

Health Risk and the Value of Life^{*}

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Abstract

We develop a stochastic life-cycle framework for valuing health and longevity improvements and apply it to data on mortality, quality of life, labor earnings, and medical spending for adults with different comorbidities. We find that sick adults are willing to pay nearly twice as much per quality-adjusted life-year (QALY) to reduce mortality risk than healthy adults, and that reducing the risk of serious illness is valued similarly to reducing the risk of mild illness. Our results provide a rational explanation for why people oppose a single threshold value for rationing care and why they invest less in prevention than in treatment.

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1 Introduction

The economic analysis of risks to life and health has made enormous contributions to academic discussions and public policy. Economists have used the standard tools of life-cycle consumption theory to propose a transparent framework that measures the value of improvements to both health and longevity ([Murphy and Topel, 2006](#)). Economic concepts such as the value of statistical life (VSL) play central roles in discussions surrounding public and private investments in medical care, public safety, environmental hazards, and countless other arenas.

However, the conventional life-cycle framework used to study the value of life includes only a single health state, with a preordained mortality rate that depends on age alone. As a result, it is ill-equipped to investigate how VSL varies with a person’s health status, and it cannot distinguish between illness and death. These shortcomings limit its explanatory power and policy relevance. For example, an array of evidence suggests that society invests less in prevention than treatment, even when both have the same health benefits ([Weisbrod, 1991](#); [Dranove, 1998](#); [Pryor and Volpp, 2018](#)). The conventional framework’s failure to explain this apparent underinvestment in prevention has led researchers to posit alternative behavioral or market failure explanations, although the evidence remains inconclusive ([Fang and Wang, 2015](#); [Bai et al., 2021](#); [Newhouse, 2021](#)).

Likewise, the conventional model’s reliance on a single health state hampers its ability to engage in the ongoing policy debates regarding whether and how healthcare reimbursements should vary with health shocks of differing severity. In many countries, a medical treatment is covered by insurance only if its price does not exceed a “cost-effective” threshold, and the underlying theory implies that this threshold should not vary with patient health or with the severity of the illness being treated ([Hammitt, 2013](#); [Lucarelli et al., 2022](#)).¹ However, survey research finds scant public support for a constant threshold ([Nord et al., 1995](#); [Linley and Hughes, 2013](#)), and several countries that rely on cost-effectiveness to allocate healthcare resources—including Norway, Sweden, the Netherlands, and the UK—have defied the underlying economic theory by codifying ad hoc approaches that increase the cost-effectiveness threshold for treatments of severe diseases.²

¹US law prohibits federal agencies such as Medicare from using conventional cost-effectiveness methods ([Lakdawalla and Phelps, 2022](#)).

²Norway and Sweden identify disease severity as a core determinant of treatment value ([Defechereux et al., 2012](#); [Persson, 2012](#)). The Netherlands relies on “proportional shortfall” methods that increase reimbursements for treatments of diseases that cause greater relative reductions in quality-adjusted life expectancy ([Reckers-Droog et al., 2018](#)). The UK has recently formalized similar ad hoc approaches in its

This paper studies how a rational individual’s marginal value of reducing mortality and illness risk varies with baseline health status and with the severity of those risks.³ We develop a new stochastic life-cycle framework that accommodates multiple health states with different mortality, quality of life, and financial risk profiles. We derive the value of statistical illness (VSI), which measures the willingness to pay to reduce illness risk and includes VSL as the special case where that risk is death. We then use the model to investigate the effect of health shocks on the value of life. Our findings indicate that, under standard risk assumptions, the willingness to pay for life-extension is higher for individuals with shorter baseline life expectancy.

This insight hinges on two countervailing effects. Consider an individual newly diagnosed with cancer. On the one hand, the attendant reduction in her life expectancy reduces VSL by reducing lifetime utility. On the other hand, this unexpected reduction in longevity increases VSL by encouraging her to spend down her wealth more quickly. To clarify when one effect dominates, we derive a sufficient condition for VSL to rise following an adverse shock to longevity. This condition, which depends on how prudence (Kimball, 1990) compares to the elasticity of intertemporal substitution, is satisfied by many standard utility functions as well as current estimates of those two parameters.

Our theoretical result that adverse shocks to longevity increase VSL holds constant other factors such as wealth. However, a health shock is often accompanied by financial shocks such as job loss or recurring medical expenses. We show that these financial effects can reduce the value of life, implying that the net effect of health shocks on the value of life is ambiguous.

To quantify the overall effect of health shocks on the value of life, we apply our model to individual-level data from a representative cohort of US adults ages 50–80. The data are obtained from the Future Elderly Model (FEM), which provides detailed information on how mortality, quality of life, labor earnings, and medical spending evolve over the life cycle for people over age 50 with different comorbidities. The data underlying the FEM include more comprehensive information than any single national survey and have been widely used to study elderly health and medical spending (e.g., Goldman et al., 2010, 2013; Leaf et al., 2021; Reif et al., 2021). We quantify VSL and VSI for individuals in twenty different health states with varying mortality, quality of life, and financial risk profiles. We measure health improvements in units of quality-adjusted life-years (QALYs), a widely used metric that combines the quality and quantity of life-years into a

latest health technology evaluation manual (NICE, 2022).

³In the remainder of the paper, “health” refers to both longevity and health-related quality of life. We use the terms “health risk” and “illness risk” interchangeably. A “unit of health” measures both longevity and health-related quality of life.

single index.

We find that VSL per QALY rises on average by \$70,000 (21%) in the year following an adverse health shock, and by over \$150,000 (45%) following the worst five percent of shocks. Among 70-year-olds, those in the sickest health state are willing to pay 1.9 times more per QALY to reduce mortality risk than healthy people, representing a wide gap in VSL across health states. These differences become even larger if people are fully insulated from the financial risks of the shocks. We conclude that while financial losses associated with health shocks do reduce the value of life, the effect is generally not large enough to offset the increase resulting from a reduction in longevity.

We also provide novel estimates of VSI by quantifying the risk-reduction value of different illnesses for a healthy consumer in a fixed health state. We find that VSI rises only a little with disease severity: a healthy 70-year-old is willing to pay up to \$23,000 (10%) more per QALY to reduce extreme risks such as serious cancer or death than to reduce mild risks such as developing hypertension. This result indicates that although the value of reducing health risk does rise significantly for those in worse health, it varies little with the severity of the risk itself.

We perform several sensitivity analyses to assess the robustness of our results. While the absolute values of our estimates are moderately sensitive to alternative assumptions about consumer risk preferences or the presence of a bequest motive, our two qualitative conclusions—that the value of reducing a health risk increases dramatically with baseline health but only slightly with severity of the risk—hold up across a number of alternative parameterizations. We also confirm that a health state’s mortality risk profile is the key factor driving our numerical results: we obtain the same patterns if we omit quality of life from our model.

Our primary contribution is the development and application of a new, more general life-cycle model of the value of life. While an individual’s value of health risk reduction in the conventional model depends on her age and wealth, in our model this value depends also on her current state of health and on the characteristics of the health risks she faces. Our finding that the value of a QALY is up to 1.9 times higher among the sick than the healthy helps explain puzzles such as why consumers invest less in prevention than treatment, why end-of-life spending is high ([Zeltzer et al., 2023](#)), why cost-effectiveness analysis appears out-of-step with policies promoting more generous reimbursement of care for more severe diseases ([Lakdawalla et al., 2014](#)), and why preventive care interventions frequently fail to improve health ([Jones et al., 2019](#)), without needing to resort to alternative explanations such as market inefficiencies or irrational behaviors. Our findings also explain consumer and voter opposition to the use of a single threshold value

when making decisions about health resource allocation, and provides further support for “top-up” insurance policies, which allow patients who value health more highly to pay incremental prices for more expensive treatments (Einav et al., 2016; Shepard et al., 2020; Lucarelli et al., 2022; Baicker et al., 2023). As a related matter, our theory also provides a novel rationale for firms offering more and less generous health insurance plans, which are taken up by the sick and the healthy, respectively. Moreover, with its (health) states appropriately redefined, our stochastic framework can also be applied to a number of other distinct questions, such as why societies appear to invest less in preventing pandemics than in mitigating them and how to value insurance in a setting with shocks to health, longevity, and spending (Kowalski, 2015; Lakdawalla et al., 2017; Fang and Shephard, 2019; Atal et al., 2020). Finally, while a stochastic framework can be challenging for practitioners to use in policy analysis, we provide a closed-form solution and an accompanying open-source tool to rapidly calculate VSL and VSI.

The economic literature on the value of life includes seminal studies by Arthur (1981), Rosen (1988), Murphy and Topel (2006), and Hall and Jones (2007). Shepard and Zeckhauser (1984) and Ehrlich (2000) note the important role played by insurance markets. Aldy and Smyth (2014) use microsimulation to assess heterogeneity in VSL by race and sex. Córdoba and Ripoll (2016) and Bommier et al. (2021) use Epstein-Zin-Weil preferences to study the implications of state non-separable utility on the value of life. The models used in these prior studies include only a single health state for living individuals and focus exclusively on the value of preventing death, setting illness aside. Our study increases the scope and relevance of standard economic theory for understanding health risk by, for example, allowing researchers to compare VSL across health states and to quantify the relative values of reducing the risk of different illnesses.⁴

Our model also reconciles the standard life-cycle framework with results from a distinct literature that uses one-period models to study the value of mortality risk-reduction (Raiffa, 1969; Weinstein et al., 1980; Pratt and Zeckhauser, 1996; Hammitt, 2000). These static models predict that an increase in baseline health risk must raise VSL when insurance markets are incomplete, a result often referred to as the “dead-anyway” effect. We show that in a dynamic life-cycle setting, this theoretical result holds only under certain risk preferences.

The remainder of this paper is organized as follows. Section 2 presents the model, derives key results, and discusses welfare. Section 3 applies the model to data and shows how VSL varies across people with different health histories and how the value of pre-

⁴Several empirical papers have already demonstrated the relevance of valuing illness-prevention (Cameron and DeShazo, 2013; Hummels et al., 2016).

venting illness varies with the degree of illness risk. Section 4 concludes.

2 Model

Consider an individual who faces a health risk such as illness or death. We are interested in analyzing the value of a marginal reduction in that risk. Section 2.1 solves a lifetime utility model that accommodates multiple health states with varying degrees of financial, illness, and mortality risk. We then present analytic expressions for the value of statistical life (VSL) and the value of statistical illness (VSI). Section 2.2 provides conditions under which VSL falls or rises following a health shock. Section 2.3 discusses welfare.

Like prior studies on the value of life, we focus throughout this paper on the demand for health and longevity. Quantifying optimal health spending requires additionally modeling the supply of health care (Hall and Jones, 2007). In light of all the institutional differences across health care delivery systems, a wide variety of plausible approaches can be taken to this modeling problem, which we leave to future research.

2.1 The value of health and longevity

Let Y_t denote the consumer's health state at time t . We assume Y_t is a continuous-time Markov chain with finite state space $Y = \{1, 2, \dots, n, n+1\}$, where state $i \in \{1, \dots, n\}$ represents different possible health states while alive, and state $i = n+1$ represents death. We assume Y_t is right-continuous with left limits (RCLL) and denote the transition rates by:

$$\begin{aligned}\lambda_{ij}(t) &= \lim_{h \rightarrow 0} \frac{1}{h} \mathbb{P}[Y_{t+h} = j | Y_t = i], \quad j \neq i, \\ \lambda_{ii}(t) &= - \sum_{j \neq i} \lambda_{ij}(t)\end{aligned}$$

For analytical convenience and without meaningful loss of generality, we assume that individuals can transition only to higher-numbered states, i.e., $\lambda_{ij}(t) = 0 \quad \forall j < i$.⁵ The probability that a consumer in state i at time 0 remains in state i at time t is then equal to:

$$\tilde{S}(i, t) = \exp \left[- \int_0^t \sum_{j > i} \lambda_{ij}(s) ds \right]$$

⁵That is, a person can transition from state i to j , $i < j$, but not vice versa. This restriction does not meaningfully limit the generality of our model because one can always define a new state $k > j$ with properties similar to state i .

For expositional purposes we shall often refer to transitions as a “health shock,” “falling ill,” or “dying,” but our model also accommodates transitions from sick states to healthy states. We denote the stochastic mortality rate at time t as:

$$\mu(t) = \sum_{i=1}^n \lambda_{i,n+1}(t) \mathbf{1}\{Y_t = i\}$$

where $\mathbf{1}\{Y_t = i\}$ is an indicator variable equal to 1 if the individual is in state i at time t and 0 otherwise. When the number of states is equal to $n = 1$, we obtain the setting with deterministic health risk studied in prior literature (e.g., [Shepard and Zeckhauser, 1984](#); [Rosen, 1988](#); [Murphy and Topel, 2006](#)). The maximum lifespan of an individual is T , and we denote her stochastic probability of surviving until $t \leq T$ as:

$$S(t) = \exp\left[-\int_0^t \mu(s) ds\right]$$

Let $c(t)$ be consumption at time t , W_0 be baseline wealth, and ρ be the rate of time preference. Health-related quality of life at time t , $q_{Y_t}(t)$, is exogenous and depends on the health state, Y_t . Normalizing the utility of death to zero, the consumer’s maximization problem for $Y_0 \in \{1, \dots, n\}$ is:

$$V(0, W_0, Y_0) = \max_{c(t)} \mathbb{E} \left[\int_0^T e^{-\rho t} S(t) u(c(t), q_{Y_t}(t)) dt \middle| Y_0, W_0 \right] \quad (1)$$

Let the state variable $W(t)$ represent current wealth at time t . We assume $W(t)$ evolves according to:

$$\begin{aligned} W(0) &= W_0, \\ W(t) &\geq 0, \\ dW(t) &= (r_{Y_t} W(t) - c(t)) dt - h_{Y_{t-1}, Y_t} W(t) dY_t \end{aligned} \quad (2)$$

The no-debt constraint, $W(t) \geq 0$, means the consumer cannot borrow. We allow state transitions to affect wealth in two ways: via a state-dependent interest rate, r_{Y_t} , and via a one-time, immediate reduction in wealth of proportion h_{Y_{t-1}, Y_t} .⁶ These two channels let us model different types of financial consequences, including reduced wage growth, recurring health costs, expenses when falling sick, and expected loss of future income.

⁶Equation (2) is a stochastic differential equation driven by the RCLL Markov process Y_t . Here, $dY_t = 0$ for all t except for the (finite number of) transition times.

We assume that $|h_{Y_{t-}, Y_t} dY_t| < 1$, so that the fraction of wealth following the transition, $\tilde{h}_{ij} \equiv 1 - h_{ij}(j - i)$, is positive.⁷

The utility function, $u(c, q)$, is time-separable and depends on both consumption and health-related quality of life. We assume throughout that $u(\cdot)$ is strictly increasing and concave in its first argument, and twice continuously differentiable. Hence, we must have $W(T) = 0$, since it cannot be optimal to have wealth remaining at the maximum possible age. We denote the marginal utility of consumption as $u_c(\cdot)$ and assume that this function diverges to positive infinity as consumption approaches zero, so that optimal consumption is always positive.

Define the consumer's objective function at time t as:

$$J(t, w, i) = \mathbb{E} \left[\int_0^{T-t} e^{-\rho v} \exp \left\{ - \int_0^v \mu(t+s) ds \right\} u(c(t+v), q_{Y_{t+v}}(t+v)) dv \middle| Y_t = i, W(t) = w \right]$$

Define the optimal value function as:

$$V(t, w, i) = \max_{c(s), s \geq t} \{J(t, w, i)\}$$

subject to the wealth dynamics above and $V(t, w, n+1) = 0$. Under conventional regularity conditions, if V and its partial derivatives are continuous, then V satisfies the following Hamilton-Jacobi-Bellman (HJB) system of equations:

$$\begin{aligned} \rho V(t, w, i) = \max_{c(t)} & \left\{ u(c(t), q_i(t)) + \frac{\partial V(t, w, i)}{\partial w} [r_i w - c(t)] + \frac{\partial V(t, w, i)}{\partial t} \right. \\ & \left. + \sum_{j>i} \lambda_{ij}(t) [V(t, w, \tilde{h}_{ij}, j) - V(t, w, i)] \right\}, \quad i = 1, \dots, n \quad (3) \end{aligned}$$

where $c(t) = c(t, w, i)$ is the optimal rate of consumption. The first-order condition in the HJB (3) implies that the marginal utility of wealth in state Y_t at time t is equal to:

$$\frac{\partial V(t, W(t), Y_t)}{\partial W(t)} = u_c(c(t), q_{Y_t}(t)) \quad (4)$$

In order to apply our value of life analysis, we exploit recent advances in the systems and control literature. [Parpas and Webster \(2013\)](#) show that one can reformulate a stochastic finite-horizon optimization problem as a deterministic problem that takes

⁷It is straightforward to let r and h also depend on age/time. For ease of exposition we do not do so here, but we allow for this dependence in the numerical model we present in Section 3.

$V(t, w, j)$, $j \neq i$, as exogenous. More precisely, we focus on the path of Y_t that begins in state i and remains in state i until time T . We denote optimal consumption and wealth in that path by $c_i(t)$ and $W_i(t)$, respectively.⁸ A key advantage of this method is that it allows us to apply the standard deterministic Pontryagin maximum principle and derive analytic expressions.

Lemma 1. *Consider the following deterministic optimization problem for $Y_0 = i$ and $W(0) = W_0$:*

$$V(0, W_0, i) = \max_{c_i(t)} \left[\int_0^T e^{-\rho t} \tilde{S}(i, t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t) \tilde{h}_{ij}, j) \right) dt \right] \quad (5)$$

subject to:

$$\begin{aligned} W_i(0) &= W_0, \\ W_i(t) &\geq 0, \\ \frac{\partial W_i(t)}{\partial t} &= r_i W_i(t) - c_i(t) \end{aligned}$$

where $V(t, w, j)$, $j \neq i$, are taken as exogenous. Then the optimal value function, $V(t, W_i(t), i)$, satisfies the HJB equation given by (3), for all $i \in \{1, \dots, n\}$.

Proof. See Appendix A ■

Because the value function $V(t, w, i)$ corresponding to (5) satisfies the HJB equation given by (3), it must also be equal to the consumer's optimal value function (Bertsekas, 2005, Proposition 3.2.1). The present value Hamiltonian corresponding to (5) is:

$$H(W_i(t), c_i(t), p_t^{(i)}) = e^{-\rho t} \tilde{S}(i, t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t) \tilde{h}_{ij}, j) \right) + p_t^{(i)} [r_i W_i(t) - c_i(t)]$$

where $p_t^{(i)}$ is the costate variable for state i . The necessary costate equation is:

$$\dot{p}_t^{(i)} = -\frac{\partial H}{\partial W_i(t)} = -p_t^{(i)} r_i - e^{-\rho t} \tilde{S}(i, t) \sum_{j>i} \lambda_{ij}(t) \tilde{h}_{ij} \frac{\partial V(t, W_i(t) \tilde{h}_{ij}, j)}{\partial w} \quad (6)$$

⁸Consumption, $c(t)$, is a stochastic process. We occasionally denote it as $c(t, W(t), Y_t)$ to emphasize that it depends on the states $(t, W(t), Y_t)$. When we reformulate our stochastic problem as a deterministic problem and focus on a single path $Y_t = i$, consumption is no longer stochastic because there is no uncertainty in the development of health states. We emphasize this point in our notation here by writing consumption as $c_i(t)$, and wealth as $W_i(t)$.

