calculating the velocity distribution of dark halos based on the Press-Schechter (Press & Schechter 1974) formalism have been proposed in the past. Narayan & White (1988) used the assumption that halos have just virialized at the epoch of observation to map the Press-Schechter distribution of halos in mass into a distribution in velocity for an $\Omega = 1$ cold dark matter (CDM) universe; Kochanek (1995) extended their analysis to other common cosmological models. However, the assumption that objects observed are only just virializing is clearly inappropriate in many cases, particularly for lower mass halos; once objects virialize and break away from the cosmological expansion, their circular velocities evolve little if at all save in major mergers (Ghigna et al. 2000). To account for this, Kitayama & Suto (1996a) proposed that the distribution of X-ray temperatures of clusters ($T \propto v_c^2$) be calculated by totaling the rate at which objects of the desired temperature that survive without merging into a much more massive object through the epoch of interest form, using the results of Lacey & Cole (1993). As the authors of that paper noted, however, there remain some ambiguities in their formulation.

We have attempted to improve on these prior methods for calculating the velocity distribution of dark matter halos, taking advantage of the progress made over the past few years in

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extending the Press-Schechter formalism. It has been repeatedly demonstrated that standard Press-Schechter calculations over-predict the abundance of low-mass halos compared with equivalent simulations; a number of remedies have been suggested recently (e.g., Bond & Myers 1996). One of the simplest to implement is that of Sheth et al. (2000), which accounts for the average ellipticities of dark matter halos by making the critical overdensity for their collapse dependent on mass. The resulting analytic expression for the number density of halos of given mass (given by their eqs. [1] and [6]) provides an excellent fit to the results of a variety of cosmological simulations. The results of Sheth et al. allow a prediction of halo abundances that is much improved over the standard Press-Schechter formula with only minimal complications to the calculation. We therefore use their equations to determine the distribution of halos in mass at a given redshift.

Because the density of the universe was greater at early times, objects that virialized at high redshift are more compact than those that virialized more recently, and therefore they have a higher circular velocity for the same mass. Furthermore, once it has collapsed, the circular velocity of a halo changes only minimally (Ghigna et al. 2000). Therefore, in determining the abundance of galaxy-scale halos with a given circular velocity today, it is preferable to take into account the time of formation of those halos in addition to their mass. Lacey & Cole (1993) derived a method for the distribution of formation times of halos with a given mass that exist as virialized objects at a given redshift (defining the formation time as the epoch when the object first had at least half its final mass). This formula provides an excellent fit to the results of simulations (Lacey & Cole 1994), even though it assumes spherical collapse while the simulated halos could be ellipsoidal. We therefore have used their semianalytic formula (eq. [2.19] of Lacey & Cole 1994) to determine dp/dt_f, the probability distribution of the epoch of collapse t_f of halos observed to have mass M at time t_0.

In practice, we are interested in calculating the abundance of halos at redshift z that have circular velocity v_c; for each halo mass, we therefore need only evaluate dp/dt_f at that formation epoch for which a halo of the given mass would virialize with the desired circular velocity. We may obtain that time from the relationship between the circular velocity and the mass M, which is most conveniently parameterized by using the length scale $r_0(h^{-1}\ \mathrm{Mpc}) \equiv (3M/4\pi\rho_0)^{1/3}$, where ρ_0 is the present-day mass density of the universe. By combining the definitions of circular velocity, of r_0, of Δ_{vir} (the ratio of the mean density of a virialized halo to the critical density of the universe when it virialized), and of the dimensionless evolution of the Hubble parameter $E(z)$, we find (cf. Narayan & White 1988; Kochanek 1995)

$$v_c = \Delta_{\mathrm{vir}}^{1/6}\Omega_m^{1/3}E(z)^{1/3}\frac{H_0 r_0}{\sqrt{2}\,2^{1/3}}, \qquad (1)$$

where we have determined the circular velocity for a halo of mass $M/2$, which is more appropriate given our definition of the formation time. The objection might be raised that we are using a formula for circular velocity that implicitly assumes spherical geometry but that we are applying it to potentially ellipsoidal dark matter halos; however, Cole & Lacey (1996) found that the velocity dispersions of dark matter halos calculated using a singular isothermal sphere model (for which $\sigma \equiv \sqrt{2}v_c$) matched the properties of N-body–simulated halos

quite well. In our calculations, we have used the fitting formulae for Δ_{vir} given by Bryan & Norman (1998).

