Journal of Business & Economic Statistics

2区

ISSN: 0735-0015 (Print) 1537-2707 (Online) Journal homepage: https://www.tandfonline.com/loi/ubes20

Measuring Asset Market Linkages: Nonlinear Dependence and Tail Risk

Juan Carlos Escanciano & Javier Hualde

To cite this article: Juan Carlos Escanciano & Javier Hualde (2019): Measuring Asset Market Linkages: Nonlinear Dependence and Tail Risk, Journal of Business & Economic Statistics, DOI: 10.1080/07350015.2019.1668797

To link to this article: https://doi.org/10.1080/07350015.2019.1668797

View supplementary material

Accepted author version posted online: 19 Sep 2019.

Submit your article to this journal

View related articles

View Crossmark data

Full Terms & Conditions of access and use can be found at
https://www.tandfonline.com/action/journalInformation?journalCode=ubes20

Check for updates

Measuring Asset Market Linkages: Nonlinear Dependence and Tail Risk

JUAN CARLOS ESCANCIANO*

Universidad Carlos III de Madrid

JAVIER HUALDE†

Universidad Pública de Navarra

*Research funded by the Spanish Programa de Generación de Conocimiento, reference number PGC2018-096732-B-I00.

†Research funded by the Spanish Plan Nacional de I+D+I, reference number ECO2015-64330-P.

Corresponding author JUAN CARLOS ESCANCIANO jescanci@eco.uc3m.es

Abstract

Traditional measures of dependence in time series are based on correlations or periodograms. These are adequate in many circumstances but, in others, especially when trying to assess market linkages and tail risk during abnormal times (e.g., financial contagion), they might be inappropriate. In particular, popular tail dependence measures based on exceedance correlations and Marginal Expected Shortfall (MES) have large variances and also contain limited information on tail risk. Motivated by these limitations, we introduce the (tail-restricted) Integrated Regression Function (IRF), and we show how it characterizes conditional dependence and persistence. We propose simple estimates for these measures and establish their asymptotic properties. We employ the proposed methods to analyze the dependence structure of some of the major international stock market indices before, during and after the 2007-2009 financial crisis. Monte Carlo simulations and the application show that our new measures are more reliable and accurate than competing methods based on MES or exceedance correlations for testing tail dependence.

Keywords and Phrases: Nonlinear dependence; Tail risk; Expected Shortfall; Market crashes.

Accepted Manuscript

JEL CLASSIFICATION: C14, C58, G15, G17.

1 Introduction

Asset market linkages, in particular those related to tail market events, have been widely studied. These analyses mainly focus on assessing the hypothesis that market correlation is higher in crisis periods, phenomenon also known as financial contagion. For this purpose several testing and estimation strategies have been introduced, including the modelization of the multivariate distribution of tail returns (Longin and Solnik, 2001), the analysis of the coincidence of tail returns across different countries (Bae, Karolyi and Stulz, 2003), the estimation of the expected number of market crashes given that at least one market crashes (Hartmann, Straetmans and de Vries, 2004) or the introduction of tools like the Marginal Expected Shortfall (MES) as an instrumental measure of systemic risk (Acharya et al., 2010, 2017).

Undoubtedly, these proposals, among others, have led to significant improvements in the difficult exercise of measuring tail dependence. Thus, within this setting, the main purpose of the present paper is to define general measures of dependence and persistence which can easily accommodate previous approaches to assess tail dependence, while at the same time offering a richer characterization. Specifically, we introduce the (tail-restricted) Integrated Regression Function (IRF) and its relation to the Marginal Expected Shortfall Function (MESF), and we show how they characterize conditional (tail) mean dependence and persistence. Unlike the previously mentioned approaches, our measures will not just focus on the occurrence of single tail market events, the so-called hits, which are only informative about whether the market is below a certain threshold. Indeed, both the IRF and the MESF are functions of a continuum of thresholds. The former is the covariance of a dependent variable of interest, often a firm ’s return, with the continuum of hits in the left tail, while the latter is the mean return conditional on the hit. In fact, measures of tail dependence based on single tail market events lead to a relatively narrow characterization of tail risk because they are not informative about the magnitude of market losses (i.e. how far is the market below the

Accepted Manuscript

fixed threshold). As will be seen, our proposed measures can address this limitation by accounting for a continuum of tail market events, thereby providing a more complete picture of tail dependence and systemic risk.

Additionally, the established link between the IRF and the MES makes inference based on estimators of the MES natural competitors for measuring and testing tail dependence. A popular method is Scaillet's (2005) smoothed MES estimator, which is an integral ingredient in Brownless and Engle's (2012, 2016) model of dynamic systemic risk. Other popular tools for measuring and testing tail dependence in empirical research are exceedance correlations, see, e.g., Longin and Solnik (2001), Ang and Bekaert (2002), Ang and Chen (2002), and Bae, Karolyi and Stulz (2003). Monte Carlo simulations and an empirical application show that, for the purpose of testing tail dependence, our measures lead to more accurate and robust inferences on tail dependence than competing methods based on MES or various versions of exceedance correlations, which can be explained by the large variances of these competing methods due to *small denominator* problems.

More generally, our proposal can be viewed as an attempt to provide useful analytical tools capable of assessing appropriately the dependence and persistence behavior of nonlinear time series (cf. Terasvirta, Tjøstheim, and Granger, 2011). In fact, while in the linear set-up there exists a complete machinery of tools for studying the dynamic properties of such series, mainly based on autocorrelations and periodograms, there are still today relatively few studies on how to nonparametrically measure the persistence properties of nonlinear time series. This problem is even more accentuated when the interest focuses on inherently nonlinear aspects such as tail dependence. Extreme value theory and copula theory have been successfully applied to measure *asymptotic* tail dependence; see tail dependence measures based on very extreme events, such as the extremogram of Davis and Mikosch (2009) and Davis, Drees, Segers and Warchol (2018), the MES estimator of Cai, Einmahl, de Haan and Zhou (2015) or the tail dependence measures based on copulas in, e.g., Oh and Patton (2017). Our proposal is complementary to these methods. While in extreme value

Accepted Manuscript

theory the threshold that defines the extreme event diverges to infinity (or negative infinity), in our approach a continuum of thresholds within a fixed interval is considered (which may include the whole tail of the distribution).

More closely related are the recent proposals for measuring and testing nonparametric dependence based on local Gaussian correlations (see, e.g., Berentsen and Tjøstheim, 2014 and Lacal and Tjøstheim, 2017, 2018) or the auto-distance correlations (see, e.g., Szekely et al., 2009, Zhou, 2012 and Davis et al., 2018). The distinctive features of our proposed measures are their simplicity of estimation, implementation and interpretation. They are easier to estimate than local Gaussian correlations, as they do not require optimization, while also avoiding bandwidth choices. They are also easier to interpret than auto-distance correlations, having direct links with popular measures of tail dependence and systemic risk.

From a theoretical perspective, our measures are rooted on the classical literature of nonparametric significance testing in Bierens (1982), Stute (1997), Hong (1999), Escanciano (2006), Escanciano and Velasco (2006a,b) and Linton and Whang (2007). Relative to this literature the paper provides four contributions: (i) it introduces new measures of tail dependence based the tail restricted IRF; (ii) it relates the IRF with new measures of dynamic systemic risk (the MESF); (iii) it presents theoretical guarantees that our measures characterize nonlinear dependence and persistence; and (iv) it provides new asymptotic theory for our nonparametric measures under weak dependence, including uncertainty quantification.

From an empirical perspective, the main contribution of this paper is that the proposed measures overcome the large variances and limited information on tail risk of MES and exceedance correlations. This is not to suggest that MES and exceedance correlations are not useful measures. While MES and exceedance correlations offer simple interpretations, our measures prove to be convenient for testing purposes and for obtaining a more complete characterization of tail risk. Thus, we view our measures and MES and exceedance correlations as complementary rather than competing.

Accepted Manuscript

The layout of the article is as follows. Section 2 introduces our dependence measures, illustrating their usefulness to capture many different forms of dependence and persistence, with special emphasis on those related to tail events. This section also makes connections with existing measures of systemic risk. Section 3 provides theoretical support for our measures, relating them to previous attempts to measure nonlinear dependence by Granger (1995, 2003). A natural extension to capture distributional dependence is introduced. Section 4 discusses estimation issues and establishes the asymptotic distribution of the new measures. Monte Carlo simulations are presented in Section 5. Section 6 reports the results of the empirical application, which focuses on the tail dependence and the persistent effects of the 2007-2009 financial crisis on some of the major international stock market indices. Finally, Section 7 concludes and describes further research. A Supplemental Appendix gathers all proofs, computational aspects, and further theoretical and empirical results.

2 New Measures of Nonlinear Dependence

Let $(Y_t, X_t)_{t\in\mathbb{Z}}$, with $Y_t \in \mathbb{R}, X_t \in \mathbb{R}^p$, be a time series vector, where X_t is allowed, but not required, to contain lagged values of Y_t. We assume Y_t is integrable, and let $E\left(Y_t\right) = \mu_t$ denote its mean for all t. Extensions to multivariate Y_t are straightforward. A natural way of characterizing persistence and dependence is to define the h-step regression function $m_{t,h}(x) = E(Y_{t+h} \,|\, X_t = x),\ \ x \in \mathbb{R}^p,\ \ h \geq 0$, almost surely (a.s.), which can be estimated nonparametrically. For example, Robinson (1983) has studied the large sample properties of kernel estimators of $m_{t,h}$ for various lags h, see also Auestad and Tjøstheim (1990). It is well known that the performance of smoothed estimators of $m_{t,h}$ is very sensitive to the dimension p of X_t, a feature that is referred to as the *curse of dimensionality* .

Our proposed measures of dependence avoid smoothing by considering the so-called (centered) Integrated Regression Function (IRF)

$$\gamma_{t,h}(x) := Cov(Y_{t+h}, 1(X_t \leq x)) = E[(Y_{t+h} - \mu_{t+h})1(X_t \leq x)],$$

Accepted Manuscript

where $1(A)$ denotes the indicator function of the event *A* and $X_t \leq x$ is understood componentwise. These measures were studied in Escanciano and Velasco (2006a) for time series; see also Stute (1997) for the different purpose of nonparametric significance testing in regression with independent data. Note that

$$\gamma_{t,h}(x) = \int_{(-\infty,x]} \left(m_{t,h}(z) - \mu_{t+h}\right) F_t(dz), \qquad (1)$$

where F_t is the cumulative distribution function (cdf) of X_t, which justifies the name for $\gamma_{t,h}$, and $(-\infty, x] = \prod_{j=1}^{p}(-\infty, x_j]$, with $x = (x_1, ..., x_p)'$. A "large" value of $\gamma_{t,h}$ suggests a large difference between the conditional mean $m_{t,h}$ and the unconditional one μ_{t+h}, which in turn indicates that Y_{t+h} and X_t are dependent in mean. In order to measure the "size" of $\gamma_{t,h}$ we introduce the uniform ($sup-$) norm $\|\gamma_{t,h}\|_\infty = \sup_{x\in\mathbb{R}^p}|\gamma_{t,h}(x)|$. We illustrate the behavior of our measures by means of several examples.