The solution to the costate equation can be obtained using the variation of the constant method:

$$p_t^{(i)} = \left[\int_t^T e^{(r_i - \rho)s} \tilde{S}(i, s) \sum_{j>i} \lambda_{ij}(s) \tilde{h}_{ij} \frac{\partial V(s, W_i(s) \tilde{h}_{ij}, j)}{\partial w} ds \right] e^{-r_i t} + \theta^{(i)} e^{-r_i t}$$

where $\theta^{(i)} > 0$ is a constant. The necessary first-order condition for consumption is:

$$p_t^{(i)} = e^{-\rho t} \tilde{S}(i, t) u_c(c_i(t), q_i(t)) \quad (7)$$

where the marginal utility of wealth at time $t = 0$ is $\frac{\partial V(0, W_0, i)}{\partial W_0} = p_0^{(i)} = u_c(c_i(0), q_i(0))$. Since the Hamiltonian is concave in $c_i(t)$ and $W_i(t)$, the necessary conditions for optimality are also sufficient (Seierstad and Sydsaeter, 1977).

To analyze the value of health and longevity, we follow Rosen (1988). Let $\delta_{ij}(t)$ be a perturbation on the transition rate, $\lambda_{ij}(t)$, $0 \leq t \leq T$, where $\sum_{j>i} \int_0^T \delta_{ij}(t) dt = 1$. The impact of a small (ε) perturbation on the likelihood of exiting state i is:

$$\tilde{S}^\varepsilon(i, t) = \exp \left[- \int_0^t \sum_{j>i} (\lambda_{ij}(s) - \varepsilon \delta_{ij}(s)) ds \right], \text{ where } \varepsilon > 0 \quad (8)$$

The marginal value of preventing illness or death is equal to $\frac{\partial V / \partial \varepsilon}{\partial V / \partial w} \Big|_{\varepsilon=0}$, the marginal rate of substitution between longer life and wealth. The next lemma provides the numerator of this marginal value expression.

Lemma 2. *The marginal utility of preventing illness or death in state i is given by:*

$$\begin{aligned} \frac{\partial V(0, W_0, i)}{\partial \varepsilon} \Big|_{\varepsilon=0} &= \int_0^T e^{-\rho t} \tilde{S}(i, t) \left[\left(\int_0^t \sum_{j>i} \delta_{ij}(s) ds \right) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t) \tilde{h}_{ij}, j) \right) \right. \\ &\quad \left. - \sum_{j>i} \delta_{ij}(t) V(t, W_i(t) \tilde{h}_{ij}, j) \right] dt \end{aligned}$$

Proof. See Appendix A ■

Lemma 2 pertains to a marginal reduction in transition rates for all states and times. Consider as a special case perturbing only $\lambda_{i,n+1}(t)$, the mortality rate in state i , and set the perturbation $\delta(\cdot)$ in Equation (8) equal to the Dirac delta function, so that the mortality rate is perturbed at $t = 0$ and remains unaffected otherwise (Rosen, 1988). This perturbation yields an expression commonly known as the value of statistical life (VSL).

Proposition 3. Set $\delta_{ij}(t) = 0 \forall j < n + 1$ in the marginal utility expression given in Lemma 2 and let $\delta_{i,n+1}(t)$ equal the Dirac delta function. Dividing by the marginal utility of wealth yields:

$$VSL(i) = \mathbb{E} \left[\int_0^T e^{-\rho t} S(t) \frac{u(c(t), q_{Y_t}(t))}{u_c(c(0), q_{Y_0}(0))} dt \middle| Y_0 = i, W(0) = W_0 \right] = \frac{V(0, W_0, i)}{u_c(c_i(0), q_i(0))} \quad (9)$$

Proof. See Appendix A ■

VSL is the value of a marginal reduction in the risk of death in the current period. Put differently, it is the amount that a large group of individuals are collectively willing to pay to eliminate a current risk that is expected to kill one of them, where the identity of the decedent is unknown *ex ante*. Proposition 3 shows that VSL is proportional to expected lifetime utility, and inversely proportional to the marginal utility of consumption.

We can also value a marginal reduction in the risk of falling ill. As before, it is helpful to choose the Dirac delta function for $\delta(t)$, so that the transition rates are perturbed at $t = 0$ only. Consider a reduction in the transition rate for a single alternative state, $j \leq n + 1$, so that $\delta_{ik}(t) = 0 \forall k \neq j$. Dividing the result by the marginal utility of wealth then yields what we term the value of statistical illness, $VSI(i, j)$:

$$VSI(i, j) = \frac{V(0, W_0, i) - V(0, W_0, \tilde{h}_{ij}, j)}{u_c(c_i(0), q_i(0))} \quad (10)$$

The interpretation of VSI is analogous to VSL: it is the amount that a large group of individuals are collectively willing to pay in order to eliminate a current disease risk that is expected to befall one of them, where the identity of the sick person is unknown *ex ante*. If health state j corresponds to death, so that $V(0, w, j) = 0$, then $VSI(i, j) = VSL(i)$. Thus, VSI is a generalization of VSL.

The values of statistical life and illness depend on how substitutable consumption is at different ages and states. Intuitively, if present consumption is a good substitute for future consumption, then living a longer life is less valuable. Define the elasticity of intertemporal substitution, σ , as:

$$\frac{1}{\sigma} \equiv -\frac{u_{cc}c}{u_c}$$

In addition, define the elasticity of quality of life with respect to the marginal utility of consumption as:

$$\eta \equiv \frac{u_{cq}q}{u_c}$$

When η is positive, the marginal utility of consumption is higher in healthier states, and vice-versa. Taking logarithms of Equation (7), differentiating with respect to t , plugging in the result for the costate equation and its solution, and rearranging yields an expression for the life-cycle profile of consumption:

$$\frac{\dot{c}_i}{c_i} = \sigma(r_i - \rho) + \sigma\eta \frac{\dot{q}_i}{q_i} - \sigma\lambda_{i,n+1}(t) - \sigma \sum_{j=i+1}^n \lambda_{ij}(t) \left[1 - \tilde{h}_{ij} \frac{u_c(c(t, W_i(t)\tilde{h}_{ij}, j), q_j(t))}{u_c(c(t, W_i(t), i), q_i(t))} \right] \quad (11)$$

The first two terms in Equation (11) relate the growth rate of consumption to the consumer rate of time preference and to life-cycle changes in the quality of life. The third term shows that consumption growth is a declining function of the current mortality rate, $\lambda_{i,n+1}(t)$. Because the consumer cannot purchase annuities to insure against her uncertain lifetime, higher rates of mortality depress the rate of consumption growth over the life-cycle (Yaari, 1965).

The fourth term in Equation (11)—which is absent from the conventional deterministic setting—accounts for the possibility that the consumer might transition to a different health state in the future. The possibility of transitioning to a state with low marginal utility of consumption shifts life-cycle consumption earlier still, whereas transitioning to a state with high marginal utility shifts consumption to later in the life-cycle. The effect on current consumption of potential financial shocks, as captured by \tilde{h}_{ij} , is ambiguous. On the one hand, the possibility of losing wealth in the future increases the incentive to consume today. On the other hand, the possibility of low future wealth—and thus high marginal utility of consumption—increases the incentive to save. The net effect depends on the curvature of the utility function.

Equation (11) describes consumption dynamics conditional on the individual's health state i . It is not readily apparent from (11) whether modeling health as stochastic causes consumption to shift forward, *on average* across all states, relative to modeling health as deterministic. We confirmed in numerical exercises that modeling health as stochastic has an ambiguous effect on consumption (and VSL), even when holding quality of life constant across states and time.⁹

⁹Modeling health as stochastic has a positive effect on lifetime utility because a stochastic environment allows the consumer to adjust consumption after a health shock. Put differently, a deterministic model is equivalent to a stochastic model where the consumer is forced to keep consumption constant across states.

2.2 The effect of health shocks on VSL

In our model, an adverse health shock can reduce a person's longevity and quality of life; increase her recurring health costs, as captured by the health-related interest rate, r_{Y_t} ; and produce an up-front financial shock, as captured by h_{Y_t, Y_t} . This section isolates each of these effects in a series of simple, transparent two-state settings and shows that they can have opposing effects on VSL.¹⁰ To quantify their net effect, we turn to a numerical model in Section 3.

We first consider a setting where a transition reduces wealth but otherwise has no effect. Because the willingness-to-pay for life extension increases with wealth, this transition reduces VSL.

Proposition 4. *Assume utility is positive. Let there be $n = 2$ states with identical healthcare spending and quality of life, so that $r_1 = r_2$ and $q_1(s) = q_2(s) \forall s$, and identical probabilities of dying, so that $\lambda_{13}(s) = \lambda_{23}(s) \forall s$. Assume that transitioning from state 1 to state 2 is associated with a one-time financial shock, $h_{12} > 0$. Suppose the consumer transitions from state 1 to state 2 at time t , and that $\lambda_{12}(\tau) = 0 \forall \tau > t$. Then $VSL(1, t) > VSL(2, t)$.*

Proof. See Appendix A ■

Next, we consider a transition that reduces the health-related interest rate, r_{Y_t} . Because this transition reduces future wealth, it will reduce expected lifetime utility. At the same time, a reduction in the interest rate may encourage individuals to increase current consumption, which raises their willingness to pay for life-extension, all else equal. The net effect depends on the elasticity of intertemporal substitution, σ . As we discuss later in this section, prior studies generally assume that $\sigma < 1$. Our next proposition shows that this assumption is a sufficient condition for VSL to decrease following a reduction in r_{Y_t} .

Proposition 5. *Assume utility is positive and satisfies the condition $\sigma < 1$. Let there be $n = 2$ states with identical quality of life and identical probabilities of dying, so that $q_1(s) = q_2(s)$ and $\lambda_{13}(s) = \lambda_{23}(s) \forall s$, and assume that there are no one-time financial shocks when transitioning, so that $h_{12} = 0$. Assume that transitioning from state 1 to state 2 is associated with a decrease in the health-related interest rate such that $r_1 > r_2$. Suppose the consumer transitions from state 1 to state 2 at time t , and that $\lambda_{12}(\tau) = 0 \forall \tau > t$. Then $VSL(1, t) > VSL(2, t)$.*

¹⁰While we focus here on how VSL changes following a state transition, our results have similar implications for VSI. Recall that $VSL(i)$ is a special case of $VSI(i, j)$, where the health state j corresponds to death. In a two-state model, the only relevant values to compare are $VSI(1, 3) = VSL(1)$ and $VSI(2, 3) = VSL(2)$. In a model with more than two states, one could also compare the values of preventing illness.

Proof. See Appendix A ■

Finally, we consider transitions that affect quality of life or longevity. An adverse shock to quality of life will lower lifetime utility, $V(\cdot)$. However, inspection of Equation (9) shows that the shock's effect on VSL will also depend on the relationship between quality of life and the marginal utility of consumption, a phenomenon often referred to as "health state dependence." Quantifying this relationship is a longstanding topic in health economics, but there is still no consensus regarding its sign or magnitude.¹¹ Thus, the effect of a quality of life shock on VSL cannot be signed without making further assumptions.

The effect on VSL of a change in longevity will depend on consumer preferences and expectations of future mortality. Intuitively, adverse longevity shocks have two countervailing effects on VSL. On the one hand, a shorter lifespan reduces the lifetime utility of life-extension. On the other hand, a shorter lifespan increases current consumption, which lowers marginal utility and thus increases the willingness to pay for health and longevity. The net effect will depend on the curvature of the utility function relative to the curvature of the marginal utility function.

We first show that consumption increases when transitioning to a state where current and future mortality are high, holding wealth constant. If wealth falls following the transition, then the proposition may not hold, depending on the size of the wealth shock. However, in our quantitative analysis—where wealth can fall after a health shock because of medical spending and income loss—we find that non-medical consumption still generally rises. This result is consistent with Smith (1999), who finds that the reduction in wealth following an adverse health shock is larger than its combined effects on out-of-pocket medical spending and income.

Proposition 6. *Let there be $n = 2$ states with identical constant quality of life and health-related interest rates, so that $q_1(s) = q_2(s) = q$ and $r_1 = r_2 \forall s$, and no financial shocks upon transition, so that $h_{1,2} = 0$. Assume that the transition rates $\lambda_{12}(s)$ are uniformly bounded (finite), and that the mortality rate is uniformly higher in state 2: $\lambda_{13}(s) < \lambda_{23}(s) \forall s$. Suppose the consumer transitions from state 1 to state 2 at time t . Then $c_1(t, W(t), 1) < c_2(t, W(t), 2)$.*

Proof. See Appendix A ■

¹¹Viscusi and Evans (1990), Sloan et al. (1998), and Finkelstein et al. (2013) find evidence of negative state dependence, meaning that the marginal utility of consumption falls when health deteriorates. In contrast, Lillard and Weiss (1997) and Edwards (2008) find evidence of positive state dependence. Evans and Viscusi (1991) find no evidence of state dependence. Murphy and Topel (2006) assume negative state dependence when performing their empirical exercises, while Hall and Jones (2007) assume state independence.

To analyze the effect of this transition on VSL, we focus on the case where $\dot{c}_i/c_i \leq 0$, so that consumption does not grow for people who stay in the same health state. Empirical evidence suggests this case accurately describes the typical consumer nearing retirement.¹² According to Equation (11), and under the assumptions of Proposition 6, a sufficient condition for a declining consumption profile is $r_i \leq \rho$.

Our analysis will compare a persistently healthy individual to someone who suffers an adverse shock to life expectancy but is otherwise identical. To make headway we must introduce the notion of prudence. The elasticity of intertemporal substitution, σ , measures utility curvature (i.e., the change in its slope). Prudence, π , is the analogous measure for the curvature of marginal utility (Kimball, 1990):

$$\pi \equiv -\frac{c u_{ccc}(\cdot)}{u_{cc}(\cdot)}$$

It will also be convenient to define the elasticity of the flow utility function:

$$\epsilon \equiv \frac{c u_c(\cdot)}{u(\cdot)}$$

The utility elasticity, ϵ , is positive when utility is positive. Positive utility ensures well-behaved preferences, and is often enforced by adding a constant to the utility function. Although adding a constant to the utility function does not affect the solution to the consumer's maximization problem, this constant matters for the value of life.¹³

The following proposition provides sufficient conditions for VSL to rise following an adverse shock to longevity.

Proposition 7. *Consider a two-state setting with assumptions set out in Proposition 6. Assume that $r_1 = r_2 \leq \rho$, and that utility is positive and satisfies the condition:*

$$\pi < \frac{2}{\sigma} + \epsilon \tag{12}$$

Suppose that the consumer transitions from state 1 to state 2 at time t , and that $\lambda_{12}(\tau) = 0 \forall \tau > t$. Then, $VSL(1, t) < VSL(2, t)$.

Proof. See Appendix A ■

¹²A typical consumption profile is constrained by low income at early ages, increasing during middle ages when income is high, and then declines during retirement until consumption equals the consumer's pension. This inverted U-shape for the age profile of consumption has been widely documented across different countries and goods (Carroll and Summers, 1991; Banks et al., 1998; Fernandez-Villaverde and Krueger, 2007).

¹³Rosen (1988) was the first to point out that the level of utility is an important determinant of the value of life. See also additional discussion on this point in Hall and Jones (2007) and Córdoba and Ripoll (2016).

Proposition 7 shows that the effect of longevity shocks on VSL depends on both prudence and the elasticity of intertemporal substitution. Consumers with inelastic demand for current consumption (low σ) prefer to smooth consumption over time because consumption expenditures at different ages are poor substitutes. They therefore have a high willingness to pay for life-extension and, all else equal, are more likely to exhibit a rise in VSL following an adverse longevity shock than consumers with more elastic demand. Likewise, consumers with low levels of prudence, π , have marginal utility that decreases rapidly with consumption and produces a high willingness to pay for life-extension following a shock that increases consumption.

Prior studies on the value of life generally assume that 0.5 to 0.8 is a reasonable range for the value of σ (Murphy and Topel, 2006; Hall and Jones, 2007), and recent empirical studies suggest that π is about 2 (Noussair et al., 2013; Christelis et al., 2020). Under these parameterizations, condition (12) will hold whenever utility is positive. Condition (12) is always satisfied by isoelastic utility, provided that utility is positive. That said, the condition is not innocuous: one can easily find linear combinations of isoelastic and polynomial utility functions where VSL declines following an illness.

Thus, VSL can in general rise or fall following an increase in baseline mortality risk. In static models commonly used in prior studies, however, VSL can only rise with baseline risk (Weinstein et al., 1980; Pratt and Zeckhauser, 1996; Hammitt, 2000). This discrepancy arises because these prior studies focus on a one-period setting with two states, alive and dead. In that context, if the marginal utility of consumption is lower in the dead state, then an increase in baseline mortality risk must lower the expected marginal utility of consumption and thus raise the willingness to pay for survival (the “dead-anyway” effect).¹⁴ Proposition 6 confirms that an increase in the risk of death also reduces marginal utility in our dynamic context. However, unlike in the highly simplified static setting, the resulting effect on VSL is ambiguous because of an offsetting decrease in lifetime utility.

The value of a health unit

Our results on the effect of health shocks on VSL extend naturally to the value of a health unit, like a life-year or quality-adjusted life-year. Let $D_i(t)$ denote some measure of remaining lifetime health for an individual in state i at time t . For example, $D_i(t)$ could be measured in units of simple life expectancy, quality-adjusted life expectancy, or generalized risk- and quality-adjusted life expectancy (Lakdawalla and Phelps, 2020, 2023).

¹⁴Let expected utility be equal to $EU = p u(0, c) + (1 - p) u(1, c)$, where $p \in (0, 1)$ is the probability of death and the states $\{0, 1\}$ represent death and life, respectively. The willingness to pay for a marginal reduction in the probability of dying is given by $VSL = \frac{u(1, c) - u(0, c)}{p u_c(0, c) + (1 - p) u_c(1, c)}$, which increases with p if $u_c(1, c) > u_c(0, c)$.

We assume this measure is non-negative, equals 0 only when dead, and is independent of consumption, but otherwise impose no restrictions on its form. When VSL rises following a transition from state i to some state j with lower health (e.g., as in Proposition 7), the value per unit of health must naturally rise as well:

$$\frac{VSL(i, t)}{D_i(t)} < \frac{VSL(j, t)}{D_j(t)}$$

This result contrasts with traditional cost-effectiveness analysis, which assumes that a quality-adjusted life-year is equally valuable regardless of baseline health (Drummond et al., 2015, Chapter 5). But in fact, a constant value arises only when the utility of consumption is constant, which will not generally be true (Bleichrodt and Quiggin, 1999). One exception is the special case where financial markets are complete, the rate of time preference equals the interest rate, and quality of life is constant.

The result also sheds light on the value of prevention versus treatment. Consider a preventive intervention that improves health for the healthy, and a treatment that improves health for the sick. These definitions accord with the common sense notion that vaccines are preventive, whereas drugs administered after an illness has occurred are considered treatments. From this conceptual framework, our result implies that preventive interventions generate less value per health unit gained. This result helps explain lower demand for preventive investments, even when prevention and treatment produce the same expected health gain.

