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Towards a non-linear trading strategy for financial time series

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Abstract

A new trading strategy based on state space reconstruction techniques is proposed. The technique uses the state space volume evolution and its rate of change as indicators. This methodology has been tested off-line using eighteen high-frequency foreign exchange time series with and without transaction costs. In our analysis an optimum mean value of approximately 25% gain may be obtained in those series without transaction costs and an optimum mean value of approximately 11% gain assuming 0.2% of costs in each transaction.

1. Introduction

Recently, with the development of nonlinear systems theory and the availability of high frequency intra-day data sets, there has been an increasing interest in the application of concepts and methods developed there to problems of economics and finance [10,8]. Between them, several techniques for forecasting nonlinear time series have been applied. The Farmer and Sidorowich [4] algorithm is an example of such nonlinear forecasting methods. In this case, delay coordinates are used to reconstruct a representation of the original state space that generated the dynamics. The state at a time t of a measured variable $s(t)$ is given by $S(t) = \{s(t), s(t-\Delta t), s(t-2\Delta t), \ldots, s(t-(d_E-1)\Delta t)\}$, whereas Δt is the time delay or the lag between data when reconstructing the state space, and d_E is the embedding dimension or the dimension of the space required to unfold the dynamics. Assuming that a relationship exists between the current state and the future state then it is possible to write: $s(t+1) = f[S(t)]$. Nonlinear forecasting methodologies try to construct an approximation function of f and they have been applied to financial time series, see for example Lisi and Medio [9] and Cao and Soofi [3]. However, state space reconstruction techniques assume stationarity in the time series which for financial time series does not hold [15]. In the context of nonstationarity, the notion of a "correct" embedding or delay is inappropriate as has been demonstrated by Grassberger et al. [5]. Instead it becomes important to remember that a sufficiently large embedding be chosen which will "contain" the relevant dynamics (as it may change from one

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dimensionality to another) as well as account the effects of noise, which tend to inflate dimension. The approach to ''overembed'' the time series to capture the dynamics as its dimension changes has been justified by Hegger et al. [6]. Similar considerations govern the choice of the time delay. As the system changes from one dimension to another the effects of the time delay are changed. Thus a so-called ''optimal'' time delay in one embedding, becomes less so as the relevant dimension changes [20]. In this context the use of local properties to characterise the underlying dynamical system becomes fundamental since the asymptotic behaviour of the system—by asymptotic behaviour, we mean the properties that prevail when time t is sufficiently large, $t \to \infty$—is not guaranteed. In this sense, we have been using

Table 1
Best parameters and predictability results without transaction costs for the currency exchange time series considered

Currency	Δt^{opt}	d_E^{opt}	$\sum g^{opt}$	% Gain
AUD	385	1	0.21	75.9
BEF	184	6	0.27	65.8
CAD	276	6	0.10	51.6
CHF	67	3	0.28	90.2
DEM	89	7	0.20	75.2
DKK	2	3	0.20	71.4
ESP	273	1	0.39	64.5
FIM	59	10	0.25	55.8
FRF	192	11	0.20	69.5
GPB	295	4	0.20	94.3
ITL	189	1	0.16	36.8
JPY	212	12	0.21	86.0
MYR	242	3	0.04	37.5
NLG	218	2	0.19	73.5
SEK	182	1	0.28	56.4
SGD	330	1	0.08	38.8
XEU	29	1	0.18	58.3
ZAR	191	7	0.37	95.5

Δt^{opt} and d_E^{opt} indicate, respectively, the optimum time delays and embedding dimensions for state space reconstruction in the sense of higher gain–loss function, i.e. $\sum g^{opt}$ % gain refers to the number of times in which there was a net gain for all combinations of reconstruction parameters, i.e. (Δt between 2 and 400 and d_E between 1 and 15).

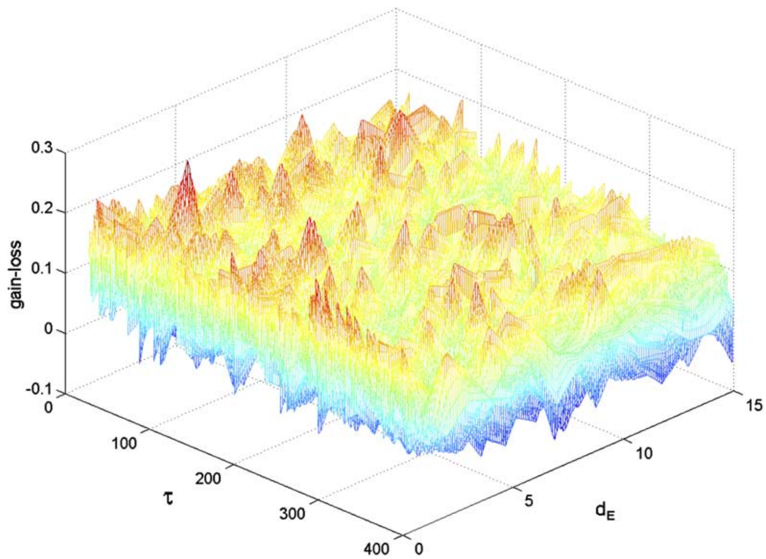

Fig. 1. Distribution of the gain–loss function, Eq. (6), for the Swiss Frank (CHF). Time delay between 2 and 400; embedding dimension between 1 and 15.

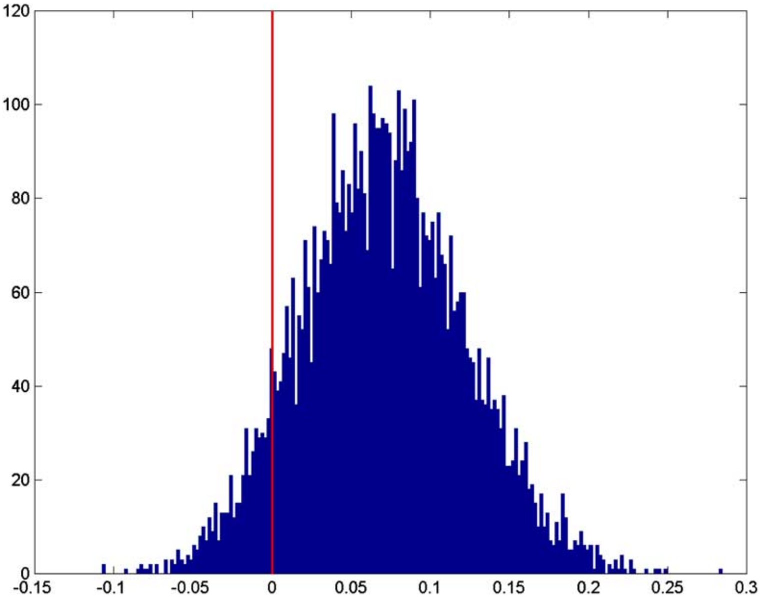

Fig. 2. Histogram of the gain–loss function, Eq. (6), for the Swiss Frank (CHF). Time delay between 2 and 400; embedding dimension between 1 and 15.