Thus, to calculate the comoving abundance of halos with a given circular velocity at an epoch of interest, we simply calculate the distribution of halos in mass on a 500-element grid, determine the probability that those halos have the desired circular velocity, and then integrate numerically over that grid. This procedure takes ∼5 s to calculate the abundance at a single circular velocity and epoch, far faster than simulations could be run, and has none of the limitations of resolution or sample size that affect even the largest N-body simulations today. Our technique seems to be similar to the one employed by Kitayama & Suto (1996b) to check their methods, save for the use of the ellipsoidal-collapse mass function rather than a traditional Press-Schechter calculation; however, details on the latter are minimal.

Our procedures could certainly be improved. For instance, the connection between circular velocity, formation time, and mass could be made using a more realistic model than a singular isothermal sphere (Navarro, Frenk, & White 1995; Bullock et al. 2000). Calculation of formation times using an ellipsoidal-halo abundance paradigm rather than the Press-Schechter method of Lacey & Cole might also yield modest improvements. Finally, we must note that the mass function of Sheth et al. was based on a fit to mass functions of halos identified in N-body simulations using the "friends-of-friends" algorithm with selection parameter $b = 0.2$; in such a method, as in standard Press-Schechter calculations, smaller subhalos within a more massive halo (e.g., galaxies within a cluster) are not counted separately in the mass function. Unless an improved method taking this into account is used, we can only calculate the abundance of relatively *isolated* dark matter halos (i.e.,
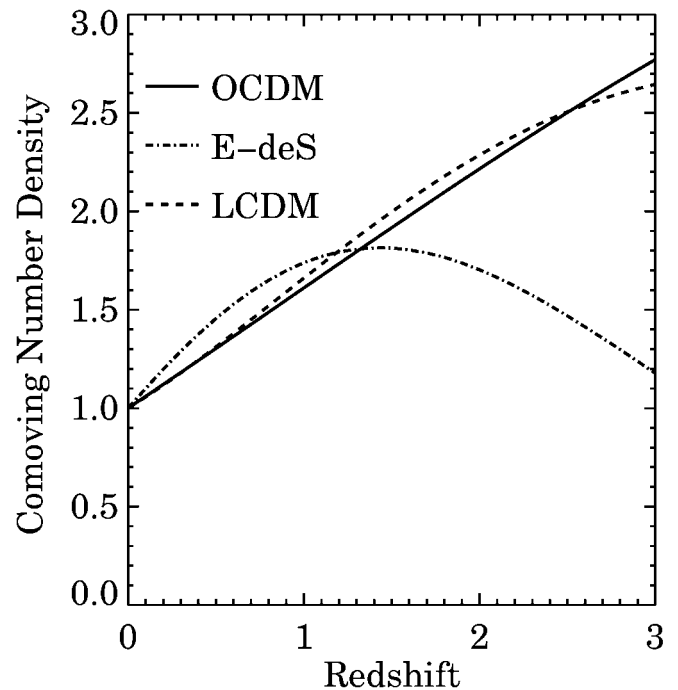

FIG. 1.—Evolution of the differential comoving number density of halos with a circular velocity of 200 km s^{-1} normalized to the value at $z = 0$. We consider three common cosmological models: an open CDM model with $\Omega = 0.3$ today (OCDM), an Einstein–De Sitter model with $\Omega = 1$ (E–deS), and a flat model having $\Omega_m = 0.3$ today and a cosmological constant (LCDM). Note the similarity of the predictions at $z \sim 1$.

those that are not part of a larger, virialized group or cluster). Such objects are readily identified observationally in well-sampled redshift surveys, but this is still a significant limitation. Despite these caveats, we believe that our technique provides a substantial advance over previous methods used; all of them suffer from similar flaws and have fewer advantages. Our method may also be useful for a variety of cosmological calculations; for instance, the circular velocity distribution of dark halos is required for calculations of the abundance of strong gravitational lenses and hence for the limits placed on Ω_Λ thereby.