While a rise in VSL following an adverse health shock is a sufficient condition for a rise in the value of a health unit, it is not necessary. Indeed, our quantitative analysis will show that even though VSL sometimes falls after a health shock, the value of a health unit still generally rises.

2.3 Welfare

This paper studies the willingness to pay for health and longevity, which helps us understand puzzles such as why sick individuals are often willing to pay more to reduce mortality risk than healthy people. Often, however, policymakers must decide how to allocate resources across different people. Who should receive limited supplies of a vaccine against a pandemic? Should a payer with a fixed budget focus resources on the elderly or the young, on the sick or the poor?

In such contexts, economists frequently rely on comparisons of aggregate social surplus, that is, the aggregate sum of willingness to pay. For example, Murphy and Topel

(2006) employ this approach in the framework of the standard life-cycle VSL model. [Garber and Phelps \(1997\)](#) rely on it to develop the theory of cost-effectiveness for health interventions. [Einav et al. \(2010\)](#) use it to study the welfare effects of health insurance. Industrial organization economists use it, in the form of deadweight loss comparisons, to evaluate the welfare consequences of market power ([Martin, 2019](#)).

While popular among applied economists and policymakers, the aggregate surplus approach has been criticized by welfare theorists for several reasons ([Boadway, 1974](#); [Blackorby and Donaldson, 1990](#)). Equity concerns arise because each dollar of surplus is weighted equally, regardless of differences in wealth or income across people; this implicitly places more weight on the utility of wealthier individuals. Aggregation can also produce intransitive rankings of alternative allocations. Heterogeneity in marginal utility across consumers can break the necessary link between growth in aggregate surplus and increases in utility ([Martin, 2019](#)). These points matter little when valuing the prevention of different illnesses, which can be accomplished from the perspective of a single individual, but it does suggest a need for caution when making welfare inferences across individuals inhabiting different health states.

One alternative solution is to aggregate utilities rather than monetized surplus, but debate persists about how to aggregate in situations involving risk ([Fleurbaey, 2010](#)). In a foundational study, [Harsanyi \(1955\)](#) shows that a social welfare function satisfying both rationality and the Pareto principle must be a weighted sum of ex ante individual utilities. However, this utilitarian approach ignores ex post distributional concerns ([Diamond, 1967](#)). As a result, one cannot simultaneously satisfy both rationality and the Pareto principle while still pursuing ex post equity. Theorists have argued for abandoning one or the other of these principles. [Diamond \(1967\)](#) advocates minimizing ex ante inequality, but this violates rationality. [Adler and Sanchirico \(2006\)](#) advocate minimizing ex post inequality, but this violates the Pareto principle. In the specific context of VSL, [Pratt and Zeckhauser \(1996\)](#) advocate maximizing ex ante utility, but this ignores ex post equity concerns in light of Diamond’s result. We do not aim to resolve this long-standing debate in welfare economics, but instead note that our stochastic model can be incorporated into these different welfare frameworks as desired.

3 Quantitative Analysis

This section quantifies the value of health improvements. While our model provides useful insights on its own, some of our theoretical results require imposing restrictions on the consumer’s setting, such as holding wealth constant. Therefore, we complement

our theory with quantitative analysis calculating the value of health improvements for a consumer with standard preferences and whose mortality, quality of life, labor earnings, and medical spending can vary across 20 different health states.¹⁵ We calculate both VSI and VSL but focus on their normalized values, VSI per QALY and VSL per QALY, which are more easily compared. All of our data and code are publicly available online.¹⁶

3.1 Framework

We employ a discrete time analogue of the model from Section 2. There are n health states (excluding death). Denote the transition probabilities between health states by:

$$p_{ij}(t) = \mathbb{P}[Y_{t+1} = j | Y_t = i]$$

The mortality rate at time t , $d(t)$, depends on the individual's health state:

$$d(t) = \sum_{j=1}^n \bar{d}_j(t) \mathbf{1}\{Y_t = j\}$$

where $\{\bar{d}_j(t)\}$ are given and $\mathbf{1}\{Y_t = j\}$ is an indicator equal to 1 if the individual is in state j at time t and 0 otherwise.¹⁷ The maximum lifespan of a consumer is T , so $d(T) = 1$. We denote the stochastic probability of surviving from time t to time $s \leq T$ as $S_t(s)$, where:

$$\begin{aligned} S_t(t) &= 1, \\ S_t(s) &= S_t(s-1)(1 - d(s-1)), \quad s > t \end{aligned}$$

Let $c(t)$ and $W(t)$ denote non-medical consumption and wealth in period t , respectively. Quality of life at time t , $q_{Y_t}(t)$, depends on the individual's health state, Y_t . Let ρ denote the rate of time preference. We measure health in state i and time t using discounted quality-adjusted life expectancy, defined as:

$$D_i(t) = \mathbb{E} \left[\sum_{j=t}^T e^{-\rho(j-t)} q_{Y_j}(j) S_t(j) \middle| Y_t = i \right] \quad (13)$$

¹⁵Our empirical framework is related to a number of papers that study the savings behavior of the elderly (Kotlikoff, 1988; Palumbo, 1999; De Nardi et al., 2010). These prior studies allow health to affect wealth accumulation by including two or three different health states in the model.

¹⁶They are available at: <https://julianreiff.com/research/reiff.wp.healthrisk.replication.zip>.

¹⁷Because our mortality data are distinct from our health state transition data, we denote the probability of dying in state i as $\bar{d}_i(t)$ rather than $p_{i,n+1}(t)$, which differs slightly from the notation used in Section 2.

We assume annuity markets are absent. This simplification allows us to calculate the value of life using an analytical solution to the consumer's problem. It is possible to incorporate partial annuitization in this setting. However, generalization requires numerical optimization, which may necessitate limiting the number of health states included in the model. In our sensitivity analysis, we model the effects of a bequest motive and of decreasing the substitutability of consumption over time, both of which—similar to annuitization—reduce consumption at earlier ages.

The consumer's maximization problem is:

$$\max_{c(t)} \mathbb{E} \left[\sum_{t=0}^T e^{-\rho t} S_0(t) u(c(t), q_{Y_t}(t)) \middle| Y_0, W_0 \right]$$

subject to:

$$\begin{aligned} W(0) &= W_0, \\ W(t) &\geq 0, \\ W(t+1) &= (W(t) - c(t)) e^{r(t, Y_t, Y_{t+1})} \end{aligned}$$

We account for out-of-pocket medical spending and other health-related financial shocks by allowing the individual's effective interest rate, $r(t, Y_t, Y_{t+1})$, to depend on her health states at the beginning and end of the period, Y_t and Y_{t+1} , respectively. Our baseline model sets $r(t, Y_t, Y_{t+1}) = r + \ln[1 - m(t, Y_t) + f(t, Y_t, Y_{t+1})]$, where r is the rate of interest, $m(t, Y_t)$ is the average share of an individual's wealth spent on medical and long-term care in health state Y_t at time t , and $f(t, Y_t, Y_{t+1})$ is the average change in the expected present value of future income when transitioning from health state Y_t to Y_{t+1} between time t and $t+1$, expressed as a share of current wealth.¹⁸ Instead of deducting financial losses from wealth directly, we treat them as modifying the interest rate. This approach allows us to capture the financial effects of changes in health, while preserving the closed-form solution that facilitates quantitative analysis, similar to [Krebs \(2003\)](#). We assume throughout that $r = \rho = 0.03$ ([Siegel, 1992](#); [Moore and Viscusi, 1990](#)).

Finally, we assume that utility takes the following isoelastic form:

$$u(c, q) = q \left(\frac{c^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma} \right) \quad (14)$$

¹⁸We calculate $m(t, Y_t)$ by dividing out-of-pocket spending in health state Y_t at time t by average wealth at time t . We calculate $f(t, Y_t, Y_{t+1})$ by dividing the change in the expected present value of future income between t and $t+1$ by wealth at time t .

The quality of life measure has non-negative values of $q \leq 1$, where $q = 1$ indexes perfect health. Utility is positive when non-medical consumption, $c \geq 0$, exceeds the subsistence level, \underline{c} . Our main specification sets $\gamma = 1.25$ and $\underline{c} = \$5,000$, consistent with the parameterization employed in [Murphy and Topel \(2006\)](#). Under these parameters, the utility function (14) satisfies condition (12) from Proposition 7. As discussed previously, there is no consensus regarding the sign or magnitude of health state dependence, $u_{cq}(\cdot)$. Here, we assume a multiplicative relationship where the marginal utility of non-medical consumption increases with the health-related quality of life (negative state dependence), which is consistent with the methods used to measure quality-adjusted life-years (QALYs) in our data, and with prior theoretical literature on VSL ([Garber and Phelps, 1997](#); [Bleichrodt and Quiggin, 1999](#); [Murphy and Topel, 2006](#)).

The value function for the consumer's maximization problem is defined as:

$$V(t, w, i) = \max_{c(s)} \mathbb{E} \left[\sum_{s=t}^T e^{-\rho(s-t)} S_t(s) u(c(s), q_i(s)) \middle| Y_t = i, W(t) = w \right]$$

We reformulate this optimization problem as a recursive Bellman equation:

$$V(t, w, i) = \max_{c(t)} \left[u(c(t), q_i(t)) + \frac{1 - \bar{d}_i(t)}{e^\rho} \sum_{j=1}^N p_{ij}(t) V(t+1, (w - c(t)) e^{r(t,i,j)}, j) \right]$$

We solve for consumption analytically and then use the formulas derived in Section 2 to calculate the value of life (see Appendix C for details).

There is significant uncertainty among economists regarding the proper values of many of the parameters in our model. The goal of the subsequent analyses is to quantify the economic significance of our insights by applying our model to real-world data using reasonable parameterizations. To investigate the sensitivity of our results to the parameterization of our utility function, we consider specifications with alternative assumptions regarding the elasticity of intertemporal substitution, $\sigma = 1/\gamma$. We also consider an alternative specification that includes a bequest motive. Rather than setting the utility of death to zero, our bequest motive specification follows [Fischer \(1973\)](#) and sets it equal to $u(W(t) - c(t), b(t))$, where $u(\cdot)$ takes the form given in (14), $W(t) - c(t)$ is wealth at death, and the parameter $b(t)$ governs the strength of the bequest motive. We conservatively set $b(t) = 1.2$, the largest value considered in [Fischer \(1973\)](#), for all t .

3.2 Data

We obtain individual-level data on mortality, disease incidence, quality of life, labor earnings, and out-of-pocket medical spending from the Future Elderly Model (FEM), a widely published microsimulation model that combines nationally representative information from the Health and Retirement Study (HRS), the Medical Expenditure Panel Survey (MEPS), the Panel Study of Income Dynamics, and the National Health Interview Survey. The FEM provides a uniquely rich set of information about the US elderly. For instance, while the HRS provides detailed data on health and wealth, it lacks survey questions that would allow us to calculate quality of life using standard survey instruments. To solve this problem, the FEM weaves together validated quality of life estimates from the MEPS and maps them to the HRS using variables common to both databases.

The FEM, which has been released into the public domain, produces estimates for individuals ages 50–100 with different comorbid conditions (see Appendix B). It accounts for six different chronic conditions (cancer, diabetes, heart disease, hypertension, chronic lung disease, and stroke) and six different impaired activities of daily living (bathing, eating, dressing, walking, getting into or out of bed, and using the toilet). We divide the health space within the FEM into $n = 20$ states. Each state corresponds to the number (0, 1, 2, 3 or more) of impaired activities of daily living (ADL) and the number (0, 1, 2, 3, 4 or more) of chronic conditions, for a total of $4 \times 5 = 20$ health states. Health states are ordered first by number of ADLs and then by number of chronic diseases, so that state 1 corresponds to 0 ADLs and 0 chronic conditions, state 2 corresponds to 0 ADLs and 1 chronic condition, and so on (see Column (1) of Table 1). This aggregation provides a parsimonious way of incorporating information about functional status and several major diseases. All ADLs and chronic conditions are treated as permanent, so they should be interpreted as the highest number of ADLs and chronic conditions that an individual has experienced. Hence, an individual’s transition probabilities are non-zero only for higher-numbered states that have more ADLs and/or more chronic conditions than her current state.

The quality of life data in the FEM are measured using the EuroQol five dimensions questionnaire (EQ-5D), which has been fielded by the MEPS (Stewart et al., 2014). These five dimensions are based on five survey questions that elicit the extent of a respondent’s problems with mobility, self-care, daily activities, pain, and anxiety/depression. These questions are then combined using weights derived from stated preference data.¹⁹ The result is a single quality of life index, the EQ-5D, which is anchored at 0 (equivalent to

¹⁹The five dimensions of the EQ-5D are weighted using estimates from Shaw et al. (2005). Additional technical details about the FEM are available in our Supplementary Materials.

death) and 1 (perfect health).

We calculate population-weighted averages by health state and age, and then use those means as inputs for our model. Table 1 provides summary means for ages 50 and 70, by health state. At age 50, life expectancy ranges from 30.9 years to 9.1 years, quality of life ranges from 0.88 to 0.54, out-of-pocket medical spending ranges from \$686 to \$2,759 per year, and labor earnings range from \$36,111 to \$8,375 per year. Columns (12) and (13) report the probability that an individual exits her health state but remains alive, i.e., acquires at least one new ADL or chronic condition within the following year. The permanence of ADLs and chronic conditions imposes some natural restrictions on state transitions. For example, an individual in state 14 has 2 ADLs and 3 chronic conditions. She can only transition to states 19 and 20, because all others involve fewer ADLs and/or fewer chronic conditions. Health states are relatively persistent, with empirical exit rates never exceeding 15 percent at ages 50 or 70. State 20 is an absorbing state with an exit rate of 0 percent.

Figure 1a plots average out-of-pocket medical spending by age for three selected health states. These data include all inpatient, outpatient, prescription drug, and long-term care spending not paid for by insurance. Spending is higher in sicker health states and, consistent with De Nardi et al. (2010), increases greatly at older ages, when long-term care expenses arise. The effect of sickness on out-of-pocket spending is modest in comparison to long-term care costs, causing the overall gap in spending across states to shrink with age.²⁰

Figure 1b plots corresponding estimates for labor earnings. For healthy individuals in state 1, earnings peak at age 55 and then fall to zero by age 80. By contrast, earnings decline continuously for individuals in state 20, the sickest possible health state. When calculating the health-related interest rate, $r(t, i, j)$, we assume income is equal to labor earnings plus a Social Security payment equal to \$15,100 per year beginning at age 65.²¹

We evaluate our life-cycle model using FEM data for ages 50–100 but focus our discussion below on ages 50–80, where the FEM estimates are more precise and labor earnings are still quite relevant for consumption decisions. We assume that the distribution of initial wealth across health states is proportional to labor earnings at age 50. Finally, we calibrate the level of initial wealth by assuming that the population-weighted average VSL at age 50 is \$6 million, which matches the value from Murphy and Topel (2006) and

²⁰FEM medical spending estimates have been validated by comparing them to estimates from the National Health Expenditure Accounts (see Section 8.2, Appendix B of National Academies of Sciences, Engineering, and Medicine, 2015).

²¹The FEM earnings data are based on HRS data from 2000–2012. In 2010, the average Social Security benefit received by retired workers was \$15,100.

is within the range estimated by empirical studies of VSL for working-age individuals (O’Brien, 2018). Our calibration implies that a healthy 50-year-old in state 1 has a VSL of \$6.8 million.

3.3 Explaining variation in the value of life

We begin with a simple example. The solid red and dashed blue lines in Figure 2 report VSL and non-medical consumption for a healthy individual who experiences a modest health shock at age 60 that reduces her life expectancy by 3.0 years, suffers a severe health shock at age 70 that reduces life expectancy by 6.8 years, and then dies at age 75. Each shock also produces sudden changes to quality of life, medical spending, and labor earnings, as estimated by the FEM. The black X at age 60 reports what this person’s VSL would have been absent the first health shock; the second X reports what her VSL at age 70 would have been absent the second shock. The vertical difference between the X and the red VSL line thus represents the effect of the health shock on VSL.

Consumption increases sharply following the two health shocks depicted in Figure 2. VSL is unaffected by the shock at age 60, but rises by 5 percent at age 70. Overall, this simple example suggests that our results from Propositions 6 and 7—which predict that consumption and VSL will rise following an adverse shock to life expectancy—are relevant in a more general setting where health shocks also reduce quality of life and wealth.

To characterize the effects of health shocks among the US elderly population more generally, we conduct a Monte Carlo exercise that begins with 50,000 nationally representative individuals at age 50. Each person’s health path then evolves at random according to the nationally representative health transition probabilities estimated by the FEM. At age 50, VSL ranges from \$0.8 million for individuals in poor health to \$6.8 million for those in excellent health. This dispersion results from differences in both initial health and wealth. Traditional theory accounts for variation in wealth, but it is not configured to analyze variation in baseline health states or future health risks. In order to abstract away from the effects of differences in initial wealth on VSL, the remainder of this section focuses on the 22,214 healthy individuals who were initially in health state 1 at age 50.

Figure 3a illustrates how the distribution of VSL varies over the life cycle for these 22,214 individuals. The figure plots all nineteen ventiles and the bottom and top five percentiles of the distribution in light blue, with the mean in bold. Figure 3b provides a fuller picture of the distribution at age 70. There is a long left-tail of sick individuals who, expecting an imminent death, have spent down their wealth. Individuals with above-

average VSL are a mix of healthy individuals and newly diagnosed sick individuals who have begun rapidly spending down their wealth. By age 70, the VSL inter-vigintile range spans \$1.7 to \$2.9 million, with the 95th percentile consumer willing to pay 70% more for life-extension than the 5th.

This cohort of 22,214 individuals experiences about 58,000 health shocks between the ages of 50 and 80. Figure 4a displays the distribution of the change in VSL following each of those shocks.²² On average, a health shock increases VSL by \$16,000 (< 1% of VSL prior to the shock), though there is some considerable heterogeneity: 3 percent of shocks increase VSL by over \$50,000, while 5 percent cause VSL to fall. Figure 4b normalizes these results by the individual's quality-adjusted life expectancy. This second plot shows that health shocks increase VSL per QALY by \$70,000 (21%), on average. The distribution is skewed to the right, with the value of a QALY rising by over \$150,000 (45%) in 5 percent of cases. Although some shocks do reduce VSL, none of them reduce the value of a QALY. This rise in the willingness to pay for a QALY helps explain why people state a preference for prioritizing the health of the severely ill (Linley and Hughes, 2013).

Figure 5 characterizes variation in the value of life for individuals at age 70.²³ The positive slope of the blue dashed line indicates that, on average, VSL rises with life expectancy, consistent with recent work finding that VSL is higher for people in better health (Ketcham et al., 2020). However, the solid red line in Figure 5 indicates that VSL per QALY falls rapidly with quality-adjusted life expectancy. Individuals in the worst health state have an average VSL per QALY of \$500,000, about 1.9 times higher than individuals in the healthiest state, where VSL per QALY is \$260,000. These results contradict conventional cost-effectiveness theory—which implies that the value of a QALY is constant—but are consistent with generalized approaches that predict cost-effectiveness thresholds that vary with income and disease severity (Lakdawalla and Phelps, 2021; Phelps and Cinatl, 2021).

