Chapter 2. The Radar Range Equation

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2.1 Introduction

As introduced in Chapter 1, the three fundamental functions of radar systems are to search for targets, track targets that are found, and in some cases develop an image of the target. In all of these functions the radar performance is influenced by the strength of the signal coming into the radar receiver from the target of interest, and the strength of the signals that interfere with the target signal. The ratio of the target signal to the interfering signal is the signal-to-interference ratio *SIR*. In the special case of noise being the interfering signal, the ratio is called the signal-to-noise ratio *SNR,* and if the interference is from a clutter signal, then the ratio is called signal-to-clutter ratio *SCR*.

In the search mode, the radar system is programmed to reposition the antenna beam in a given sequence to "look" at each possible position in space for a target. If the signal at any spatial position exceeds the interference by sufficient margin, then a *detection* is made, and a target is deemed to be at that position. In this sense, *detection* is a process by which for every possible position for a target, the signal is compared to some threshold level, to determine if the signal is large enough to be deemed a target of interest. The probability that a target will be *detected* is dependent on the SIR and the threshold level to which the signal is compared. The detection process is discussed in more detail in Chapters 3 and 15, and special processing techniques designed to perform the detection process automatically are discussed in Chapter 16.

In the tracking mode, the accuracy or precision with which a target is tracked also depends on the *SIR*. The higher the *SIR*, the more accurate and precise the track will be. Chapter 19 describes the tracking process, and the relationship between tracking precision and the SIR.

In the imaging mode, the *SIR* determines the fidelity of the image. It determines the dynamic range of the image - the ratio between the "brightest" spots and the dimmest on the target. The *SIR* also determine to what extent false scatterers are seen in the target image.

The tool that the radar system designer or analyst has to compute the *SIR* is the *radar range equation* (RRE). It is a relatively simple formula, or a family of formulae, that predicts the received power of the radar's radio waves "reflected"[1] from a target and the interfering noise power level, and, when these are combined, the signal-to-noise ratio (SNR). In addition, it can be used to calculate the power received from surface and volumetric clutter, which can be considered a target, or an interfering signal. When the system application calls for detection of the clutter, the clutter signal becomes the target. When the clutter signal is deemed to be an interfering signal, then the signal-to-interference ratio (SIR) is determined by dividing the target signal by the clutter signal. One-way analysis is of the propagating signal is called the one-way link equation, and can determine the received signal resulting from a jammer, a beacon transponder, or a communications system.

This chapter includes a discussion of several forms of the radar range equation, including those forms that are most often used in predicting radar performance. It begins with predicting the power density at a distance *R*, and extends to the two-way case for monostatic radar for targets, surface clutter, and volumetric clutter. Then radar receive thermal noise power is determined, providing the SNR. Equivalent, but specialized forms of the RRE are developed for a search radar and then for a tracking radar. Initially, an idealized approach is presented, limiting the introduction of terms to the ideal radar parameters. After the basic RRE is derived, non-ideal effects are introduced. Specifically, the component, propagation, and signal processing losses are introduced, providing a more realistic value for the received target signal power.

2.2 Power Density at a Distance R

Although the radar range equation is not formally derived here from first principles, it is informative to develop the equation in several steps. The total peak power developed by

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[1] It will be seen in Chapter 6 that the signal illuminating a target induces currents on the target, and the target reradiates these electromagnetic fields, some of which are directed toward the illuminating source. For simplicity, this process is often termed "reflection".

the radar transmitter, P_t, is applied to the antenna system. If the antenna had an isotropic, or omnidirectional radiation pattern, the power density Q_i at a distance R from the radiating antenna would be the total power divided by the surface area of a sphere of radius R,

$$Q_i = \frac{P_t}{4\pi R^2} \qquad 2.1$$

as depicted in Figure 2.1.

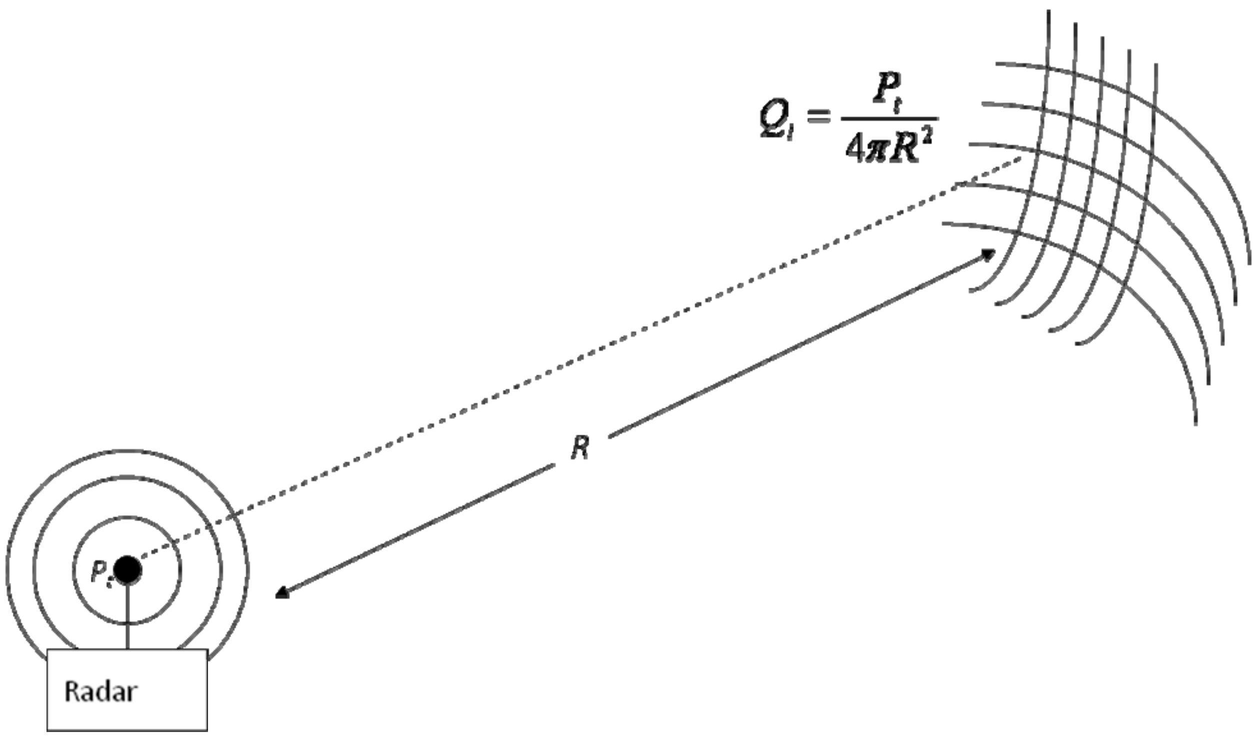


Figure 2.1. Power density at range R from the radar transmitter.