Example 1 (Mean Dependence). Take $Y_t = X_t = Z_t$, where $\{Z_t\}_{t\in\mathbb{Z}}$ is a sequence of strictly stationary, mean-zero, unit-variance, Gaussian random variables (r.v.'s) with correlation $\rho_h = E[Z_t Z_{t+h}]$, $h \geq 1$. By a change of variables, it can be shown that the IRF is

$$\gamma_{t,h}(x) = -\rho_h \phi(x),$$

where $\phi(\cdot)$ is the standard Gaussian density. Therefore we have $\|\gamma_{t,h}\|_\infty = |\rho_h| / \sqrt{2\pi}$. Thus, under Gaussianity and mild regularity conditions, our measure of dependence reduces, as expected, to the traditional linear one based on correlations. Note that in this example the significance and sign of $\gamma_{t,h}$ is fully determined by the correlation (so $\gamma_{t,h}(x) > 0$ for all *x* iff $\rho_h < 0$), the discrepancy between both signs being natural in view of the definition of $\gamma_{t,h}(x)$. ■

In other situations the object of interest is not the conditional mean of the process but some higher order moments, e.g. volatility. An example in the Supplemental Appendix shows how our measures can be easily adapted to cover this case. Additionally, we

Accepted Manuscript

might be interested in other parts of the distribution. Of special interest in risk management is the tail and the impact of systemic tail events on individual firms. In fact, as we show next, it is possible to connect our measures with well known measures of systemic risk and contagion, and in doing so, we extend existing measures to more informative measures that better account for tail risk.

From the definition of conditional probability

$$\gamma_{t,h}(x) = \left(MES_{t,h}(x) - \mu_{t+h}\right) F_t(x), \qquad (2)$$

where $MES_{t,h}(x) = E\left(Y_{t+h} \middle| X_t \leq x\right)$ is the MESF introduced in this paper. The MES of Acharya et al. (2010, 2017) corresponds to $MES_{t,0}(x)$ with *x* equals the $\alpha-$quantile of the distribution $F_t,$ for the market return index X_t. As seen below, by focusing on a continuum of values of *x*, rather than just one, our measures better capture tail dependence, and as such, they contain more information on tail risk than existing measures; see Example 2 below for illustration.

From (2), when $F_t(x) > 0,$ we can compute $MES_{t,h}(x)$ as a function of our measure as

$$MES_{t,h}(x) = \mu_{t+h} + \frac{\gamma_{t,h}(x)}{F_t(x)}. \qquad (3)$$

If $F_t(x) = 0,$ then by the absolute continuity of $\gamma_{t,h}(x)$ with respect to $F_t(x),$ it must hold that $\gamma_{t,h}(x) = 0.$ Some conditions are then needed to solve the indeterminacy, and we provide below some sufficient conditions for this. More generally, a *small denominator* $F_t(x)$ may lead to problems for estimating MES—problems that are not present for estimating our measures—and these difficulties accentuate when the whole tail is of interest.

Specifically, let $\bar{x}$ denote some threshold with $F_t(\bar{x})$ potentially small but positive. For example, $\bar{x}$ may denote an unconditional 5% quantile when *p* = 1 or a vector of quantiles when $p > 1.$ Then, the restricted $\sup$ (pseudo)-norm over the whole tail is

Accepted Manuscript

$$\| \gamma_{t,h} \|_{\bar{x}} = \sup_{x \le \bar{x}} \left| \gamma_{t,h}(x) \right|,$$

which defines measures of tail dependence (precisely, how X_t being in the left tail, as specified by $X_t \le \bar{x}$, affects the mean of Y_{t+h}). Similar measures applied to $MES_{t,h}$ would inevitably face the *small denominator* problems mentioned above, since the region $x \le \bar{x}$ contains values x for which $F_t(x)$ gets arbitrarily small. Our measures avoid such problems. The motivation to consider the supremum over $x \le \bar{x}$ is to account for tail risk, as illustrated in the following example.

Example 2 (Tail Risk). Consider two hypothetical scenarios: in scenario 1, the profit of a bank (in billion US$), say Y_t, and a market index, denoted as X_t, follow a bivariate normal distribution with mean zero, unit variances and correlation 0.25. The MES at 5%, i.e the bank's expected profit conditional on the market being smaller than its 5% quantile, is a loss of 515.7 million US$. In scenario 2, the marginal distribution of the market index is the same as in scenario 1, but the bank losses 14.157 billion US$ if the market index is below its 0.5% quantile and it gains 1 billion otherwise. The MES at 5% of scenario 2 is the same as scenario 1, despite tail risk being rather different under these two scenarios. In contrast, our measure with $h = 0$ and $\bar{x}$ equals the 5% quantile is 192.5% larger in scenario 2 than in scenario 1. It is in this sense that MES provides limited information on tail risk. Details on the computations in this example are provided in the Supplemental Appendix to this paper. ■

The next example shows how we can adapt our methodology to examine the predictability and persistent properties of market crashes. The phenomenon of how market crashes spill over to other countries was first systematically studied by Morgenstern (1959) and it has been extensively studied after that. The new measures provide useful tools to study crashes and their dynamic and systemic properties. We will particularize them to two stocks (whose returns are denoted as $r_{t,1}$ and $r_{t,2}$, respectively), although generalizations are straightforward.

Accepted Manuscript

Example 3 (Persistence of Crashes). Choose $r_{crash}^{(1)}$ so that we define a market crash at time t whenever $r_{t,1} \le r_{crash}^{(1)}$ ($r_{crash}^{(1)}$ could be, e.g., the 1% or 5% empirical quantile). Set $Y_t = 1(r_{t,1} \le r_{crash}^{(1)})$ and $X_t = r_{t,2}$. Then, noting (3),

$$E\left(Y_{t+h} \middle| X_t \le x\right) = \Pr\left(r_{t+h,1} \le r_{crash}^{(1)} \middle| r_{t,2} \le x\right) = F_{t+h}^{(1)}\left(r_{crash}^{(1)}\right) + \frac{\gamma_{t,h}(x)}{F_t^{(2)}(x)}, \qquad (4)$$

where $F_t^{(1)}, F_t^{(2)}$, are the cdf's of $r_{t,1}, r_{t,2}$, respectively. Thus the conditional probability of a crash equals the unconditional one plus a correction term which captures the tail dependence between $r_{t+h,1}$ and $r_{t,2}$. ■

To conclude this section, these examples and others reported in the Supplemental Appendix illustrate the versatility of our measures by applying them to different definitions of Y_t for studying dependence in mean, volatility and market crashes, among others. In all these cases, the relation between MES and our measures in (3) or (4) favours our approach due to the benefits of avoiding the close-to-zero denominator. As will be seen in Section 5, this advantage translates to smaller variances and more accurate inference in comparison to alternative procedures. This justifies our decision to not consider correlation versions of our measures which would lead to the denominator problems mentioned above, particularly so for measures that account for tail risk. We lose the simple interpretation of correlations, but in our opinion the gains in inference substantially out-weight the loss of interpretation.

3 Characterization of Dependence and Persistence

In this section we present some theoretical properties of our measures and, additionally, we provide a natural generalization to distributional dependence. We start introducing new concepts of independence, which are well motivated in, e.g., risk management.

DEFINITION 1. Y_{t+h} *is* $\bar{x}-$ *tail–mean independent of* X_t if $m_{t,h}(x) = \mu_{t+h}$ *a.s. for all* $x \le \bar{x}$.

Accepted Manuscript

This concept of independence is weaker than full independence. It is also weaker than mean independence ($m_{t,h}(x) = \mu_{t+h}$ a.s.) when $F_t(\bar{x}) < 1.$ Our first result characterizes tail-mean independence in terms of our measures.

PROPOSITION 1. Y_{t+h} *is* $\bar{x}-$ *tail–mean independent of* X_t iff $\left\| \gamma_{t,h} \right\|_{\bar{x}} = 0.$

Interestingly, this result implies that our unconditional measures characterize a type of conditional dependence (tail-mean dependence). A special case of this result was obtained in Escanciano and Velasco (2006a,b), who showed that

$$m_{t,h}(X_t) = \mu_{t+h} \text{a.s.} \Leftrightarrow \left\| \gamma_{\text{t,h}} \right\|_{\infty} = 0. \qquad (5)$$

That is, for $\bar{x} = \infty$ the IRF characterizes conditional mean independence. We note that a necessary but not sufficient condition for $\bar{x}-$tail–mean independence is $MES_{t,h}(\bar{x}) = \mu_{t+h}$. Tail-mean independence involves the whole tail, i.e. all thresholds x with $x \leq \bar{x}$, and not just the single threshold $\bar{x}$. A characterization with the MESF is possible, but more involved due to denominator problems. The following result provides primitive conditions for the univariate case.

PROPOSITION 2. *For all t, assume: (i)* $E\left|Y_t\right| < \infty;$ *(ii)* X_t is absolutely continuous with density $f_t(x) > 0$ *for all* $x \in \mathbb{R};$ *and (iii)* $\lim_{x \to -\infty} E\left(Y_{t+h} \middle| X_t = x\right)$ *exists. Then,*

1. $\lim_{x \to -\infty} MES_{t,h}(x) = \lim_{x \to -\infty} E\left(Y_{t+h} \middle| X_t = x\right)$.
2. $\left\| MES_{t,h} - \mu_{t+h} \right\|_{\bar{x}} = 0 \Leftrightarrow \left\| \gamma_{t,h} \right\|_{\bar{x}} = 0$ for any $\bar{x} \in \mathbb{R}$.

Thus, an interpretation of $\bar{x}-$tail–mean independence is that none of the tail events $1(X_t \leq x)$ help to predict Y_{t+h} when $x \leq \bar{x}.$ This type of independence restriction can be useful to identify parameters for MES models, similar to conditional mean (median) restrictions that are used for identification of conditional mean (median) models. Indeed, the case $\bar{x} = \infty$ corresponds to conditional mean independence. Indirectly, the results above show formally that the MESF characterizes mean independence.

Accepted Manuscript

Additionally, we explore the connection between our measures and a very general notion of nonparametric persistence in mean proposed by Granger (1995). Following Granger (1995), we define the concepts of *Short Memory in Mean* (SMM) and *Extended Memory in Mean* (EMM) as follows.

DEFINITION 2. $(Y_t, X_t)_{t\in\mathbb{Z}}$ *is called SMM if for all* $t \in \mathbb{Z}$

$$\lim_{h\to\infty} E\left(m_{t,h}\left(X_t\right) - \mu_{t+h}\right)^2 = 0.$$

If $(Y_t, X_t)_{t\in\mathbb{Z}}$ *does not satisfy the previous condition is called EMM.*

Granger (1995) allowed for $p = \infty$, but to make this definition practically operative $p < \infty$ is convenient, although potentially *p* can be large. Granger and Hallman (1991) replaced the name of EMM by long memory in mean, which could be slightly misleading because, in general, a stationary linear long memory process is SMM. Gourieroux and Jasiak (1999) referred to SMM and EMM as nonlinear integrated and nonlinear integrated of order zero, respectively. As noted by Granger (1995) the concepts of SMM and EMM are related to a kind of "mixing in mean" property, or, more precisely, to the concept of mixingale (cf. McLeish, 1974); see Davidson (1994, Chapter 16).