Next, we quantify how the value of reducing illness risk varies with the severity of the illness. The dashed blue line in Figure 6 reports VSI's for different illnesses from the perspective of a healthy 70-year-old with a quality-adjusted life expectancy of 11.0 QALYs. Each value represents the healthy individual's marginal willingness to pay for a reduction in the risk of death or of transitioning to one of the 19 other health states in

²²The change is equal to the difference between actual VSL and a counterfactual VSL, where the counterfactual assumes the individual did not experience the health shock. See Figure 2 for a visual example.

²³Figure 4a described changes in VSL for people who had just experienced a shock. Because health states are persistent (see Columns (10)–(11) of Table 1), the typical 70-year-old in Figure 5 has not experienced a recent health shock. For individuals who survive sufficiently long, an adverse shock to longevity must eventually reduce VSL, relative to no shock, because it causes them to spend down their wealth more quickly.

our model. The values are inversely related to life expectancy in the sick state because it is more valuable in absolute terms to reduce the risk of a severe illness than a mild one. A marginal reduction in the risk of transitioning to the worst health state, where quality-adjusted life expectancy is 2.6 QALYs, is worth about \$2.1 million. VSL, which is a special case of VSI where life expectancy is 0 years in the sick state, is \$2.8 million.

The solid red line in Figure 6 reports VSI per QALY. The negative slope indicates that the values assigned to reducing the risk of more severe diseases are higher. For instance, reducing the risk of death (\$260,000 per QALY) is worth 10 percent more per QALY than reducing the risk of transitioning to health state 2 (\$237,000 per QALY), the mildest possible illness in our model (life expectancy of 9.7 QALYs). Notably, the solid red line in Figure 6 is positioned lower and features a flatter slope than the solid red line in Figure 5. This difference indicates that while consumers place significantly less value on QALYs gained in healthier states, the value of a QALY is relatively insensitive to the severity of the illness prevented.

Figure 7 shows how different utility function parameterizations and the presence of a bequest motive affect our estimates. Setting $\gamma = 1.5$, which makes demand for current consumption more inelastic, flattens the life-cycle consumption profile and increases the value of a QALY. Setting $\gamma = 0.8$, by contrast, pulls consumption forward in time and reduces the value of life-extension because consumption at early ages provide a good substitute for consumption at later ages. A bequest motive encourages individuals to delay consumption, because money saved for consumption in old age has the added benefit of increasing bequests in the event of death (Figure 7a). Likewise, it reduces the value of life-extension because death is less costly (Figure 7b).

Finally, Figure 8 shows what happens to our estimates when we omit quality of life or health-related financial shocks from our model.²⁴ Omitting financial shocks accentuates the increase in consumption and VSL following an adverse health shock (Panels a–b), and widens the gap in the value of QALY between the sickest and healthiest states (Panel c), but these effects are relatively small in magnitude. Omitting quality of life heightens the sensitivity of the value of a QALY to the severity of the illness (Panel d), but has little effect on other outcomes.

Overall, while some of these alternative specifications produce meaningful shifts in the absolute values of VSL and VSI, they do not affect our qualitative conclusions. In all cases, the value of a QALY is significantly larger for sicker individuals (Figures 7c and 8c) and rises with the severity of illness risk (Figures 7d and 8d).

²⁴Omitting quality of life is equivalent to setting $q = 1$ in Equation (14). Omitting financial losses is equivalent to setting the effective interest rate $r(t, Y_t, Y_{t+1}) = r = 0.03$.

4 Conclusion

The economic theory surrounding the value of life has many important applications. Yet, a number of limitations have surfaced over time. The conventional model does not distinguish between illness and death, and fails to explain several empirical facts, such as the apparent preferences of consumers to pay more for life-extension when survival prospects are bleaker.

Our model overcomes these limitations by allowing for multiple health states. Our framework provides a practical tool for policymakers and health agencies, since patient health varies dramatically across diseases and because many health investments involve preventing the deterioration of health rather than reducing an immediate mortality risk. By providing a closed-form solution, we ease the burden on practitioners, enabling them to easily calculate VSL and VSI in real-world applications. Indeed, our model has meaningful practical implications: Using nationally representative data, we find that the value of a QALY is substantially higher for people in poorer health states.

These findings provide a rational explanation for why many people state preferences for prioritizing the health of the severely ill over other patients and for why it has proven so difficult for policymakers to encourage investments in preventive care ([Linley and Hughes, 2013](#); [Reif et al., 2020](#)). [Kremer and Snyder \(2015\)](#) show that heterogeneity in consumer valuations distorts R&D incentives by allowing firms to extract more consumer surplus from treatments than preventives. Our results suggest that differences in private VSL may reinforce this result and further disadvantage incentives to develop preventives.

Regarding healthcare resource allocation, our results provide support for the notion of a “severity premium” that increases reimbursements for treatments of more severe diseases ([Skedgel et al., 2022](#)). They also provide an additional justification for “top-up” insurance policies: because the ex post willingness to pay for treatment when sick can significantly exceed the ex ante willingness to pay for health insurance coverage when healthy, consumers benefit from a mechanism that permits them to make supplemental purchases in the sick state. Likewise, our results provide a novel rationale for insurers offering health care plans that are more generous alongside those that are less generous. Because US private insurance contracts are typically short-term arrangements, consumers’ health status varies at the time of purchasing insurance. Our results imply that sicker consumers are inclined to buy more generous (and expensive) policies that cover costly treatments, whereas healthier consumers are not, even if the treatments are equally effective for both groups.

Our analysis raises a number of important questions for further research. First, what

are our model's implications for the value of health insurance? While it is clear that willingness to pay for medical treatments may increase following a health shock, it is less clear what the effect on insurance valuations is *ex ante*. Second, what are the most practical strategies for incorporating our insights into the practice of cost-effectiveness? Practitioners have long assumed that a QALY possesses a constant value, even though many people indicate otherwise on surveys ([Linley and Hughes, 2013](#); [Sullivan, 2023](#)). While flawed, this conventional approach is simpler to implement than allowing the value to depend on health histories and illness severity. Translational research should focus on practical strategies for aligning cost-effectiveness practice with the generalized theory of the value of life. Finally, what are the implications for the empirical literature on VSL? Prior studies have assumed that health histories can be ignored when estimating VSL ([Hirth et al., 2000](#); [Mrozek and Taylor, 2002](#); [Viscusi and Aldy, 2003](#)), but more recent research suggests otherwise ([Ketcham et al., 2020](#)). This missing insight may be one reason for the widely disparate empirical estimates of VSL.

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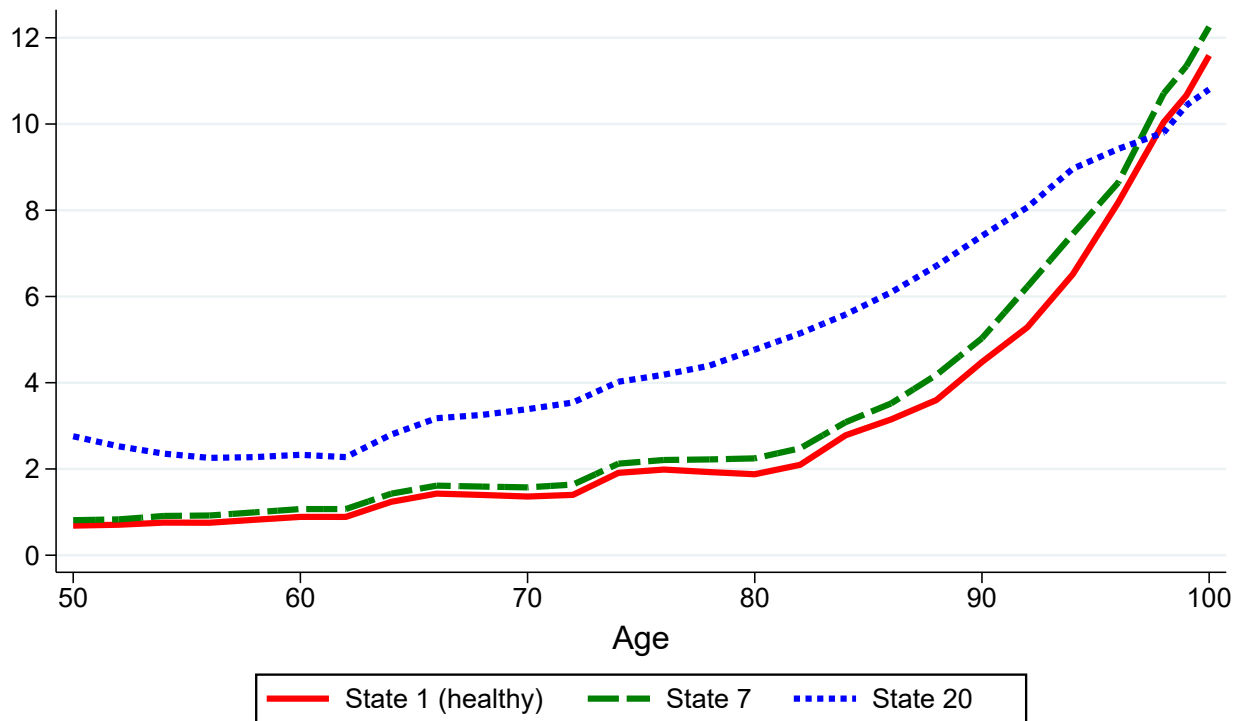
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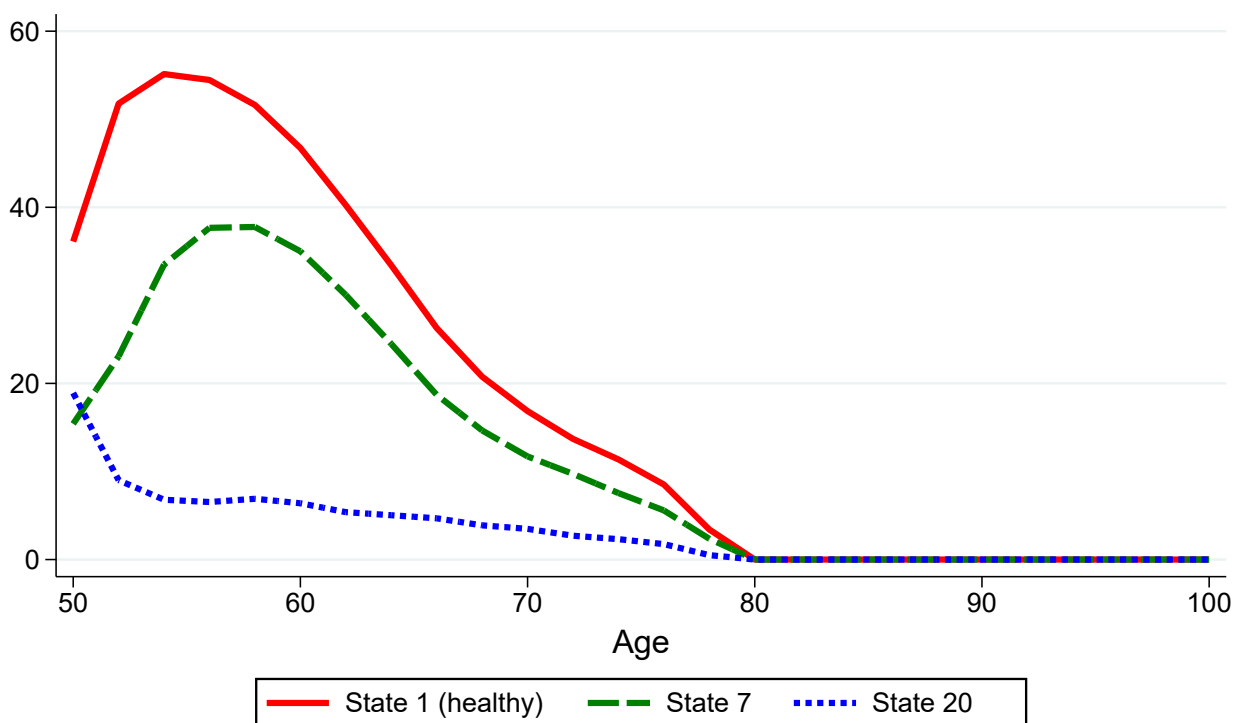
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Figure 1: Average annual medical spending and labor earnings, by age

(a) Out-of-pocket medical spending (thousands of 2010 dollars)

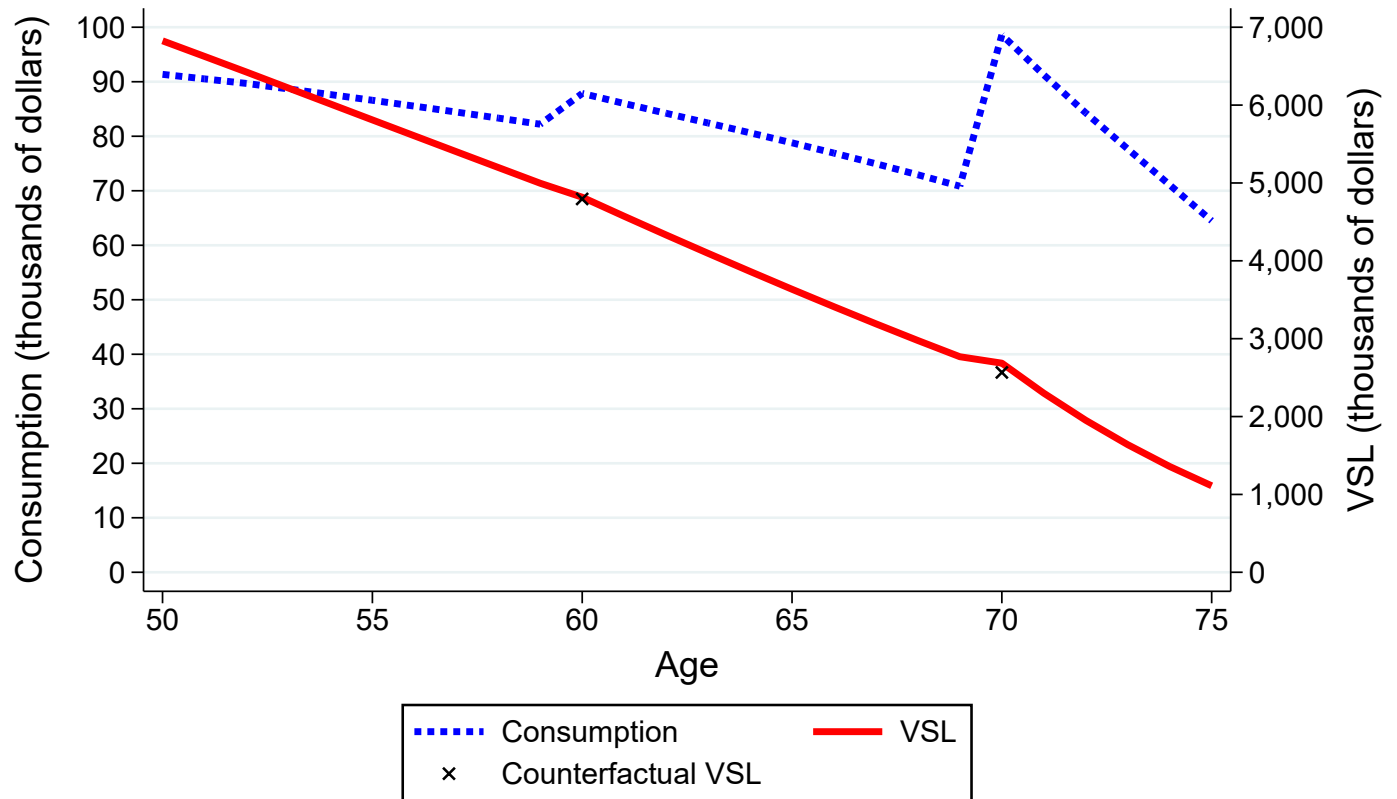


(b) Labor earnings (thousands of 2010 dollars)



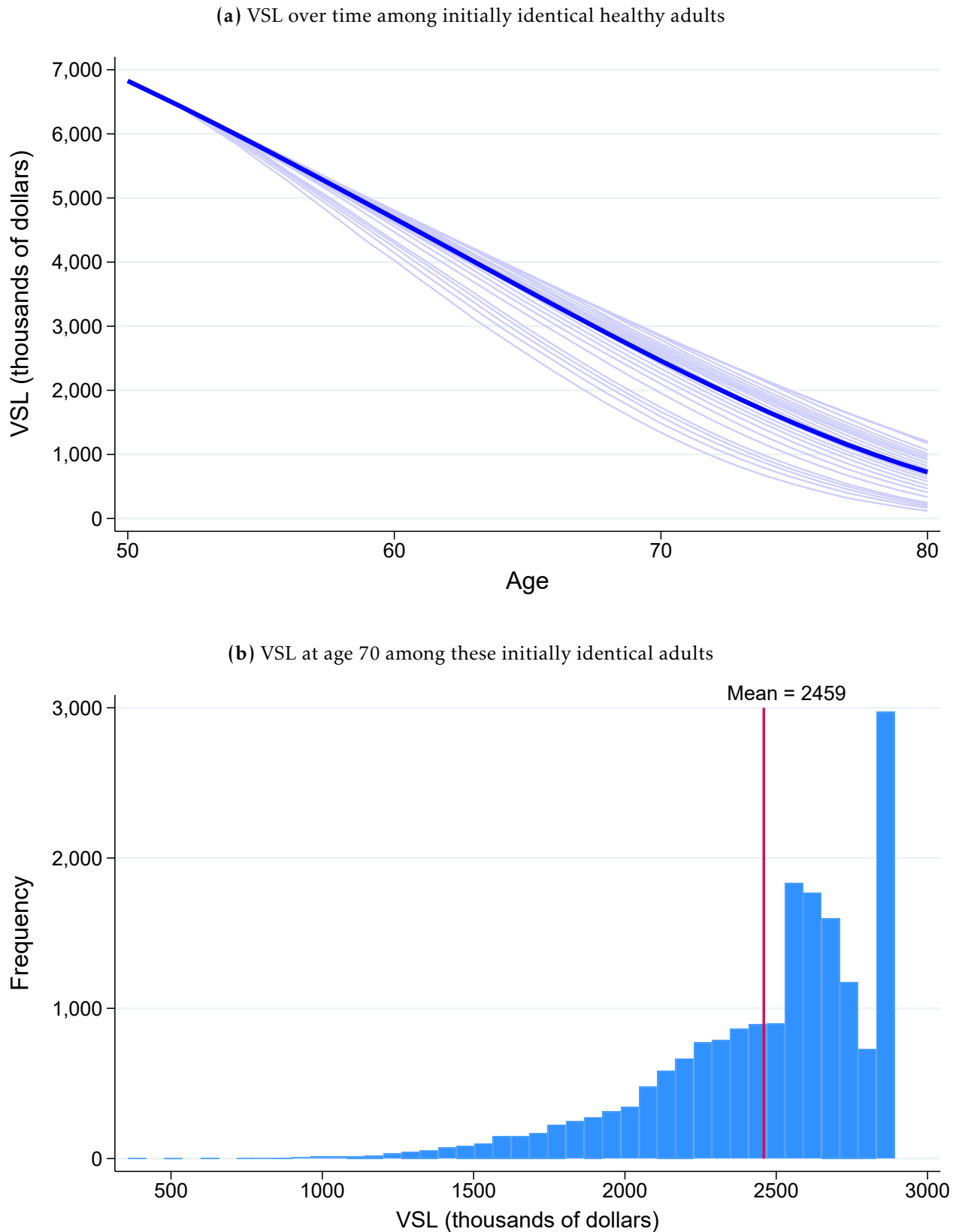
Notes: All estimates are produced by the Future Elderly Model (FEM). Out-of-pocket medical spending includes all inpatient, outpatient, prescription drug, and long-term care spending not paid for by insurance. Health state 1 describes healthy individuals with no impaired activities of daily living (ADL) and no chronic conditions. State 7 describes individuals with 1 impaired activity of daily living and 1 chronic condition. State 20 describes very ill individuals with three or more impaired ADLs and four or more chronic conditions. Additional characteristics for these health states are provided in Table 1.