Essentially all radar systems use an antenna that has a directive beam pattern, rather than an isotropic beam pattern. This means that the transmitted power is concentrated into a finite angular extent, usually having a width of several degrees in both the azimuthal and elevation planes. In this case, the power density at the center of the antenna beam pattern is higher than that from an isotropic antenna, because the transmit power is concentrated onto a smaller area on the surface of the sphere, as depicted in Figure 2.2. The power density in the grey ellipse depicting the antenna beam is increased from that of an isotropic antenna. The ratio between the power density for a conventional antenna and an isotropic antenna is termed the directivity, or antenna gain *G*. The subscript *t* is used to denote a transmit antenna, so the gain is G_t. Given the increased power density due to use of a directive antenna, [2]

$$Q_i = \frac{P_t G_t}{4\pi R^2} \qquad 2.2$$

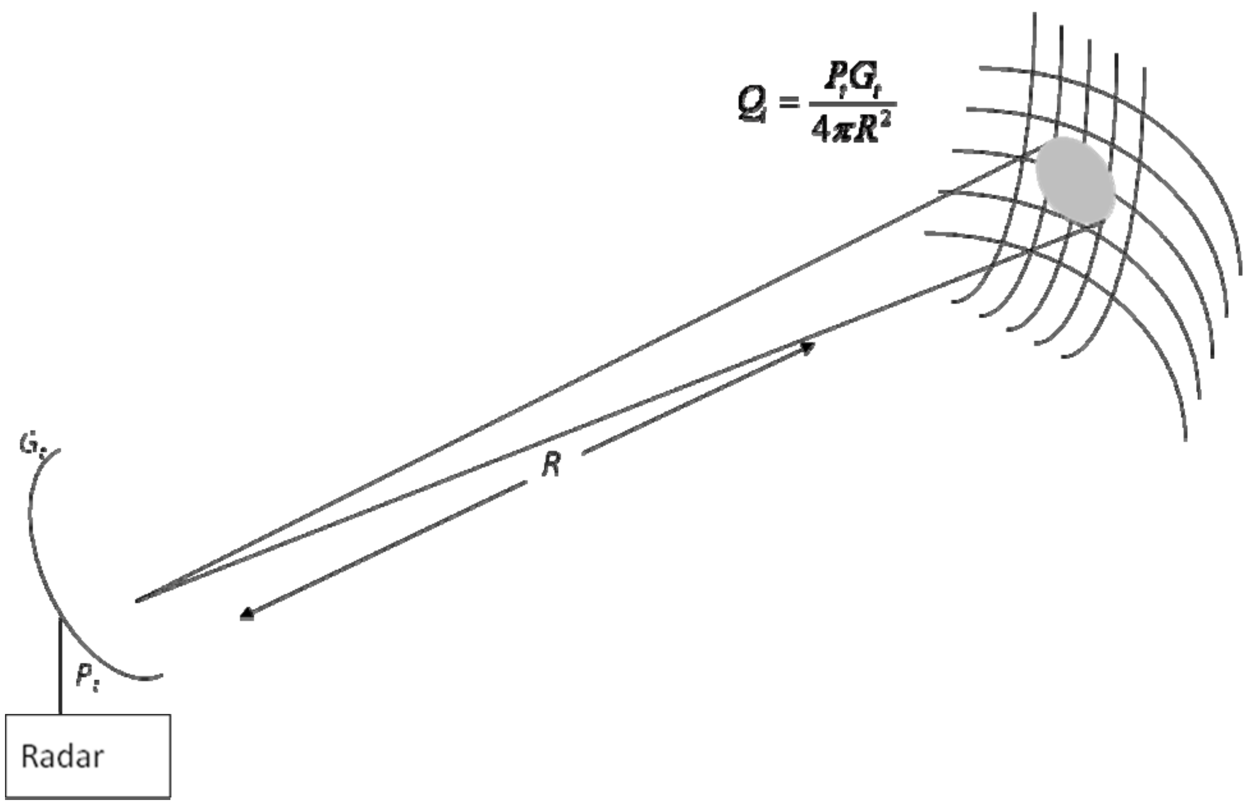


Figure 2.2. Power density at range *R*, given transmit antenna gain G_t.

[2] Antenna engineers often use the term *directivity* to describe the ratio in radiated power density between a realistic radar antenna and an isotropic antenna. The difference between gain and directivity is the dissipative losses in the hardware associated with getting the transmit signal from the antenna input port to the radiating portion [1].

2.3 Received Power from a Target

Next, consider that there is a radar "target" at range *R*, illuminated by the signal from a radiating antenna. The incident transmitted signal is reflected in a variety of directions, as depicted in Figure 2.3. Some part of the incident radio wave will be "reflected" back toward the radar. The incident radar signal induces time-varying currents on the target so that the target now becomes a source of radio waves, part of which will propagate back to the radar. The power reflected by the target back toward the radar (P_{refl}) is expressed as the product of the incident power density times and a factor called the *radar cross section* (RCS) σ of the target. The units for RCS are square meters (m^2). The radar cross section of a target is determined by the physical size of the target, the shape of the target, and the materials from which the target is made, particularly the outer surface.[4] The expression for the power reflected back toward the radar P_{refl} from the target is:

$$P_{refl} = Q_i \sigma = \frac{P_t G_t \sigma}{4\pi R^2} \tag{2.3}$$

[4] A more formal definition and additional discussion of radar cross section are given in Chapter 6.