We shall extend the characterization (5) to the case of persistence. That is, we show that, under a mild moment condition,

$$\lim_{h\to\infty} E\left(m_{t,h}\left(X_t\right) - \mu_{t+h}\right)^2 = 0 \Leftrightarrow \lim_{h\to\infty} \left\| \gamma_{t,h} \right\|_\infty = 0.$$

This new characterization justifies measuring nonparametric persistence with the measures $\gamma_{t,h}$. As mentioned before, this result also shows that although our measures are unconditional means, which is convenient for estimation, they characterize persistence of *conditional* measures of dependence.

THEOREM 1. *If* $\sup_t EY_t^2 < \infty$, *then* $(Y_t, X_t)_{t\in\mathbb{Z}}$ *is SMM iff* $\lim_{h\to\infty} \left\| \gamma_{t,h} \right\|_\infty = 0$ *.*

Accepted Manuscript

This theorem together with (5) formalizes the use of $\gamma_{t,h}$ to measure nonlinear mean dependence and persistence. These results also justify introducing new concepts of tail persistence. We will say $(Y_t, X_t)_{t\in\mathbb{Z}}$ is $\bar{x}-$tail–SMM iff $\lim_{h\to\infty}\left\|\gamma_{t,h}\right\|_{\bar{x}}=0$. Concepts such as these may be useful to answer questions such as: does a market tail event have a persistent long run effect on a firm's return?

3.1 Distributional Dependence

In some instances, the interest is in the joint distribution of (Y_{t+h}, X_t). Granger (2003) defines a persistent process in distribution using the joint and marginal densities at different lags. Gourieroux and Jasiak (2002) considered series expansions estimators for the nonlinear canonical analysis of the series, cf. Buja (1990). In this paper we propose alternative measures that avoid existence and estimation of densities, and the corresponding bandwidth choices. Following the logic above, we propose using the cumulative measure

$$\begin{aligned}\eta_{t,h}(y,x) &= Cov(1(Y_{t+h}\le y), 1(X_t \le x)) \\ &= K_{t,h}(y,x) - G_{t+h}(y)F_t(x),\end{aligned}$$

where $K_{t,h}(y,x)$, $G_{t+h}(y)$, and $F_t(x)$ are, respectively, the joint and marginal cdf's of Y_{t+h} and X_t, so for the choice $X_t = Y_t$, $\eta_{t,h}(y,x)$ corresponds to the Laplace cross-covariance kernel of Dette et al. (2015). Under Gaussianity and additional regularity conditions, our concept of persistence in distribution captures the same dependence as linear measures. More generally, by Hoeffding's identity (see Hoeffding, 1940), when p = 1

$$Cov(Y_{t+h}, X_t) = \int \eta_{t,h}(y,x)dydx,$$

from which is clear that $\eta_{t,h}$ also contains the information from linear dependence.

We could tailor $\eta_{t,h}(y,x)$ to specific values of y and x to focus on different parts of the distribution. For example, when y and x are set in the left tail of the distributions of Y_{t+h}

Accepted Manuscript

and X_t, respectively, the covariances $\eta_{t,h}(y,x)$ measure distributional tail dependence. Specifically, we define the tail dependence measure

$$\|\eta_{t,h}\|_{\bar{y},\bar{x}} = \sup_{y \le \bar{y}} \sup_{x \le \bar{x}} \left|\eta_{t,h}(y,x)\right|.$$

This extends the linear correlation measures defined in Longin and Solnik (2001), Ang and Bekaert (2002), Ang and Chen (2002), and Bae, Karolyi and Stulz (2003), which have been used to define contagion in financial markets, and which correspond to setting y and x to a single value equal to an empirical quantile, such as the 5%. Considering a continuum of values of y and x brings much more information on distributional tail dependence. Our measure of tail dependence is different from the extremogram in which $\bar{y}$ and $\bar{x}$ diverge to negative infinity (see Davis and Mikosch 2009). It is also different from tail dependence measures defined in the literature on copulas. To see this, define $\alpha_y = G_{t+h}(\bar{y})$ and $\alpha_x = F_t(\bar{x})$, and let $C_{t,h}$ denote the copula function of Y_{t+h} and X_t (with $p=1$), i.e. $C_{t,h}(u,v) = K_{t,h}(G_{t+h}^{-1}(u), F_t^{-1}(v))$ when G_{t+h} and F_t are strictly monotone (see Malevergne and Sornette 2006 for an extensive discussion of copulas and tail dependence measures). Then, our measure can be written as

$$\|\eta_{t,h}\|_{\bar{y},\bar{x}} = \sup_{0 \le u \le \alpha_y} \sup_{0 \le v \le \alpha_x} \left|C_{t,h}(u,v) - uv\right|, \quad (6)$$

which is different from the commonly used copula tail dependence measure

$$\lambda_{t,h} := \lim_{u \to 0} \frac{C_{t,h}(u,u)}{u}.$$

The quantity $\lambda_{t,h}$ is asymptotic and its nonparametric estimation requires smoothing of the data in the tails along the diagonal $u = v$ as $u \to 0$. In contrast, our nonparametric estimators of $\|\eta_{t,h}\|_{\bar{y},\bar{x}}$ do not require smoothing. In conection with the representation (6) in terms of the joint copula, we note that Schweizer and Wolff (1981) have studied the properties of the measure $\|\eta_{t,h}\|_{\bar{y},\bar{x}}$, as a measure of full statistical dependence, for the special case $\bar{y} = \bar{x} = \infty$ (so $\alpha_y = \alpha_x = 1$).

Accepted Manuscript

Recently, alternative measures of distributional dependence, called auto-distance covariances, have been proposed in Szekely et al. (2009), Zhou (2012) and Davis et al. (2018). These measures were introduced to measure and test full independence, and they are given, for a suitable measure $\mu,$ by

$$\int_{\mathbb{R}^{p+1}} \left|Cov(\exp(iyY_{t+h}), \exp(ixX_t))\right|^2 d\mu(y,x).$$

Unlike with our measures, it is not clear how to taylor the auto-distance covariances to focus on tail-dependence rather than full independence, and thus to relate them with popular measures of tail dependence and systemic risk.

4 Estimation and Asymptotic Theory

We discus now how the new measures are estimated and establish their asymptotic properties for strictly stationary time series. The analysis for general non-stationary data is more complicated and beyond the scope of this paper. The natural estimator for $\gamma_h \equiv \gamma_{t,h}$ based on a sample $\{Y_t, X_t\}_{t=1}^n$ is

$$\hat{\gamma}_h(x) = \frac{1}{n-h}\sum_{t=1}^{n-h}(Y_{t+h} - \bar{Y}_{n-h})1(X_t \le x), \qquad (7)$$

with $\bar{Y}_{n-h} = (n-h)^{-1}\sum_{t=1}^{n-h} Y_t.$

Thus, a natural estimator for MESF based on our measures when $\hat{F}_h(x) > 0$ is

$$MES_h(x) = \bar{Y}_{n-h} + \frac{\hat{\gamma}_h(x)}{\hat{F}_h(x)}, \qquad (8)$$

where

$$\hat{F}_h(x) = \frac{1}{n-h}\sum_{t=1}^{n-h} 1(X_t \le x).$$

An alternative smoothed MES estimator proposed in Scaillet (2005) is

Accepted Manuscript

$$\hat{S}_h(x) = \frac{\sum_{t=1}^{n-h} Y_{t+h} \Phi(\frac{x - X_t}{a})}{\sum_{t=1}^{n-h} \Phi(\frac{x - X_t}{a})}, \qquad (9)$$

where $a > 0$ is a bandwith parameter. Under suitable regularity conditions, the unsmoothed and smoothed estimators are asymptotically equivalent. One advantage of $\hat{S}_h(x)$ over $MES_h(x)$ is that the denominator is always positive for $a > 0$.

Distributional measures of dependence are similarly estimated as

$$\hat{\eta}_h(y, x) = \frac{1}{n-h} \sum_{t=1}^{n-h} 1_c(Y_{t+h}, y) 1_c(X_t, x), \qquad (10)$$

where $1_c(Z_t, x) = 1(Z_t \le x) - (n-h)^{-1} \sum_{s=1}^{n-h} 1(Z_s \le x)$, for a generic series $\{Z_t\}_{t=1}^{n}$.

We investigate the asymptotic properties of the estimated measures $\hat{\gamma}_h$ and $\hat{\eta}_h$. Throughout the rest of this section the lag $h \ge 1$ is fixed. For a generic *Z*, define $1_e(Z, x) \equiv 1(Z \le x) - E[1(Z \le x)]$. By extending the definition of $\hat{\gamma}_h$ as $\hat{\gamma}_h(+\infty) = 0 = \hat{\gamma}_h(-\infty)$, we can view $\hat{\gamma}_h$ as a process in $\ell^\infty(\overline{\mathbb{R}}^p)$, the space of uniformly bounded functions on $\overline{\mathbb{R}}^p$, where $\overline{\mathbb{R}}^p := [-\infty, \infty]^p$. The same is done with $\hat{\eta}_h$. We denote $\Rightarrow$, the weak convergence in $(\ell^\infty(\overline{\mathbb{R}}^p), d_\infty)$ or $(\ell^\infty(\overline{\mathbb{R}}^{p+1}), d_\infty)$ in the sense of J. Hoffmann-Jørgensen (see Definition 1.3.3 in van der Vaart and Wellner, 1996).

Under strictly stationarity and ergodicity, the uniform consistency of $\hat{\gamma}_h(\cdot)$ or $\hat{\eta}_h(\cdot)$ follows from the Ergodic Theorem and a Glivenko-Cantelli's argument, see e.g. Koul and Stute (1999). Thus, this section focusses on the weak convergence of

$$\upsilon_{n,h}(\cdot) = \sqrt{n-h}(\hat{\gamma}_h(\cdot) - \gamma_h(\cdot)) \qquad \text{and} \qquad \omega_{n,h}(\cdot) = \sqrt{n-h}(\hat{\eta}_h(\cdot) - \eta_h(\cdot)).$$

We establish the asymptotic theory for $\upsilon_{n,h}(\cdot)$ and $\omega_{n,h}(\cdot)$ for β-mixing processes; see Escanciano and Hualde (2017) for extensions to mixingale classes. With these

Accepted Manuscript

asymptotic results, we can compute uniform confidence bands for $\hat{\gamma}_h, \hat{\eta}_h$, and the hypothesis of restricted (e.g. tail) independence can be tested, as shown below.