Figure 2: Consumption and the value of statistical life for an individual who suffers two health shocks



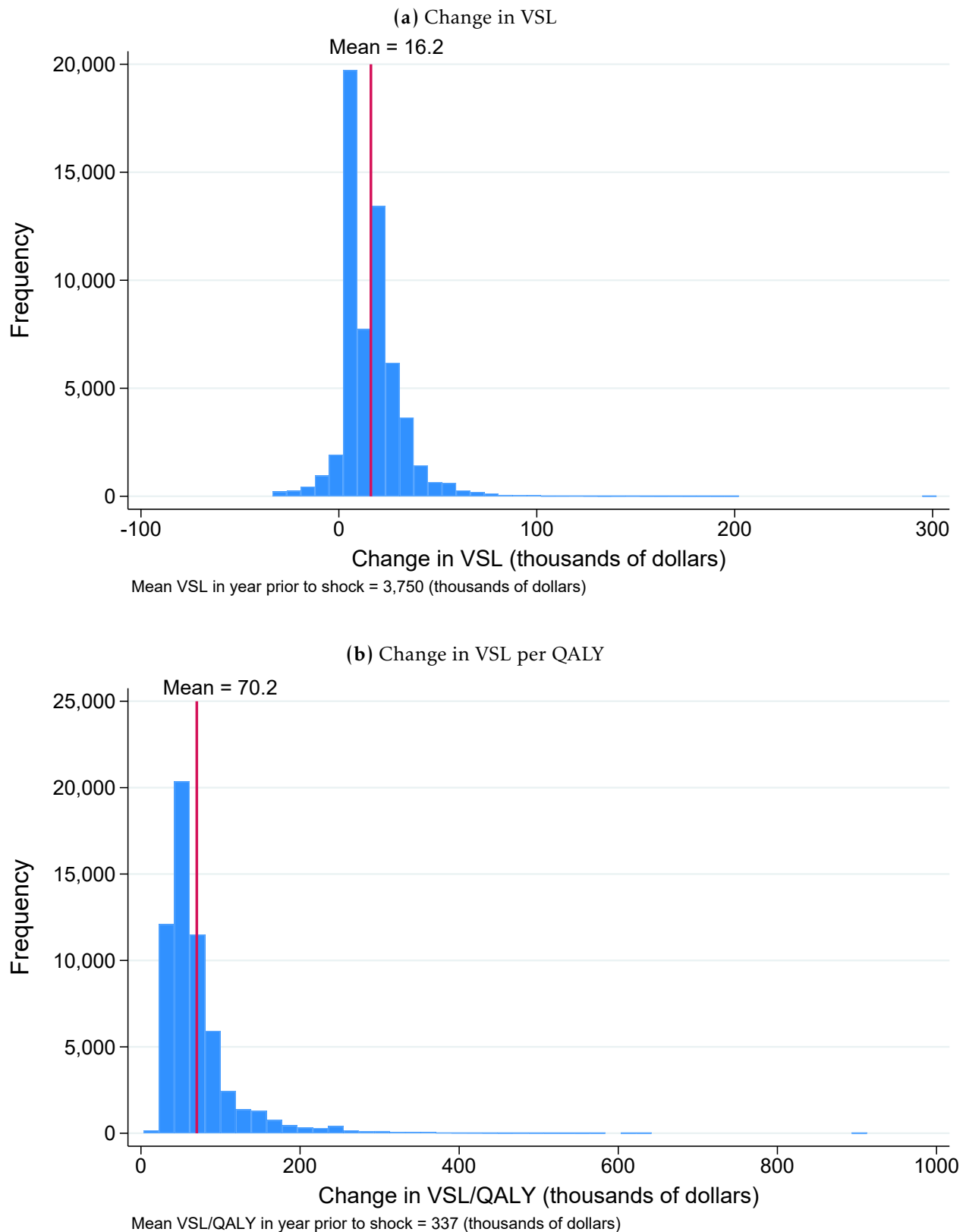
Notes: This figure plots an individual's non-medical consumption (dashed blue line) and value of statistical life (VSL, solid red line) in a setting where mortality, quality of life, labor earnings, and medical spending are stochastic. The individual is healthy at age 50, but then falls ill twice, once at age 60 and then again at age 70. At age 60, the illness impairs one activity of daily living (ADL). At age 70, she is diagnosed with three chronic conditions and one additional impaired ADL. Equivalently, she transitions from state 1 to state 6 at age 60, and then from state 6 to state 14 at age 70 (see Table 1). The individual dies at age 75. The two black X's report VSL for the counterfactual where the individual did not suffer a health shock at age 60 or age 70, respectively. The vertical difference between the black X and the solid red VSL line equals the effect of the health shock on VSL.

Figure 3: Health risk produces significant heterogeneity in the value of statistical life



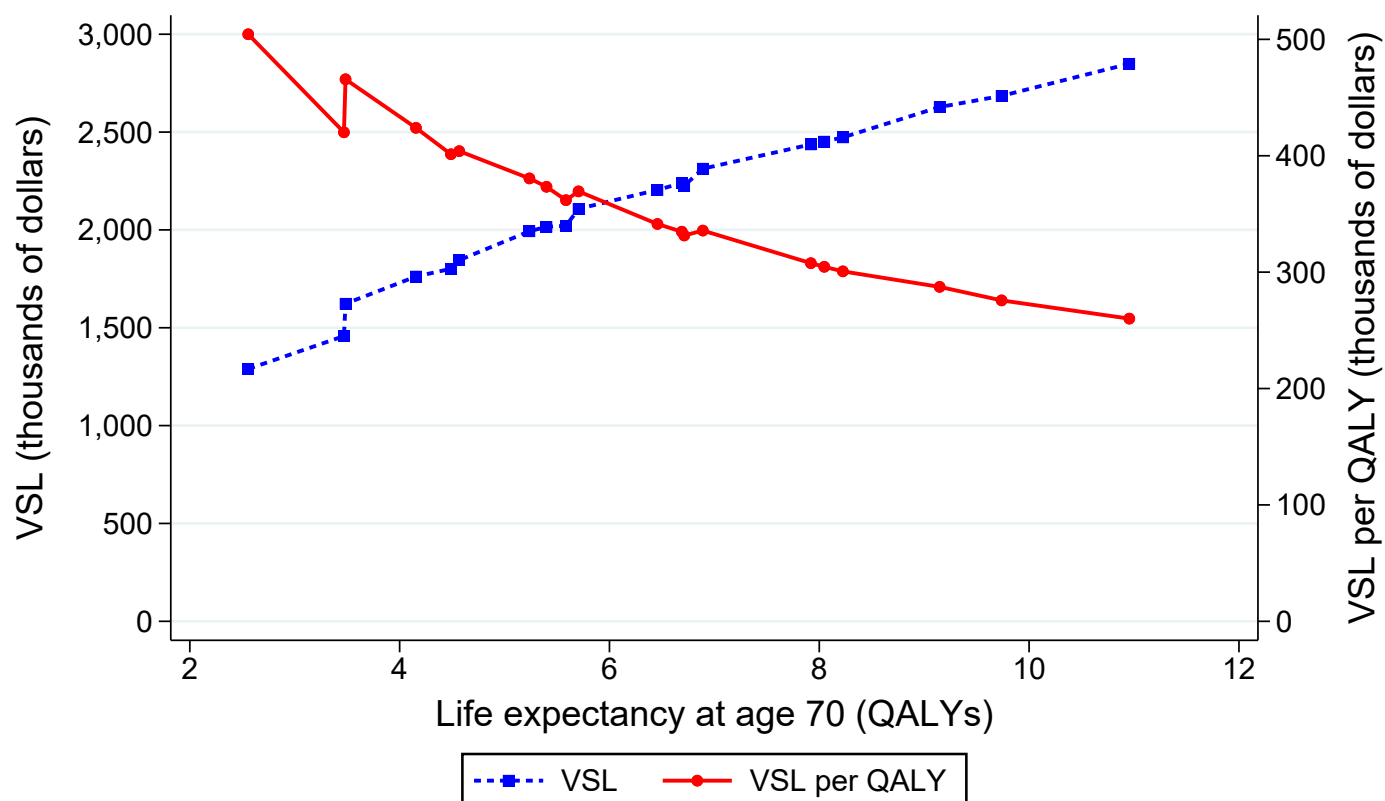
Notes: This figure reports VSL statistics for 22,214 initially identical individuals who at age 50 were all healthy and had the same wealth. These individuals then suffer health shocks as they age. Panel (a) plots the nineteen ventiles and the bottom and top five percentiles of VSL for this population in light blue, and the mean in dark blue. Panel (b) plots the VSL distribution at age 70. Probabilities and characteristics of the health shocks are estimated by the Future Elderly Model (FEM). FEM summary statistics are available in Table 1.

Figure 4: Distribution of changes in VSL following a health shock, ages 50–80



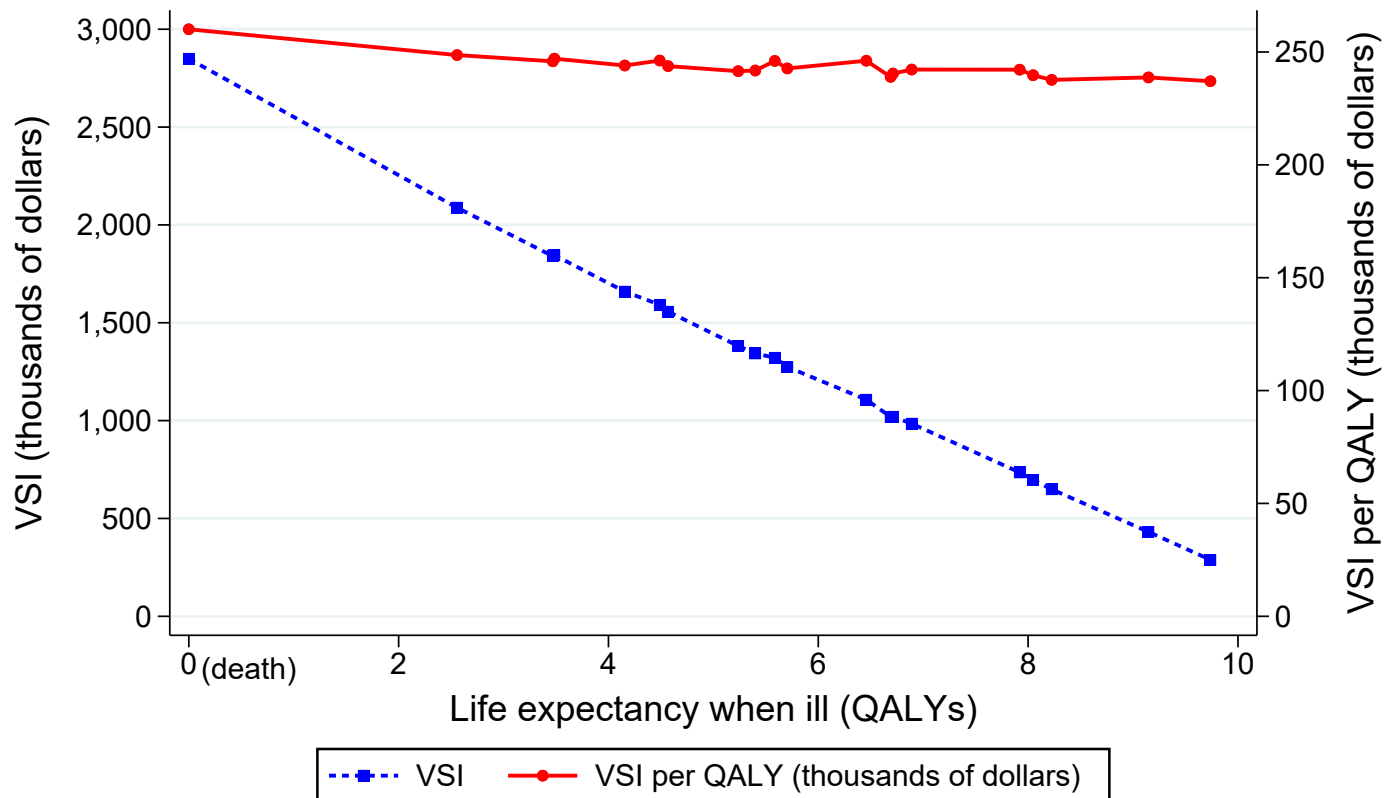
Notes: This figure illustrates the change in VSL following a health shock, relative to a counterfactual with no shock, among the sample of 22,214 initially healthy adults from the Future Elderly Model. These individuals experience 58,008 shocks between the ages of 50–80. Panel (a) plots the distribution of the change in VSL following a health shock. Panel (b) plots the distribution of the change in VSL per quality-adjusted life year (QALY). The vertical red lines report the means of the distributions. QALYs are discounted at a rate of 3 percent. Figure 3a reports how average VSL evolves over the life cycle for this cohort of individuals.

Figure 5: VSL per QALY declines with remaining life expectancy



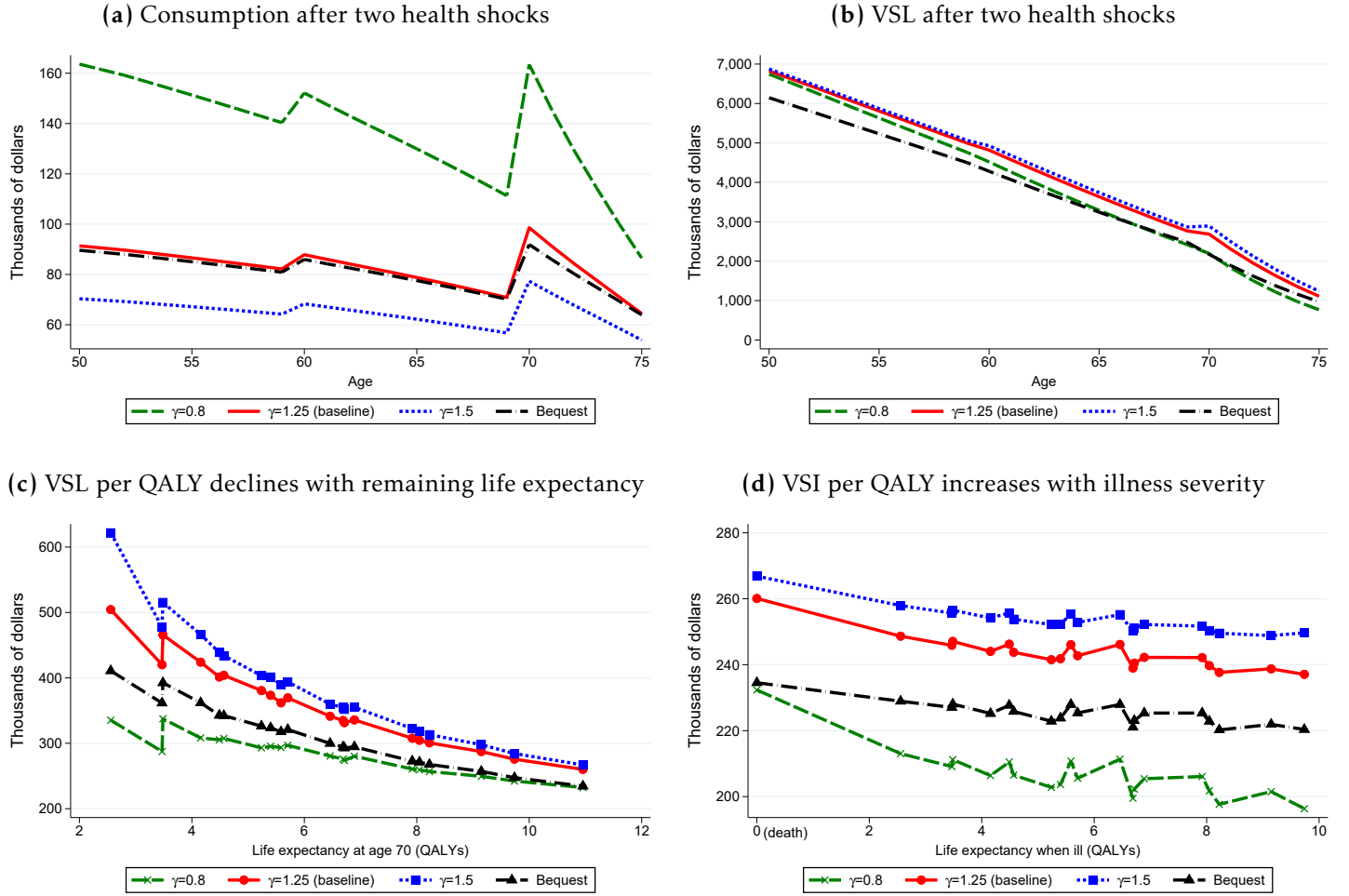
Notes: This figure presents VSL calculations among the sample of 22,214 initially healthy adults from the Future Elderly Model. The dashed blue line reports average VSL at age 70 for each of the 20 health states described in Table 1. The solid red line normalizes that value by the life expectancy for a person in that health state. Life expectancy is measured in units of quality-adjusted life-years (QALYs) and discounted at a rate of 3 percent. The negative slope of the VSL per QALY line indicates that, on average, sick individuals have a higher willingness to pay for a fixed health gain than healthier individuals.

