11.5/20

1) $P = (2, 3, -1)$, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -25 \end{bmatrix} + t\begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}$

$P = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$, $P' = \begin{pmatrix} 5+3t \\ 2t \\ -25-2t \end{pmatrix}$ $\Rightarrow \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 3+3t \\ -3+2t \\ -24-2t \end{pmatrix} = 0$

$\overrightarrow{PP'} = P' - P = \begin{pmatrix} 3+3t \\ -3+2t \\ -24-2t \end{pmatrix}$

$\Rightarrow (9+9t) + 2(-3+2t) - 2(-24-2t) = 0$

$\Rightarrow -45 + 17t = 0 \rightarrow t = 45/17$

$\Rightarrow PP' = \begin{pmatrix} 3 + 3(45/17) \\ -3 + 2(45/17) \\ -24 - 2(45/17) \end{pmatrix}$

$$\boxed{\Rightarrow |PP'| = \sqrt{\left(\tfrac{186}{17}\right)^2 + \left(\tfrac{39}{17}\right)^2 + \left(-\tfrac{398}{17}\right)^2}}$$

I'm not clear what you are doing. need to give some explanation, esp when answer is a bit off

+1

2)

a) $A = \begin{bmatrix} 1 & 1 & -2 \\ 1 & 3 & -1 \\ 2 & 1 & 5 \end{bmatrix} \xrightarrow[R_3+2R_1]{R_2-R_1} \begin{bmatrix} 1 & 1 & -2 \\ 0 & 2 & 1 \\ 0 & -1 & -9 \end{bmatrix} \xrightarrow[2R_3+R_2]{} \begin{bmatrix} 1 & 1 & -2 \\ 0 & 2 & 1 \\ 0 & 0 & -17 \end{bmatrix}$

$\boxed{\text{rows are linearly independent}}$

b) col A = row A^T

$A^T = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 1 \\ -2 & -1 & 5 \end{bmatrix} \xrightarrow[R_3+2R_1]{R_2-1R_1} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & -1 \\ 0 & 1 & 9 \end{bmatrix} \xrightarrow[2R_3-1R_2]{} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 19 \end{bmatrix}$

$\boxed{\text{cols are linearly independent}}$

+2

c) $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 1 & 3 & 1 & 0 \\ -2 & -1 & 5 & 0 \end{array}\right] \xrightarrow[R_3-2R_1]{R_2-1R_1} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & -1 & -9 & 0 \end{array}\right] \xrightarrow[2R_3+R_2]{} \left[\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & -17 & 0 \end{array}\right]$

$x_1 + x_2 - 2x_3 = 0 \rightarrow x_1 = 0$

$2x_2 + x_3 = 0 \rightarrow x_2 = 0$

$-17x_3 = 0 \rightarrow x_3 = 0$

$$\boxed{N(A) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$$

3) a) $A\vec{v} = \lambda\vec{v}$ for some λ

b) $A - \lambda I = \begin{bmatrix} 1-\lambda & 2 \\ 2 & 1-\lambda \end{bmatrix} \Rightarrow (1-\lambda)^2 - 4 = 0$

$(1-2\lambda+\lambda^2)-4=0$

$\lambda^2 - 2\lambda - 3 = 0$

$\lambda = -3, -1$

$E_3 = N\left(\begin{bmatrix} 3-1 & 2 \\ 2 & 3-1 \end{bmatrix}\right) = N\left(\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\right)$

$\boxed{E_3 = \left\{ \begin{bmatrix} x_1 \\ -x_1 \end{bmatrix} \cancel{\}} : x_1 \in \mathbb{R} \right\}}$

$E_{-1} = N\left(\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}\right)$

$\boxed{E_{-1} = \left\{ \begin{bmatrix} x_1 \\ x_1 \end{bmatrix} \cancel{\}} : x_1 \in \mathbb{R} \right\}}$

+4

4) $W = c_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

what is c_1, c_2?

$\vec{v}_1 = \begin{bmatrix} c_1 + c_2 \\ c_2 \\ c_1 - c_2 \end{bmatrix}$ $\vec{v}_2 = \begin{bmatrix} a \\ b \\ k \end{bmatrix}$ st

$a + k = 0$

$\vdots$

$a + b - k = 0$

$a + k = a + b - k$

$2k = b$

$\cancel{\vec{v}_2 = \begin{bmatrix} -k \\ 2k \\ k \end{bmatrix}}$

+0.5

$\vec{v}_1 \cdot \vec{v}_2 = \begin{bmatrix} c_1 + c_2 \\ c_2 \\ c_1 - c_2 \end{bmatrix} \cdot \begin{bmatrix} -k \\ 2k \\ k \end{bmatrix}$

$= -kc_1 - kc_2 + 2kc_2 + kc_1 - kc_2 = 0$ ✓

5) a) True

b) True

c) False, $\vec{z}$ could be an eigenvector

+4

d) False, a 4×3 matrix can have independent rows, but not cols bc it has more rows than cols