The following regularity conditions are needed for the subsequent asymptotic analysis. Let $\mathcal{F}_s^t \equiv \mathcal{F}_s^t(X,Y)$ denote the σ-algebra generated by $\{(X_j, Y_j),\ j = s,\ldots,t\},\ s \le t,\ s,t \in \mathbb{Z}$. Define the β-mixing coefficients as (see, e.g., Doukhan (1994))

$$\beta_t = \sup_{m\in\mathbb{Z}} \sup_{A\in\mathcal{F}_{t+m}^{\infty}} E\left|P(A|\mathcal{F}_{-\infty}^{m}) - P(A)\right|.$$

Define $\xi_{t,h} = (Y_{t+h}, X_t)$ and for a measurable function f of $\xi_{t,h}$ define the generic long-run variance function

$$C_h(f) = \sum_{j=-\infty}^{\infty} E[\left\{ f(\xi_{0,h}) - E[f(\xi_{0,h})]\right\}\left\{ f(\xi_{j,h}) - E[f(\xi_{j,h})]\right\}].$$

Define the classes $\mathcal{F}_w = \left\{ f_{(y,x)}(\xi_{t,h}) = 1_e(Y_{t+h} \le y) 1_e(X_t \le x) : (y,x) \in \overline{\mathbb{R}}^{p+1}\right\}$ and $\mathcal{F}_v = \left\{ f_x(\xi_{t,h}) = (Y_{t+h} - \mu) 1_e(X_t \le x) : x \in \overline{\mathbb{R}}^{p}\right\}$. Let $w_{\infty,h}$ denote a Gaussian process in $\ell^\infty(\overline{\mathbb{R}}^{p+1})$ with zero-mean, continuous sample paths, and variance function $C_h(f_{(y,x)})$, with $f_{(y,x)} \in \mathcal{F}_w$, and let $v_{\infty,h}$ denote a Gaussian process in $\ell^\infty(\overline{\mathbb{R}}^{p})$ with zero-mean, continuous sample paths, and variance function $C_h(f_x),\ f_x \in \mathcal{F}_v$. We need the following assumption on temporal dependence and moments:

A1: $\{Y_t, X_t\}_{t\in\mathbb{Z}}$ is a strictly stationary and absolutely regular (β-mixing), with mixing coefficients of order $O(t^{-b})$, for some b such that: (a) $b > 1$ or (b) $b > p/(p-2)$ and $E(|Y_1|^p) < \infty$ for some $2 < p < \infty$.

THEOREM 2: (i) *Under A1(a),* $w_{n,h} \Rightarrow w_{\infty,h}$. (ii) *Under A1(b),* $v_{n,h} \Rightarrow v_{\infty,h}$.

Weak convergence for $w_{n,h}$ requires weaker conditions than for $v_{n,h}$; a result due to indicator functions being bounded. Theorem 2 and the continuous mapping theorem imply, under the corresponding assumptions, the convergences

Accepted Manuscript

$$\left\| w_{n,h} \right\|_\infty \Rightarrow \left\| w_{\infty,h} \right\|_\infty \text{ and } \left\| v_{n,h} \right\|_\infty \Rightarrow \left\| v_{\infty,h} \right\|_\infty,$$

with similar convergences holding for restricted norms. The limiting distributions $\left\| w_{\infty,h} \right\|_\infty$ and $\left\| v_{\infty,h} \right\|_\infty$ depend on the data generating process in a complicated way. To approximate quantiles of these distributions we use the moving block boostrap (MBB) of Kunsch (1989) and Liu and Singh (1992). To describe the bootstrap denote the original sample by $Z_1,...,Z_n$, where $Z_t = (Y_t, X_t)$. Let b denote the block size, $1 \le b < n$, and define $Z_{n+i} = Z_i$ for $i = 1,...,b$, and the block $B_{i,b} = \{Z_i,...,Z_{i+b-1}\}$, $i \le n$. If $I_1,...,I_k$, where $k = [n/b]$, are *iid* uniform on $\{1,...,n\}$, then the MBB sample consists of all data points that belong to the blocks $B_{I_1,b},...,B_{I_k,b}$, i.e. $Z_1^* = Z_{I_1},...,Z_b^* = Z_{I_1+b-1}, Z_{b+1}^* = Z_{I_2},...,Z_l^* = Z_{I_k+b-1}$, where $l = kb$. Without loss of generality we can assume $l = n$. Alternative bootstrap methods could be used, including the stationary bootstrap of Politis and Romano (1994), or subsampling; see the monograph by Politis, Romano and Wolf (1999). When Y_t is a martingale difference sequence with respect to $\mathcal{F}_{-\infty}^{t-1}$, a wild bootstrap will also work, see e.g. Escanciano and Velasco (2006a,b). In this paper we focus on the more general dependence case. The validity of the MBB approximations follows from Radulovic (1996). We use these results to construct nonparametric tests of tail (mean and distributional) independence.

We compute bootstrap p-values for testing tail–mean independence as follows. Fix a lag $h \ge 1$ and $\bar{x} \in \mathbb{R}^p$, and compute the Kolmogorov-Smirnov (KS) statistic

$$K_{n,\bar{x}}(h) := \left\| \hat{\gamma}_h \right\|_{\bar{x}} .$$

An algorithm to compute $\hat{\gamma}_h(x)$, $MES_h(x)$, $K_{n,\bar{x}}(h)$ and related quantities is given in the Supplemental Appendix. This algorithm is also used to compute bootstrap test statistics $K_{n,\bar{x}}^{(r)}(h) = \left\| \hat{\gamma}_h^{(r)} - \hat{\gamma}_h \right\|_{\bar{x}}$, for $r = 1,...,R$, based on R bootstrap samples from the MBB. Centering the bootstrap statistics by the original measures $\hat{\gamma}_h$ is key for the validity of the bootstrap approximation (cf. Radulovic (1996)). Then, we compute the bootstrap p-value as

Accepted Manuscript

$$\hat{p}_{n,\bar{x}}(h) = \frac{1}{R}\sum_{r=1}^{R} 1(K_{n,\bar{x}}^{(r)}(h) \geq K_{n,\bar{x}}(h)). \quad (11)$$

We reject the hypothesis of $\bar{x}-$tail–mean independence at lag h (i.e. $\|\gamma_h\||_{\bar{x}} = 0$) and significance level α iff $\hat{p}_{n,\bar{x}}(h) < \alpha.$ Bootstrap tests for distributional dependence are discussed in the Supplemental Appendix.

5 Monte Carlo Simulations

In this section we report simulations to illustrate the finite sample performance of the proposed measures in the context of testing for $\bar{x}-$ tail-mean independence and ($\bar{y},\bar{x}$)-tail-distributional independence for several choices of $\bar{y}$ and $\bar{x}$. To save space, we report the simulation results for distributional dependence in the Supplemental Appendix, while those for $\bar{x}-$tail-mean dependence are reported in the main text. We evaluate the performance of our bootstrap test in (11) relative to a test based on the MES estimator suggested in Scaillet (2005), and used in Brownlees and Engle (2012, 2016), and also to linear exceedance correlation tests.

Regarding the test based on the smoothed MES, we define the test statistic as

$$\hat{T}_{n,\bar{x}}(h) = \max_{1\leq s\leq n}\left|\hat{S}_h(X_s) - \bar{Y}_{n-h}\right| 1(X_s \leq \bar{x}),$$

where $\hat{S}_h(X_s)$ is defined in (9). To carry out this test we also use a MBB approximation, and compute the bootstrap p-value as

$$\hat{s}_{n,\bar{x}}(h) = \frac{1}{R}\sum_{r=1}^{R} 1(\hat{T}_{n,\bar{x}}^{(r)}(h) \geq \hat{T}_{n,\bar{x}}(h)),$$

where $\hat{T}_{n,\bar{x}}^{(r)}(h),\ r = 1,...,R,$ are R centered bootstrap realizations of $\hat{T}_{n,\bar{x}}(h),$ i.e.

$$\hat{T}_{n,\bar{x}}^{(r)}(h) = \max_{1\leq s\leq n}\left|\hat{S}_h^{(r)}(X_s) - \bar{Y}_{n-h}^{(r)} - \hat{S}_h(X_s) + \bar{Y}_{n-h}\right| 1(X_s \leq \bar{x}).$$

We follow Scaillet (2005) and choose the bandwidth as $a = \hat{\sigma}\times(n-h)^{-1/5},$ where $\hat{\sigma}$ is the sample standard deviation of $\{X_t\}_{t=1}^{n-h}.$

Accepted Manuscript

We also compare our proposal with a test based on the exceedance correlation measure

$$\hat{L}_{n,\bar{x}}(h) = \left|Corr(Y_{t+h}, X_t 1(X_t \le \bar{x})\right|.$$

The linear correlation test is implemented with the bootstrap p-value $\hat{l}_{n,\bar{x}}(h) = \sum_{r=1}^{R} 1(\hat{L}^{(r)}_{n,\bar{x}}(h) \ge \hat{L}_{n,\bar{x}}(h))/R,$ where $\hat{L}^{(r)}_{n,\bar{x}}(h)$ is a centered bootstrap version of $\hat{L}_{n,\bar{x}}(h)$.

We simulate data from the following data generating processes (DGPs), with further details and DGPs provided in the Supplemental Appendix. Henceforth, ε_t is an independent $N(0, 1)$ shock and $\tau \in (0,1]$.

- DGP1: Y_t, X_t independent $N(0,1)$.
- DGP2: Y_t, X_t independent $GARCH(1,1)$.
- DGP3: $X_t \sim U[0,1]$ and $Y_t = 0.5\left[\left(X_t - 1\right) + \left(X_t - \tau\right)\right]1(X_t > \tau) + \varepsilon_t$.
- DGP4: $X_t \sim N(0,1)$ and $Y_t = 0.5X_t + \varepsilon_t$.
- DGP5: $X_t \sim U[0,1]$ and $Y_t = 0.5\left[\left(X_t - 1\right) + \left(X_t - \tau\right)\right]1(X_t \le \tau) + \varepsilon_t$.
- DGP6: $X_t \sim N(0,1)$ and $Y_t = X_t^2 + \varepsilon_t$.

As mentioned before, we focus on testing for $\bar{x}$-tail-mean independence $H_0 : \|\gamma_h\|_{\bar{x}} = 0$ for $\bar{x} = F_h^{-1}(\tau)$, $\tau = 0.1, 0.5, 1$, and $h = 0$. We compare the three tests described above, $K_{n,\bar{x}}, \hat{T}_{n,\bar{x}}$ and the linear correlation test $\hat{L}_{n,\bar{x}}$. The number of Monte Carlo simulations is 500 and the bootstrap replications $R = 500$. To select the size of the blocks in the MBB we use the automatic procedure implemented in the b.star function of the np R package (Racine 2018). The R code to implement all the simulations and the empirical application will be provided at the journal website. Rejection Probabilities (RP), multiplied by 100, are reported in Table 1. DGP1 and DGP2 fall under H_0, as the two processes are fully independent. An interesting feature of DGP3 is that although Y_t and X_t are dependent, they are $\bar{x}$ -tail-mean independent, since $F_h^{-1}(\tau) = \tau$ and $1(X_t \le x)1(X_t > \tau) = 0$ for all $x \le \tau$ (thus $\|\gamma_h\|_{\bar{x}} = 0$). DGP4 to DGP6 fall under the alternative. Our test is able to control the empirical size even for a small sample size as

Accepted Manuscript

$n = 100,$ despite considering small quantile levels such as 0.1. In contrast, the Scaillet's (2005) MES based test and correlation tests present some size distortions (overrejections). For the former, these distortions persist even for larger sample sizes as n = 300 and larger quantile levels. The empirical power of our test is satisfactory, in most cases either the best or comparable to the best performance among the three tests, despite the overrejections observed in the competing tests. The linear correlation test presents low power for DGP5 at the median with n = 100 and at the tail for n = 300, and no power against DGP6 at τ = 1 even for large sample sizes. Summarizing, these simulations illustrate that the finite sample performance of our test is satisfactory across different DGPs, sample sizes and quantiles. In general, our method has better performance than competing methods based on estimation of the MES and exceedance correlation measures.