Figure 6: The value of preventing illness at age 70 increases with illness severity



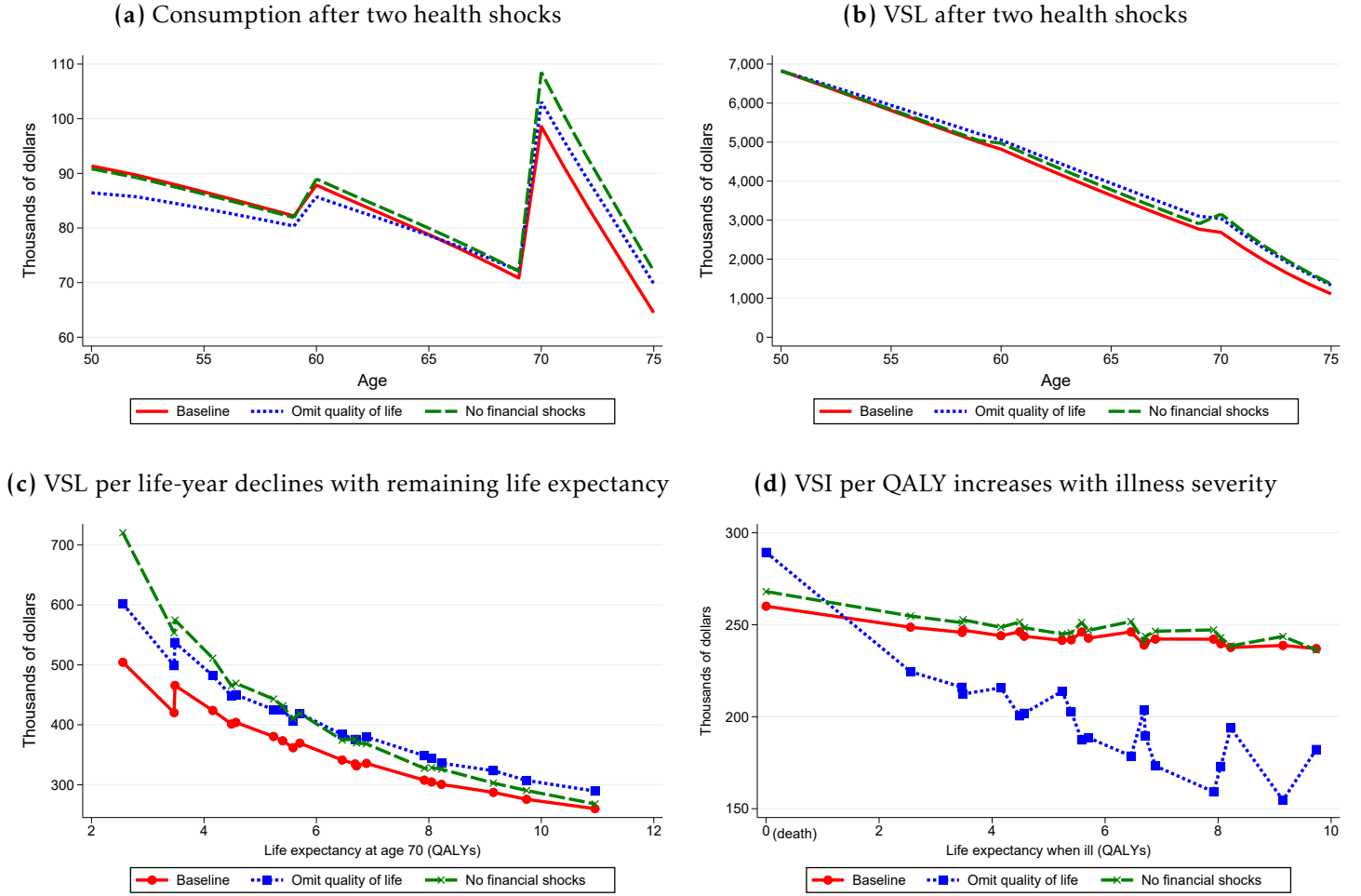
Notes: This figure presents values of statistical illness (VSI) among the sample of 22,214 initially healthy adults from the Future Elderly Model. The dashed blue line reports a healthy (health state 1) 70-year-old's marginal willingness to pay to reduce the risk of different illness, including death (value 0 on the x-axis). The solid red line normalizes that value by the change in life expectancy caused by the illness. Life expectancy is measured in units of quality-adjusted life-years (QALYs) and discounted at a rate of 3 percent. Life expectancy for a 70-year-old in health state 1 is equal to 11.0 QALYs (see Table 1). The negative slope of the VSI per QALY line indicates that individuals are willing to pay more per QALY to reduce severe illness risks than mild illness risks.

Figure 7: Sensitivity of results to different parameterizations of utility and to presence of bequest motive



Notes: The solid red lines in panels (a), (b), (c), and (d) replicate the baseline results from Figure 2 (consumption and VSL), Figure 5, and Figure 6, respectively. The dashed green and dashed blue lines present results under the alternative parameter assumptions $\gamma = 0.8$ and $\gamma = 1.5$, respectively, for the utility function (14). The bequest motive specification, depicted by the black dashed line, is based on Fischer (1973) and sets the bequest motive parameter $b(t) = 1.2$ (see Appendix C). Life expectancy is measured in quality-adjusted life-years (QALYs) and discounted at a rate of 3 percent.

Figure 8: Sensitivity of results to quality of life and financial shocks



Notes: The solid red lines in panels (a), (b), (c), and (d) replicate the baseline results from Figure 2 (consumption and VSL), Figure 5, and Figure 6, respectively. The dashed blue lines present results when omitting quality of life from the model, which is equivalent to setting $q_{Y_t} = q = 1$. The dashed green lines present results when omitting financial losses from health transitions, which is equivalent to setting the effective interest rate $r(t, Y_t, Y_{t+1}) = r = .03$. Life expectancy is measured in units of quality-adjusted life-years (QALYs) and discounted at a rate of 3 percent.

Table 1: Summary means for the Future Elderly Model data, by health state

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
		Life expectancy (years)		Life expectancy (QALYs)		Quality of life (EQ-5D)		Medical spending (2010 \$)		Labor earnings (2010 \$)		Exit probability (%)	
Health state	ADLs / CCs	Age 50	Age 70	Age 50	Age 70	Age 50	Age 70	Age 50	Age 70	Age 50	Age 70	Age 50	Age 70
1 (healthy)	0 / 0	30.9	17.6	16.3	11.0	0.88	0.87	686	1,361	36,111	16,858	4.2	12.6
2	0 / 1	28.2	15.8	14.8	9.7	0.85	0.84	866	1,578	34,517	13,309	3.6	10.8
3	0 / 2	24.6	13.6	12.8	8.2	0.81	0.80	1,145	1,925	32,483	10,875	3.6	10.2
4	0 / 3	20.5	11.2	10.7	6.7	0.77	0.76	1,487	2,366	28,718	8,537	3.9	10.2
5	0 / 4+	16.1	9.0	8.3	5.2	0.73	0.72	2,318	3,193	25,689	6,503	3.9	7.9
6	1 / 0	26.6	15.3	13.5	9.1	0.83	0.82	598	1,378	15,216	15,576	6.3	14.7
7	1 / 1	24.0	13.7	12.1	8.0	0.80	0.78	812	1,573	15,410	11,711	5.7	12.7
8	1 / 2	20.5	11.6	10.2	6.7	0.75	0.75	1,129	1,940	15,766	9,398	6.1	12.2
9	1 / 3	16.8	9.5	8.3	5.4	0.72	0.71	1,394	2,439	22,850	7,033	6.4	11.7
10	1 / 4+	13.2	7.5	6.5	4.2	0.67	0.66	2,098	3,287	11,009	4,904	6.1	8.6
11	2 / 0	24.3	13.8	11.9	7.9	0.78	0.77	585	1,314	14,853	15,383	7.3	14.3
12	2 / 1	21.5	12.3	10.4	6.9	0.75	0.73	797	1,600	12,282	11,488	7.5	14.3
13	2 / 2	18.1	10.4	8.7	5.7	0.71	0.69	1,043	1,934	17,616	8,915	7.5	13.8
14	2 / 3	15.0	8.5	7.1	4.6	0.67	0.66	1,348	2,412	10,848	6,507	7.5	13.1
15	2 / 4+	11.5	6.7	5.4	3.5	0.63	0.61	1,997	3,322	8,375	4,111	7.3	10.6
16	3+ / 0	21.9	11.8	10.3	6.5	0.70	0.69	693	1,358	9,369	13,618	3.4	11.1
17	3+ / 1	19.0	10.4	8.9	5.6	0.66	0.66	948	1,567	12,421	10,821	2.8	8.5
18	3+ / 2	15.7	8.6	7.3	4.5	0.62	0.62	1,105	1,965	10,130	7,604	2.3	7.1
19	3+ / 3	12.7	6.9	5.8	3.5	0.58	0.58	1,671	2,472	9,044	5,376	1.4	5.3
20	3+ / 4+	9.1	5.3	4.1	2.6	0.54	0.54	2,759	3,388	18,917	3,481	0.0	0.0

Notes: This table reports selected means for the health data obtained from the Future Elderly Model (FEM). Column (1) reports the number of impaired activities of daily living (ADLs) and the number of chronic conditions (CCs), which together define a health state. Columns (2)–(3) report life expectancy in years. Columns (4)–(5) report life expectancy in QALYs, which is calculated using Equation (13) with a 3% discount rate. Columns (6)–(7) report average quality of life as measured by the EQ-5D, where 1 indexes perfect health. Columns (8)–(9) report average annual out-of-pocket medical spending, which includes all inpatient, outpatient, prescription, and long-term care spending not covered by insurance. Columns (10)–(11) report average labor earnings. Columns (12)–(13) report the percentage probability that an individual transitions to a different health state in the following year (excluding death). All impaired ADLs and chronic conditions are permanent, i.e., individuals can transition only to higher-numbered health states. Additional details about the FEM are available in Appendix B.

Online Appendix

“Health Risk and the Value of Life”

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Appendix **A**: Mathematical Proofs

Appendix **B**: Future Elderly Model

Appendix **C**: Supporting Calculations for Quantitative Analysis

A Mathematical Proofs

Proof of Lemma 1. Recall that the transition rates $\lambda_{ij}(t) = 0 \forall j < i$. The optimization problem in state n is therefore the standard problem with a single health state. We can contemplate a successive solution strategy by starting in state n and then moving sequentially to state $n - 1$, $n - 2$, etc. Thus, we can consider the deterministic optimization problem for an arbitrary state i by taking $V(t, w, j)$, $j > i$, as given (exogenous):

$$V(0, W_0, i) = \max_{c_i(t)} \left\{ \int_0^T e^{-\rho t} \tilde{S}(i, t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t) \tilde{h}_{ij}, j) \right) dt \right\}$$

subject to:

$$\begin{aligned} \frac{\partial W_i(t)}{\partial t} &= r_i W_i(t) - c_i(t), \\ W_i(0) &= W_0 \end{aligned}$$

Optimal consumption and wealth in state i are denoted by $c_i(t)$ and $W_i(t)$, respectively. Denote the optimal value-to-go function as:

$$\tilde{V}(u, W_i(u), i) = \max_{c_i(t)} \left\{ \int_u^T e^{-\rho t} \tilde{S}(i, t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t) \tilde{h}_{ij}, j) \right) dt \right\}$$

Setting $\tilde{V}(t, W_i(t), i) = e^{-\rho t} \tilde{S}(i, t) V(t, W_i(t), i)$ then demonstrates that $V(\cdot)$ satisfies the HJB (3) for i . See Theorem 1 and the proof of Theorem 2 in [Parpas and Webster \(2013\)](#) for additional details and intuition behind this result. ■

Proof of Lemma 2. From (5), the marginal utility of preventing an illness or death is:

$$\begin{aligned} \left. \frac{\partial V}{\partial \varepsilon} \right|_{\varepsilon=0} &= \left. \frac{\partial}{\partial \varepsilon} \int_0^T e^{-\rho t} \exp \left\{ - \int_0^t \sum_{j>i} (\lambda_{ij}(s) - \varepsilon \delta_{ij}(s)) ds \right\} \left(u(c_i^\varepsilon(t), q_i(t)) + \sum_{j>i} (\lambda_{ij}(t) - \varepsilon \delta_{ij}(t)) V(t, W_i^\varepsilon(t) \tilde{h}_{ij}, j) \right) dt \right|_{\varepsilon=0} \\ &= \int_0^T e^{-\rho t} \tilde{S}(i, t) \left[\left(\int_0^t \sum_{j>i} \delta_{ij}(s) ds \right) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t) \tilde{h}_{ij}, j) \right) - \sum_{j>i} \delta_{ij}(t) V(t, W_i(t) \tilde{h}_{ij}, j) \right] dt \\ &\quad + \int_0^T e^{-\rho t} \tilde{S}(i, t) \left(u_c(c_i^\varepsilon(t), q_i(t)) \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} + \sum_{j>i} \lambda_{ij}(t) \tilde{h}_{ij} \frac{\partial V(t, W_i(t) \tilde{h}_{ij}, j)}{\partial w} \frac{\partial W_i^\varepsilon(t)}{\partial \varepsilon} \right) dt \Big|_{\varepsilon=0} \end{aligned}$$

where $c_i^\varepsilon(t)$ and $W_i^\varepsilon(t)$ represent the equilibrium variations in $c_i(t)$ and $W_i(t)$ caused by this perturbation.

We conclude the proof by showing that the second term in the last equality is equal to 0. Note that along this path, wealth at time t is equal to:

$$W_i(t) = W_0 e^{r_i t} - \int_0^t e^{r_i(t-s)} c_i(s) ds$$

which implies $\frac{\partial W_i^\varepsilon(t)}{\partial \varepsilon} = - \int_0^t e^{r_i(t-s)} \frac{\partial c_i^\varepsilon(s)}{\partial \varepsilon} ds$. From the solution to the costate equation, we know that:

$$e^{-\rho t} \tilde{S}(i, t) u_c(c_i(t), q_i(t)) = \left[\int_t^T e^{(r_i - \rho)s} \tilde{S}(i, s) \sum_{j>i} \lambda_{ij}(s) \tilde{h}_{ij} \frac{\partial V(s, W_i(s) \tilde{h}_{ij}, j)}{\partial w} ds \right] e^{-r_i t} + \theta^{(i)} e^{-r_i t}$$

Thus, we can rewrite the second term in the expression for $\frac{\partial V}{\partial \varepsilon} \Big|_{\varepsilon=0}$ above as:

$$\begin{aligned}
& \int_0^T \left[\int_t^T e^{(r_i - \rho)s} \tilde{S}(i, s) \sum_{j>i} \lambda_{ij}(s) \tilde{h}_{ij} \frac{\partial V(s, W_i(s) \tilde{h}_{ij}, j)}{\partial w} ds + \theta(i) \right] e^{-r_i t} \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} dt - \int_0^T e^{-\rho t} \tilde{S}(i, t) \sum_{j>i} \lambda_{ij}(t) \tilde{h}_{ij} \frac{\partial V(t, W_i(t) \tilde{h}_{ij}, j)}{\partial w} \int_0^t e^{r_i(t-s)} \frac{\partial c_i^\varepsilon(s)}{\partial \varepsilon} ds dt \Big|_{\varepsilon=0} \\
&= \int_0^T \left[\int_t^T e^{(r_i - \rho)s} \tilde{S}(i, s) \sum_{j>i} \lambda_{ij}(s) \tilde{h}_{ij} \frac{\partial V(s, W_i(s) \tilde{h}_{ij}, j)}{\partial w} ds \right] e^{-r_i t} \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} dt - \int_0^T \left[\int_t^T e^{(r_i - \rho)s} \tilde{S}(i, s) \sum_{j>i} \lambda_{ij}(s) \tilde{h}_{ij} \frac{\partial V(s, W_i(s) \tilde{h}_{ij}, j)}{\partial w} ds \right] e^{-r_i t} \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} dt + \int_0^T \theta(i) e^{-r_i t} \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} dt \Big|_{\varepsilon=0} \\
&= \theta(i) \frac{\partial}{\partial \varepsilon} \underbrace{\int_0^T e^{-r_i t} c_i^\varepsilon(t) dt}_{W_0} \Big|_{\varepsilon=0} = 0
\end{aligned}$$

where the last equality follows from application of the budget constraint. ■

Proof of Proposition 3. Choosing the Dirac delta function for $\delta(t)$ in Lemma 2 yields:

$$\begin{aligned}
\frac{\partial V}{\partial \varepsilon} \Big|_{\varepsilon=0} &= \int_0^T \left[e^{-\rho t} \tilde{S}(i, t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) \right] dt \\
&= \mathbb{E} \left[\int_0^T e^{-\rho t} S(t) u(c(t), q_{Y_t}(t)) dt \Big| Y_0 = i, W(0) = W_0 \right]
\end{aligned}$$

Dividing the result by the marginal utility of wealth at time $t = 0$, given by Equation (4), then yields the expression for VSL given by Equation (9):

$$VSL(i) = \mathbb{E} \left[\int_0^T e^{-\rho t} S(t) \frac{u(c(t), q_{Y_t}(t))}{u_c(c(0), q_{Y_0}(0))} dt \Big| Y_0 = i, W(0) = W_0 \right]$$
■

Proof of Proposition 4. Since the characteristics of state 1 and state 2 are identical and a transition after time t is no longer possible, the value functions and consumption levels for these two states are identical at time τ , holding wealth constant: $V(\tau, w, 1) = V(\tau, w, 2)$, $c(\tau, w, 1) = c(\tau, w, 2) \forall w, \tau \geq t$. Since a transition from state 1 to state 2 at time t decreases wealth:

$$W(t) = W(t-)(1 - h_{1,2}) < W(t-)$$

it suffices to show that VSL at time t :

$$\frac{V(t, W, 1)}{u_c(c(t, W, 1), q_1(t))}$$

is increasing in wealth. Taking derivatives, we obtain:

$$\frac{\partial}{\partial W} \frac{V(t, W, 1)}{u_c(c(t, W, 1), q_1(t))} = \frac{V_w(t, W, 1) u_c(c(t, W, 1), q_1(t)) - V(t, W, 1) u_{cc}(c(t, W, 1), q_1(t)) \frac{d}{dw} c(t, W, 1)}{(u_c(c(t, W, 1), q_1(t)))^2} > 0$$

where the inequality follows since $V_w, u_c, \frac{d}{dw} c(t, W, 1) > 0$ and $u_{cc} < 0$. ■

Proof of Proposition 5. Without loss of generality, we will prove the proposition for the case where the consumer transitions from state 1 to state 2 at time $t = 0$. Let $c_1(t)$ denote consumption in state 1 and $c_2(t)$ denote consumption in state 2 after the transition. The interest rates in states 1 and 2 are denoted as r_1 and r_2 , respectively. By assumption, we have $r_1 > r_2$.

Suppose that initial consumption in state 1 is not lower than consumption in state 2: $c_1(0) \geq c_2(0)$. By assumption, the two states have identical probabilities of dying ($\lambda_{13}(s) = \lambda_{23}(s) \forall s$), and there are no transitions from state 1 to state 2 after

time $t > 0$ ($\lambda_{12}(t) = 0 \forall t > 0$). Thus, the first-order condition (7) implies:

$$\frac{p_t^{(1)}}{p_t^{(2)}} = \frac{u_c(c_1(t), q_1(t))}{u_c(c_2(t), q_2(t))} = \frac{\theta^{(1)}}{\theta^{(2)}} e^{(r_2 - r_1)t} = \frac{u_c(c_1(0), q_1(0))}{u_c(c_2(0), q_2(0))} e^{(r_2 - r_1)t} < 1$$

where $\theta^{(i)} > 0$ is a constant obtained from the solution to the costate equation in a two-state setting with $\lambda_{12}(t) = 0 \forall t > 0$. The final inequality follows because $u_{cc}(\cdot) < 0$, $q_1(0) = q_2(0)$, and $r_1 > r_2$. Thus, since $q_1(t) = q_2(t)$, we have that $c_1(t) > c_2(t) \forall t > 0$. Because utility is increasing in consumption, that implies:

$$VSL(1) = \frac{V(0, W_0, 1)}{u_c(c_1(0), q_1(0))} > \frac{V(0, W_0, 2)}{u_c(c_2(0), q_2(0))} = VSL(2)$$

Therefore, it suffices to prove that $c_1(0) \geq c_2(0)$.