3. THE PROPOSED TEST

In order to study the evolution of the volume element of the universe, we require some identifiable tracer whose number density evolution is well known. The dark halos that contain galaxies are an obvious candidate; the results of semianalytic calculations (e.g., Sheth et al. 2000) and *N*-body simulations (e.g., Jenkins et al. 1997) for the behavior of dark matter are by now fairly well understood. Of course, directly observing the properties of dark halos at high redshift is impossible; however, some of their parameters may be determined by studying galaxies lying within them. In particular, it is well known that the motion of material in the outer parts of typical galaxies is dominated by the gravitational influence of dark matter (Fischer et al. 2000); the depth of the halo potential well may thus be directly derived from observed line widths or velocity dispersions of galaxies. Unlike mass measurements, such observations are relatively insensitive to distance from a galaxy's center

(for instance, the rotation curves of many spiral galaxies are observed to be fairly flat over a great range of radii). Under modest assumptions (e.g., about their radial profiles), the expected abundance of dark matter halos as a function of observable line width may be determined from simulations or from the semianalytic method described in § 2.

The results of our semianalytic calculations for the evolution of the differential comoving abundance of isolated halos with a circular velocity of 200 km s^{-1} in some commonly used cosmological models are depicted in Figure 1. A CDM-like power spectrum with a shape parameter of $\Gamma = 0.25$ and normalized to match the value of σ_8 derived by Borgani, Plionis, & Kolokotronis (1999) from X-ray observations of galaxy clusters was employed in all cases. Since it is likely that selection effects will be somewhat similar for galaxies at $z = 0$ and
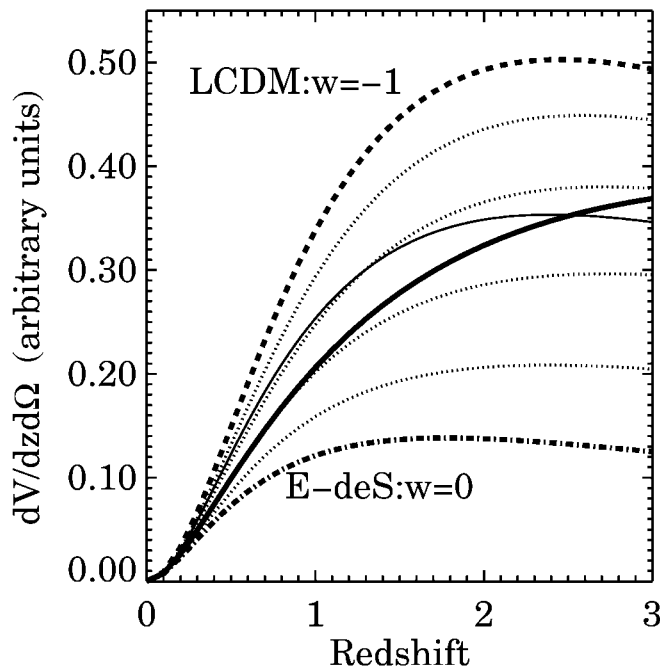
Fig. 3.—Predicted 95% confidence level contours for application of the *dN/dz* test to the DEEP Redshift Survey (*solid curves*) along with other existing or predicted constraints. These have been obtained by conservatively assuming that the DEEP Redshift Survey will obtain useful line width information on 10,000 galaxies distributed as $(1 + z)^{-2}$ among 8 bins between $z = 0.7$ and 1.5 (i.e., that the great majority of observed galaxies are excluded from the test because of potential systematic errors or incompleteness). *Top*: Confidence contours in the Ω_m-Ω_Λ plane (i.e., under the assumption that $w = -1$) about the point ($\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$). Overplotted are the current 68% confidence constraint on SNe Ia from Perlmutter et al. (1999, *dashed curve*), the target 95% confidence interval (statistical errors only) for the proposed *SNAP* satellite (*dot-dashed curve*, taken from figures on the SNAPSAT Web site at http://snap.lbl.gov), and a sample CMB constraint (Melchiorri et al. 1999) with errors reduced to simulate a determination of $\Omega_{curvature}$ to ± 0.1, representative of what may be expected from experiments performed before DEEP is completed, such as BOOMERANG and *MAP* (*dotted lines*). Our estimated constraints are similar in strength to those predicted for SNAPSAT, although less sensitive to the present-day curvature and more sensitive in the orthogonal direction (i.e., more complementary to CMB measurements). The solid curves in the bottom panel show the predicted 95% confidence contour from the DEEP Redshift Survey in the Ω_m-w plane (under the assumption of a zero-curvature universe) about the points ($\Omega_m = 0.3$, $w = -0.7$) and ($\Omega_m = 0.3$, $w = -1$). Also plotted are 95% confidence constraints for the latter model from the SNAPSAT (*dot-dashed curve*) and observations of abundances of clusters by the proposed *COSMEX* satellite (*gray region*; see Haiman, Mohr, & Holder 2000). The techniques involved (and thus any systematic errors) differ greatly among these methods, making each a valuable check on the others. Since Ω_m will be increasingly constrained from other experiments in the near future (including other analyses of the DEEP data set), the test we propose has the potential to determine w to ~3%, even without other complementary measurements.

