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2.20 | PROBLEMS

1. Target received power: Using equation (2.8), determine the single-pulse received power level from a target for a radar system having the following characteristics:

Transmitter:	100 kilowatt peak
Frequency:	9.4 GHz
Antenna Gain:	32 dB
Target RCS:	0 dBsm
Target Range:	50 km

2. Using equation (2.10), determine the receiver noise power (in dBm) for a receiver having a noise figure of 2.7 dB and an instantaneous bandwidth of 1 MHz.

3. Using equation (2.11), determine the single-pulse SNR for the target described in problem 1 if the receiver has a noise figure of 2.7 dB and an instantaneous bandwidth of 1 MHz.

4. Ignoring any losses, using equation (2.8), determine the single-pulse received power level (in dBm) for a 1 square meter target at a range of 36 km for radar systems with the following characteristics.

	P_t (watts)	G	Freq
Radar a	25,000	36 dB	9.4 GHz
Radar b	250,000	31 dB	9.4 GHz
Radar c	250,000	31 dB	2.8 GHz
Radar d	250,000	36 dB	9.4 GHz

5. Using equation (2.11), determine the SNR for the four conditions described in problem 4 for the following noise-related characteristics. Bandwidth for both frequencies is 10 MHz, the noise figure for 9.4 GHz systems is 3.2 dB, and the noise figure for the 2.8 GHz system is 2.7 dB.

6. Using equation (2.17), determine the four answers in problems 4 and 5 for the following loss conditions:

$L_{tx} = 2.1$ dB

$L_{rx} = 4.3$ dB.

7. If atmospheric propagation losses of 0.12 dB per km (two-way) are also considered, determine the resulting SNR values in problem 6.

8. If we desire the SNR in problem 7 to be the same as in problem 5, we can increase the SNR in problem 7 by transmitting, receiving, and processing multiple pulses. Use equation (2.14) to determine how many pulses we have to transmit to recover from the losses added in problems 6 and 7. (Hint: instead of solving the problem from the beginning, merely determine the relationship between the number of pulses transmitted and the SNR improvement.)

Appendix B: Answers to Selected Problems

Chapter 1

1. $0.15x$ km.
3. 10.73 μs, 6.67 μs, 666.67 μs, 2.68 ms, 40.64 ns.
5. (a) 11.81 GHz (b) 85.714 GHz. (c) 34.88 GHz. (d) 333.33 MHz. (e) 3.33 GHz. (f) 984.25 MHz.
7. (a) 66.67 m. (b) 20 m. (c) 0.67 m. (d) 20 m. (e) 1125 m.
10. 1 kW = 30 dBW.
11. $150/x$ km.
13. (a) 2980 Hz (b) 22.0 kHz (c) 1.00 kHz (d) 707.1 Hz (e) 212.1 Hz (f) 500 Hz.
15. (a) 150 m (b) 150 m (c) 15 m (d) 1.5 m.
17. (a) 15.7 degrees (b) 4.25 degrees (c) 361.2 feet (d) 97.9 feet

Chapter 2

1. 2.06×10^{-14} W $= 2.06 \times 10^{-11}$ mW, -106.9 dBm.
3. 2.765, or 4.42 dB.
5. (a) 1.65 dB (b) 1.65 dB (c) 12.6 dB (d) 11.65 dB.
7. (a) -9.07 dB (b) -9.07 dB (c) $+1.88$ dB (d) $+0.93$ dB.
9. (a) 42.05 km (b) 28.12 km.
11. 17.5 ms.
13. 90 m^2, and -19.54 dB.
15. 62.3 dB.
17. 32.3 dB.
19. 158 kW.

Chapter 3

1. 90.
3. 1.0 seconds.
5. 1.11×10^{-2}.
7. 3 verifications.
9. 912 ms.

where η_a is the antenna efficiency. Antenna efficiency is a value between 0 and 1; however, it is seldom below 0.5 and seldom above 0.8.

Solving (2.7) for A_e and substituting into equation (2.6), the following expression for the received power, P_r results:

$$P_r = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4} \tag{2.8}$$

In this expression,

P_t is the peak transmitted power in watts.
G_t is the gain of the transmit antenna.
G_r is the gain of the receive antenna.
λ is the carrier wavelength in meters.
σ is the mean[3] RCS of the target in square meters.
R is the range from the radar to the target in meters.

This form is found in many existing standard radar texts, including [2–6].

For many monostatic radar systems, particularly those using mechanically scanned antennas, the transmit and receive antennas gains are the same, so in those cases the two gain terms in (2.8) are replaced by G^2. However, for bistatic systems and in many modern radar systems, particularly those that employ electronically scanned antennas, the two gains are generally different, in which case the preferred form of the radar range equation is that shown in (2.8), allowing for different values for transmit and receive gain.

For a bistatic radar, one for which the receive antenna is not colocated with the transmit antenna, the range between the transmitter and target, R_t, may be different from the range between the target and the receiver, R_r. In this case, the two different range values must be independently specified, leading to the bistatic form of the equation

$$P_r = \frac{P_t G_t G_r \lambda^2 \sigma_{bistatic}}{(4\pi)^3 R_t^2 R_r^2} \tag{2.9}$$

Though in the following discussions the monostatic form of the radar equation is described, a similar bistatic form can be developed by separating the range terms and using the bistatic radar cross section, $\sigma_{bistatic}$, of the target.