To gain further insights into the overrejections of MES based tests we report in Figure 1 block bootstrap standard errors for marginal measures $\hat{S}_h(x)$ and $\hat{\gamma}_h(x)$ estimated from the data in our empirical application for x at the order statistics of $X_t =$ S&P500 for predicting $Y_t =$ FTSE at lag $h = 1$. We use daily data for these two stocks from our application below for the period August 8th 2009 to February 3rd 2018. The results show that the uncertainty involved in estimating the MES is much larger than for estimating our measures. Furthermore, this uncertainty increases significantly with the severity of losses (the smaller the quantile level, the larger the uncertainty). We conjecture that this explains the overrejections observed for MES based tests throughout the simulations and the application.

Next we test for predictability of crashes. This corresponds to testing conditional tail-mean independence with $Y_t = 1(r_t \le r_{crash}^{(1)})$ where $r_{crash}^{(1)}$ is the 5% empirical quantile of $\{r_t\}_{t=1}^n$. We consider the same DGPs above for generating $(r_t, X_t),$ with r_t replaced by Y_t in the DGPs, and we test for $H_0 : \| \gamma_h \|_{\bar{x}} = 0$ for $\bar{x} = F_h^{-1}(\tau),$ $\tau = 0.1,$ 1, and $h = 0.$ We compare the three tests described above, $K_{n,\bar{x}}, \hat{T}_{n,\bar{x}}$ and the linear correlation test $\hat{L}_{n,\bar{x}}$. Unreported simulations suggest that the combination of rare events such as crashes at 5% and small quantile levels such as $\tau = 0.1$ require much larger samples for accurate

Accepted Manuscript

finite sample performance than for cases with larger τ (or for more frequent events than 5%). For this reason we consider sample sizes of n = 500 and n = 1000. The RP are reported in Table 2. We observe that while our test is able to control the empirical size below 10% for DGP1 and DGP2, Scaillet (2005) MES based test and correlation tests fail to do so dramatically, with very large size distortions in some cases (up to 35% for Scaillet test). DGP3 presents larger size distortions, suggesting that larger sample sizes than 1000 are needed (afterall, the effective sample size corresponding to the joint tail event with $\tau = 0.1$ and $n = 1000$ is only of 5 observations, $1000 \times 0.05 \times 0.1 = 5$). The empirical power of our test is again either the highest or close to the highest in general. The power of Scaillet's test is low for DGP4 and DGP5, while the power of the correlation test is low for DGP5 at $\tau = 0.1$ and for DGP6 at τ = 1. Overall, these simulations show a satisfactory finite sample performance of our test relative to competing methods based on estimation of the MES and linear correlation measures.

Summarizing, these simulations and others reported in the Supplemental Appendix show that our tests for tail mean and distributional dependence present in general a more accurate size performance than alternative methods based on estimating the MES in Scaillet (2005), exceedance correlations or local Gaussian correlation measures in Lacal and Tjøstheim (2018). In contrast to linear correlation measures, our tests present an omnibus power performance, with an empirical power that is very competitive relative to competing methods. These clear advantages are accompained by a simplicity in implementation, without concerns on close-to-zero denominators, large variances, or solving nonlinear optimization problems.

6 Asset Market Linkages and Tail Risk Before, During and After the Financial Crisis.

In this section we apply our methodology to study the nonlinear dependence and persistence properties of some of the major international stock market indices before, during and after the recent financial crisis. We are particularly concerned with the dependence link structure from the US to European markets. A previous version of this paper (see Escanciano and Hualde, 2017) looked at a more extensive group of

Accepted Manuscript

markets, including Asian markets, as well as other features of the distribution such as persistence in conditional variances. The data consist in daily closed stock returns for S&P500 (SP500), the London FTSE-100 Index (FTSE), Frankfurt DAX Index (DAX), and the French CAC (CAC). The daily data are taken from January 1, 2005 to March 2, 2018. The daily closed values for these stock indices are obtained from Datastream. We consider the returns of the indices obtained as the log differences of the data, multiplied by 100, which excludes dividend payments.

Within this setting, we aim to investigate two different features of these data sets. First, we study the nonlinear tail-mean dependence properties before, during and after the financial crisis. We try to answer questions such as: did the tail events of the financial crisis in the US have a significant effect on the European returns? Did the dependence structure change before, during and after the crisis? How do our measures compare with alternative measures based on MES? Were the tail events during the crisis persistent in mean? Second, we employ our results to analyze the persistence of market crashes during and after the financial crisis.

Table A4 in the Supplemental Appendix provides basic descriptive statistics for each stock in the three periods: Before Crisis (from January 1st, 2005 until August 9th, 2007), During Crisis (from August 9th, 2007 until August 8th, 2009) and After Crisis (from August 8th, 2009 until March 2nd, 2018). All stock returns are characterized by having negative average returns during the crisis with also large variances and kurtosis. Stock returns are typically linearly unpredictable at a daily frequency, but results for linear correlations reveal that the SP500 presents negative correlations at the first lag during the crisis. This is confirmed with the data-driven Box-Pierce test in Escanciano and Lobato (2009). The null of lack of correlation is rejected for the SP500 during the crisis with a p-value of 1%, but the data driven test fails to reject the white noise hypothesis for other stocks and periods. Thus, classical linear dependence measures do not provide much information on the dependence structure in the data. Additionally, it is known that linear correlations are biased upwards in periods of high volatility (cf. Forbes

Accepted Manuscript

and Rigobon, 2002), which make them inappropriate to study market dependencies in periods such as the recent financial crisis.

6.1 Market Linkages and persistence in conditional mean

The martingale properties of individual stocks returns have been extensively investigated in the literature, see e.g. Lo and MacKinlay (1999) and references therein. These earlier studies showed that stock price changes are not martingale difference sequences (MDS), although evidence supporting or refuting the martingale hypothesis with some stock indices seems mixed; see e.g. Escanciano and Velasco (2006b) with the S&P500. Our results below contribute to this literature by providing empirical evidence of substantial cross-country mean predictability from the US stock market to European markets. Beyond this case, we find little or no persistence in mean and tail-mean dependence.

To gain insight on the asset market linkages before, during and after the financial crisis, we report in Table 3 the bootstrap p-values computed with the block bootstrap approximation for testing the significance of the measure γ_h at the first lag $h = 1$ and all combinations of stocks. We consider mean dependence ($\bar{x} = F_h^{-1}(\tau)$ for τ = 1) and tail-mean dependence ($\bar{x} = F_h^{-1}(\tau)$ for $\tau = 0.05$), for the same tests statistics used in the Monte Carlo simulations. We reject the hypothesis of tail-mean independence from the US to Europe at lag $h = 1$ across all time periods at 10% significance level with our tests and with correlation based tests. MES tests fail to reject tail-mean independence during and after the crisis, despite their systematic overrejections observed in simulations. When focussing on the serial dependence of the SP500, our test provides evidence of tail-mean serial dependence at 10% before the crisis, while the other tests fail to reject at this nominal level. This case highlights the benefits our approach. The SP500 becomes highly serially predictable during the crisis, but much less so afterwards. None of the European stocks seem to have an impact on the US at lag $h = 1$ or larger lags (unreported). Among European stocks we find no dependence relation, with the exception of the CAC before the crisis, which seems to affect other European stocks according to our test results, with other tests failing to reject tail-mean independence.

Accepted Manuscript

These results draw a mapping of nonlinear directional predictability that might be useful for international diversification and hedging; see, e.g., Poon, Rockinger and Tawn (2004) and Christoffersen et al. (2012) for the importance of accounting for nonlinear tail dependence in international diversification.

Of particular concern to international investors is international systemic risk in times when a market crashes. Table 3 shows that inference based on popular measures such as MES might be missleading for testing tail dependence at quantile levels such as 5%. Not only these measures may lead to rejection of a correct null hypothesis of independence, as shown in simulations, but they also may lead to a lack of power to detect tail dependence.

To further understand the discrepancy between our measures and MES based measures, and the dependence structure between US and European stocks before and during the crisis we plot in Figure 2 the IRF when the lagged S&P500 is the conditioning variable, together with bootstrap uniform confidence bands (see the Supplemental Appendix for the corresponding figure for the MESF) for the different quantiles. We standardize the dependent variables for a better comparison. Several observations result from these plots. First, the dependence structure between the S&P500 and its lagged value is fundamentally different from the dependence structure with European stocks. The IRF plots suggest a negative expectation dependence between the S&P500 and its lagged value, but positive dependence between European stocks and lagged values of S&P500. This finding has important consequences for portfolio choice and diversification (see Wright 1987). This behaviour persists accross different periods (results can be obtained from the authors upon request). Second, once we standardize the dependent variable by its sample standard deviation, the dependence structure is qualitatively similar before and during the crisis, albeit with larger uncertainty during the crisis. From the plot in the Supplemental Appendix we see that the uniform bands for the MESF are much wider than for the IRF, which is consistent with the Monte Carlo simulations. These plots also show that the maximum discrepancy between MES and the unconditional mean is often achieved at the very extreme quantiles where the

Accepted Manuscript

uncertainty is largest, which may explain the lack of power of MES based tests in the application.

We also computed the functions $K_n(h) = \| \hat{\gamma}_h \|_\infty$ as a function of *h*, when we condition on the SP500 for lags up to 100, see also Escanciano and Hualde (2017). These unreported results suggest very little persistence in mean and in tail-mean. Thus, we conclude that although the tail events that occur in the US had a short term effect on European stocks, they did not have a persistent effect in the expected returns.