Fig. 2.—Comoving volume per unit redshift per steradian, $dV/dz\, d\Omega$, in a variety of cosmological models (see Peebles 1993). The thickest curves indicate the models considered in Fig. 1; the thick solid curve depicts the volume element in an $\Omega_m = 0.3$ open universe, the thick dot-dashed curve shows an Einstein–de Sitter model, and the thick dashed curve depicts an LCDM model. The thin dotted curves show the evolution of the volume element in standard quintessence models with $\Omega_m = 0.3$, $\Omega_Q = 0.7$, and constants $w = -0.2, -0.4, -0.6$, and -0.8 (listed from lowest volume to highest). Finally, the thin solid line depicts a model in which w is proportional to the expansion parameter, $w = -0.8a$.
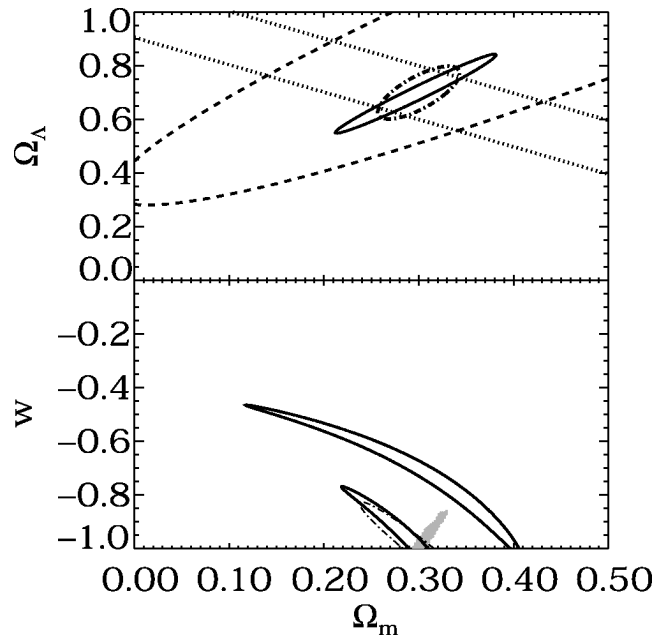
$z \sim 1$, we plot the ratio of the abundance of galaxies at a given redshift to that at $z = 0$. The abundance of halos at low circular velocities is smaller now than in the recent past because of merging of smaller halos into new, larger objects and because smaller halos lose their individual identities in a Press-Schechter framework when they are included in a larger virialized object such as a group or cluster. Qualitatively, the predictions of N-body simulations are similar (e.g., Ghigna et al. 2000). Our calculations demonstrate that the comoving abundance of halos with fixed line width at $z \sim 1$ relative to today is very insensitive to the background cosmological model under reasonable assumptions about the matter power spectrum; e.g., for models with a low matter density $\Omega_m = 0.3$, the relative abundance is only 2% different in a flat model from an open one. Thus, observations of the apparent numbers of such objects per unit redshift per steradian in comparison with their abundance today directly yields a measurement of the cosmological volume element and hence of fundamental cosmological parameters.