2.4 | RECEIVER THERMAL NOISE

In the ideal case, the received target signal, which usually has a very small amplitude, could be amplified by some arbitrarily large amount until it could be visible on a display or within the dynamic range of an analog-to-digital converter (ADC). Unfortunately, as discussed in Chapter 1 and in the introduction to this chapter, there is always an interfering signal described as having a randomly varying amplitude and phase, called *noise*, which is produced by several sources. Random noise can be found in the environment, mostly due

[3]The target RCS is normally a fluctuating value, so the mean value is usually used to represent the RCS. The radar equation therefore predicts a mean, or average, value of received power and, when noise is taken into consideration, SNR.

to solar effects. Noise entering the antenna comes from several sources. Cosmic noise, or galactic noise, originates in outer space. It is a significant contributor to the total noise at frequencies below about 1 GHz but is a minor contributor above 1 GHz. Solar noise is from the sun. The sun's proximity makes it a significant contributor; however, its effect is reduced by the antenna sidelobe gain, unless the antenna main beam is pointed directly toward the sun. Even the ground is a source of noise, but not at as high a level as the sun, and usually enters the receiver through antenna sidelobes.

In addition to antenna noise, thermally agitated random electron motion in the receiver circuits generates a level of random noise with which the target signal must compete. Though there are several sources of noise, the development of the radar range equation in this chapter will assume that the internal noise in the receiver dominates the noise level. This section presents the expected noise power due to the active circuits in the radar receiver. For target detection to occur, the target signal must exceed the noise signal and, depending on the statistical nature of the target, sometimes by a significant margin before the target can be detected with a high probability.

Thermal noise power is essentially uniformly distributed over all radar frequencies; that is, its *power spectral density* is constant, or uniform. It is sometimes called "white" noise. Only noise signals with frequencies within the range of frequencies capable of being detected by the radar's receiver will have any effect on radar performance. The range of frequencies for which the radar is susceptible to noise signals is determined by the receiver bandwidth, B. The thermal noise power adversely affecting radar performance will therefore be proportional to B. The power, P_n, of the thermal noise in the radar receiver is given by [4]

$$P_n = kT_S B = kT_0 F B \tag{2.10}$$

where

k is Boltzmann's constant (1.38×10^{-23} watt-sec/K).
T_0 is the standard temperature (290 K).
T_s is the system noise temperature ($T_s = T_0 F$).
B is the instantaneous receiver bandwidth in Hz.
F is the noise figure of the receiver subsystem (unitless).

The *noise figure*, F, is an alternate method to describe the receiver noise to system temperature, T_s. It is important to note that noise figure is often given in dB; however, it must be converted to linear units for use in equation (2.10).

As can be seen from (2.10), the noise power is linearly proportional to receiver bandwidth. However, the receiver bandwidth cannot be made arbitrarily small to reduce noise power without adversely affecting the target signal. As will be shown in Chapters 8 and 11, for a simple unmodulated transmit signal, the bandwidth of the target's signal in one received pulse is approximated by the reciprocal of the pulse width, τ (i.e., $B \approx 1/\tau$). If the receiver bandwidth is made smaller than the target signal bandwidth, the target power is reduced, and range resolution suffers. If the receiver bandwidth is made larger than the reciprocal of the pulse length, then the signal to noise ratio will suffer. The optimum bandwidth depends on the specific shape of the receiver filter characteristics. In practice, the optimum bandwidth is usually on the order of $1.2/\tau$, but the approximation of $1/\tau$ is very often used.

As a result of incorporating the losses into (2.14), the RRE becomes

$$SNR = \frac{P_t G_t G_r \lambda^2 \sigma n_p}{(4\pi)^3 R^4 k T_0 F B L_s} \tag{2.17}$$

The following sections describe the most common of these losses individually.

2.7.1 Transmit Loss

The radar equation (2.14) is developed assuming that all of the transmit power is radiated out an antenna having a gain G_t. In fact, there is some loss in the signal level as it travels from the transmitter to the antenna, through waveguide or coaxial cable, and through devices such as a circulator, directional coupler, or transmit/receive (T/R) switch. For most conventional radar systems, the loss is on the order of 3 or 4 dB, depending on the wavelength, length of transmission line, and what devices are included. For each specific radar system, the individual losses must be accounted for. The best source of information regarding the losses due to components is a catalog sheet or specification sheet from the vendor for each of the devices. In addition to the total losses associated with each component, there is some loss associated with connecting these components together. Though the individual contributions are usually small, the total must be accounted for. The actual loss associated with a given assembly may be more or less than that predicted. If maximum values are used in the assumptions for loss, then the total loss will usually be somewhat less than predicted. If average values are used in the prediction, then the actual loss will be quite close to the prediction. It is necessary to measure the losses to determine the actual value.

There is some loss between the input antenna port and the actual radiating antenna; however, this term is usually included in the specified antenna gain value provided by the antenna vendor. The analyst must determine if this term is included in the antenna gain term and, if not, must include it in the loss calculations.

2.7.2 Atmospheric Loss

Chapter 4 provides a thorough discussion of the effects of propagation through the environment on the SNR. The EM wave experiences attenuation in the atmosphere as it travels from the radar to the target, and again as the wave travels from the target back to the radar. Atmospheric loss is caused by interaction between the electromagnetic wave and oxygen molecules and water vapor in the atmosphere. Even clear air exhibits attenuation of the EM wave. The effect of this attenuation generally increases with increased carrier frequency; however, in the vicinity of regions in which the wave resonates with the water or oxygen molecules, there are sharp peaks in the attenuation, with relative nulls between these peaks. In addition, fog, rain, and snow in the atmosphere add to the attenuation caused by clear air. These and other propagation effects (diffraction, refraction, and multipath) are discussed in detail in Chapter 4.

Range-dependent losses are normally expressed in units of dB/km. Also, the absorption values reported in the technical literature are normally expressed as one-way loss. For a monostatic radar system, since the signal has to travel through the same path twice, two-way loss is required. In this case, the values reported need to be doubled on a dB scale (squared on a linear scale). For a bistatic radar, the signal travels through two different paths on transmit and receive, so the one-way loss value is used for each path.