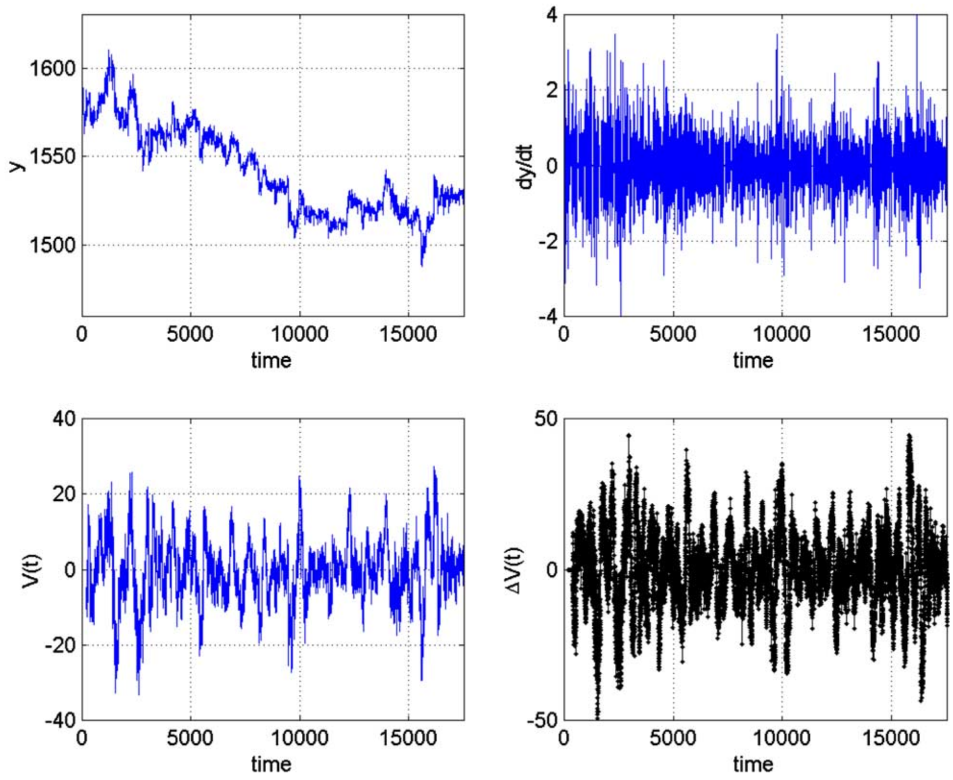

Fig. 3. No transaction costs, no state space volume limitations: (a) Italian Lira–US dollar time series (bid); (b) first derivative; (c) state space volume; (d) state space volume change. Reconstruction parameters: $\Delta t = 189$, $d_E = 1$.

the divergence of a dynamical system for the characterisation and analysis of chemical transient reactors [16,18,2]. The divergence of the flow, which is locally equivalent to the trace of the Jacobian, measures the rate of change of an infinitesimal state space volume $V(t)$ following an orbit of the original state space $\boldsymbol{x}(t)$. The original state space is related to our measured signal by $s(t) = h[\boldsymbol{x}(t)]$.

In this work, we have applied state space reconstruction techniques to estimate the state space volume, $V(t)$, and its variation, $\Delta V(t)$, for high-frequency currency exchange data from the HFDF96 data set provided by Olsen & Associates [12]. The time series studied are the exchange rates between the US Dollar and 18 other foreign currencies from the Euro zone; i.e. Belgium Franc (BEF), Finnish Markka (FIM), German Mark (DEM), Spanish peseta (ESP), French Frank (FRF), Italian Lira (ITL), Dutch Guilder (NLG), and finally ECU (XEU); and from outside the Euro zone: Australian Dollar (AUD), Canadian Dollar (CAD), Swiss Frank (CHF), Danish Krone (DKK), British Pound (GBP), Malaysian Ringgit (MYR), Japanese Yen (JPY), Swedish Krona (SEK), Singapore Dollar (SGD), and South African Rand (ZAR). These values, $V(t)$ and $\Delta V(t)$, have allowed us to define a trading methodology by considering a sort of acceleration in a high-dimensional state space system as a kind of momentum indicator similar to those used in financial technical analysis [13,14]. Our interest was to develop a general trading strategy to determine and quantify the amount of predictability in these time series. This strategy is quite general and may be applied to other financial series.

2. State space volume calculation

State space reconstruction preserves certain information on the original system that originated the time series we are measuring. However, all this information applies to the asymptotic behaviour of the system. By asymptotic behaviour, we mean the properties that prevail when time t is sufficiently large, $t \to \infty$. In our case as financial time series are transient, we need a local measure, not a global one that reflects the actual status of the system. In this sense, the divergence of the system is a local property which is preserved under state space reconstruction [1]. The divergence of the flow,

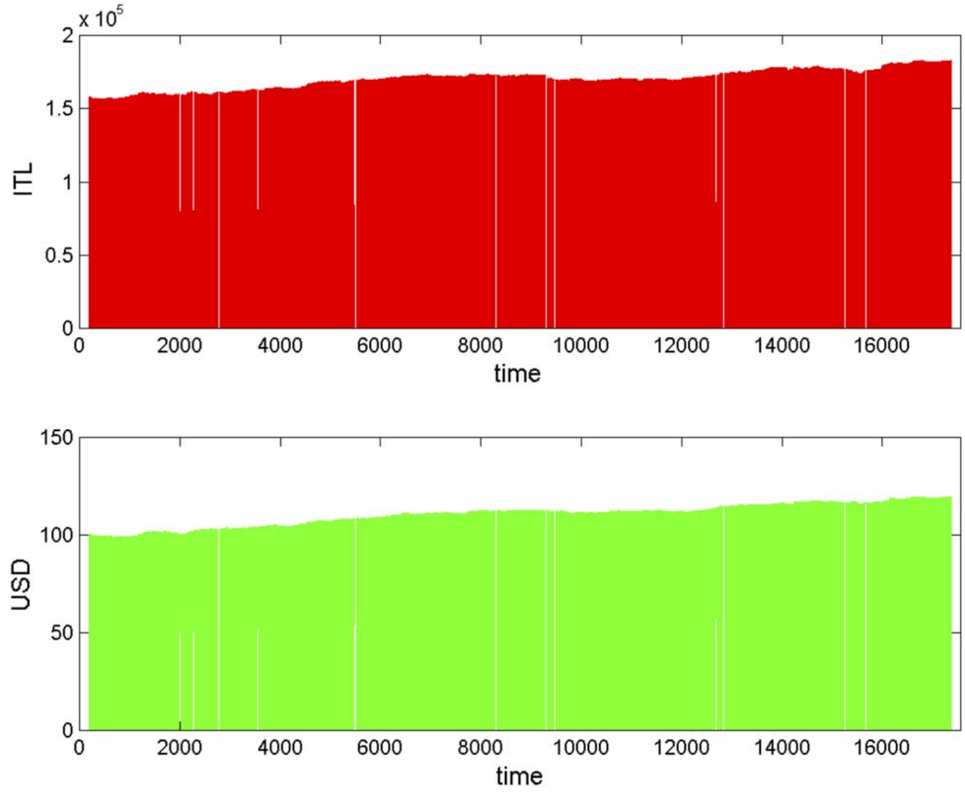

Fig. 4. Exchanges between ITL and USD using no transaction costs and no state space volume limitations trading strategy and starting with 100 units in USD.