Suppose by way of contradiction that $c_1(0) < c_2(0)$, so that $\theta^{(2)} < \theta^{(1)}$, where $\theta^{(i)} > 0$ is again a constant obtained from the solution to the costate equation. Consider the following consumption plan:

$$c_1^*(t) = u_c^{-1} \left[\theta^{(2)} e^{-r_1 t} e^{\rho t} e^{\int_0^t \lambda_{13}(s) ds}, q_1(t) \right] > u_c^{-1} \left[\theta^{(1)} e^{-r_1 t} e^{\rho t} e^{\int_0^t \lambda_{13}(s) ds}, q_1(t) \right] = c_1(t)$$

where the inequality follows because $u_{cc}(\cdot) < 0$, and hence $u_{cc}^{-1}(\cdot) < 0$. We will show that $\{c_1^*(t)\}$ satisfies the budget constraint, which contradicts the optimality of the plan $\{c_1(t)\}$.

Consider the function:

$$g(x) = x \cdot u_c^{-1} \left[\theta^{(2)} x e^{\rho t} e^{\int_0^t \lambda_{13}(s) ds}, q(t) \right]$$

Then:

$$\begin{aligned} g'(x) &= u_c^{-1} \left[\theta^{(2)} x e^{\rho t} e^{\int_0^t \lambda_{13}(s) ds}, q(t) \right] - \frac{x \cdot \theta^{(2)} e^{\rho t} e^{\int_0^t \lambda_{13}(s) ds}}{-u_{cc} \left[u_c^{-1} \left[\theta^{(2)} x e^{\rho t} e^{\int_0^t \lambda_{13}(s) ds}, q(t) \right], q(t) \right]} \\ &= \underbrace{u_c^{-1} \left[\theta^{(2)} x e^{\rho t} e^{\int_0^t \lambda_{13}(s) ds}, q(t) \right]}_y - \frac{u_c[y, q(t)]}{-u_{cc}[y, q(t)] y} y = y(1 - \sigma) \\ &\geq 0 \end{aligned}$$

where the last inequality follows because we have assumed utility satisfies the condition $\sigma < 1$. Thus:

$$\int_0^T e^{-r_1 t} c_1^*(t) dt = \int_0^T g(e^{-r_1 t}) dt \leq \int_0^T g(e^{-r_2 t}) dt = \int_0^T e^{-r_2 t} c_2(t) dt = W_0$$

where the last equality follows because $\{c_2(t)\}$ satisfies the budget constraint. Thus, $\{c_1^*(t)\}$ also satisfies the budget constraint, which yields the contradiction. ■

Proof of Proposition 6. Without loss of generality, we will prove the proposition for the case where the consumer transitions from state 1 to state 2 at time $t = 0$. Because we hold quality of life constant, we omit $q_i(t)$ in the notation below in order to keep the presentation concise.

We want to prove that $c_2(0) > c_1(0)$. Assume by way of contradiction that $c_2(0) \leq c_1(0)$. We will show that this assumption implies $c_2(t) < c_1(t)$ for all $t > 0$, which is a contradiction since the feasible consumption plan $c_1(\cdot)$ dominates $c_2(\cdot)$.

We proceed by inductively constructing a sequence $0 < t_1 < t_2 \dots$ where for each element in the sequence:

$$\begin{aligned} c_2(t_i) &< c_1(t_i) \\ W_1(t_i) &< W_2(t_i) \\ p_{t_i}^{(1)} &\leq \exp\left\{-\int_0^{t_i} \lambda_{12}(s)ds\right\} p_{t_i}^{(2)} \end{aligned}$$

To construct the sequence, for the base case $i = 1$, we first note that from the first-order condition (7), we obtain:

$$p_0^{(1)} = u_c(c_1(0)) \leq u_c(c_2(0)) = p_0^{(2)}$$

The costate equation (6) then implies:

$$\begin{aligned} \dot{p}_0^{(1)} &= -p_0^{(1)} r - \lambda_{12}(0) u_c(c_2(0)) \\ &= -p_0^{(1)} \left[r + \lambda_{12}(0) \underbrace{\frac{u_c(c_2(0))}{u_c(c_1(0))}}_{\geq 1} \right] \\ &\leq -p_0^{(1)} [r + \lambda_{12}(0)] = \left. \frac{\partial g(t)}{\partial t} \right|_{t=0} \end{aligned}$$

where $g(t) = p_0^{(1)} \exp\left\{-\int_0^t r + \lambda_{12}(s)ds\right\}$. Hence, there exists a $t_1 > t_0 = 0$ such that:

$$p_t^{(1)} \leq g(t) \leq p_0^{(2)} \exp\left\{-\int_0^t (r + \lambda_{12}(s))ds\right\} = p_t^{(2)} \exp\left\{-\int_0^t \lambda_{12}(s)ds\right\}, \quad 0 \leq t \leq t_1$$

which together with the first-order condition (7) implies:

$$e^{-\rho t} \exp\left\{-\int_0^t (\lambda_{12}(s) + \lambda_{13}(s))ds\right\} u_c(c_1(t)) \leq e^{-\rho t} \exp\left\{-\int_0^t (\lambda_{12}(s) + \lambda_{23}(s))ds\right\} u_c(c_2(t)), \quad 0 \leq t \leq t_1$$

so that $c_1(t) > c_2(t)$, $0 < t \leq t_1$. The first inequality in turn implies $W_1(t_1) < W_2(t_1)$.

For the induction step, suppose that the following properties also hold for $i \geq 1$:

$$\begin{aligned} c_2(t_i) &< c_1(t_i) \\ W_1(t_i) &< W_2(t_i) \\ p_{t_i}^{(1)} &\leq \exp\left\{-\int_0^{t_i} \lambda_{12}(s)ds\right\} p_{t_i}^{(2)} \end{aligned}$$

The induction hypothesis implies:

$$c(t_i, W_1(t_i), 2) \leq c(t_i, W_2(t_i), 2) = c_2(t_i) < c_1(t_i)$$

so that:

$$\begin{aligned}
\dot{p}_{t_i}^{(1)} &= -p_{t_i}^{(1)} r - e^{-\rho t_i} \tilde{S}(1, t_i) \lambda_{12}(t_i) u_c(c(t_i, W_1(t_i), 2)) \\
&= -p_{t_i}^{(1)} \left[r + \lambda_{12}(t_i) \underbrace{\frac{u_c(c(t_i, W_1(t_i), 2))}{u_c(c_1(t_i))}}_{>1} \right] \\
&< -p_{t_i}^{(1)} [r + \lambda_{12}(t_i)] = \frac{\partial \tilde{g}(t)}{\partial t} \Big|_{t=0}
\end{aligned}$$

with $\tilde{g}(t) = p_{t_i}^{(1)} \exp\left\{-\int_{t_i}^t (r + \lambda_{12}(s)) ds\right\}$. Hence, there exists a $t_{i+1} > t_i$ such that:

$$\begin{aligned}
p_t^{(1)} &\leq \tilde{g}(t) \\
&\leq \exp\left\{-\int_0^{t_i} \lambda_{12}(s) ds\right\} p_{t_i}^{(2)} \exp\left\{-\int_{t_i}^t (r + \lambda_{12}(s)) ds\right\} \\
&= p_t^{(2)} \exp\left\{-\int_0^t \lambda_{12}(s) ds\right\}, \quad t_i \leq t \leq t_{i+1}
\end{aligned}$$

Applying again the first-order condition (7) for all $t_i \leq t \leq t_{i+1}$ yields:

$$\exp\left\{-\int_0^t (\lambda_{12}(s) + \lambda_{13}(s)) ds\right\} u_c(c_1(t)) \leq \exp\left\{-\int_0^t (\lambda_{12}(s) + \lambda_{23}(s)) ds\right\} u_c(c_2(t))$$

which in turn implies $u_c(c_1(t)) < u_c(c_2(t))$ and $c_2(t) < c_1(t)$. Once again, this inequality implies $W_1(t_{i+1}) < W_2(t_{i+1})$.

Thus, we have proven the existence of the sequence. We then obtain $c_2(t) < c_1(t)$, $t > 0$, by noting that $\{t_i\}_{i \geq 0}$ strictly increases due to the uniformly boundedness condition on $\lambda_{12}(t)$, which is the desired contradiction.

We note that this proof implies that the consumption paths $c_1(t)$ and $c_2(t)$ cross (at most) once. As soon as $c_1(t)$ exceeds $c_2(t)$ for some time t_0 , $c_1(t)$ will exceed $c_2(t)$ for $t > t_0$. However, we have that $c_2(t)$ exceeds $c_1(t)$ prior to t_0 . In particular, consumption jumps up upon transition at time zero. ■

For the proof of Proposition 7, we need the following two lemmas (Lemma A.1 and A.2). Since these lemmas hold in more general situations than the assumptions of Proposition 7 and may be of independent interest, we present them separately.

Lemma A.1. *Consider a setting without health shocks, i.e., $h_{ij} \equiv 0 \forall i, j$. Then the marginal utility of wealth in state i is equal to:*

$$\begin{aligned}
\frac{\partial V(0, W_0, i)}{\partial W_0} &= u_c(c_i(0), q_i(0)) \\
&= \mathbb{E} \left[e^{\int_0^t (r_{Y_s} - \rho) ds} \exp\left\{-\int_0^t \mu(s) ds\right\} u_c(c(t, W(t), Y_t), q_{Y_t}(t)) \Big| Y_0 = i, W(0) = W_0 \right], \forall t > 0
\end{aligned}$$

Proof of Lemma A.1. The first equality in Lemma A.1 follows immediately from the first-order condition in state i in the HJB (3) (see Equation (4)). Our proof derives the second equality, which shows that the consumer sets the expected discounted marginal utility of consumption at time t equal to the current marginal utility of wealth. This result is the stochastic analogue of the first-order condition from a conventional (deterministic health risk) model.

We show the result at an arbitrary time t and a future time $\tau > t$:

$$\frac{\partial V(t, W_i(t), i)}{\partial W_i(t)} = u_c(c_i(t), q_i(t)) = \mathbb{E} \left[e^{\int_t^\tau (r_{Y_s} - \rho) ds} \exp \left\{ - \int_t^\tau \mu(s) ds \right\} u_c(c(\tau, W(\tau), Y_\tau), q_{Y_\tau}(\tau)) \right] \Big| Y_t = i, W(t) = W_i(t), \forall \tau > t$$

The proof proceeds by induction on $i \leq n$. For the base case $i = n$, in which no state transitions are possible, the solution to the costate equation (6) simplifies to:

$$\begin{aligned} p_\tau^{(n)} &= \theta^{(n)} e^{-r_n \tau} \\ &= \exp \left\{ - \int_0^\tau \rho + \lambda_{n,n+1}(s) ds \right\} u_c(c_n(\tau), q_n(\tau)) \\ &= \theta^{(n)} e^{-r_n t} e^{-r_n(\tau-t)} \\ &= p_t^{(n)} e^{-r_n(\tau-t)} \\ &= \exp \left\{ - \int_0^t \rho + \lambda_{n,n+1}(s) ds \right\} u_c(c_n(t), q_n(t)) e^{-r_n(\tau-t)} \end{aligned}$$

where the second equality makes use of the first-order condition (7). Using the expressions in the second and the last lines then gives:

$$u_c(c_n(t), q_n(t)) = e^{r_n(\tau-t)} e^{-\rho(\tau-t)} \exp \left\{ - \int_t^\tau \lambda_{n,n+1}(s) ds \right\} u_c(c_n(\tau), q_n(\tau))$$

which shows that the result holds for $i = n$.

For the induction step, suppose the lemma is true for $j > i$, $1 \leq i \leq n-1$. For any subinterval $[0, \tau]$, the solution of the costate equation can be written as:

$$p_t^{(i)} = \left[\int_t^\tau e^{(r_i - \rho)s} \exp \left\{ - \int_0^s \sum_{j>i} \lambda_{ij}(u) du \right\} \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-r_i t} + \theta(\tau, i) e^{-r_i t} \quad (\text{A.1})$$

where $\theta(\tau, i)$ is a constant that depends on the choice of τ and i . (Take the derivative of $p_t^{(i)}$ with respect to t to verify.) Evaluating Equation (A.1) at $t = \tau$ and combining with Equation (7) from the main text yields:

$$p_\tau^{(i)} = \theta(\tau, i) e^{-r_i \tau} = \exp \left\{ - \int_0^\tau \rho + \sum_{j>i} \lambda_{ij}(s) ds \right\} u_c(c_i(\tau), q_i(\tau))$$

which implies:

$$\theta(\tau, i) = e^{(r_i - \rho)\tau} \exp \left\{ - \int_0^\tau \sum_{j>i} \lambda_{ij}(s) ds \right\} u_c(c_i(\tau), q_i(\tau)) \quad (\text{A.2})$$

Plugging Equations (7) and (A.2) into Equation (A.1) yields:

$$\begin{aligned} u_c(c_i(t), q_i(t)) \exp \left\{ - \int_0^t \rho + \sum_{j>i} \lambda_{ij}(s) ds \right\} &= \left[\int_t^\tau e^{(r_i - \rho)s} \exp \left\{ - \int_0^s \sum_{j>i} \lambda_{ij}(u) du \right\} \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-r_i t} \\ &\quad + e^{-r_i t} e^{(r_i - \rho)\tau} \exp \left\{ - \int_0^\tau \sum_{j>i} \lambda_{ij}(s) ds \right\} u_c(c_i(\tau), q_i(\tau)) \end{aligned}$$

Since $\frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} = u_c(c(s, W_i(s), j), q_j(s))$ from the first-order condition in the HJB for state j , we obtain:

$$\begin{aligned}
u_c(c_i(t), q_i(t)) &= \int_t^\tau e^{(r_i - \rho)(s-t)} \exp \left\{ - \int_t^s \sum_{j>i} \lambda_{ij}(u) du \right\} \sum_{j>i} \lambda_{ij}(s) u_c(c(s, W_i(s), j), q_j(s)) ds + e^{(r_i - \rho)(\tau-t)} \exp \left\{ - \int_t^\tau \sum_{j>i} \lambda_{ij}(s) ds \right\} u_c(c_i(\tau), q_i(\tau)) \\
&= \int_t^\tau e^{(r_i - \rho)(s-t)} \exp \left\{ - \int_t^s \sum_{j>i} \lambda_{ij}(u) du \right\} \sum_{j>i} \lambda_{ij}(s) \mathbb{E} \left[e^{(r_i - \rho)(\tau-s)} \exp \left\{ - \int_s^\tau \mu(s) ds \right\} u_c(c(\tau, W(\tau), Y_\tau), q_{Y_\tau}(\tau)) \middle| Y_s = j, W(s) = W_i(s) \right] ds \\
&\quad + e^{(r_i - \rho)(\tau-t)} \exp \left\{ - \int_t^\tau \sum_{j>i} \lambda_{ij}(s) ds \right\} u_c(c_i(\tau), q_i(\tau)) \\
&= \mathbb{E} \left[e^{\int_t^\tau (r_{Y_s} - \rho) ds} \exp \left\{ - \int_t^\tau \mu(s) ds \right\} u_c(c(\tau, W(\tau), Y_\tau), q_{Y_\tau}(\tau)) \middle| Y_t = i, W(t) = W_i(t) \right]
\end{aligned}$$

where the second equality follows from the induction hypothesis. ■

We can use Lemma A.1 to obtain the following expression for VSL:

Lemma A.2. *Applying the second equality given in Lemma A.1 to Equation (9) and rearranging yields the following, equivalent expression for VSL in state i :*

$$VSL(i) = \int_0^T \tilde{v}(i, t) dt$$

where $\tilde{v}(i, t)$ represents the discounted value of a one-period change in survival from the perspective of current time:

$$\tilde{v}(i, t) = \frac{\mathbb{E} \left[S(t) u(c(t), q_{Y_t}(t)) \middle| Y_0 = i, W(0) = W_0 \right]}{\mathbb{E} \left[e^{\int_0^t r_{Y_s} ds} S(t) u_c(c(t), q_{Y_t}(t)) \middle| Y_0 = i, W(0) = W_0 \right]}$$

Proof of Lemma A.2. Applying Lemma A.1 for $t = 0$ allows us to rewrite VSL as:

$$\begin{aligned}
VSL(i) &= \mathbb{E} \left[\int_0^T e^{-\rho t} \frac{S(t) u(c(t), q_{Y_t}(t))}{\mathbb{E} \left[e^{\int_0^t (r_{Y_s} - \rho) ds} \exp \left\{ - \int_0^t \mu(s) ds \right\} u_c(c(t), q_{Y_t}(t)) \middle| Y_0 = i, W(0) = W_0 \right]} dt \middle| Y_0 = i, W(0) = W_0 \right] \\
&= \mathbb{E} \left[\int_0^T \frac{S(t) u(c(t), q_{Y_t}(t))}{\mathbb{E} \left[e^{\int_0^t r_{Y_s} ds} \exp \left\{ - \int_0^t \mu(s) ds \right\} u_c(c(t), q_{Y_t}(t)) \middle| Y_0 = i, W(0) = W_0 \right]} dt \middle| Y_0 = i, W(0) = W_0 \right]
\end{aligned}$$

Exchanging expectation and integration then yields:

$$VSL(i) = \int_0^T \tilde{v}(i, t) dt$$
■

Proof of Proposition 7. Without loss of generality, consider the case $t = 0$. Applying our assumptions that $r_1 = r_2 \leq \rho$, $q_1(s) = q_2(s) = q \forall s$, $\lambda_{13}(s) < \lambda_{23}(s) \forall s$, and no financial shocks to Equation (11) implies that $c_1(t)$ and $c_2(t)$ are decreasing in t . In addition, from Proposition 6 we have that $c_1(0) < c_2(0)$, $c_1(t) < c_2(t)$ for $t \leq t_0$, and $c_1(t) > c_2(t)$ for $t > t_0$, where t_0 was defined in the proof of Proposition 6.

