ECE 09.303 Fall 2019
Homework 3 Solutions
Chapter 3/21 – Gauss's Law

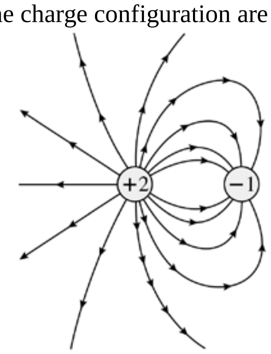
1.

Charges $+2q$ and $-q$ are near each other. Sketch some field lines for this charge distribution, using eight lines for a charge of magnitude q.

INTERPRET This problem is an exercise in drawing electric field lines to represent the field strength of a charge configuration.

DEVELOP We follow the methodology illustrated in Figure 21.3. There are 16 lines emanating from charge $+2q$ (eight for each unit of $+q$). Similarly, we have 8 lines ending on $-q$.

EVALUATE The field lines of the charge configuration are shown in the figure below.



2.

The net charge shown in Fig. 1 is $+Q$. Identify each of the charges A, B, and C shown.

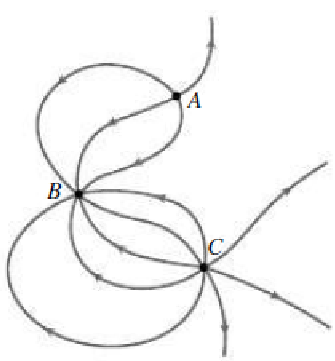


FIGURE 1

INTERPRET In this problem we are asked to identify the charges based on the pattern of the field lines and the given net charge.

DEVELOP From the direction of the lines of force (away from positive and toward negative charge) one sees that *A* and *C* are positive charges and *B* is a negative charge. Eight lines of force terminate on *B*, eight originate on *C*, but only four originate on *A*, so the magnitudes of *B* and *C* are equal, while the magnitude of *A* is half that value.

EVALUATE Based on the reasoning above, we may write $Q_C = -Q_B = 2Q_A$. The total charge is $Q = Q_A + Q_B + Q_C = Q_A$, so $Q_C = 2Q = -Q_{B.}$

ASSESS The magnitude of the charge is proportional to the number of field lines emerging from or terminating at the charge.

3.

A point charge of $-2Q$ is at the center of a spherical shell of radius R carrying charge Q spread uniformly over its surface. Find the electric field at (a) $r = \frac{1}{2}R$ and (b) $r = 2R$. (c) How would your answers change if the charge on the shell were doubled?

INTERPRET The given charge distribution has spherical symmetry, so we can apply Gauss's law to find the electric field. In addition, because the charge distribution consists of two distinct charge distributions, we can apply the principle of superposition to find the net electric field.

DEVELOP The total electric field is the superposition of the fields due to the point charge and the spherical shell.

$$E = E_{\text{pt}} + E_{\text{shell}}$$

The field is spherically symmetric about the center. Inside the shell, the contribution E_{shell} to the electric field is zero because there is no charge enclosed (remember, this is just for the shell with no point charge inside). Outside the shell, the contribution E_{shell} is like that of a point with charge Q located at the center of the sphere of the shell. Both inside and outside the shell, the contribution E_{pt} is that of a point charge with charge $-2Q$.

EVALUATE **(a)** At $r = R/2 < R$ (inside the shell), the electric field is

$$E = E_{\text{pt}} + E_{\text{shell}} = \frac{k(-2Q)}{(R/2)^2} + 0 = -\frac{8\,kQ}{R^2}$$

Note that the minus sign means the direction is radially inward.

(b) At $r = 2R > R$ (outside shell), the field strength is

$$E = E_{\text{pt}} + E_{\text{shell}} = \frac{k(-2Q + Q)}{(2R)^2} = -\frac{kQ}{4R^2}$$

Again the direction of the field is radially inward.

(c) If $Q_{\text{shell}} = 2Q$, the field inside would be unchanged, but the field outside would be zero by Gauss's law (Equation 21.3) since $q_{\text{enclosed}} = q_{\text{shell}} + q_{\text{pt}} = 2Q - 2Q = 0$.

ASSESS By Gauss's law, the shell produces no electric field in its interior. The field outside a spherically symmetric distribution is the same as if all the charges were concentrated at the center of the sphere.

4.

A charged slab extends infinitely in two dimensions and has thickness d in the third dimension, as shown in Fig. 2 . The slab carries a uniform volume charge density ρ. Find expressions for the electric field (a) inside and (b) outside the slab, as functions of the distance x from the center plane. (Although the infinite slab is impossible, your answer is a good approximation to the field of a finite slab whose width is much greater than its thickness.)

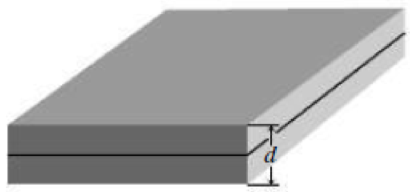
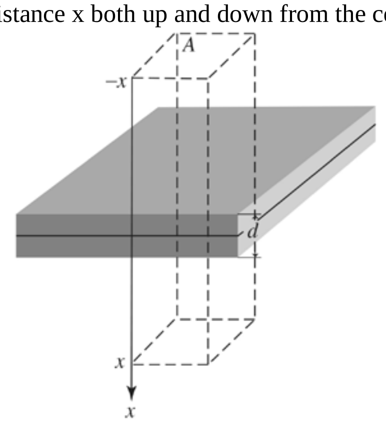


FIGURE 2

INTERPRET The infinitely large slab has plane symmetry, and we can apply Gauss's law to compute the electric field.

DEVELOP When we take the slab to be infinitely large, the electric field is everywhere normal to the slab's surface and symmetrical about the center plane. We follow the approach outlined in Example 21.6 to compute the electric field. As the Gaussian surface, we choose a box that has area A on its top and bottom and that extends a distance x both up and down from the center of the slab. See figure below.



EVALUATE **(a)** For points inside the slab $|x| \le d/2$, the charge enclosed by our Gaussian box is

$$q_{\text{enclosed}} = \rho V_{\text{enclosed}} = \rho A(2x)$$

Thus, Gauss's law gives

$$\Phi = \int \vec{E} \cdot d\vec{A} = E(2A) = \frac{q_{\text{enclosed}}}{\epsilon_0} \quad \rightarrow \quad E = \frac{\rho x}{\epsilon_0}$$

The direction of $\vec{E}$ is away from (toward) the central plane for positive (negative) charge density.

(b) For points outside the slab $|x| > d/2$, the enclosed charge is

$$q_{\text{enclosed}} = \rho V_{\text{enclosed}} = \rho A d$$

Applying Gauss's law again gives

$$E = \frac{\rho d}{2\epsilon_0}$$

ASSESS Inside the slab, the charge distribution is equivalent to a sheet with $\sigma = 2\rho x$. On the other hand, outside the slab, it is equivalent to a sheet with $\sigma = \rho d$.