which is the trace of the Jacobian, and measures the rate of change of an infinitesimal state space volume $V(t)$, following an orbit $\mathbf{x}(t)$. Furthermore, Liouville's theorem [1] states that:

$$V(t) = V(0) \cdot \exp\left[\int_0^t \mathrm{div}\{\mathbf{F}[\mathbf{x}(\tau)]\}\,\mathrm{d}\tau\right] \tag{1}$$

where

$$\mathrm{div}\{\mathbf{F}[\mathbf{x}(t)]\} = \frac{\partial F_1[\mathbf{x}(t)]}{\partial x_1} + \frac{\partial F_2[\mathbf{x}(t)]}{\partial x_2} + \cdots + \frac{\partial F_d[\mathbf{x}(t)]}{\partial x_d} \tag{2}$$

From Eq. (1), it is possible to write [16]:

$$\mathrm{div}[J(x)] = \frac{\dot{V}(t)}{V(t)} \tag{3}$$

In dissipative systems the state space volume contracts and therefore $V(t)$ and $\dot{V}(t)$ rapidly tend to zero and produce artefacts when introduced as denominator in Eq. (3), for this reason we have used separately $V(t)$ and $\Delta V(t) = V(t) - V(t - \Delta t)$ avoiding the need to divide the two small numbers.

Numerically, $V(t)$ may be calculated, assuming that the time step from one point to another in the time series is short enough that the Jacobian of the system has not substantially changed, using the determinant between close points in state space as [2]:

$$V(t) = \det\begin{bmatrix} s(t) - s(t-\Delta t) & 0 & \ldots & 0 \\ 0 & s(t-\Delta t) - s(t-2\Delta t) & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & s(t-(d_{\mathrm{E}}-1)\Delta t) - s(t-d_{\mathrm{E}}\Delta t) \end{bmatrix} \tag{4}$$

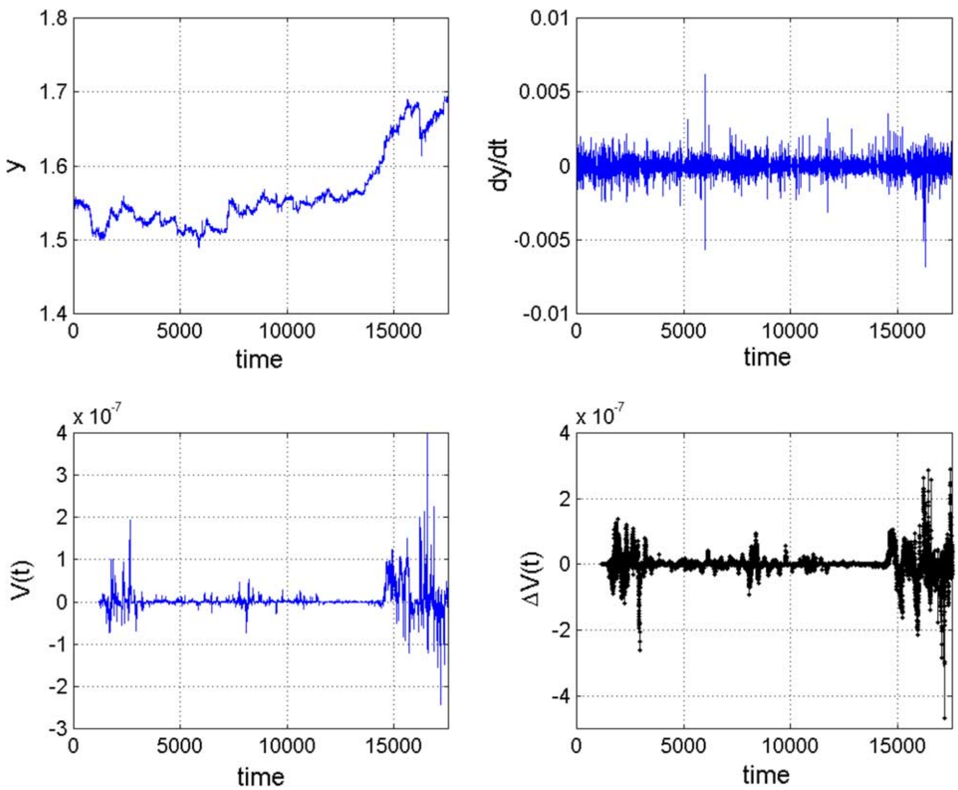

Fig. 5. No transaction costs, no state space volume limitations: (a) US dollar—British Pound time series (bid); (b) first derivative; (c) state space volume; (d) state space volume change. Reconstruction parameters: $\Delta t = 295$, $d_{\mathrm{E}} = 4$.