Applying our assumption that there are no state transitions for $t > 0$ to Lemma A.2 yields:

$$VSL(2, 0) = \int_0^T \frac{S_2(t) u(c_2(t))}{e^{rt} S_2(t) u_c(c_2(t))} dt = \int_0^T e^{-rt} \frac{u(c_2(t))}{u_c(c_2(t))} dt$$

and:

$$VSL(1,0) = \int_0^T e^{-rt} \frac{u(c_1(t))}{u_c(c_1(t))} dt$$

Let $Y(x) \equiv \frac{u(x)}{u_c(x)}$. Under the stated assumptions on preferences, we have that:

$$Y'(x) = 1 - \frac{u(x)u_{cc}(x)}{(u_c(x))^2} > 0,$$

$$Y''(x) = \frac{2(u_{cc}(x))^2 u(x) - u_c^2(x)u_{cc}(x) - u_c(x)u(x)u_{ccc}(x)}{(u_c(x))^3} > 0$$

Employing Taylor's theorem and making use of the assumption that $c(t)$ is weakly declining in t then implies that for some $\xi(t)$ that lies in-between $c_1(t)$ and $c_2(t)$:

$$\begin{aligned} VSL(2,0) &= \int_0^T e^{-rt} Y(c_2(t)) dt \\ &= \int_0^T e^{-rt} \left[Y(c_1(t)) + [c_2(t) - c_1(t)] Y'(c_1(t)) + \underbrace{\frac{1}{2}[c_2(t) - c_1(t)]^2 Y''(\xi(t))}_{>0} \right] dt \\ &> \int_0^T e^{-rt} Y(c_1(t)) dt + \int_0^{t_0} e^{-rt} Y'(c_1(t)) \underbrace{[c_2(t) - c_1(t)]}_{\geq 0} dt + \int_{t_0}^T e^{-rt} Y'(c_1(t)) \underbrace{[c_2(t) - c_1(t)]}_{\leq 0} dt \\ &\geq \int_0^T e^{-rt} Y(c_1(t)) dt + \int_0^{t_0} e^{-rt} Y'(c_1(t_0)) [c_2(t) - c_1(t)] dt + \int_{t_0}^T e^{-rt} Y'(c_1(t_0)) [c_2(t) - c_1(t)] dt \\ &= \int_0^T e^{-rt} Y(c_1(t)) dt + Y'(c_1(t_0)) \underbrace{\left[\int_0^T e^{-rt} c_2(t) dt - \int_0^T e^{-rt} c_1(t) dt \right]}_{=0} \\ &= \int_0^T e^{-rt} Y(c_1(t)) dt \\ &= VSL(1,0) \end{aligned}$$

where the final step follows from the budget constraint. ■

B Future Elderly Model

The empirical exercises presented in Section 3 employ data obtained from the Future Elderly Model (FEM). The FEM is a microsimulation model that projects future health and medical spending for Americans ages 50 and over. It has been used by a variety of researchers and policy analysts to understand the implications of population aging, health trends, new medical technologies, pandemics, and possible health policy interventions in the US, Europe, and Asia (Goldman et al., 2005; Lakdawalla et al., 2005, 2008; Goldman et al., 2009, 2010; Michaud et al., 2011, 2012; Goldman et al., 2013; Goldman and Orszag, 2014; National Academies of Sciences, Engineering, and Medicine, 2015; Chen et al., 2016; Gonzalez-Gonzalez et al., 2017; Leaf et al., 2021; Reif et al., 2021). Detailed technical information about its data sources and methods is included in our Supplementary Materials, and is also available online at:

<https://healthpolicy.usc.edu/future-elderly-model/fem-technical-specifications/>

The FEM has three core modules. The first is the Replenishing Cohorts module, which predicts economic and health outcomes of new cohorts of 50-year-olds using data from the Panel Study of Income Dynamics, and incorporates trends in disease and other outcomes based on data from external sources, such as the National Health Interview Survey and the American Community Survey. This module generates new cohorts as the simulation proceeds, so that we can measure outcomes for the age 50+ population in any given year.

The second component is the Health Transition module, which uses the longitudinal structure of the Health and Retirement Study (HRS) to calculate transition probabilities across various health states, including chronic conditions, functional status, body-mass index, and mortality. These transition probabilities depend on a battery of predictors: age, sex, education, race, ethnicity, smoking behavior, marital status, employment and health conditions. FEM transitions produce a large set of simulated outcomes, including diabetes, high-blood pressure, heart disease, cancer (except skin cancer), stroke or transient ischemic attack, and lung disease (either or both chronic bronchitis and emphysema), disability, and body-mass index. Disability is measured by limitations in instrumental activities of daily living, activities of daily living, and residence in a nursing home.

Finally, the Policy Outcomes module estimates medical spending, including payments made by insurers (Medicare, Medicaid and Private) and out-of-pocket payments made by individuals. Medical spending for an individual is predicted as a function of health status (chronic conditions and functional status), demographics (age, sex, race, ethnicity, and education), nursing home status, and mortality. Estimates are based on spending data from the Medical Expenditure Panel Survey for individuals ages 64 and younger and the Medicare Current Beneficiary Survey for individuals ages 65 and older.

The following example illustrates how the three modules interact. For the year 2014, the model begins with the population of Americans ages 50 and over based on nationally representative data from the HRS. Individual-level health and economic outcomes for the next two years are predicted using the Policy Outcomes module. The cohort is then aged two years using the Health Transition Module. Aggregate health and functional status outcomes for those years are then calculated. At that point, a new cohort of 50-year-olds is introduced into the 2016 population using the Replenishing Cohort module, and they join those who survived from 2014 to 2016. This forms the age 50+ population for 2016. The transition model is then applied to this population. The same process is repeated until reaching the last year of the simulation. For our study, we ran the simulation until the year 2064, which gives us complete life-cycle data for ages 50–100 for all people who were ages 50 and over as of 2014.

The projections produced by the FEM have been extensively validated. Mortality forecasts line up closely with published death counts and achieve lower error rates than alternative forecasts used by the Social Security Administration (Leaf et al., 2021). Population, smoking behavior, cancer, diabetes, heart disease, hypertension, lung disease, and stroke forecasts perform well in cross-validation exercises. Medical spending data have been comprehensively tested against national aggregates.

C Supporting Calculations for Quantitative Analysis

This appendix provides the solution to the discrete-time dynamic programming problem described in Section 3.1. This model is solved analytically and provides exact solutions for optimal consumption.

The consumer's problem is:

$$\max_{c(t)} \mathbb{E} \left[\sum_{t=0}^T e^{-\rho t} S_0(t) u(c(t), q_{Y_t}(t)) + e^{-\rho t} ((S_0(t) - S_0(t+1)) u(W(t) - c(t), b(t))) \right] \Big| Y_0, W_0$$

subject to:

$$\begin{aligned} W(0) &= W_0, \\ W(t) &\geq 0, \\ W(t+1) &= (W(t) - c(t)) e^{r(t, Y_t, Y_{t+1})} \end{aligned}$$

where all variables are defined as in the main text. The strength of the bequest motive is governed by the parameter $b(t)$. We set $b(t) = 0$ in our baseline specification, which assumes no bequest motive (and normalizes utility of death to zero). The utility function is given by Equation (14) from the main text:

$$u(c, q) = q \left(\frac{c^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma} \right)$$

where \underline{c} is the subsistence level of consumption for a healthy person with no bequest motive. Because optimal consumption is unaffected by affine transformations of utility, we shall initially assume $u(c, q) = q c^{1-\gamma} / (1-\gamma)$ when solving the model for consumption.

Define the value function as:

$$V(t, W(t), Y_t) = \max_{c(s)} \mathbb{E} \left[\sum_{s=t}^T e^{-\rho(s-t)} S_t(s) u(c(s), q_{Y_s}(s)) + e^{-\rho(s-t)} (S_t(s) - S_t(s+1)) u(W(s) - c(s), b(s)) \right] \Big| Y_t, W(t)$$

subject to:

$$W(s+1) = (W(s) - c(s)) e^{r(s, Y_s, Y_{s+1})}, \quad s > t, \quad W(s) \geq 0$$

We reformulate this optimization problem as a recursive Bellman equation:

$$V(t, w, i) = \max_{c(t)} \left\{ u(c(t), q_i(t)) + \bar{d}_i(t) u((w - c(t)), b(t)) + e^{-\rho} (1 - \bar{d}_i(t)) \sum_{j=1}^n p_{ij}(t) V(t+1, (w - c(t)) e^{r(t, i, j)}, j) \right\}$$

Proposition C.1. *The value function and the optimal consumption level satisfy:*

$$\begin{aligned} V(t, w, i) &= \frac{w^{1-\gamma}}{1-\gamma} K_{t,i}, \\ c^*(t, w, i) &= w \times c_{t,i} \end{aligned}$$

where $K_{t,i}$ satisfies the recursion:

$$\begin{aligned} K_{t,i} &= \left[q_i(t)^{\frac{1}{\gamma}} + \left[\bar{d}_i(t) b(t) + e^{-\rho} (1 - \bar{d}_i(t)) \left(\sum_{j=1}^n p_{ij}(t) e^{r(t, i, j)(1-\gamma)} K_{t+1, j} \right) \right]^{\frac{1}{\gamma}} \right]^{\gamma}, \quad t < T, \\ K_{T,i} &= \left[q_i(T)^{\frac{1}{\gamma}} + b(T)^{\frac{1}{\gamma}} \right]^{\gamma} \end{aligned}$$

and:

$$\begin{aligned}
c_{t,i} &= \left[1 + \left(\frac{\left[\bar{d}_i(t) b(t) + e^{-\rho} (1 - \bar{d}_i(t)) \left(\sum_{j=1}^n p_{ij}(t) e^{r(t,i,j)(1-\gamma)} K_{t+1,j} \right) \right]}{q_i(t)} \right)^{\frac{1}{\gamma}} \right]^{-1} \\
&= \left[1 + \left(\bar{d}_i(t) \frac{b(t)}{q_i(t)} + e^{-\rho} (1 - \bar{d}_i(t)) \left(\sum_{j=1}^n p_{ij}(t) e^{r(t,i,j)(1-\gamma)} \frac{q_j(t+1)}{q_i(t)} \frac{1}{c_{t+1,j}^\gamma} \right) \right)^{\frac{1}{\gamma}} \right]^{-1}, t < T, \\
c_{T,i} &= \left[1 + \left(\frac{b(t)}{q_i(t)} \right)^{\frac{1}{\gamma}} \right]^{-1}
\end{aligned}$$

Proof. See Appendix C.1 ■

When calculating VSL, we incorporate subsistence consumption back into the utility function. In this case, the value function is:

$$\begin{aligned}
V(0, w, i) &= \sum_{t=0}^T e^{-\rho t} \mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} \left(q_{Y_t}(t) \frac{c(t)^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma} \right) \middle| Y_0 = i, W(0) = w \right] \\
&\quad + e^{-\rho t} \mathbb{E} \left[\left(\exp \left\{ - \int_0^t \mu(s) ds \right\} - \exp \left\{ - \int_0^{t+1} \mu(s) ds \right\} \right) \left(b(t) \frac{(W(t) - c(t))^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma} \right) \middle| Y_0 = i, W(0) = w \right] \quad (C.1)
\end{aligned}$$

Rearranging yields:

$$\begin{aligned}
V(0, w, i) &= \sum_{t=0}^T e^{-\rho t} \mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} q_{Y_t}(t) \frac{c(t)^{1-\gamma}}{1-\gamma} \middle| Y_0 = i, W(0) = w \right] \\
&\quad + e^{-\rho t} b(t) \mathbb{E} \left[\left(\exp \left\{ - \int_0^t \mu(s) ds \right\} - \exp \left\{ - \int_0^{t+1} \mu(s) ds \right\} \right) \frac{(W(t) - c(t))^{1-\gamma}}{1-\gamma} \middle| Y_0 = i, W(0) = w \right] \\
&\quad - \frac{\underline{c}^{1-\gamma}}{1-\gamma} \left[q_{Y_0}(0) + b(0) + \sum_{t=1}^T e^{-\rho t} \mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} (q_{Y_t}(t) + b(t) - e^\rho b(t-1)) \middle| Y_0 = i \right] \right] \\
&= \frac{1}{1-\gamma} \left[w^{1-\gamma} K_{0,i} - \underline{c}^{1-\gamma} \left[q_{Y_0}(0) + b(0) + \sum_{t=1}^T e^{-\rho t} \mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} (q_{Y_t}(t) + b(t) - e^\rho b(t-1)) \middle| Y_0 = i \right] \right] \right]
\end{aligned}$$

We can then calculate VSL in state i using the following formula:

$$VSL(i) = \frac{V(0, w, i) - b(0) \left(\frac{w^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma} \right)}{u_c(w c_{0,i}, q_i(0))} \quad (C.2)$$

The second term in the numerator of (C.2) is the utility at death (the bequest function). When the bequest motive is absent ($b(t) \equiv 0$), the value function simplifies to:

$$V(0, w, i) = \frac{1}{1-\gamma} \left[w^{1-\gamma} K_{0,i} - \underline{c}^{1-\gamma} \underbrace{\sum_{t=0}^T e^{-\rho t} \mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} q_{Y_t}(t) \middle| Y_0 = i \right]}_{\text{discounted quality-adjusted life expectancy in state } i} \right]$$

and the expression for VSL simplifies to Equation (9) from the main text.

Once one has calculated VSL, it is straightforward to calculate VSI:

Corollary C.2. *The value of a marginal reduction in the probability of transitioning from state i to state j is equal to:*

$$VSI(i, j) = VSL(i) - VSL(j) \frac{q_j(0) c_{0,j}^{-\gamma}}{q_i(0) c_{0,i}^{-\gamma}} = VSL(i) - \left(\frac{q_j(0)}{q_i(0)} \right) \left(\frac{c_{0,i}}{c_{0,j}} \right)^{\gamma} VSL(j)$$

Proof. See Appendix C.1 ■

C.1 Proofs

Proof of Proposition C.1. The proof proceeds by induction on $t \leq T$. For the base case $t = T$, note that $\bar{d}_i(t) = 1$, so that the first-order condition from the Bellman equation gives:

$$q_i(T) c(T)^{-\gamma} = b(T) (w - c(T))^{-\gamma}$$

Rearranging this first-order condition yields:

$$c(T) = \frac{w \left(\frac{q_i(T)}{b(T)} \right)^{\frac{1}{\gamma}}}{1 + \left(\frac{q_i(T)}{b(T)} \right)^{\frac{1}{\gamma}}} = w \underbrace{\left[1 + \left(\frac{b(T)}{q_i(T)} \right)^{\frac{1}{\gamma}} \right]^{-1}}_{c_{T,i}}$$

Hence, we obtain:

$$\begin{aligned} V(T, w, i) &= \frac{w^{1-\gamma}}{1-\gamma} \left(q_i(T) c_{T,i}^{1-\gamma} + b(T) (1 - c_{T,i})^{1-\gamma} \right) \\ &= \frac{w^{1-\gamma}}{1-\gamma} \left[q_i(T)^{\frac{1}{\gamma}} + b(T)^{\frac{1}{\gamma}} \right]^{\gamma} \end{aligned}$$

For the induction step, suppose the proposition is true for case $t + 1$. We have:

$$V(t, w, i) = \max_c \left\{ q_i(t) \frac{c^{1-\gamma}}{1-\gamma} + b(t) \bar{d}_i(t) \frac{(w-c)^{1-\gamma}}{1-\gamma} + e^{-\rho} (1 - \bar{d}_i(t)) \sum_{j=1}^n p_{ij}(t) \frac{K_{t+1,j}}{1-\gamma} \left[(w-c) e^{r(t,i,j)} \right]^{1-\gamma} \right\}$$

From the first-order condition we obtain:

$$q_i(t) c^{-\gamma} = b(t) \bar{d}_i(t) (w-c)^{-\gamma} + e^{-\rho} (1 - \bar{d}_i(t)) (w-c)^{-\gamma} \sum_{j=1}^n p_{ij}(t) K_{t+1,j} e^{r(t,i,j)(1-\gamma)}$$

Rearranging yields:

$$q_i(t) c^{-\gamma} = (w-c)^{-\gamma} \left[\bar{d}_i(t) b(t) + (1 - \bar{d}_i(t)) e^{-\rho} \sum_{j=1}^n p_{ij}(t) K_{t+1,j} e^{r(t,i,j)(1-\gamma)} \right]$$

which implies:

$$q_i(t)^{-1/\gamma} c = (w-c) \left[\bar{d}_i(t) b(t) + (1 - \bar{d}_i(t)) e^{-\rho} \sum_{j=1}^n p_{ij}(t) K_{t+1,j} e^{r(t,i,j)(1-\gamma)} \right]^{-1/\gamma}$$

Rearranging further yields:

$$c = w \underbrace{\left[1 + \left(\frac{\left[\bar{d}_i(t)b(t) + (1 - \bar{d}_i(t))e^{-\rho} \sum_{j=i}^n p_{ij}(t) K_{t+1,j} e^{r(t,i,j)(1-\gamma)} \right]}{q_i(t)} \right)^{\frac{1}{\gamma}} \right]^{-1}}_{c_{t,i}}$$

Thus we obtain:

$$\begin{aligned} V(t, w, i) &= q_i(t) c_{t,i}^{1-\gamma} \frac{w^{1-\gamma}}{1-\gamma} + b(t) \bar{d}_i(t) \frac{w^{1-\gamma}}{1-\gamma} (1 - c_{t,i})^{1-\gamma} + e^{-\rho} (1 - \bar{d}_i(t)) \frac{w^{1-\gamma}}{1-\gamma} (1 - c_{t,i})^{1-\gamma} \sum_{j=i}^n p_{ij}(t) K_{t+1,j} e^{r(t,i,j)(1-\gamma)} \\ &= \frac{w^{1-\gamma}}{1-\gamma} \left[q_i(t) c_{t,i}^{1-\gamma} + (1 - c_{t,i})^{1-\gamma} \left[\bar{d}_i(t) b(t) + e^{-\rho} (1 - \bar{d}_i(t)) \sum_{j=i}^n p_{ij}(t) K_{t+1,j} e^{r(t,i,j)(1-\gamma)} \right] \right] \end{aligned}$$

Note that we can write:

$$c_{t,i} = \frac{q_i(t)^{1/\gamma}}{q_i(t)^{1/\gamma} + \left[\bar{d}_i(t) b(t) + (1 - \bar{d}_i(t)) e^{-\rho} \sum_{j=i}^n p_{ij}(t) K_{t+1,j} e^{r(t,i,j)(1-\gamma)} \right]^{1/\gamma}}$$

which implies:

$$1 - c_{t,i} = \frac{\left[\bar{d}_i(t) b(t) + (1 - \bar{d}_i(t)) e^{-\rho} \sum_{j=i}^n p_{ij}(t) K_{t+1,j} e^{r(t,i,j)(1-\gamma)} \right]^{1/\gamma}}{q_i(t)^{1/\gamma} + \left[\bar{d}_i(t) b(t) + (1 - \bar{d}_i(t)) e^{-\rho} \sum_{j=i}^n p_{ij}(t) K_{t+1,j} e^{r(t,i,j)(1-\gamma)} \right]^{1/\gamma}}$$

Plugging this into the expression for $V(t, w, i)$ from above yields:

$$\begin{aligned} V(t, w, i) &= \frac{w^{1-\gamma}}{1-\gamma} \frac{q_i(t)^{1/\gamma} + \left[\bar{d}_i(t) b(t) + (1 - \bar{d}_i(t)) e^{-\rho} \sum_{j=i}^n p_{ij}(t) K_{t+1,j} e^{r(t,i,j)(1-\gamma)} \right]^{1/\gamma}}{\left[q_i(t)^{1/\gamma} + \left[\bar{d}_i(t) b(t) + (1 - \bar{d}_i(t)) e^{-\rho} \sum_{j=i}^n p_{ij}(t) K_{t+1,j} e^{r(t,i,j)(1-\gamma)} \right]^{1/\gamma} \right]^{1-\gamma}} \\ &= \frac{w^{1-\gamma}}{1-\gamma} \left[q_i(t)^{1/\gamma} + \left[\bar{d}_i(t) b(t) + (1 - \bar{d}_i(t)) e^{-\rho} \sum_{j=i}^n p_{ij}(t) K_{t+1,j} e^{r(t,i,j)(1-\gamma)} \right]^{1/\gamma} \right]^\gamma \end{aligned}$$

■

Proof of Corollary C.2. The proof follows immediately from the expression for VSI, given by Equation (10), and from noting that $u_c(c_i(0), q_i(0)) = q_i(0) c_{i,0}^{-\gamma} w^{-\gamma}$. ■