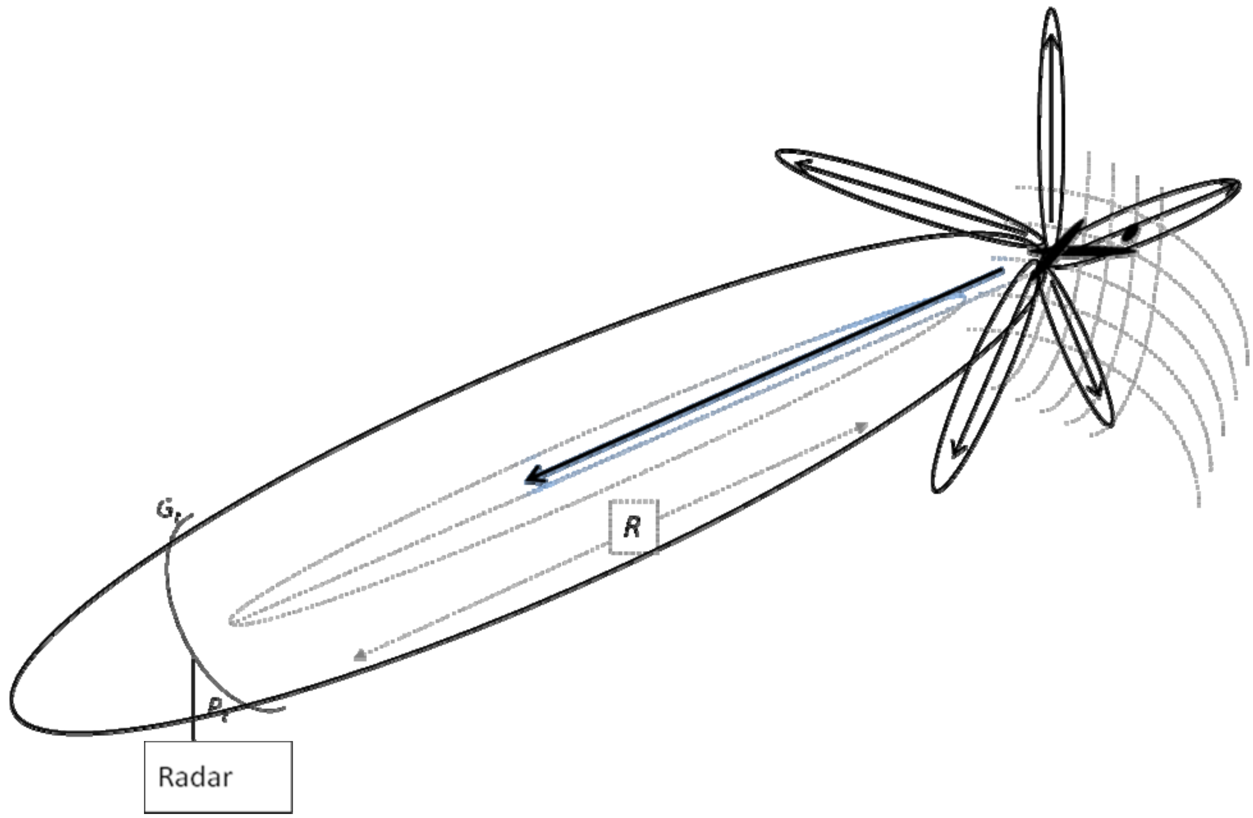


Figure 2.3. Power density, Q_r back at the radar antenna.

The signal reflected from the target propagates back toward the radar system over a distance R so that the power density back at the radar receiver Q_r is:

$$Q_r = \frac{P_{refl}}{4\pi R^2} \tag{2.4}$$

Combining Eqs. (2.3) and (2.4) the power density of the radio wave received back at the radar receiver antenna is given by:

$$Q_r = \frac{Q_t \sigma}{4\pi R^2} = \frac{P_t G_t \sigma}{(4\pi)^2 R^4} \tag{2.5}$$

Notice that the radar-target range R appears in the denominator raised to the fourth power! As an example of its significance, if the range from the radar to the target doubles, the received power density of the reflected signal from a target decreases by a factor of 16(12 dB)!

The radar wave reflected from the target, which has propagated through a distance R and results in the power density given by Eq. (2.5), is received (gathered) by a radar receiver antenna having an effective aperture area of A_e . The power received (S) from a target at range R at a receiving aperture of area A_e is found from the power density at the antenna, times the effective area of the antenna:

$$S = Q_r A = \frac{P_t G_t A_e \sigma}{(4\pi)^2 R^4} \tag{2.6}$$

It is customary to replace the antenna effective area term A_e with the value of receiver antenna gain G_r that is produced by that area. Too, as described in Chapter 9, because of the effects of tapering and losses, the *effective* area of an antenna is somewhat less than the physical area. The relationship between an antenna gain G and its effective area A_e is [1]

$$G = \eta_a \frac{4\pi A}{\lambda^2} = \frac{4\pi A_e}{\lambda^2} \tag{2.7}$$

Where: η_a is the aperture efficiency.

Solving (2.7) for A_e and substituting into Eq. (2.6), the following expression for the received power results:

$$S = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4} \tag{2.8}$$

where

P_t is the peak transmitted power in watts,
G_t is the gain of the transmit antenna,

G_r is the gain of the receive antenna,

λ is the carrier wavelength in meters,

σ is the mean[5] radar cross section (RCS) of the target in square meters, and

R is the range from the radar to the target in meters.

This form is found in many standard radar texts, including [2]–[5].

For a monostatic radar system using a mechanically scanned antenna, the transmit and receive antennas gains are the same, so the two gain terms in (2.8) are usually replace by G^2. However, in many modern radar systems, particularly those that employ electronically scanned antennas, the two gains are generally different, in which case the preferred form of the radar range equation is that shown in (2.8), allowing for different values for transmit and receive gain.

2.4 Receiver noise

In the ideal case, the received target signal, which usually has a very small amplitude, could be amplified by some arbitrarily large amount until it was visible on a display or detectable by an analog-to-digital converter. Unfortunately, as discussed in Chapter 1, there is always an interfering signal described as having a randomly varying amplitude and phase, called noise, which is produced by several sources. As discussed in Chapter 1, there exists a small, but measurable level of random noise in the environment, mostly due to solar effects. In addition, random electron motion in the receiver circuits generates a level of random noise, with which the target signal must compete. The internal noise in the receiver tends to dominate the noise level; in special cases the galactic noise may be significant. This section presents the expected noise power due to the active circuits in the radar receiver. For target *detection* to occur, the target signal must exceed the noise signal and, depending on the statistical nature of the target and interfering signals,

[5] The target RCS is normally not a constant value, so the mean value is usually used to represent the RCS.The radar equation, therefore predicts a mean, or average value of SNR, since the received power likewise varies.

sometimes by a significant margin before the target can be detected with a high probability.

Thermal noise power is essentially uniformly distributed over all radar frequencies, *i.e.*, its *power spectral density* is constant, or *uniform*. Therefore, only those noise signals with frequencies within the range of frequencies capable of being detected by the radar's receiver will have any effect on radar performance. The range of frequencies for which the radar is susceptible to noise signals is determined by the receiver bandwidth, B. The thermal noise power adversely affecting radar performance will therefore be proportional to B. The power P_n of the thermal noise in the radar receiver is given by

$$P_n = kT_S B = kT_O BF \tag{2.9}$$

where:

k is Boltzmann's constant (1.38×10^{-23} watt-sec/°K),
T_0 is the standard temperature (290° K),
T_s is the system noise temperature ($T_s = T_0 F$),
B is the instantaneous receiver bandwidth in Hz, and
F is the noise factor of the receiver subsystem (unitless).

The *noise factor* is a measure of the amount of noise added by the receiver circuits in excess of the theoretical environmental noise value of kT_0B. It is important to note that the noise factor is a linear value (as opposed to dB). Its value is often given in dB, however, and must be converted to linear units for use in Eq. (2.9). When expressed in dB, it is often called the *noise figure.*

As can be seen from Eq. (2.9), the noise power is linearly proportional to receiver bandwidth. The receiver bandwidth cannot be made arbitrarily small, however, to reduce noise power without adversely affecting the target signal fidelity. As will be shown in Chapters 8 and 11, for an uncoded transmit signal, the bandwidth of the target's signal in one received pulse is approximated by the reciprocal of the pulse width (τ), *i.e.*, $B_t = 1/\tau$. If the receiver bandwidth is made smaller than the target bandwidth, the target power and fidelity is reduced, and range resolution suffers. If the receiver bandwidth is made significantly larger than the reciprocal of the pulse length, then the signal to noise ratio will suffer. The true optimum bandwidth depends on the specific shape of the receiver filter characteristics. In practice, the optimum bandwidth is usually on the order of $1.2/\tau$, but the approximation of $1/\tau$ is very often used.