6.2 Dependence and persistence of crashes

In contrast to the mean, there is an extensive literature documenting a high persistence in volatility; see the previous version of this paper and references therein. There is, however, relatively less evidence on the dependence and persistent effects on market crashes. We define a market crash as the indicator of the market being below its 5% empirical quantile, and take this as the dependent variable

$$Y_{t+h,i} = 1(r_{t+h,i} \leq \hat{F}_{h,i}^{-1}(0.05)),$$

where $r_{t+h,i}$ is the return of the stock $i-th$ at time $t+h$ and $\hat{F}_{h,i}^{-1}(0.05)$ the empirical 5% quantile of the sample $\{r_{t,i}\}_{t=1+h}^{n}$. This implies that estimates of

$$MES_{h,ij}(x) = E\left(Y_{t+h,i} \middle| r_{t,j} \leq x\right),$$

are estimates of the probability of a crash in market *i*, *h* periods ahead, when market *j* is below the threshold *x*. One possibility is to use $x = \hat{F}_{h,j}^{-1}(0.05),$ which relates to the co-exceedances used in Bae, Karolyi and Stulz (2003) and provides a further application of the MES measures proposed in Acharya et al. (2017). As mentioned above, a more informative measure about tail risk uses a continuum of *x* in the left tail. For example, we consider

$$C_{n,ij}(h) := \sup_{x \leq \bar{x}} MES_{h,ij}(x) \qquad h \geq 1,$$

Accepted Manuscript

where $MES_{h,ij}(x)$ is Scaillet's (2005) smoothed estimator.
Figure 3 plots the coefficients $C_{n,ij}(h)$ for all *i* when $j =$ SP500, together with 5% block bootstrap critical values for these coefficients under the null hypothesis of tail-mean independence. We observe estimated conditional probabilities of crashes well above the unconditional probability of 5%. However, despite these large probability estimates, bootstrap marginal tests for significance at 5% fail to reject crash dependence. Figure 4 plots the IRF cross-coefficients $K_n(h) = \| \hat{\gamma}_h \|_\infty$ as a function of *h* with 5% marginal bootstrap critical values. In contrast to MES, our measures lead to significant results at 5% for most lags. Thus, in view of our measures, market crashes are shown to be quite persistent.

7 Conclusions

In this paper we have introduced new measures of nonlinear dependence. They are easy to estimate and require relatively smaller sample sizes for accurate inference than competing methods. Additionally, our measures are nonparametric, but do not rely on bandwidths. Also, we find that they are particularly convenient for testing tail independence, overcoming the *small denominator* problems and computational problems of tests based on MES, conditional correlations or local Gaussian dependence measures. They compare favorably in simulations and in the empirical application to popular competing methods. Their performance is theoretically justified by a characterization of dependence and persistence, as well as by the asymptotic theory developed in this paper. The proposed measures are easily adapted to situations where tail dependence is of interest. We have established connections with existing measures of systemic risk (see Acharya et al., 2010, 2017) and shown in an application that the proposed measures provide a more accurate description of the tail risk than their counterparts based on MES.

The proposed tools can be applied and extended to a number of settings. For example, when applied to residuals from a fitted model, they can be used to develop formal lack of fit tests or backtests for out of sample forecast evaluation that focus on the tail of the

Accepted Manuscript

conditional distribution. This approach allows researchers and regulators to focus on the part of the distribution that is of interest for risk management. Another line of potential research is the developement of formal inferential procedures for testing the short and extended memory character of a given time series. Moreover, our operative measures pave the way for a formal definition of cointegration in nonlinear set-ups that can formalize some of the ideas put forward by previous studies. We leave these and other important extensions of our methods for future research.

Acknowledgments

We thank seminar participants at many institutions. We also thank Marine Carrasco, Sílvia Gonçalves, Bruce Hansen, Alex Maynard and two anonymous referees for helpful comments, and Hakon Otneim for helping with the lg R Package.

References

Acharya, V.V., L. Pedersen, T. Philippon, and Richardson, M., 2010, Measuring systemic risk. Working paper.

Acharya, V.V., L. Pedersen, T. Philippon, and Richardson, M., 2017, Measuring systemic risk. The Review of Financial Studies 30, 2-47.

Ang, A. and Bekaert, G., 2002, International asset allocation with regime shifts. Review of Financial Studies 15, 1137-1187.

Ang, A. and Chen, J., 2002, Asymmetric correlations of equity portfolios. Journal of Financial Econometrics 63, 443-494.

Arcones, M.A. and Yu, B., 1994, Central limit theorems for empirical and U-processes of stationary mixing sequences. Journal of Theoretical Probability, 7, 47-71.

Auestad, B. and Tjøstheim, D., 1990, Identification of nonlinear time series: first order characterization and order determination. Biometrika 77, 669-687.

Accepted Manuscript

Bae, K.H., Karolyi, G.A. and Stulz, R.M., 2003, A new approach to measuring financial contagion. The Review of Financial Studies, 16, 717-763.

Berentsen, G. D. and Tjøstheim, D., 2014, Recognizing and visualizing departures from independence in bivariate data using local Gaussian correlation. Statistics and Computing 24, 785-801.

Bierens, H.J., 1982, Consistent model specification tests. Journal of Econometrics 20, 105-134.

Billingsley, P., 1968, Convergence of probability measures. Wiley, New York.

Brownlees, C. and Engle, R.F., 2012, Volatility, correlations and tails for systemic risk measurement. Working paper.

Brownlees, C. and Engle, R.F., 2016, SRISK: A conditional capital shortfall measure of systemic risk. The Review of Financial Studies 30, 48-79.

Buja, A., 1990, Remarks on functional canonical variates, alternating least squares methods, and ACE. Annals of Statistics 18, 1032-1069.

Cai, J., J.H.J. Einmahl, L. de Haan and Zhou, C., 2015, Estimation of the marginal expected shortfall: the mean when a related variable is extreme. Journal of the Royal Statistical Society Series B, 77, 417-447.

Christoffersen, P., Errunza, V., Jacobs, K. and Langlois, H., 2012, Is the potential for international diversification disappearing? A dynamic copula approach. The Review of Financial Studies 25, 3711-3751.

Davidson, J., 1994, Stochastic limit theory. Oxford: Oxford University Press.

Davis, R. A. and Mikosch, T., 2009, The extremogram: a correlogram for extreme events. Bernoulli 15, 977-1009.

Accepted Manuscript

Davis, R.A., Drees, H., Segers, J. and Warchol, M., 2018, Inference on the tail process with application to financial time series modeling. Journal of Econometrics, 205, 508-525.

Davis, R., M. Matsui, T. Mikosch, and Wan, P., 2018, Applications of distance correlation to time series. Bernoulli 24, 3087-3116.

Dette, H., Hallin, M., Kley, T. and Volgushev, S., 2015, Of copulas, quantiles, ranks and spectra: an L_1-approach to spectral analysis. Bernoulli 21, 781-831.

Doukhan, P., 1994. Mixing. Properties and examples, Springer, Lecture Notes in Statistics.

Escanciano, J. C., 2006, Goodness-of-fit tests for linear and non-linear time series models. Journal of the American Statistical Association 101, 531-541.

Escanciano, J.C. and Hualde, J., 2017, Measuring Asset Market Linkages: Nonlinear Dependence and Tail Risk. December 13, 2017. CAEPR WORKING PAPER 2017-017. Available at SSRN: https://ssrn.com/abstract=3088182.

Escanciano J.C. and Lobato I.N., 2009, An automatic portmanteau test for serial correlation. Journal of Econometrics 151, 140-149.

Escanciano, J. C. and Velasco, C., 2006a, Testing the martingale difference hypothesis using integrated regression functions. Computational Statistics & Data Analysis 51, 2278-2294.

Escanciano, J. C., and Velasco, C., 2006b, Generalized spectral tests for the martingale difference hypothesis. Journal of Econometrics 134, 151-185.

Forbes, K. and Rigobon, R., 2002, No contagion, only interdependence: measuring stock market co-movements. Journal of Finance 57, 2223-2261.

Accepted Manuscript

Gourieroux, C. and Jasiak, J., 1999, Nonlinear persistence and copersistence. Preprint, Institut National de la Statistique et des Etudes Economiques.

Gourieroux, C. and Jasiak, J., 2000, Nonlinear innovations and impulse responses. Preprint, Institut National de la Statistique et des Etudes Economiques.

Gourieroux, C. and Jasiak, J., 2002, Nonlinear autocorrelograms: an application to inter-trade durations. Journal of Time Series Analysis 23, 127-154.

Granger, C. W. J., 1995, Modelling nonlinear relationships between extended-memory variables. Econometrica 63, 265-279.

Granger, C. W. J., 2003, Time series concepts for conditional distributions. Oxford Bulletin of Economics and Statistics 65 (supplement), 639-701.

Granger, C. W. J. and Hallman, J., 1991, Nonlinear transformations of integrated time series. Journal of Time Series Analysis 12, 207-224.

Han, H., Linton, O., Oka, T., and Whang, Y.-J., 2016, The cross-quantilogram: measuring quantile dependence and testing directional predictability between time series. Journal of Econometrics 193, 251-270.

Hartmann, P., Straetmans, S. and de Vries, C.G., 2004, Asset market linkages in crisis periods. The Review of Economics and Statistics, 86, 313-326.

Hoeffding, W., 1940, Scale-invariant correlation theory. In Fisher, N.I, Sen, P.K (Eds), The Collective Works of Wassily Hoeffding. Springer-Verlag, New York, 1994.

Hong, Y., 1999, Hypothesis testing in time series via the empirical characteristic function. Journal of American Statistical Association, 84, 1201-1220.

Koul, H. L. and Stute, W., 1999, Nonparametric model checks for time series. Annals of Statistics 27, 204-236.

Accepted Manuscript

Künsch, H. R., 1989, The jackknife and the bootstrap for general stationary observations. Annals of Statistics, 40, 1217-1241.

Lacal, V. and Tjøstheim, D., 2017, Local Gaussian autocorrelation and tests of serial independence. Journal of Time Series Analysis, 38, 51-71.

Lacal, V. and Tjøstheim, D., 2018, Estimating and testing nonlinear local dependence between two time series. Journal of Business and Economic Statistics, to Appear.

Liu, R. Y., and Singh, K. 1992, Moving blocks Jackknife and bootstrap Capture weak dependence, in Exploring the Limits of the Bootstrap, eds. R. LePage and L. Billard, New York: Wiley, pp. 225–248.

Linton, O. and Whang, Y.-J., 2007, The quantilogram: with an application to evaluating directional predictability. Journal of Econometrics, 141, 250-282.

Lo, A.W. and MacKinlay, A.C., 1999, A non-random walk down Wall street. Princeton.

Longin, F. and Solnik, B., 2001, Extreme correlation of international equity markets. Journal of Finance 56 (2), 649-676.

Malevergne, Y. and Sornette, D., 2006, Extreme financial risks. From dependence to risk management. Springer Science & Business Media.

McLeish, D. L., 1974, Dependent central limit theorems and invariance principles. The Annals of Probability 24, 620-628.

Morgenstern, O., 1959, International financial transactions and the business cycle. Princeton, NJ: Princeton University Press.

Oh, D. H. and Patton A.J., 2017, Modeling dependence in high dimensions with factor copulas. Journal of Business & Economic Statistics, 35, 139-154.

Accepted Manuscript

Otneim, H., 2018. Package ‘lg’. Available at https://cran.rstudio.com/web/packages/lg/lg.pd.

Politis, D., and Romano, J., 1994, The stationary bootstrap. Journal of American Statistics Association, 89, 1303-1313.

Politis, D. N., Romano, J.P. and Wolf, M., 1999, Subsampling. Springer-Verlag, New York.

Poon, S., Rockinger, M., Tawn, J., 2004, Extreme value dependence in financial markets: diagnostics, models, and financial implications. Review of Financial Studies 17, 581-610.

Racine, J.S. 2018. Package np. https://cran.r-project.org/web/packages/np/np.pdf.

Radulović, D., 1996, The bootstrap for empirical processes based on stationary observations. Stochastic Processes and their Applications, 65, 259-279.