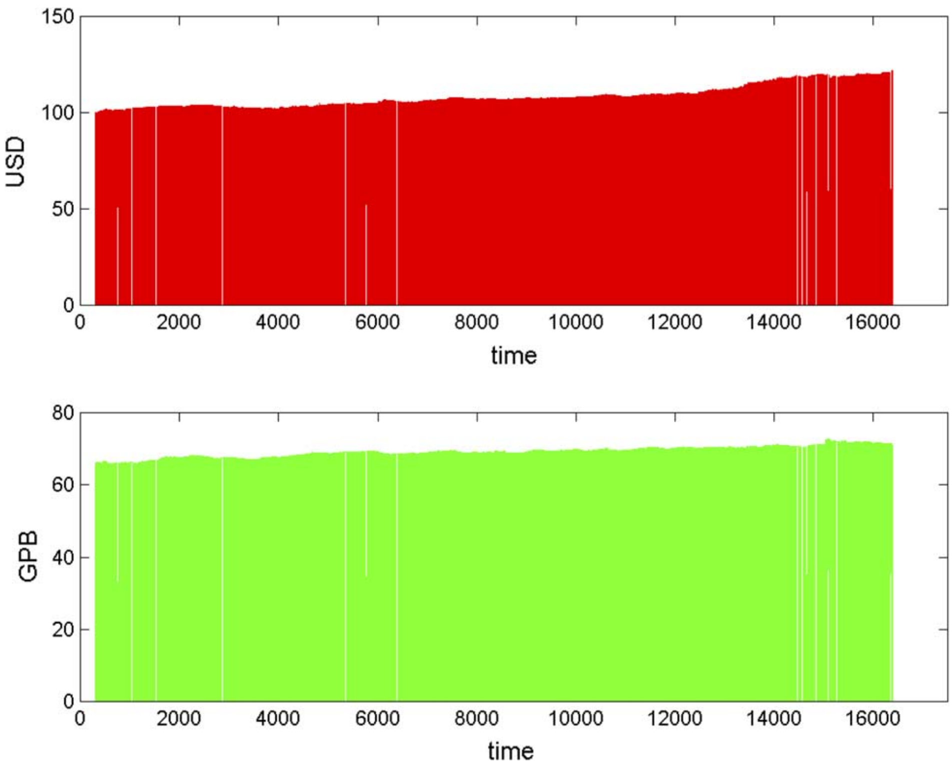

Fig. 6. Exchanges between USD and GPB no transaction costs and no state space volume limitations trading strategy and starting with 100 units in USD.

Table 2
Best parameters and predictability results without transaction costs for the currency exchange time series considered by optimising the state space volume at which a transaction is allowed

Currency	Δt^{opt}	$d_{\text{E}}^{\text{opt}}$	$\sum g^{\text{opt}}$	% Gain
AUD	385	1	0.22	83.9
BEF	133	1	0.34	99.8
CAD	2	1	0.11	73.5
CHF	47	1	0.31	97.4
DEM	2	1	0.25	96.6
DKK	4	1	0.23	99.5
ESP	13	1	0.51	99.9
FIM	2	1	0.32	99.4
FRF	3	1	0.27	99.9
GPB	4	1	0.20	96.4
ITL	42	1	0.20	93.7
JPY	37	1	0.23	100.0
MYR	2	1	0.08	83.4
NLG	36	1	0.22	97.7
SEK	2	1	0.30	99.7
SGD	3	1	0.10	72.8
XEU	2	1	0.22	84.0
ZAR	171	7	0.37	99.9

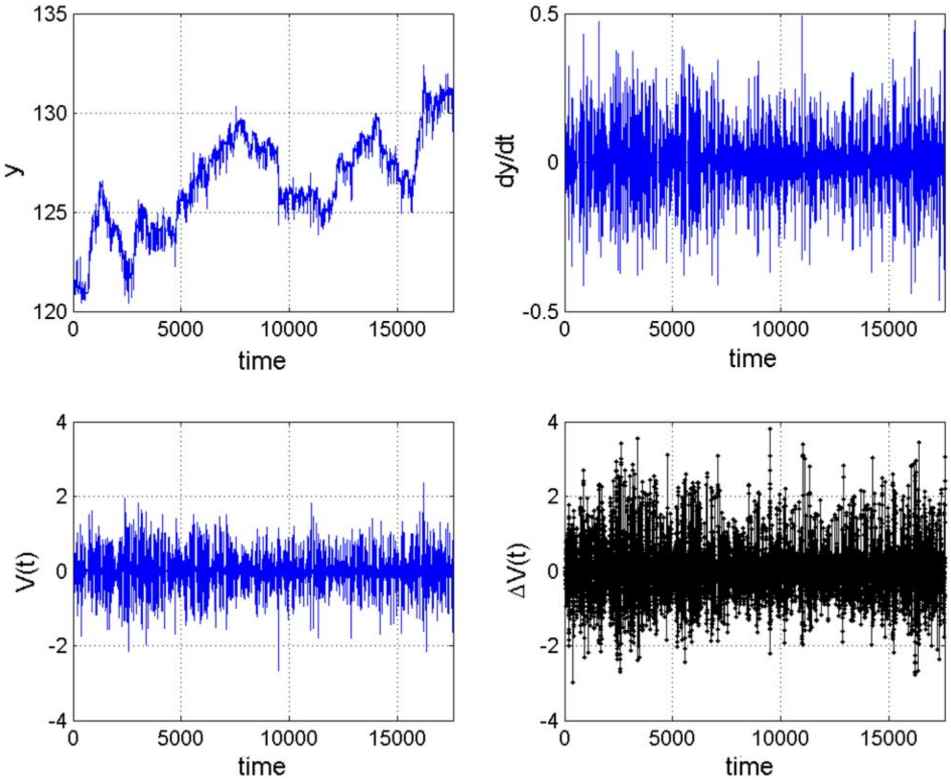

Fig. 7. No transaction costs, state space volume limitation: (a) Spanish Peseta–US dollar time series (bid); (b) first derivative; (c) state space volume; (d) state space volume change. Reconstruction parameters: $\Delta t = 13$, $d_E = 1$; and with $|V| > \lim$ (0.51) as minimum state space volume to trade.

3. Trading strategy

A three step approach has been followed in applying the trading strategy. Even though it is not realistic, we assume in a first step that we can exchange our assets at no cost. Therefore, the number of transaction is not important. Furthermore, we have only tested one-step ahead prediction, i.e. $t+1$, based on all available information at time t. In the first case, we apply the following simple rule: if the variation of state space volume decreases, i.e. $\Delta V(t) > \Delta V(t-1)$, we change all our assets into *currency*$_2$ at $t+1$, on the contrary, we exchange all our assets into *currency*$_1$. The application of this strategy is equivalent to detect if the volume has a positive acceleration. We will then consider this acceleration as an index of the strength of the stock exchange. The net ‘‘profit’’, also called return, is evaluated using the gain–loss function g [11] as:

$$g = \frac{s(t+1) - s(t)}{s(t)} \tag{5}$$

This function represents the rate of gain or loss incurred in one time step. The total gain–loss is calculated for all the time series as:

$$G = \sum_{i=d_E \cdot \Delta t}^{n} g_i \tag{6}$$

Therefore, if $\Delta V(t) > \Delta V(t-1)$, we will change all our assets into *currency*$_2$ at $t+1$, if we are confronted for the first time to a decrease in ΔV, if not then no action is performed. As we will see later on, this strategy for real financial time series produces a considerable amount of transactions since our ΔV is oscillating around zero. In case of transaction costs this strategy would fail.

To reduce the number of transactions and consider only the most relevant ones, we have introduced a second trading criterion in which we will change our assets if the previous criterion holds and when $|V(t)| > \text{limit}$. This approach reduces the number of transactions and, therefore, we have applied it, for the case of analysing the results when transaction costs are involved, a fixed 0.2% cost for each transaction which in practice means that we multiply by 0.998 our assets after each transaction.