2.5 Signal-to-Noise Ratio and the Radar Range Equation

When the target signal power, S, is divided by the noise power, P_n, the result is called the signal-to-noise ratio (SNR). The equation describing the SNR is called the radar range equation. This is the simplest form of the radar range equation that is useful in predicting target detection performance.

The ratio of the signal power to the noise power is S/P_n. For a discrete target, this is the ratio of Eq. (2.8) to (2.9):

$$SNR = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 kT_0 BFR^4} \tag{2.10}$$

Ultimately, it is the signal-to-interference ratio (SIR) that determines radar performance. The interference can be from noise (receiver or jamming), or from clutter, and/or other electromagnetic interference from motors, generators, ignitions, cell services, *etc*. If the power of the receiver thermal noise is N, from clutter is C, and from jamming noise is J, then the SIR is

$$SIR = \frac{S}{N + C + J} \tag{2.11}$$

Although, usually one of these interference sources dominates, reducing the SIR to the signal power divided by the dominant interference power, *S/N*, *S/C*, or *S/J*, a complete calculation must be made in each case to see if this simplification applies.

2.6 Summary of Losses

To this point, the radar equation has been presented in an idealized form, that is, no losses have been assumed. Unfortunately, the received signal power is usually lower than that predicted if the analyst ignores the effects of signal loss. Atmospheric absorption, component resistive losses, and non-ideal signal processing conditions lead to less than ideal SNR performance. This section summarizes the losses most often encountered in radar systems, and presents the effect on *SNR*. Included are losses due to clear air, rain, component losses, beam scanning, straddling, signal processing, and so forth. It is important to realize that the loss value, if used in the denominator of the RRE, as suggested above, must be a number greater than one (1).

Often the loss values are determined in dB space. Use of the loss effects in the RRE require that the loss be described in terms of the linear value. It is convenient to sum the losses in dB space, and finally convert to the linear value. Equation 2.12 provides the total system loss term.

$$L_s = L_t L_{atm} L_r L_{sp} \qquad 2.12$$

where:

L_s is the system loss,

L_t is the transmit loss,

L_{atm} is the atmospheric loss,

L_r is the receiver loss, and

L_{sp} is the signal processing loss.

The following sections describe the most common of these losses individually.

As a result of incorporating the losses into (2.10), the RRE becomes:

$$SNR = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 k T_0 B F L_s R^4} \qquad 2.13$$

2.6.1 Transmit loss

The radar equation (2.8) is developed assuming that all of the transmit power is radiated out the antenna having a gain *G*. In fact, there is some loss in the signal level as it travels from the transmitter to the antenna, through waveguide or coaxial cable, and through devices such as a circulator, directional coupler, and/or transmit/receive (TR) switch. For most conventional radar systems, the loss is on the order of 3 or 4 dB, depending on the wavelength, length of transmission line, and what devices are included. For each specific radar system, the individual losses must be accounted for. The best source of information regarding the losses due to components is a catalog sheet or specification sheet from the vendor for each of the devices. In addition to the total losses associated with each component, there is some loss associated with connecting these components together. Though the individual contributions are usually small, the total must be accounted for. The actual loss associated with a given assembly may be more or less than that predicted. If maximum values are used in the assumptions for loss, then the total loss will usually be somewhat less than that predicted. If average values are used in the prediction, then the actual loss will be quite close to the prediction. It is necessary to measure the losses to determine the actual value.

There is some loss between the input antenna port and the actual radiating aperture, however, this term is usually included in the specified antenna gain value provided by the antenna vendor. The analyst must determine if this term is included in the antenna gain term, and if not, include it in the loss calculations.

2.6.2 Atmospheric Loss

Chapter 4 provides a thorough discussion of the effects of propagation through the environment on the SNR. The EM wave experiences attenuation in the atmosphere as it travels from the radar to the target, and again as the wave travels from the target back to the radar. Atmospheric loss is caused by interaction between the electromagnetic wave and oxygen molecules and water vapor in the atmosphere. Even clear air exhibits attenuation of the EM wave. The effect of this attenuation generally increases with increased carrier frequency, however in the vicinity of regions in which the wave resonates with the water or oxygen molecules, there are sharp peaks in the attenuation, with relative nulls between these peaks. The attenuation is not monotonically increasing with frequency. Below about 10 GHz, the effect is monotonic with frequency. In addition, fog, rain and snow in the atmosphere add to the attenuation caused by clear air. These and other propagation effects (diffraction, refraction, and multipath) are discussed in detail in Chapter 4.

Range-dependent losses are normally expressed in units of dB/km. Also, the absorption values reported in the technical literature are normally expressed as one-way loss, whereas for the radar system, since the signal has to travel through the path twice, two-way loss is required. In this case, the values reported need to be doubled on a dB scale (squared on a linear scale).

Significant loss can be encountered as the signal propagates through the atmosphere. For example, if the two-way loss through rain is 0.8 dB/km and the target is 10 km away, then the rain-induced reduction in SNR is 8 dB compared to the SNR obtained in clear air. The quantitative effect of such a reduction in SNR is discussed in Chapter 3, however, to provide a sense of the enormity of an 8 dB reduction in SNR, usually a reduction of 1 dB will produce significant system performance reduction.

2.6.3 Receive loss

Component losses are also present in the path between the receive antenna terminal and the radar receiver. As with the transmit losses, these are caused by receive transmission line and components. In particular, waveguide and coaxial cable, the circulator, receiver protection switches, and preselection filters, if employed, contribute to this loss value. As with the transmit path, the specified receiver antenna gain may or may not include the loss between the receive aperture and the receive antenna port. All losses up to the point in the system at which the noise figure is specified must be considered. Again, the vendor data provide maximum and average values, but actual measurements provide the best information on these losses.

2.6.4 Signal Processing Loss

Most modern systems employ some form of multi-pulse processing which improves the single-pulse SNR by the factor *n* which is the number of pulses in a "*coherent processing interval*" (CPI), or dwell time. The effect of this processing gain is included in the average power form of the RRE, developed in section 2.11. If the single-pulse, peak power form is used, then typically, a gain term is included in the RRE which assumes perfect coherent processing gain. In either case, imperfections in signal processing are then accounted for by adding a signal processing loss term. Some examples of the signal processing effects which contribute to system loss are beam scan loss, straddle loss (sometimes called scalloping loss), automatic detection (CFAR) loss, and mismatch loss. Each of these is described further in the following paragraphs.