Robinson, P.M., 1983, Nonparametric estimators for time series. Journal of Time Series Analysis 4, 185-207.

Scaillet, O., 2005, Nonparametric estimation of conditional expected shortfall. Insurance and Risk Management Journal, 74, 639-660.

Schweizer, B., and Wolff, E.F., 1981, On nonparametric measures of dependence for random variables. Annals of Statistics, 4, 879-885.

Stute, W., 1997, Nonparametic model checks for regression. Annals of Statistics 25, 613-641.

Székely, G. J., and Rizzo, M. L., 2009, Brownian distance correlation. Annals of Applied Statistics, 3, 1236-1265.

Accepted Manuscript

Terasvirta, T., Tjostheim, D. and Granger, C.W.J., 2011, A review of modelling nonlinear economic time series. Oxford: Oxford University Press.

van der Vaart, A. W. and Wellner, J. A., 1996, Weak convergence and empirical processes. Springer, New York.

Wright, R., 1987, Expectation dependence of random variables, with an application in portfolio theory. Theory and Decision 22, 111-124.

Wu, C.F.J., 1986, Jacknife, bootstrap and other resampling methods in regression analysis (with discussion). Annals of Statistics 14, 1261-1350.

Zhou, Z. 2012, Measuring nonlinear dependence in time-series, a distance correlation approach. Journal of Time Series Analysis, 33, 438-457.

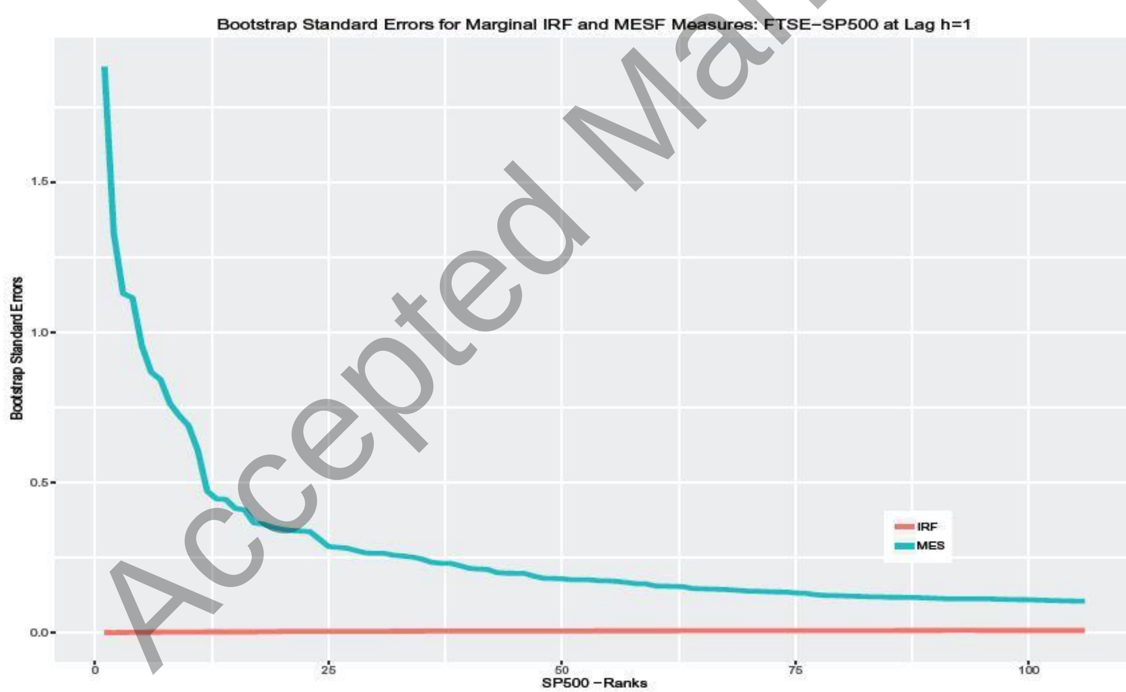

Fig. 1 Block bootstrap standard errors for IRF and MESF measures at empirical quantiles based on 500 bootstrap replications.

Accepted Manuscript

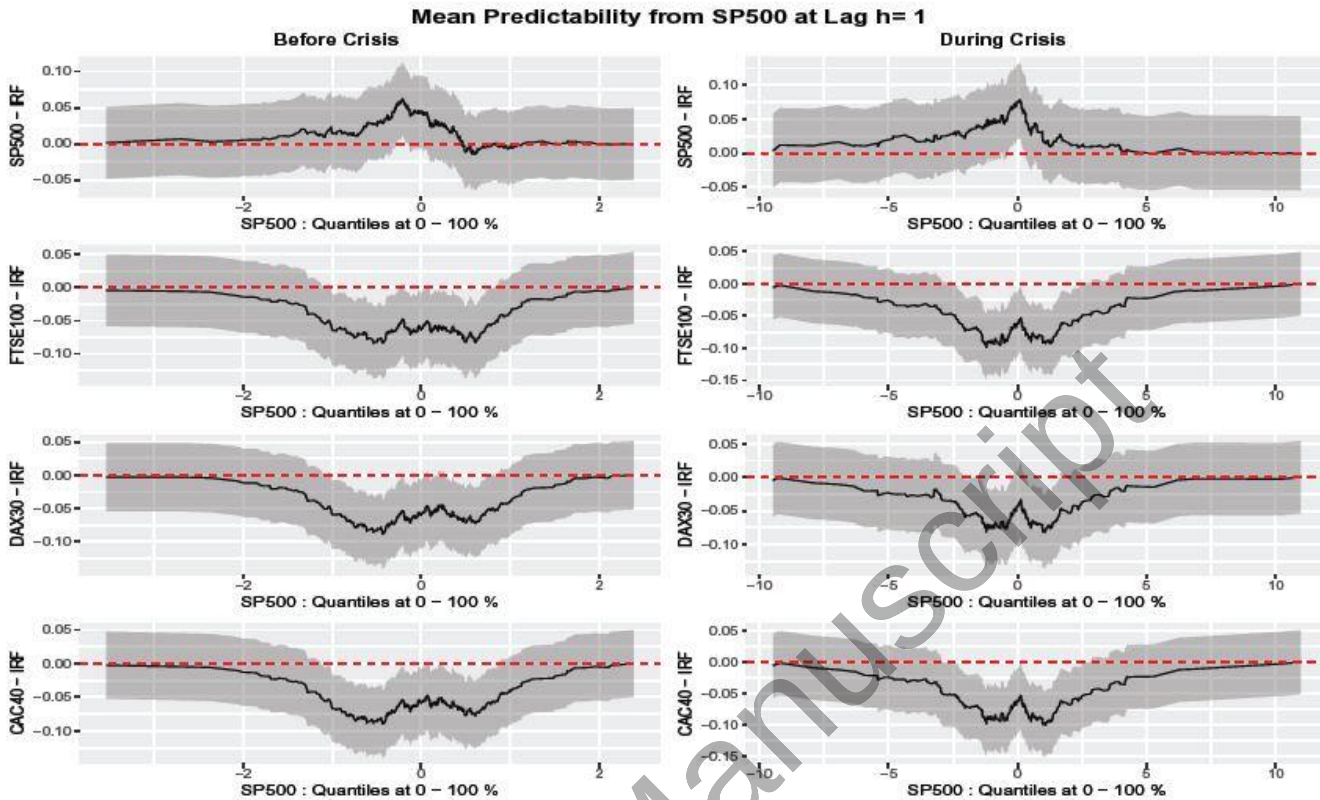

Fig. 2 IRF Before Crisis (left panel) and During Crisis (right panel). Block bootstrap confidence bands based on 300 bootstrap replications.

Accepted Manuscript

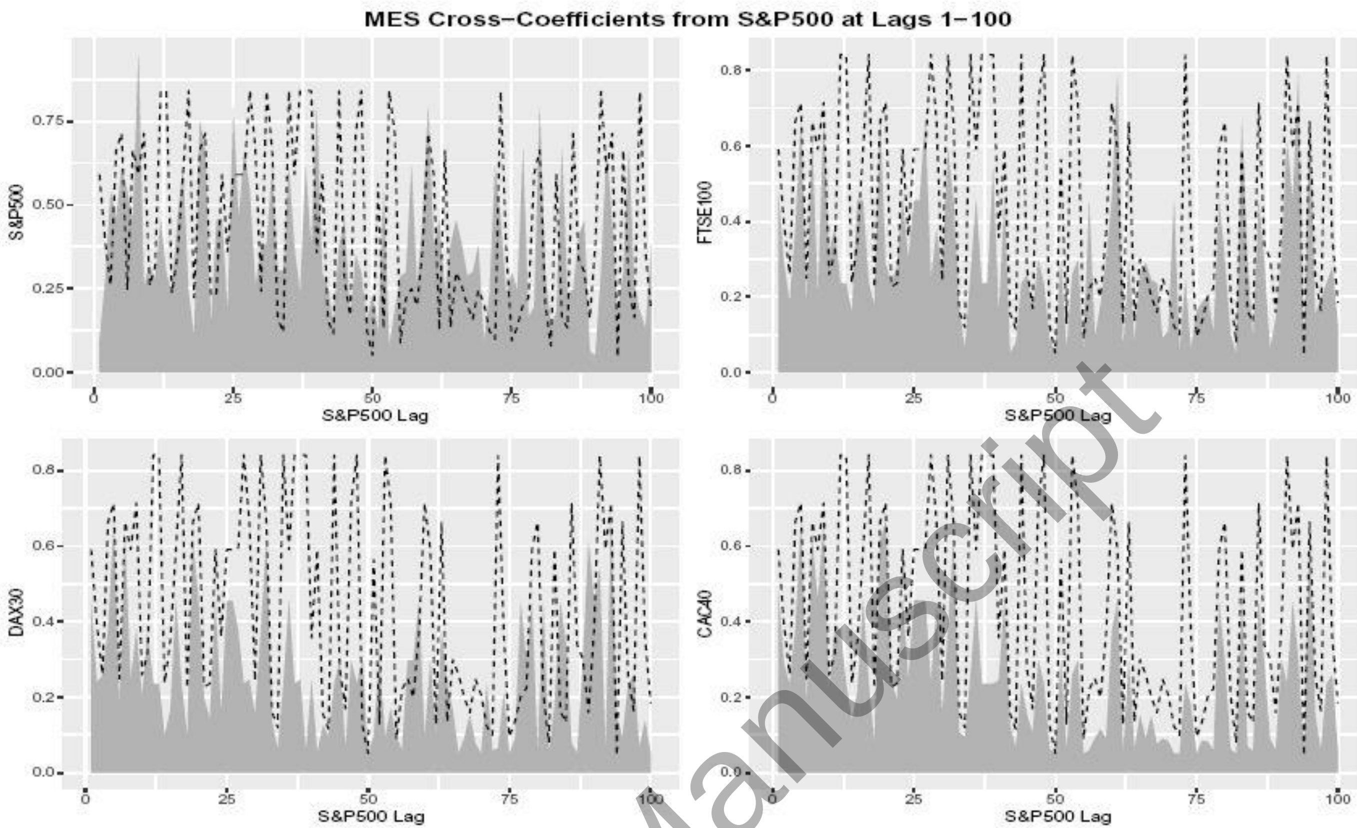

Fig. 3 MES cross-coefficients for 10%-Crashes from lagged SP500 with block 95% bootstrap critical values (dashed lines).

