b) $\alpha_n = \left\{ \frac{1-2n^2}{3n^2+n-1} \right\} \to -2/3 \qquad \beta_n = ?$

$\lim_{n\to\infty} \alpha_n = -2/3 = \alpha$

$$|\alpha_n - \alpha| = \left| \frac{1-2n^2}{3n^2+n-1} + \frac{2}{3} \right|$$

$$= \text{algebra } \text{~~(least common denominator)~~}$$

$$= \left| \frac{3-6n^2+6n^2+2n-2}{3(3n^2+n-1)} \right|$$

$$= \left| \frac{2n+1}{3(3n^2+n-1)} \right| \qquad \beta_n = \frac{1}{n^p}$$

$$\leq \left| \frac{2n}{3(3n^2+n-1)} \right| + \left| \frac{1}{3(3n^2+n-1)} \right|$$

$$= \left| \frac{2}{3(3n+1-1/n)} \right| + \left| \frac{1}{3(3n^2+n-1)} \right|$$

$$< \cancel{\tfrac{2}{3} \ldots \tfrac{2}{3}\tfrac{1}{n}}$$

$$< \frac{2}{3}\left|\frac{1}{n}\right| + \frac{1}{3}\left|\frac{1}{n}\right|$$

$$= \left|\frac{1}{n}\right|$$

$$|\alpha_n - \alpha| < \left|\frac{1}{n} - 0\right|$$

$\beta_n = \frac{1}{n}$

$\beta = 0$

* the rate of convergence of $\{\alpha_n\} = \left\{ \frac{1-2n^2}{3n^2+n-1} \right\}$ to $\alpha = -\frac{2}{3}$ is similar to the rate of convergence of $\{\beta_n\} = \{\frac{1}{n}\}$ to $\beta = 0$.

c) $\alpha_n = \left\{\frac{2^n+3}{2^n+7}\right\}$

$\beta_n = \frac{1}{2^n}$

$$\alpha = \lim_{n\to\infty} \alpha_n = \lim_{n\to\infty} \frac{2^n+3}{2^n+7} = \lim_{n\to\infty} \frac{1+\cancel{\frac{3}{2^n}}^{0}}{1+\cancel{\frac{7}{2^n}}^{0}} = 1$$

$$|\alpha_n - \alpha| = \left|\frac{2^n+3}{2^n+7} - \frac{1}{1}\right| = \left|\frac{\cancel{2^n}+3-\cancel{2^n}-7}{2^n+7}\right| = \left|\frac{-4}{2^n+7}\right|$$

$$\Rightarrow \left|\frac{-4}{2^n+7}\right| \leq 4\left|\frac{1}{2^n}\right|$$

$\beta_n = \frac{1}{2^n}$

$\beta = 0$

$k = 4$

* The rate of convergence of $\{\alpha_n\} = \left\{\frac{2^n+3}{2^n+7}\right\}$ to $\alpha = 1$ is smaller to the rate of convergence of $\{\beta_n\} = \left\{\frac{1}{2^n}\right\}$ to $\beta = 0$ where $k = 4$.

b) $\alpha_n = \left\{ \frac{1-2n^2}{3n^2+n-1} \right\} \rightarrow -2/3$ $\quad\quad \beta_n = ?$

$$\lim_{n\to\infty} \alpha_n = -2/3 = \alpha$$

$$|\alpha_n - \alpha| = \left| \frac{1-2n^2}{3n^2+n-1} + \frac{2}{3} \right|$$

= ~~algebra (least common denominator)~~

$$= \left| \frac{3-6n^2+6n^2+2n-2}{3(3n^2+n-1)} \right|$$

$$= \left| \frac{2n+1}{3(3n^2+n-1)} \right| \quad\quad \beta_n = \frac{1}{n^p}$$

$$\leq \left| \frac{2n}{3(3n^2+n-1)} \right| + \left| \frac{1}{3(3n^2+n-1)} \right|$$

$$= \left| \frac{2}{3(3n+1-1/n)} \right| + \left| \frac{1}{3(3n^2+n-1)} \right|$$

$<$ [illegible]

$$< \frac{2}{3}\left|\frac{1}{n}\right| + \frac{1}{3}\left|\frac{1}{n}\right|$$

$$= \left|\frac{1}{n}\right|$$

$$|\alpha_n - \alpha| < \left|\frac{1}{n} - 0\right|$$

$\beta_n = \frac{1}{n}$

$\beta = 0$

* the rate of convergence of $\{\alpha_n\} = \left\{\frac{1-2n^2}{3n^2+n-1}\right\}$ to $\alpha = -\frac{2}{3}$ is similar to the rate of convergence of $\{\beta_n\} = \left\{\frac{1}{n}\right\}$ to $\beta = 0$.