Beam shape loss arises because the radar equation is developed using the antenna gains (transmit and receive) as if the target is at the center of the beam pattern for every pulse processed during a CPI. In many system applications, such as a mechanically scanning search radar, the target will at best be at the center of the beam pattern for only one of the pulses processed for a given dwell. If the CPI is defined as the time for which the antenna beam scans in angle from the -3 dB point, through the center, to the other -3 dB point, the average loss in signal compared to the case in which the target is always at the beam peak for a typical beam shape, is about 1.6 dB. Of course, the precise value depends on the particular shape of the beam, as well as the scan amount during a search dwell, so a more exact calculation may be required.
Figure 2.4 depicts a scanning antenna beam, such that the beam scans in angle from left-to-right. A target is depicted as an aircraft. There are five (5) beam positions shown. (Often there would be more than five pulses for such a scan, but only five are shown here for clarity.) For the first pulse, the target is depicted at 2.8 dB below the beam peak, the second, 0.6 dB, the third at nearly beam center (-0.0 dB), the fourth at -0.3 dB and the fifth at -2.2 dB. dB, Beam shape losses are discussed in more detail in Chapter 9.

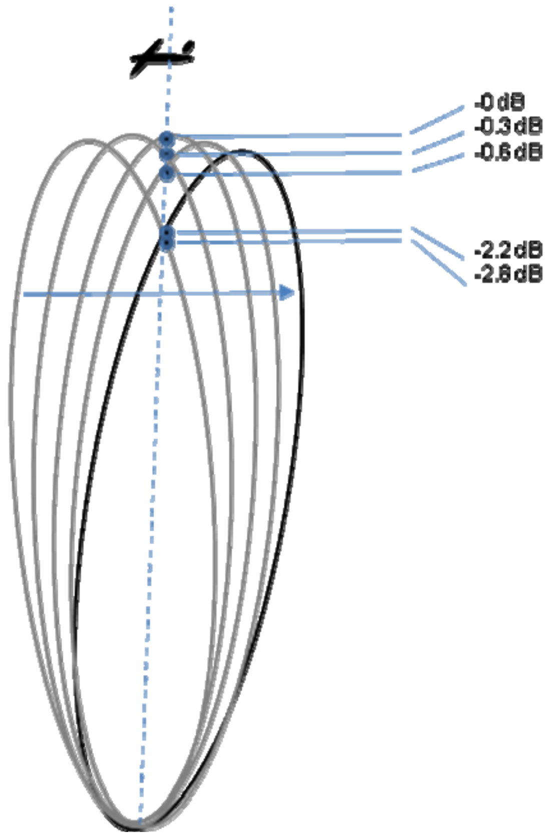


Figure 2.4. Target signal loss due to beam scan.

In a tracking mode, since the angular position of the target is known, the antenna beam can be pointed directly at the target, such that the target is in the center (or at least very close to the center) of the beam for the entire CPI. If this is the case, the SNR for track mode will not be degraded due to the beamshape loss.

Straddle loss (sometimes called scalloping loss) arises because, as with the assumption that the target is at the center of the antenna beam, it is also assumed that the target is centered in a range bin, defined by the sampling time relative to the transmit time, and is also centered in a Doppler filter, defined by the Doppler processing parameters. It may be that the centroid of the received target pulse/spectrum is somewhere between two range bins, and somewhere between two Doppler filters, reducing the target signal power. Figure 2.5 depicts a series of several Doppler filters, with a target depicted at a position in frequency such that it is not centered in any filter, but “straddling” two filters. A similar condition will occur in the range (time) dimension, that is, a target signal will, in general, appear somewhat between two range sample times. The worst-case loss due to range and Doppler straddle depends on a number of sampling and resolution parameters, but is usually no more than 3 dB each in range and Doppler. However, usually the average loss rather than the worst case is considered when predicting the SNR. The loss experienced is dependent on the extent to which successive bins overlap, that is, the depth of the dip between two adjacent bins. Thus, straddle loss can be reduced by oversampling in range and Doppler, reducing the depth of the "scallop" between bins. Depending on these details, an expected average loss of about 1 dB for range and 1 dB for Doppler is often reasonable. If the system parameters are known, a more rigorous analysis should be performed. Straddle loss is analyzed in more detail in Chapters 14 and 17. As with the beam shape loss, in the tracking mode the range and Doppler sampling can be adjusted so that the target is centered in these bins, eliminating the straddle loss.

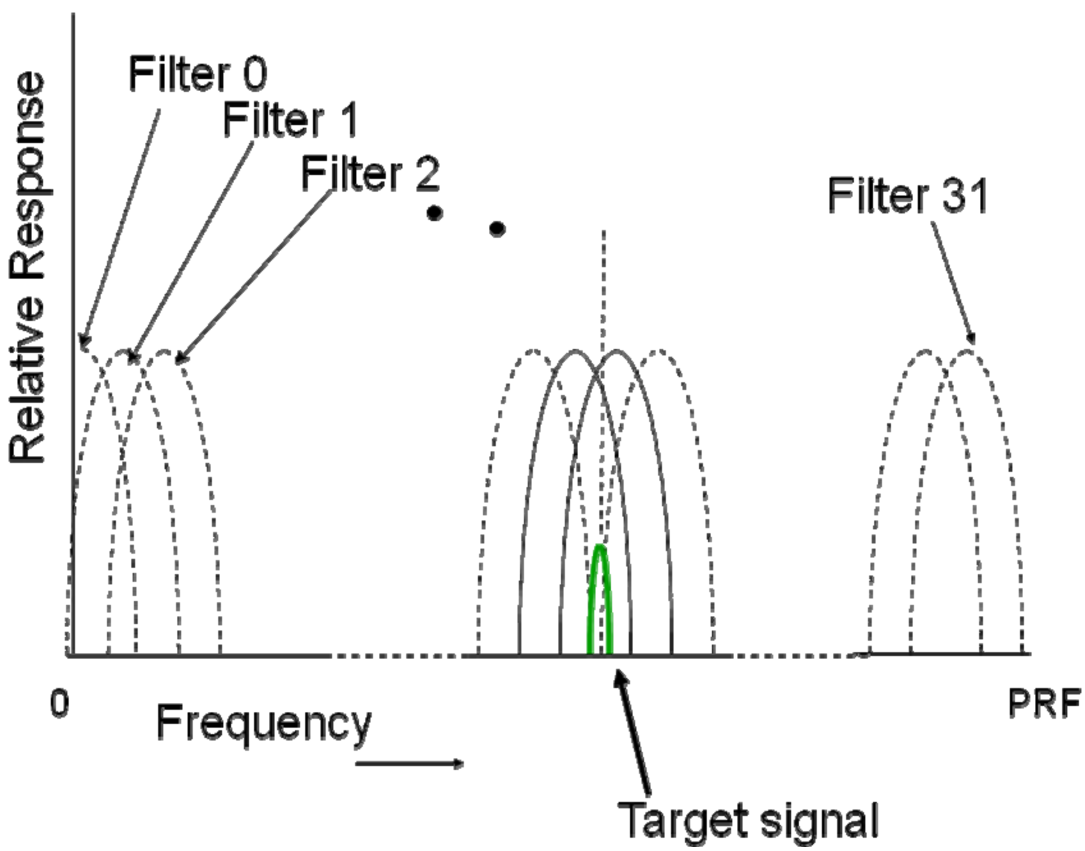


Figure 2.5. Doppler filter bank, showing a target straddling two filters.