4. Results and discussion

4.1. No transaction costs

In order to assess the level of predictability, we have tested the gain–loss function, Eq. (6), for values of time delay between 2 and 400 and embedding dimensions between 1 and 15. These values have been selected in agreement with our previous analysis [15] using nonlinear time series methods for the HFDF96 high frequency data set provided by Olsen & Associates [12]. This analysis is reminiscent of a similar approach developed by Zbilut and Webber [19] using RQA analysis to derive embeddings and delays. Table 1 summarises the results for each foreign currency. In the first column we represent the optimal time delay, the optimal embedding dimension as a function of the net gain, the gain obtained with these reconstruction parameters, and the percentage of combinations of time delays and embedding dimensions for which a positive value for the gain–loss function is obtained. Fig. 1 show the gain–losses surface as function of the embedding parameters, whereas in Fig. 2 the histogram of the gain–losses is represented for the case of the Swiss Frank–US dollar exchange time series. As can be seen the distribution shows a displacement versus net

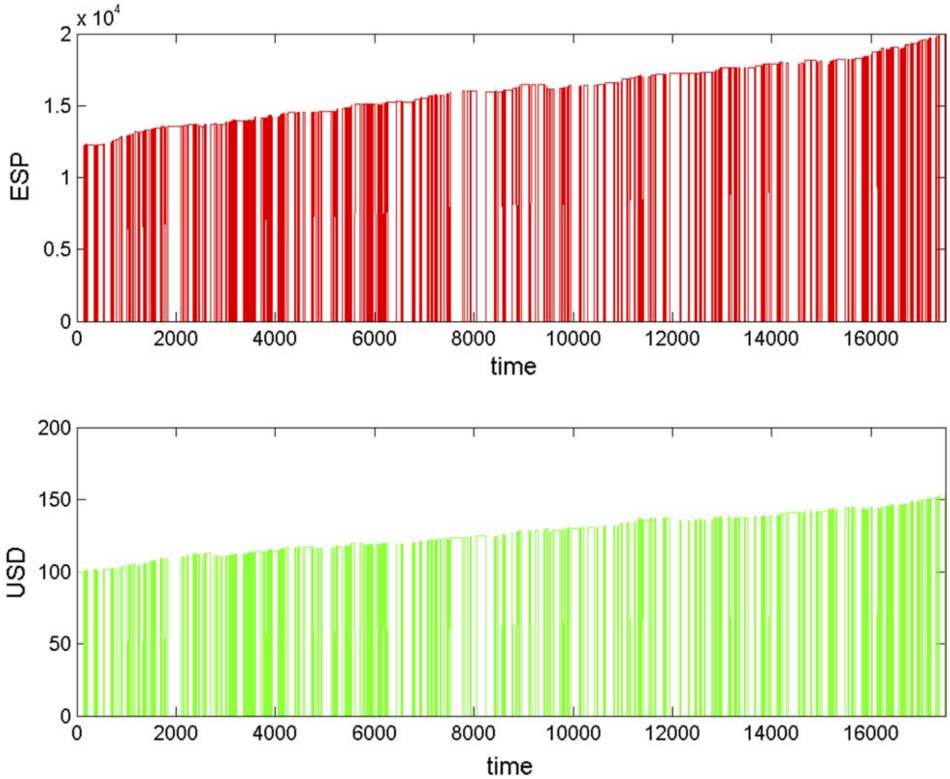

Fig. 8. Exchanges between USD and ESP with no transaction costs and state space volume limitation, i.e. $|V| > \lim$ (0.51) and starting with 100 units in USD.

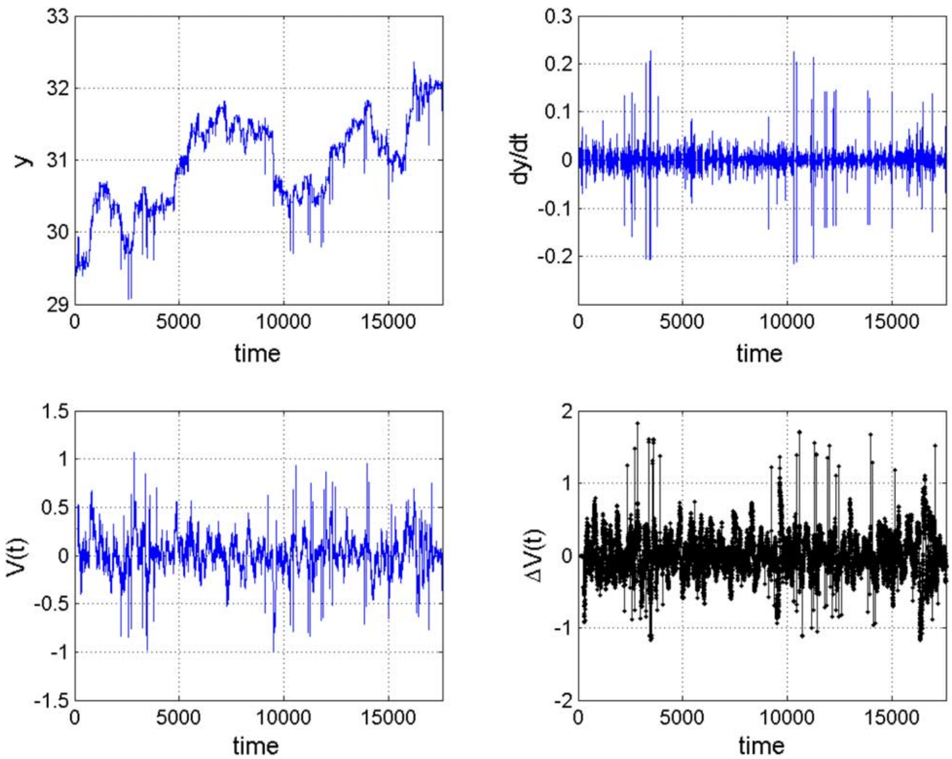

Fig. 9. (a) Belgium Franc–US dollar time series (bid); (b) first derivative; (c) state space volume; (d) state space volume change. Reconstruction parameters: $\Delta t = 133$, $d_E = 1$; and with $|V| > \lim$ (0.16) as minimum state space volume to trade.