Accepted Manuscript

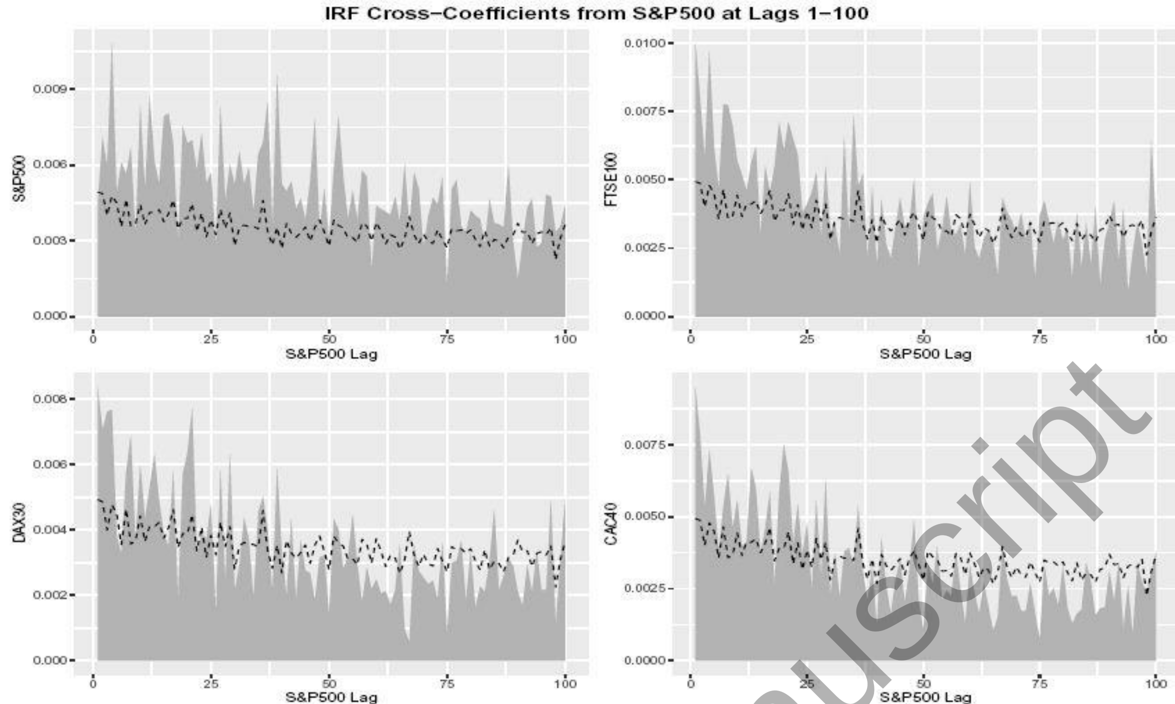

Fig. 4 IRF cross-coefficients for 10%-Crashes from lagged SP500 with block 95% bootstrap critical values (dashed lines).

TABLE 1. RP($\times 100$) FOR DGP1-DGP6 AT $\bar{x} = \hat{F}_h^{-1}(\tau)$.

$\mathbf{n = 100}$		$\tau = 0.1$			$\tau = 0.5$			τ = 1	
DGP/Test	$K_{n,\bar{x}}$	$\hat{T}_{n,\bar{x}}$	$\hat{L}_{n,\bar{x}}$	$K_{n,\bar{x}}$	$\hat{T}_{n,\bar{x}}$	$\hat{L}_{n,\bar{x}}$	$K_{n,\bar{x}}$	$\hat{T}_{n,\bar{x}}$	$\hat{L}_{n,\bar{x}}$
DGP1	5.6	11.2	11.2	4.0	11.2	5.4	5.8	11.0	7.4
DGP2	4.4	11.2	8.3	4.2	12.4	6.4	6.6	9.8	5.6
DGP3	5.8	12.6	11.5	6.8	5.4	7.0	7.2	6.2	6.6
DGP4	70.8	66.6	82.6	97.6	68.8	98.0	98.8	70.2	100
DGP5	22.6	30.0	26.8	70.0	48.8	28.0	77.4	51.6	85.6
DGP6	94.8	98.2	97.9	81.8	99.8	86.8	97.4	99.8	11.4
$\mathbf{n = 300}$		$\tau = 0.1$			$\tau = 0.5$			τ = 1	
DGP1	4.6	11.8	6.4	6.0	10.6	5.2	5.8	12.2	4.8

Accepted Manuscript

$\mathbf{n}=\mathbf{100}$		$\tau=0.1$			$\tau=0.5$			τ = 1	
DGP2	4.6	12.8	5.6	4.6	12.2	6.2	4.6	8.6	4.4
DGP3	5.8	10.6	6.0	5.4	6.6	4.8	4.8	7.0	6.4
DGP4	99.8	81.0	99.8	100	80.2	100	100	81.8	100
DGP5	64.8	66.4	47.0	99.0	87.4	55.2	99.6	87.4	100
DGP6	100	100	100	100	100	99.6	100	100	7.8

Note: 500 Monte Carlo Simulations and 500 Block Bootstrap Replications.

TABLE 2. RP($\times 100$) FOR CRASHES AT 5% FOR DGP1-DGP6 AT $\bar{x}=\hat{F}_h^{-1}(\tau)$.

$\mathbf{n}=\mathbf{500}$		$\tau=0.1$			τ = 1	
DGP/Test	$K_{n,\bar{x}}$	$\hat{T}_{n,\bar{x}}$	$\hat{L}_{n,\bar{x}}$	$K_{n,\bar{x}}$	$\hat{T}_{n,\bar{x}}$	$\hat{L}_{n,\bar{x}}$
DGP1	8.6	35.6	13.2	5.2	31.2	5.2
DGP2	8.2	33.4	13.0	4.2	32	4.8
DGP3	12.8	11.4	20.6	5.0	14.2	6.8
DGP4	76.4	11.2	82	99.6	14.8	99.8
DGP5	21.4	20.0	15.8	74.8	21.6	85.2
DGP6	99.6	100	99.8	57.0	100	4.4
$\mathbf{n}=\mathbf{1000}$		$\tau=0.1$			τ = 1	
DGP1	9.4	31.4	10.2	5.2	33.8	5.8
DGP2	9.4	33.8	12.0	3.4	33.0	5.2
DGP3	13.0	10.4	13.4	4.8	8.2	5.4
DGP4	99.2	20.6	99.2	100	17.0	100
DGP5	51.6	48.0	35.2	97.2	51.8	99.2
DGP6	100	100	100	98.6	100	4.8

Note: 500 Monte Carlo Simulations and 500 Block Bootstrap Replications.

Accepted Manuscript

TABLE 3. TAIL-MEAN DEPENDENCE BEFORE, DURING AND AFTER CRISIS

Y_{t+1} / X_t		SP500			FTSE			DAX			CAC	
Test	$K_{n,\bar{x}}$	$\hat{T}_{n,\bar{x}}$	$\hat{L}_{n,\bar{x}}$	$K_{n,\bar{x}}$	$\hat{T}_{n,\bar{x}}$	$\hat{L}_{n,\bar{x}}$	$K_{n,\bar{x}}$	$\hat{T}_{n,\bar{x}}$	$\hat{L}_{n,\bar{x}}$	$K_{n,\bar{x}}$	$\hat{T}_{n,\bar{x}}$	$\hat{L}_{n,\bar{x}}$
SP500	0.01	0.29	0.12	0.17	0.09	0.88	0.77	0.18	0.98	0.34	0.19	0.98
FTSE	0.00	0.00	0.00	0.40	0.85	0.44	0.09	0.74	0.79	0.00	0.82	0.67
DAX	0.00	0.00	0.00	0.31	0.78	0.41	0.18	0.83	0.78	0.02	0.74	0.97
CAC	0.00	0.00	0.00	0.28	0.77	0.57	0.09	0.73	0.65	0.00	0.84	0.86
SP500	0.07	0.40	0.14	0.49	0.09	0.82	0.51	0.24	0.82	0.78	0.30	0.95
FTSE	0.00	0.00	0.00	0.28	0.48	0.30	0.24	0.76	0.27	0.48	0.60	0.54
DAX	0.00	0.23	0.00	0.37	0.49	0.35	0.67	0.93	0.63	0.66	0.58	0.86
CAC	0.00	0.00	0.00	0.45	0.44	0.47	0.74	0.73	0.71	0.74	0.55	0.99
SP500	0.00	0.03	0.01	0.06	0.26	0.47	0.36	0.54	0.41	0.00	0.5	0.25
FTSE	0.00	0.47	0.00	0.18	0.14	0.22	0.74	0.14	0.18	0.08	0.39	0.07
DAX	0.00	0.42	0.01	0.12	0.34	0.14	0.82	0.46	0.34	0.08	0.52	0.03
CAC	0.00	0.57	0.00	0.2	0.07	0.15	0.78	0.15	0.26	0.2	0.27	0.06
SP500	0.08	0.03	0.05	0.56	0.27	0.57	0.68	0.65	0.68	0.53	0.63	0.41
FTSE	0.08	0.54	0.05	0.56	0.11	0.41	0.07	0.22	0.08	0.12	0.45	0.06
DAX	0.09	0.50	0.04	0.48	0.35	0.33	0.29	0.56	0.22	0.13	0.61	0.03
CAC	0.08	0.63	0.07	0.38	0.07	0.32	0.26	0.23	0.18	0.12	0.3	0.03
SP500	0.43	0.28	0.49	0.21	0.61	0.66	0.7	0.19	0.93	0.75	0.29	0.42
FTSE	0.00	0.28	0.00	0.4	0.27	0.48	0.77	0.13	0.76	0.61	0.3	0.21
DAX	0.00	0.54	0.00	0.08	0.36	0.65	0.21	0.08	0.46	0.22	0.27	0.65
CAC	0.00	0.33	0.00	0.22	0.30	0.74	0.89	0.08	0.6	0.73	0.25	0.44
SP500	0.49	0.33	0.33	0.74	0.75	0.55	0.80	0.20	1.00	0.49	0.20	0.33
FTSE	0.00	0.36	0.00	0.46	0.27	0.40	0.84	0.13	0.8	0.26	0.14	0.24

Accepted Manuscript

Y_{t+1} / X_t		SP500			FTSE			DAX			CAC	
DAX	0.00	0.62	0.00	0.62	0.37	0.71	0.64	0.04	0.4	0.74	0.15	0.72
CAC	0.00	0.44	0.00	0.58	0.27	0.88	0.8	0.05	0.51	0.59	0.13	0.52

Bootstrap p-values for tests at τ = 1 and $\tau = 0.05$: Before Crisis (top two panels, resp.)

During Crisis (middle two panels, resp.) and After Crisis (bottom two panels, resp.).

Based on 500 Moving Block Bootstrap Replications.

Accepted Manuscript