Most modern radar systems are designed to automatically detect the presence of a target in the presence of interfering signals, such as atmospheric and receiver noise, intentional interference (jamming), unintentional interference (electromagnetic interference), and clutter. A constant-false-alarm-rate (CFAR) processor might be used to determine the presence of a target. The processor compares the signal amplitude for each resolution cell to a local average, or mean of the surrounding cells, which ostensibly contain only interference signals. A threshold is established at some level (several standard deviations) above such an average, to maintain a predicted average rate of false alarm. If the interference level is constant, and known, then an optimum threshold level can be determined, that will maintain a fixed P_{fa}. But, because the interfering signal is varying, the interference may be higher than the mean is some region of the sample space, and it may be lower than the mean in other regions. To avoid a high P_{fa} in any region, the threshold will have to be somewhat higher than the optimum setting. This means that the P_d will be somewhat lower than optimum. The consequence is that the SNR must be higher than that required for an optimum detector for a given P_d. The SNR is not increased due to this effect, however, it is considered to be a *loss*. Such a loss in detection performance is called a *CFAR loss*, and is on the order of 1 to 2 dB for most standard conditions. Chapter 16 provides a complete discussion of the operation of a CFAR processor and its attendant losses.

The SNR is estimated given a matched filter in the receiver. A matched filter is a receiver frequency response designed to maximize the output SNR; see Chapters 14 and 20 for a detailed discussion of matched filters. Thus, it is assumed that most of the target signal comes through the receiver filter, and that the noise bandwidth is no more than that required for a given target signal bandwidth. For a simple (unmodulated) pulse, this occurs when the noise bandwidth is about $1.2/\tau$, depending on the spectral shape of the signal and the particular implementation of the receiver filters. If the filter bandwidth is any wider than this, though some additional target signal increase is experienced, the noise power increases proportionally with the increase of the bandwidth. That is, if the bandwidth doubles, the noise power doubles, but the signal power increases only marginally, reducing the SNR. This decrease in SNR is the mismatch loss, resulting from a receiver bandwidth that is not optimally selected for the transmitted pulse shape.

For a pulse compression system, the matched filter condition is obtained only when there is no Doppler frequency offset on the target signal, or the Doppler shift is compensated in the processing. If neither of these is the case, a Doppler mismatch loss is usually experienced.

2.7 Multiple Pulse Effects

Seldom is a radar system required to detect a target on the basis of a single transmitted pulse. Usually, several pulses are transmitted with the antenna beam pointed in the direction of the (supposed) target, and the received signals from these pulses are processed to improve the detectability of a target in the presence of clutter by performing coherent integration (averaging, see Chapter 15), or more often, spectral analysis (Doppler processing, see Chapter 17), to improve the detectability of the target in the presence of noise. This section describes the effect of such processing. Note that the Doppler processing is equivalent to coherent integration, insofar as the improvement in SNR is concerned.

Often the antenna beam is pointed in a given azimuth-elevation position while several (typically on the order of 16 or 20) pulses are transmitted and received. In this case, the integrated signal-to-noise ratio is the important factor in determining detectability. If coherent integration processing is employed, the SNR resulting from coherently integrating N pulses ($SNR_c(N)$) is N times the single-pulse SNR ($SNR(1)$):

$$SNR_c(N)=N\cdot SNR(1) \tag{2.14}$$

A more appropriate form of the RRE when N pulses are coherently combined is thus

$$SNR = \frac{P_t G_t G_r \lambda^2 \sigma N}{(4\pi)^3 kT_0 BFL_s R^4} \tag{2.15}$$

This form of the RRE is often used to determine the SNR of a system, knowing the number of pulses coherently processed.

Coherent processing uses the phase information when averaging data from multiple pulses. It is also common to use *noncoherent integration* to improve the SNR. Noncoherent integration discards the phase of the individual echo samples, averaging only the magnitude information. The integration gain that results from noncoherent integration of *N* pulses is harder to characterize than in the coherent case, but for many cases is at least $\sqrt{N}$ but less than *N*:

$$\sqrt{N} \cdot SNR(1) \le SNR(1) \le N \cdot SNR(1) \tag{2.16}$$

Chapter 15 provides additional detail on noncoherent integration.

2.8 Solving for other variables

Range as a Dependent Variable

There are occasions for which it is desirable to determine the range at which a given target RCS can be detected with a given SNR. In this case, solving Eq. (2.15) for *R* yields

$$R = \left[\frac{P_t G_t G_r \lambda^2 \sigma N}{(4\pi)^3 SNRkT_0 BFL_s} \right]^{\frac{1}{4}} \tag{2.17}$$

In using (2.17), however, bear in mind that some of the losses in L_s (primarily atmospheric attenuation) are range-dependent.

Solving for Minimum Detectable RCS

There are other occasions for which it is important to determine the minimum detectable radar cross section. This calculation is based on assuming that there is a minimum SNR,

SNR_{min}, required for reliable detection (see Chapter 15). Substituting SNR_{min} for SNR and solving Eq. (2.15) for radar cross section yields

$$\sigma_{\min} = SNR_{\min} \frac{(4\pi)^3 kT_0 BFL_s R^4}{P_t G_t G_r \lambda^2 N} \tag{2.18}$$

Clearly, Eq. (2.15) could be solved for any of the variables of interest. However, the forms given above are those most commonly used.

2.9 Decibel Form of the Equation

Many radar systems engineers use the form of the radar equation presented above, which is given in linear space. That is, the equation consists of a set of values that describe the radar parameters in watts, seconds, meters, *etc*., and the values in the numerator are multiplied, and divided by the product of the values in the denominator. Many radar systems engineers prefer to convert each term to the decibel (dB) value, and add the numerator terms and subtract the denominator terms, resulting in SNR being expressed directly in dB. The use of this form of the radar equation is strictly based on the preference of the analyst. Many of the terms in the SNR equation are naturally determined in dB space, and many are determined in linear space, so in either case, some of the terms must be converted from one space to the other. The terms which normally appear in dB space are antenna gains, RCS, noise figure, and system losses. It remains to convert the remaining values to dB equivalents, and then proceed with the summations. Equation (2.19) demonstrates the dB form of the RRE.

$$\begin{aligned} SNR\ (\text{dB}) &= 10\log_{10}(P_t) + G_t\ (\text{dB}) + G_r\ (\text{dB}) + 20\log_{10}(\lambda) + \sigma(dBsm) + 10\log_{10}(N)\ldots \\ &\ldots - 33 - (-204) - 10\log_{10}(B)\ (\text{dB}) - F\ (\text{dB}) - L_s\ (\text{dB}) - 40\log_{10}(R) \end{aligned} \tag{2.19}$$

In the above presentation the constant values (such as π and kT_0, *etc*.) have been converted to the dB equivalent. For instance, $(4\pi)^3$ equals approximately 1,984, and

$10\log_{10}(1984)$ = -33 dB. In addition to the simplicity associated with adding and subtracting, the dB form lends itself more readily to tabulation.