gains, the positive part of the axis for 90.2% of the combinations, see Table 1. Furthermore, there is a mean value of 66.5% of positive predictions, being in some currency series higher than 90%. The optimal gain–loss function oscillates between 0.04 and 0.39. This implies a gain in percentage approximately between 4 and 39%. A similar result, but with lower values, i.e. max. 0.02, was obtained by Ohira et al. [11] using a simple rule, antipersistance, for the dollar–yen exchange time series. Figs. 3–6 show two examples corresponding to the Italian Lira (ITL) and to the British Pound (GPB) of the results obtained using the optimal reconstruction parameters. As can be seen there is a continuous gain all over the year even though both time series have a completely different behaviour from the point of view of currency exchange, i.e. one series is increasing the other decreasing. However, with this trading strategy without limitations in the number of transactions, it seems evident that it will be difficult to obtain a net gain once transaction costs are considered. For this reason, we have tested a second strategy in which we use the value of the state space volume to decide if a transaction should be performed or not. We have run the same algorithm but changing at the same time the $|V(t)|$ at which transactions are allowed. The results, without transaction costs, are summarised in Table 2. As can be seen, only for one currency, i.e. Australian dollar, we have obtained the same optimum value for state space reconstruction as in the previous case. Furthermore, one should notice that in this case the optimum values are found using an embedding dimension of one which practically means we are calculating averaged derivatives of the time series, the exceptions is the South African Rand—however for this time series a value close to the optimal is found with embedding dimension of one, i.e. ZAR ($d_E = 1, \Delta t = 6, \sum g^{opt} = 0.35$). In this sense typical instruments of financial technical analysis [13,14] using by chartists seem justified. Another striking feature is that, with this strategy, the number of combinations of time delay and embedding dimension at which we will obtain a net gain, using the optimal value of $|V(t)|$ at which start a transaction, is quite high. In this case, the probability to choose a time delay and an embedding dimension for which we could obtain a net gain has an average value of 93%. These two facts, i.e. optimal embedding dimension equal to one and high gain probabilities may explain why, despite all the work on Efficient Market Hypothesis [17], there has been a considerable amount of traders that use technical analysis tools to trade in financial markets. As can be seen in the two selected examples, i.e. Spanish peseta and Belgium Franc, Figs. 7–10, the number of transactions has decreased when compared with the first trading methodology but it is still very high to be able to deal with transaction costs.

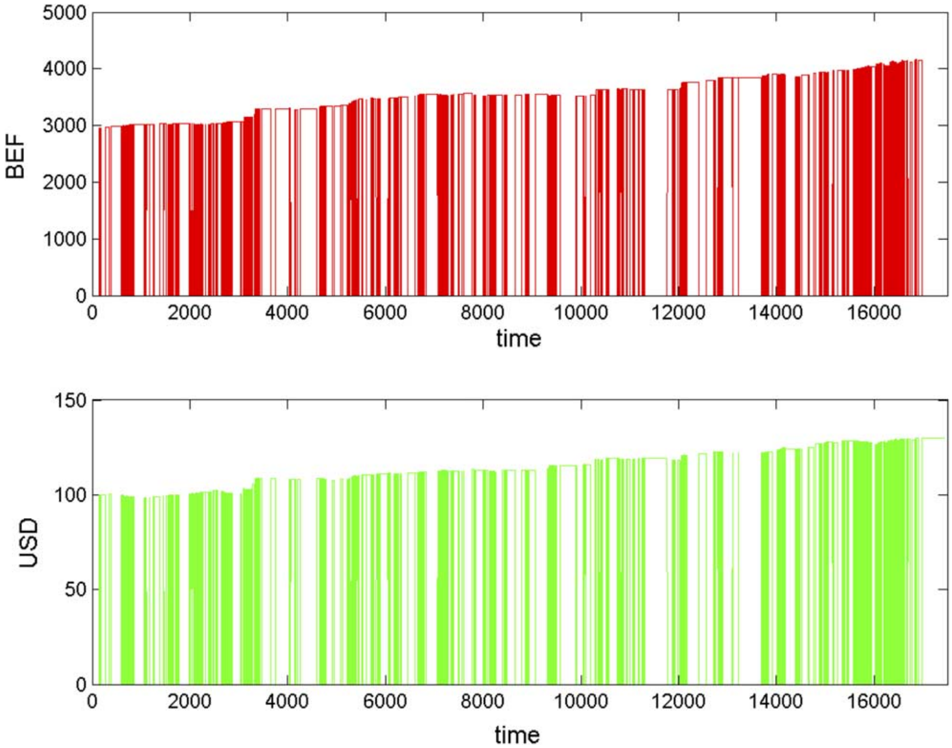

Fig. 10. Exchanges between USD and BEF with no transaction costs and state space volume limitation, i.e. $|V| > \lim$ (0.16) and starting with 100 units in USD.

Table 3
Best parameters and predictability with 0.2% transaction costs, at each transaction, for the currency exchange time series considered optimising the state space volume at which a transaction is allowed

Currency	Δt^{opt}	$d_{\text{E}}^{\text{opt}}$	$\sum g^{\text{opt}}$	% Gain
AUD	4	1	0.11	66.0
BEF	56	1	0.15	89.1
CAD	125	4	0.04	36.2
CHF	14	4	0.19	96.1
DEM	7	4	0.12	78.2
DKK	11	3	0.10	86.0
ESP	11	1	0.14	93.2
FIM	8	1	0.12	78.5
FRF	10	1	0.09	79.7
GPB	10	2	0.14	94.3
ITL	52	1	0.05	28.7
JPY	335	14	0.13	99.6
MYR	311	5	0.02	50.5
NLG	392	5	0.11	83.1
SEK	3	5	0.10	69.0
SGD	125	1	0.02	7.0
XEU	25	2	0.09	57.2
ZAR	129	2	0.26	99.4

