

# Midterm Sheet

Thursday, October 4, 2018 10:41 PM

## Vector Rules -

$$|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$$

$$\hat{a} = \frac{x}{|\vec{a}|} + \frac{y}{|\vec{a}|} + \frac{z}{|\vec{a}|}$$

$$R_{CD} = D - C$$

## Dot Product

$$\vec{a} \cdot \vec{b} = [x_a * x_b + y_a * y_b] = N_{ab}$$

## Cross Product

$$\vec{a} = x_1, y_1, z_1$$

$$\vec{b} = x_2, y_2, z_2$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

$$= (y_1 z_2 - z_1 y_2) \hat{i} + (z_1 x_2 - x_1 z_2) \hat{j} + (x_1 y_2 - y_1 x_2) \hat{k}$$

## Cylindrical Coordinates

$$\rho = \sqrt{x^2 + y^2}$$

$$\Phi = \tan^{-1} \frac{y}{x}$$

$$z = z$$

$$R_C + R_D = R_{CD}$$

$$R_{CD,\rho} = x \cos(\phi) + y \sin(\phi)$$

$$R_{CD,\phi} = -x \sin(\phi) + y \cos(\phi)$$

$$R_{CD,z} = z$$

## Work:

$$W = \vec{E} \cdot \vec{\delta}$$

Shrinking sphere is positive work

Field is doing negative work

## Potential

$$\Delta V_{AB} = \frac{\Delta E_{AB}}{q} = \frac{\Delta K_{AB}}{q} = \int_A^B \frac{E \cdot dr}{\frac{kq}{r^2}}$$

## Dipole potential

$$V(r, \theta) = \frac{kp \cos \theta}{r^2}$$

Where  $p = 2aq$

## Energy

$$KE = \frac{1}{2} mv^2$$

$$U = \frac{\text{energy}}{\text{volume}} = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} cv^2$$

## Constants

$$e = 1.6 * 10^{-19} C$$

$$k = 9.0 * 10^9 N * \frac{m^2}{C^2}$$

$$\epsilon_0 = 8.854 * 10^{-12} F/m$$

$$e_{mass} = 9.11 E^{-31} kg$$

## Miscellaneous

$$f\lambda = c$$

$$E_{pt} + E_{shell} = \frac{k(-2Q)}{r^2} + E_{out}$$

$$\text{If } r < R \text{ then } E_{out} = 0$$

Electric field at p between two points:

$$F_p = q\vec{E}$$

$$\vec{E} = \frac{kq}{r^2} \hat{r}$$

$$|E| = \sqrt{\left(\frac{kq}{r^2} \hat{i}\right)^2 + \left(\frac{kq}{r^2} \hat{j}\right)^2 + \left(\frac{kq}{r^2} \hat{k}\right)^2}$$

## Field Lines / Gauss

- Every Q has N field lines
- + attracted to -

## Flux -

$$\oint \vec{E} \cdot d\vec{A} \cos \theta$$

Largest flux is determined by the charge inside the surface (Area does not matter)

On a sphere,

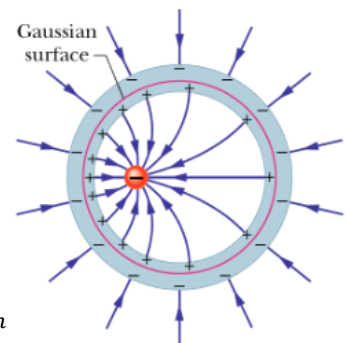
$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$

$$\phi = 4\pi r^2 E = EA \cos \theta$$

$$E = \frac{Qr}{4\pi\epsilon_0 R^3} \text{ (field inside uniformly charged sphere)}$$

An infinite line of charge

$$2\pi r L E = \frac{\lambda L}{\epsilon_0} \rightarrow E = \frac{\lambda}{2\pi\epsilon_0 r}$$



## Capacitance

$$C = \frac{Q}{V} = \frac{\epsilon_0 A}{d}$$

In parallel...

$$C = C_1 + C_2 \dots + C_n$$

In series...

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \dots \frac{1}{C_n}$$

Remember differential breakdown

Different capacitors can't handle different loads

## Current

$$\Delta Q$$

$$U = \frac{\text{energy}}{\text{volume}} = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} c v^2$$

$$\Delta V_{AB}(x) = - \int_A^B a x' dx' (\hat{i} \cdot \hat{i}) = - \int_0^x a x' dx' = - \frac{a}{2} x^2$$

$$V(x) = V(0) + \Delta V_{AB}(x) = - \frac{a}{2} x^2$$

Current

$$I = \frac{\Delta Q}{\Delta t}$$

Drift Velocity

$$I = n A q v_d$$

## Continuous Distributions: A Ring and a Disk

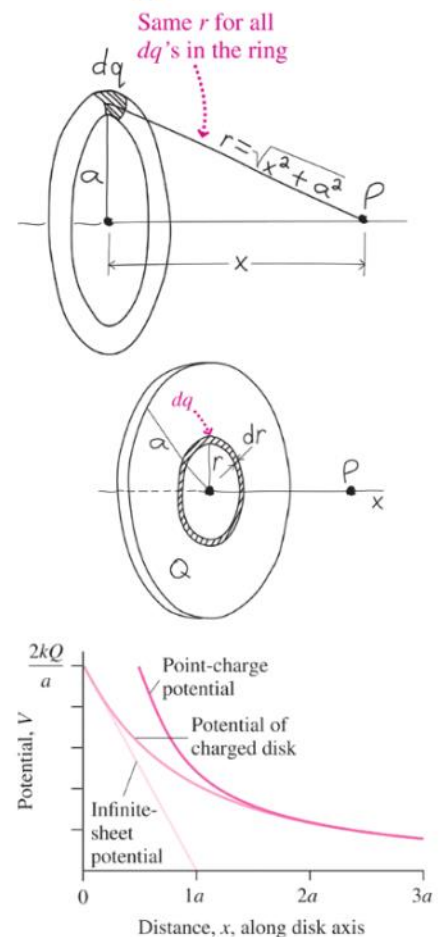
- For a uniformly charged ring of total charge  $Q$ , integration gives the potential on the ring axis:

$$V(x) = \int \frac{k dq}{r} = \frac{k}{r} \int dq = \frac{kQ}{\sqrt{x^2 + a^2}}$$

- Integrating the potentials of charged rings gives the potential of a uniformly charged disk:

$$V(x) = \frac{2kQ}{a^2} \left( \sqrt{x^2 + a^2} - |x| \right)$$

- This result reduces to the infinite-sheet potential close to the disk, and the point-charge potential far from the disk.



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