2.10 Signal Processing Gain – Intrapulse Coding

The factor of *N* in Eq. (2.15) is a form of signal processing gain resulting from coherent integration of multiple pulses. Signal processing gain can also arise from processing pulses with intra-pulse modulation, or coding. Radar systems are sometimes required to produce a given probability of detection, which might require a given SNR, while at the same time maintaining a specified range resolution. When using simple (unmodulated) pulses, the receiver bandwidth is inversely proportional to the pulse length τ, as discussed earlier. Thus, increasing the pulse length will increase the SNR. However, range resolution is also proportional to τ, so the pulse must be kept short to meet range resolution requirements. A way to overcome this conflict is to maintain the average power by transmitting a wide pulse, while maintaining the range resolution by incorporating a wide bandwidth in that pulse – wider than the reciprocal of the pulse width. This extended bandwidth can be achieved by incorporating coding (phase or frequency) within the pulse. Proper matched filtering of the received pulse is needed to achieve both goals. The use of intra-pulse coded waveforms to achieve fine range resolution while maintaining high average power is called *pulse compression*.

As presented in Chapter 20, the appropriate form of the radar range equation for this type of system is:

$$SNR_{pc} = SNR_u \tau\beta \qquad (2.20)$$

Where τ is the pulse length and *B* is the pulse bandwidth.
Substituting this into (2.15) we get:

$$SNR = \frac{P_t G_t G_r \lambda^2 \sigma N}{(4\pi)^3 kT_0 BFL_s R^4} \tau\beta \qquad (2.21)$$

While this concept is described in detail in Chapter 20, suffice it to say that the following development of the RRE is appropriate for such pulse compression systems, as well as simple pulse systems.

2.11 Average Power Form of the Radar Range Equation

Given that the radar usually transmits several pulses and processes the results of those pulses to detect a target, there is an often-used form of the radar range equation that replaces the peak power, number of pulses processed, and instantaneous bandwidth terms with average power and dwell time. This form of the equation is universally applicable, independent of the transmitted waveform, and takes into account all signal processing gain effects.

The average power form of the RRE can be obtained from the peak power form with the following series of substitutions:

$$T_d = \text{dwell time} = N \cdot PRI = N/PRF \quad (2.22)$$

Solving for N:

$$N = T_d \cdot PRF \quad (2.23)$$

$$\text{Duty cycle} = \tau \cdot \text{PRF} \quad (2.24)$$

$$P_{avg} = P_t \cdot (\text{duty cycle}) = P_t \cdot (\tau \cdot PRF) \quad (2.25)$$

And, for a simple pulse of width τ, the optimum receiver bandwidth B is:

$$B = 1/\tau \quad (2.26)$$

Combining (2.23), (2.25), and (2.26), and solving for P_t, we get:

$$P_t = P_{avg} T_d B/N \quad (2.27)$$

Substituting P_t in (2.15) for the P_t in (1), we get:

$$SNR = \left(\frac{P_{avg}T_d B}{N}\right)\frac{G_t G_r \lambda^2 \sigma N}{(4\pi)^3 kT_0 F L_s B R^4} = \frac{P_{avg}T_d G_t G_r \lambda^2 \sigma}{(4\pi)^3 kT_0 F L_s R^4} \tag{2.28}$$

Beginning with the RRE for a pulse compression system, (2.21) the same substitutions can be made, resulting in:

$$SNR = (\frac{P_{avg}T_d}{N\tau})\frac{G_t G_r \lambda^2 \sigma N}{(4\pi)^3 kT_0 B F L_s R^4}\tau\beta = \frac{P_{avg}T_d G_t G_r \lambda^2 \sigma}{(4\pi)^3 kT_0 F L_s R^4} \tag{2.29}$$

In this form of the equation, the average power–dwell time terms provide the energy in the processed waveform, while the kT_0F terms provide the noise energy.

2.12 A Graphical Example

It is instructive to present an example of a hypothetical radar system SNR analysis in tabular form and in graphical form. The example plot of SNR as a function of target range presented here is that of a ground- or air-based radar system, having the following characteristics.

Transmitter:	10 kilowatt peak
Frequency:	9.4 GHz
Pulse width:	0.1 microseconds
PRF:	1 kilohertz
Antenna:	0.8 meter diameter circular aperture
Target RCS:	0 dBsm, -10 dBsm
Processing dwell time	7.62 milliseconds
Receiver Noise Figure:	2.5 dB
Transmit Losses:	3.1 dB

Receive Losses:	2.4 dB
Signal Processing Losses:	3.2 dB
Atmospheric Losses:	0.16 dB/km (one way)
Target Range:	1 to 100 km

It is customary to plot the SNR in dB for the two target RCS values as a function of range, from the minimum range of interest to the maximum range of interest. Figure 2-6 is an example of the plot resulting from the parameters given above. If it is assumed that the target is reliably detected at an SNR of about 15 dB, then the 1 m^2 target will be detectable at a range of approximately 75 km, while the 0.1 m^2 target will be detectable at approximately 59 km.