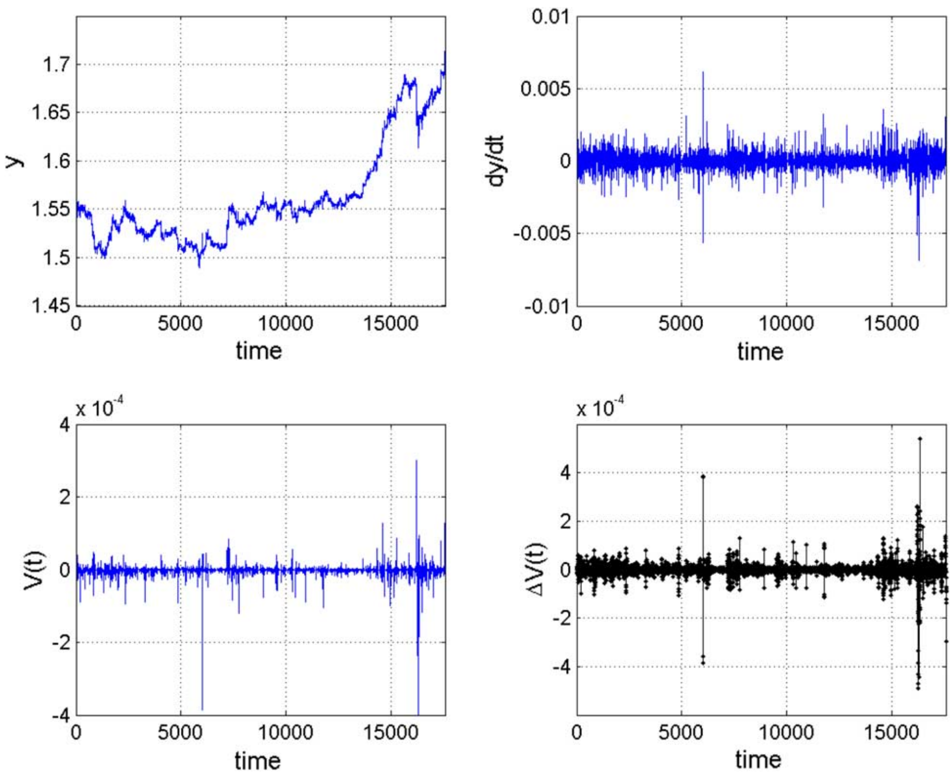

Fig. 11. Fixed transaction costs (0.2%) and optimum state space volume: (a) US dollar–British Pound time series (bid); (b) first derivative; (c) state space volume; (d) state space volume change. Reconstruction parameters: $\Delta t = 10$, $d_{\mathrm{E}} = 2$, $|V| > 1.3 \times 10^{-4}$.

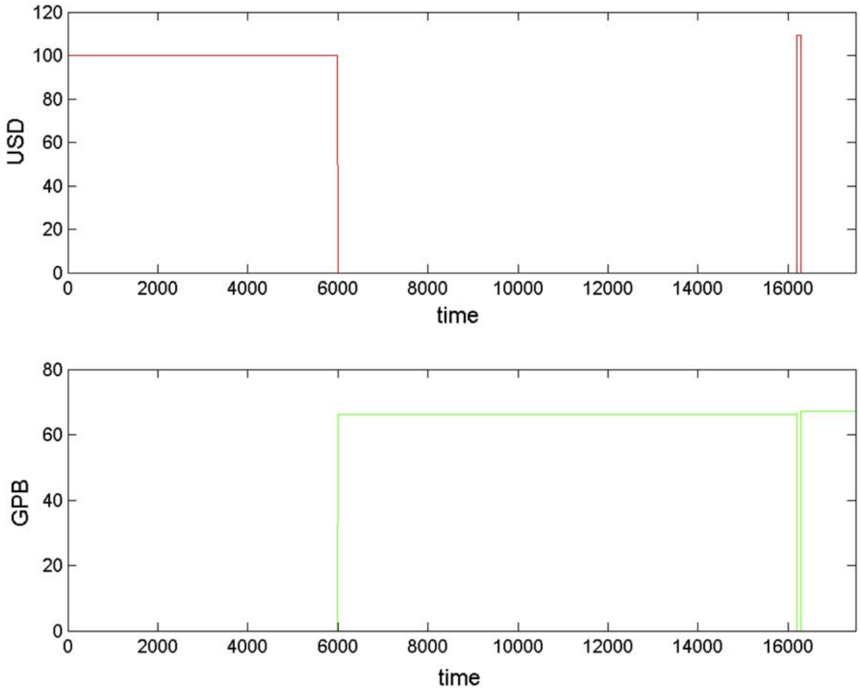

Fig. 12. Exchanges between GPB and USD with fixed transaction costs (0.2%) and state space volume limitation, starting with 100 units in USD.

Table 4
Best parameters without transaction costs for the currency exchange time series considered using the RSI momentum

Currency	b^{opt}	w^{opt}	$\sum g^{opt}$
AUD	17	5	0.28
BEF	8	10	0.42
CAD	3	30	0.14
CHF	1	24	0.32
DEM	1	68	0.27
DKK	2	10	0.25
ESP	1	14	0.47
FIM	1	30	0.33
FRF	1	5	0.31
GPB	1	12	0.26
ITL	2	25	0.30
JPY	1	28	0.28
MYR	6	24	0.08
NLG	3	58	0.19
SEK	4	29	0.36
SGD	2	9	0.11
XEU	3	10	0.32
ZAR	10	24	0.39

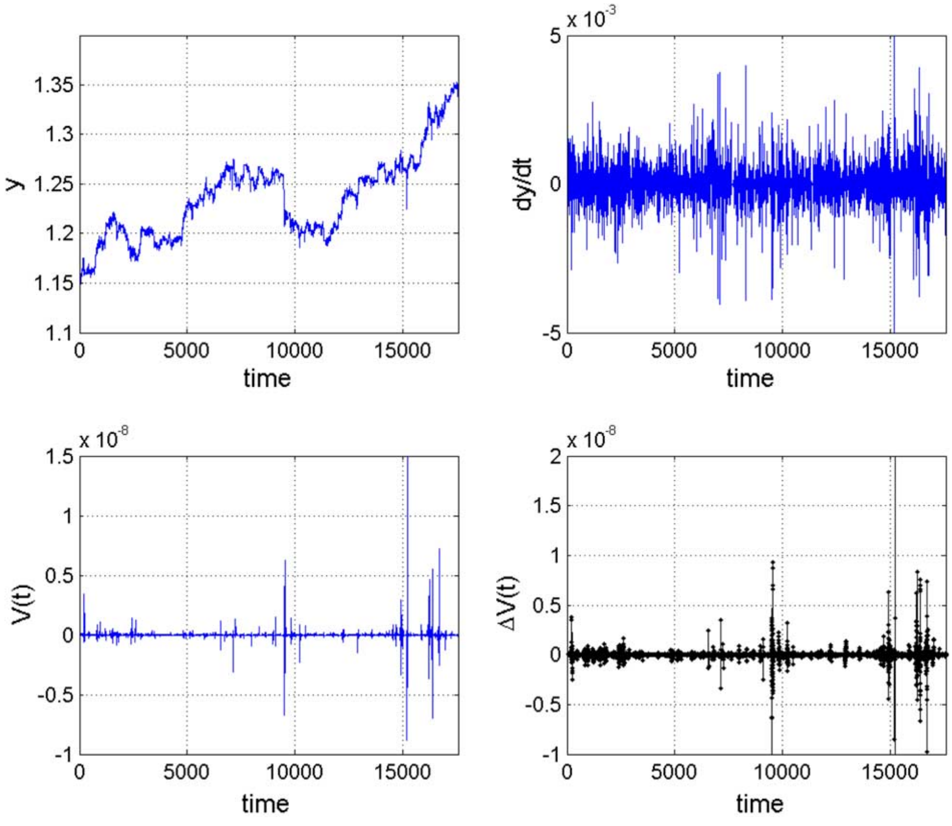

Fig. 13. Fixed transaction costs (0.2%) and optimum state space volume: (a) US dollar–Swiss Frank time series (bid); (b) first derivative; (c) state space volume; (d) state space volume change. Reconstruction parameters: $\Delta t = 14$, $d_E = 4$, $|V| > 1.5 \times 10^{-9}$.