A measurement of $dN/dz \, d\Omega$ will be quite feasible once the Deep Extragalactic Evolutionary Probe (DEEP) Redshift Survey (Davis & Faber 1998) is completed. This survey, which will begin in early 2001 and should be completed within the 5 years following, will obtain spectra of $\sim 60,000$ galaxies at redshift $z > 0.7$ with a resolution of ~ 80 km s^{-1} FWHM, allowing measurements of line widths for perhaps one-quarter to one-half of the galaxies in the sample. The number of spectra obtained will be great enough to test for selection effects in redshift or line width by comparison with calculations or simulations at the few percent level. For instance, smaller halos are predicted to be distributed in circular velocity according to a power law with a slope that is well determined by both N-body simulations and our techniques; deviation from such a distribution would be a strong indicator of observational selection effects, allowing us to correct for or avoid regions of parameter space suffering from incompleteness. Complementary information on the line width distribution of galaxies at low redshift should be available from the Sloan Digital Sky Survey (Loveday et al. 1998), the Two Degree Field Survey (Colless 1998), or other local surveys, allowing accurate normalization to the abundance of halos at $z = 0$.

The determination of the volume element afforded by such measurements can place constraints not only on the simplest models of the universe, which include only matter and a cosmological constant, but also on so-called "quintessence" models (Turner & White 1997; Caldwell, Dave, & Steinhardt 1998), in which the equation of state of a nonmaterial component, $P = w\rho$, can take an arbitrary value between -1 and 1 ($w = 0$ for matter, while $w = -1$ for a cosmological constant). The amount of comoving volume per unit redshift per steradian at $z \sim 1$ depends strongly on w, as illustrated in Figure 2. The test we propose, in combination with a determination of Ω_m (e.g., from velocity statistics of galaxies), should allow a direct determination of the equation of state of any dark energy present, as illustrated in Figure 3. In comparison, cosmic microwave background (CMB) measurements planned for the near future and ground-based observations of the apparent magnitudes of Type Ia supernovae (SNe Ia) can constrain w to $\sim 10\%$ at best (Hu et al. 1999); similar accuracy might be achieved by using the statistics of lensed sources in the Sloan Digital Sky Survey (Cooray & Huterer 1999). The technique we propose is at least as effective as these, even without including complementary information from other tests.

The dN/dz test could also detect the evolution of w, if any exists and if other cosmological parameters are well determined. It has recently been proposed that in reasonable quintessence models, w may vary with the current matter density as the universe evolves (the so-called "tracker solutions"; see Steinhardt, Wang, & Zlatev 1999) or even oscillate over time; such changes would be quite difficult to measure via methods that study integrated quantities (e.g., the luminosity distances of SNe Ia). However, the test proposed here measures a more localized quantity, the volume element; hence, we may be able to observe changes in that quantity even on relatively small scales. By studying the apparent abundances of the dark halos of galaxies, we may test the most recent cosmological models—or even those not yet envisioned—with techniques having their origin in the earliest days of extragalactic astronomy.

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