Homework #1

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— Numerical Analysis —

$\boxed{\text{Sec 1.3 } ① \pm \cancel{2}, \cancel{3}, \cancel{5}, 10}$

$E_p = |p - \hat{p}|$ (absolute)

$R_p = \dfrac{|p - \hat{p}|}{|p|}$ (relative)

significant digits

$\dfrac{|p-\hat{p}|}{|p|} < 5 \cdot 10^{-d}$

1.)

a.) $x = 2.71828182$

$\hat{x} = 2.7182$

$E_x = |2.71828182 - 2.7182|$

$= \boxed{0.00008128}$

$R_x = \dfrac{|2.71828182 - 2.7182|}{|2.71828182|}$

$= \boxed{0.00003009989}$

Significant digits:

$0.00003009989 < 5 \cdot 10^{-d} \Rightarrow \boxed{d = 5}$

b.) $y = 98,350$

$\hat{y} = 98,000$

$E_y = |98,350 - 98,000|$

$= \boxed{350}$

$R_y = \dfrac{|98,350 - 98,000|}{|98,350|}$

$= \boxed{0.003558871886}$

$0.003558871886 < 5 \cdot 10^{-d} \Rightarrow \boxed{d = 3}$

10.)

Sum: $e^h + \sin(h) = 1 + h + \frac{h^2}{2!} + \cancel{\frac{h^3}{3!}} + \frac{h^4}{4!} + O(h^5)$

$$+ h - \cancel{\frac{h^3}{3!}} + O(h^5)$$

$$= 1 + 2h + \frac{h^2}{2!} + \frac{h^4}{4!} + 2O(h^5)$$

since $O(h^5) + \frac{h^4}{4!} = O(h^5)$

and $O(h^5) + O(h^5) = O(h^5)$

$$= 1 + 2h + \frac{h^2}{2!} + \frac{h^4}{4!} + O(h^5).$$

ORDER OF APPROX => $\boxed{O(h^5)}$

Product: $e^h \sin(h)$

$$= \left(1 + h + \frac{h^2}{2!} + \frac{h^3}{3!} + \frac{h^4}{4!} + O(h^5)\right)\left(h - \frac{h^3}{3!} + O(h^5)\right)$$

= order of approx => $\boxed{O(h^5)}$

c) $z = 0.000068$

$\hat{z} = 0.00006$

$E_z = |z - \hat{z}| = \boxed{0.000008}$

$R_z = \dfrac{|z-\hat{z}|}{|z|} = 0.1176470588\,2$

Relative Error $= 0.1176470588\,2 < 5 \cdot 10^{-1} \Rightarrow \boxed{d = 1}$

2) $\int_0^{1/4} \left(1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!}\right) dx = \hat{p}$

$= \int_0^{1/4} 1\,dx + \int_0^{1/4} x^2\,dx + \int_0^{1/4} \frac{x^4}{2!}\,dx + \int_0^{1/4} \frac{x^6}{3!}\,dx$

$= x\Big|_0^{1/4} + \frac{x^3}{3}\Big|_0^{1/4} + \frac{x^5}{10}\Big|_0^{1/4} + \frac{1}{6}\left(\frac{x^7}{7}\right)\Big|_0^{1/4}$

$= \frac{1}{4} + \frac{1}{3}\left(\left(\frac{1}{4}\right)^3 - 0\right) + \frac{1}{10}\left(\left(\frac{1}{4}\right)^5 - 0\right) + \frac{1}{42}\left(\left(\frac{1}{4}\right)^7 - 0\right)$

$= \frac{1}{4} + \frac{1}{192} + \frac{1}{10240} + \frac{1}{688128}$

$= 0.2557813953\,21$

Truncation error b/c we are representing $\int_0^{1/4} e^{x^2} dx$ with 0.255781395321.

Hence, error $= |p - \hat{p}| = |0.255307406\,06 - 0.255781395321|$

$= \boxed{-0.000250649261}$

5.)

a) $\ln(x+1) - \ln(x)$ for large x

$$\left[\ln\frac{a}{b} = \ln(a) - \ln(b)\right]$$

$$\Rightarrow \ln\left(\frac{x+1}{x}\right) = \boxed{\ln\left(1+\frac{1}{x}\right)}$$

b) $\sqrt{x^2+1} - x$ for large x

$$\Rightarrow \sqrt{x^2+1} - x \cdot \left(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}\right)$$

$$= \frac{\left(\sqrt{x^2+1}\right)^2 - x^2}{\sqrt{x^2+1}+x} = \frac{\cancel{x^2}+1-\cancel{x^2}}{\sqrt{x^2+1}+x} = \boxed{\frac{1}{\sqrt{x^2+1}+x}}$$

c) $\cos^2(x) - \sin^2(x)$ for $x \approx \frac{\pi}{4}$

$$\left[\cos(2x) = \cos^2(x) - \sin^2(x)\right]$$

$$\cos^2(x) - \sin^2(x) = \boxed{\cos(2x)}$$

d) $\sqrt{\frac{1+\cos(x)}{2}}$ for $x \approx \pi$

$$= \sqrt{\frac{1+\cos(x)}{2}} = \sqrt{\frac{2\cos^2\left(\frac{x}{2}\right)}{2}} = \boxed{\cos\left(\frac{x}{2}\right)}$$