4.2. Fixed transaction costs

Furthermore, we have considered a fixed amount of transactions costs equal to 0.2% for each transaction [7]. It is clear that in this case we need to optimise the number of transactions. Therefore, we have applied the same approach

Table 5
Best parameters and gain with 0.2% fixed transaction costs for the currency exchange time series considered using the RSI momentum

Currency	b^{opt}	w^{opt}	$\sum g^{opt}$
AUD	27	104	0.013
BEF	7	359	0.018
CAD	20	88	−0.024
CHF	14	240	0.005
DEM	23	88	0.002
DKK	10	342	0.006
ESP	12	189	0.002
FIM	9	274	0.004
FRF	12	206	0.009
GPB	11	223	0.003
ITL	5	444	0.002
JPY	25	78	0.003
MYR	26	108	0.000
NLG	20	121	0.007
SEK	17	98	−0.007
SGD	32	98	0.000
XEU	10	291	0.005
ZAR	32	108	−0.004

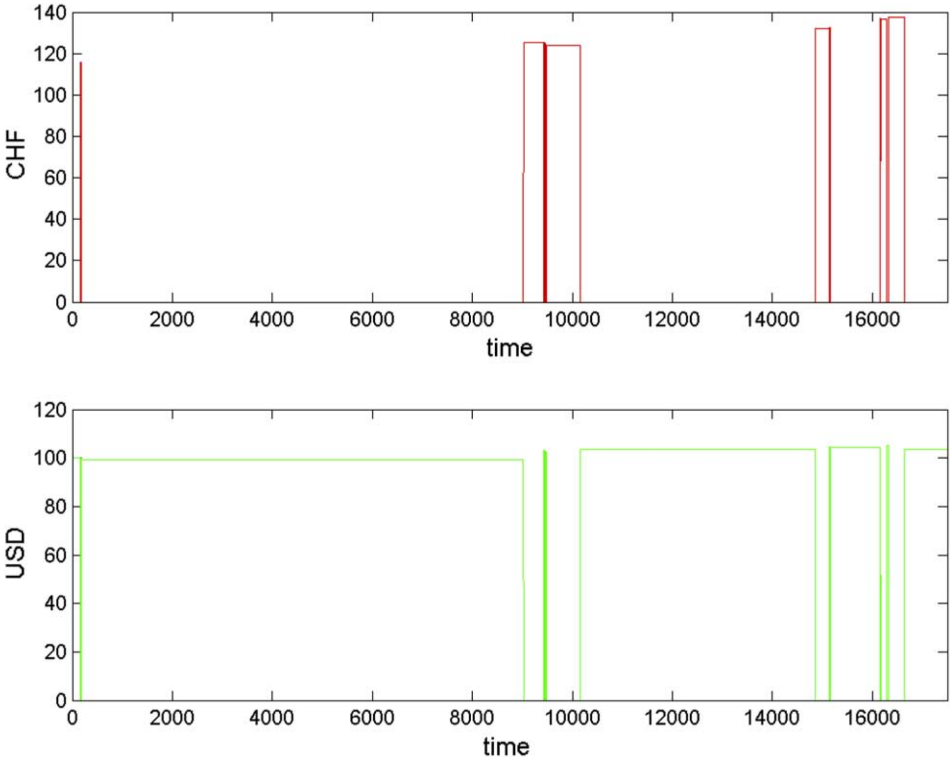

Fig. 14. Exchanges between CHF and USD with fixed transaction costs (0.2%) and state space volume limitation, starting with 100 units in USD.

described previously, i.e. if $|V(t)| > limit$, we perform the transaction. Table 3 summarises the results. As can be seen, in this case the gain decreases but still fluctuates around 11%. Furthermore, the number of combinations of time delay and embedding dimension for which it is possible to obtain a net gain also decreases considerably when compared with the results in Table 2. Moreover, considering 0.2% of transaction costs, few optimal strategies are found for embedding dimension of one—which does not imply that net gain may be still obtained. It seems that considering transaction costs trading strategies based on simple averaged first and second derivatives would produce worse results than using state space volume representations.

In Figs. 11–14 the results of two selected examples, the GPB and the CHF, are shown. As can be seen, to obtain a net gain the number of transactions decreases and only those for which state space volume is important are considered.

4.3. Comparison with the relative strength index (RSI)

Over the years, investors have developed many different indicators which attempt to measure the velocity or the acceleration of price movements and are used to determine a trading strategy. These indicators are grouped together under the heading of momentum. Some of the more popular indicators are: rate of change (ROC), relative-strength index (RSI), moving average convergence–divergence (MACD), and stochastic oscillator [13,14].

In order to compare with a well-known trading strategy, we have chosen the relative strength index (RSI) [14]. This index is a popular indicator created by US analyst J. Welles Wilder Jr. and measures the ratio of the sum of the up-moves to down-moves normalising the calculation between 0 and 100. We have taken the complete time series and optimise the trading values by modifying the percentages of the band (outside of which we will change) and window (past values used to calculate the average of up-moves and the down-moves). Table 4 summarises the results. As can be seen, the optimised RSI gives slight greater gains than the state space in fifteen out of eighteen currencies.

We have also compared with RSI for the case of fixed transaction costs 0.2%, see Table 5. Contrary to the case previous case, the state space volume method outperforms the traditional trading strategy by more than one order of magnitude. In this case, we can conclude that our trading strategy produces better results that traditional techniques usually employed by chartists.

5. Conclusions

In this work a new trading methodology based on state space volume calculation has been introduced. This methodology has been tested using eighteen high-frequency foreign exchange time series with and without transaction costs. In our analysis an optimum mean value of approximately 25% gain may be obtained in those series without transaction costs and an optimum mean value of approximately 11% gain assuming 0.2% of costs in each transaction. The trading strategy has been compared with the RSI (relative-strength index) used for trading in financial market. Even though slight better results, in terms of net gain, are obtained when transaction costs are not considered, when a fixed 0.2% transaction cost is introduced, our state space volume algorithm outperforms RSI by more than one order of magnitude.

In order to test the applicability of this proposed trading techniques, the analysis presented in this work should be done in real-time since in real markets investors' current and future forecast of payoffs affect their current and future trades which in turns affect returns, i.e. there is a feedback. Furthermore, we have used the complete time series to find optimal values of reconstruction parameters (time delay and embedding dimension) in order to show that there are values for which a net gain is possible. The high percentage of net gain cases indicates that is not difficult to find an adequate combination and update iteratively on real-time as data become available. This may be done using a similar approach as the RSI, which here we have also analysed using all the information available. Furthermore, even though we have presented results for trading at $t+1$ using information available at time t, the extension to $t+n$ is possible.

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