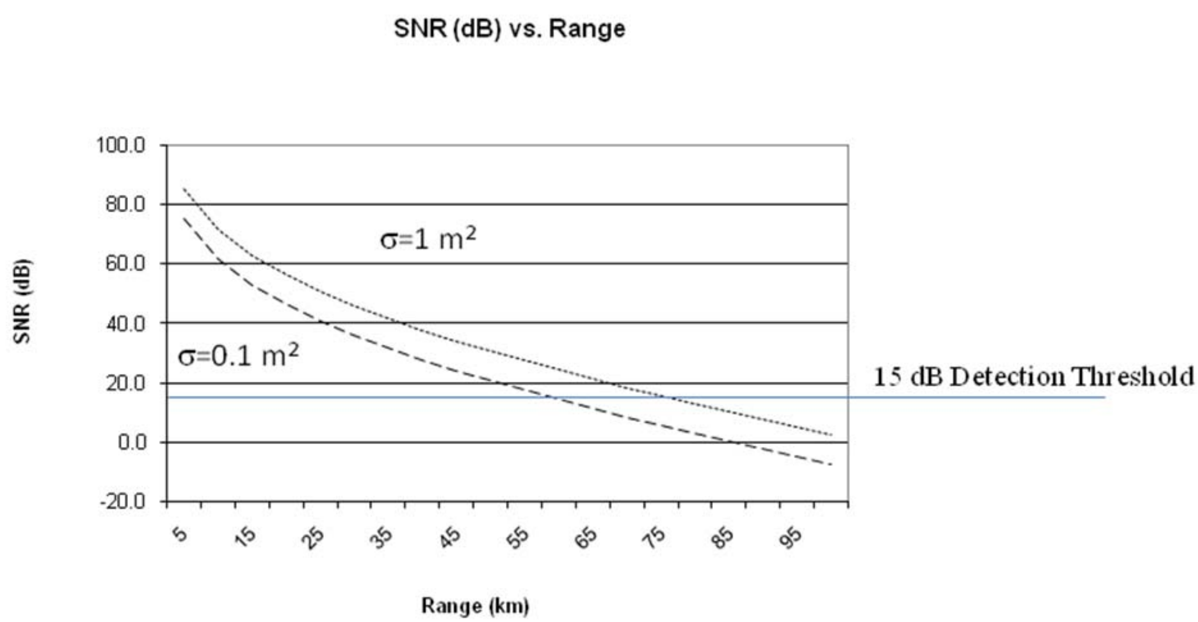


Figure 2-6. Graphical solution to radar range equation.

Once the formulas for this plotting example are developed in a spreadsheet program such as Excel®, then it is relatively easy to extend the analysis to plotting probability of detection (see Chapter 3) and tracking measurement precision (Chapter 17) as functions of range, since these are dependent primarily on SNR and additional fixed parameters.

2.13. Extended Clutter as the Target

Though the intent is usually to detect a discrete target in the presence of noise and other interference, there are often unintentional signals received from other objects in the antenna beam. Unintentional signals can result from illuminating clutter. Such clutter can be on the surface of the earth, either on land or sea, or in the atmosphere, such as rain and snow. For surface clutter, the area illuminated by the radar antenna beam determines the signal power. For atmospheric clutter, the volume is defined by the antenna beamwidths and the pulse length. The importance of the radar equation is to determine the target signal-to-interference ratio (SIR), given that the interference is (surface or atmospheric) clutter. In the case of either, the ratio is determined by dividing the target signal S by the clutter signal S_c. All of the terms in the radar equation cancel except for the RCS (σ or σ_c) terms, resulting in:

$$SIR = \frac{\sigma}{\sigma_c} \tag{2.30}$$

In some cases, as with a ground mapping radar or weather radar, the intent is to detect these objects. In other cases, the intent is to detect discrete targets in the presence of these interfering signals. In either case, it is important to understand the signal received from these "clutter" regions. The use of the RRE for the signal resulting from clutter is summarized by substituting the RCS of the clutter cell into the RRE in place of the target RCS. Chapter 5 describes characteristics and statistical behavior of clutter in detail. A summary is provided here.

2.13.1 Surface clutter

The radar cross section value for a surface clutter cell is determined from the average reflectivity σ^0 of the particular clutter type, in square meters per unit area, times the area of the clutter cell A_c illuminated by the radar:

$$\sigma_{cs} = A_c \sigma^0 \tag{2.31}$$

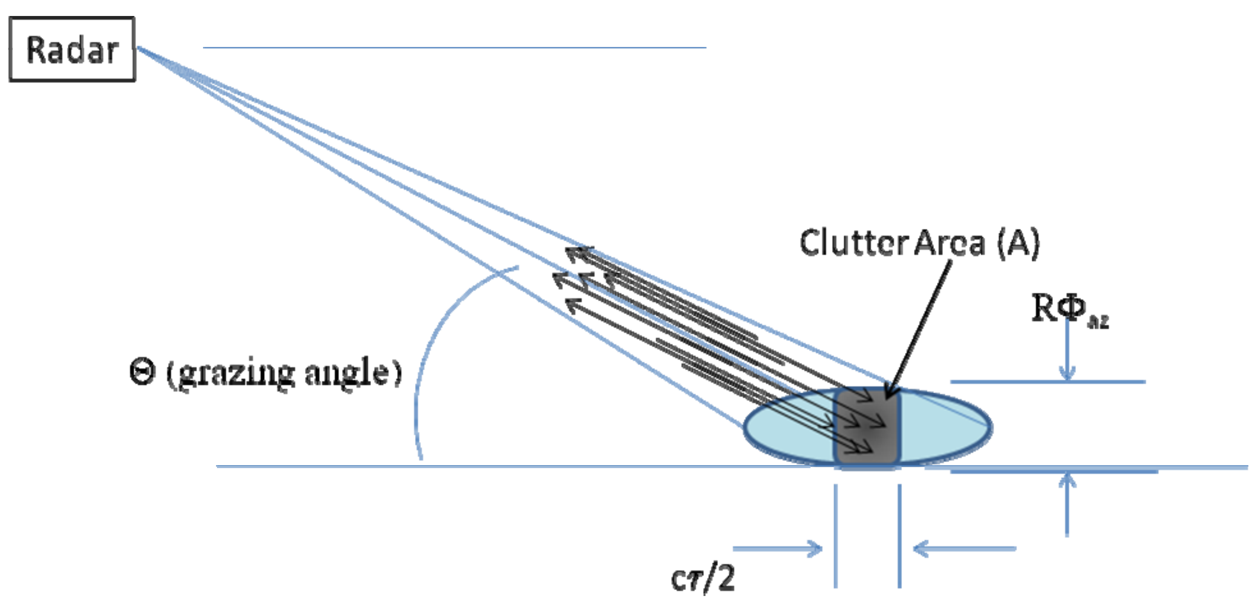
where:

σ_{cs} is the surface clutter radar cross section,

σ^0 is the surface backscatter coefficient (average reflectivity per unit area), and

A_c is the area of the illuminated (ground or sea surface) clutter cell.

The area A_c of the cell illuminated on the ground by the radar, depends on the azimuthal beamwidth of the antenna, and either the pulse width or the elevation beamwidth of the antenna, depending on the grazing angle[7]. Chapter 5 includes a thorough discussion of the area calculation, which depends on the radar scenario. Figure 2.7 depicts the area of a clutter cell illuminated on the surface. The clutter consist of a multitude of individual reflecting objects (rocks, grass, dirt mounds, twigs, branches, etc.) often called *scatterers*. The resultant of these many *scatterers* is a single net received signal back at the radar receiver.



[7] Grazing angle is sometimes called depression angle. The difference is that grazing angle is measured between the radar beam and the surface plane, while depression angle is measured between the radar beam and the horizontal. If the surface plane is level (horizontal), the two are the same.

Figure 2.7. Area (surface) clutter.

2.13.2 Volume clutter

The radar cross section value for volumetric clutter cell is determined from the average reflectivity of the particular clutter type per unit volume, η, times the volume of the clutter cell V illuminated by the radar:

$$\sigma_{cV} = V\eta \tag{2.32}$$

where:

σ_{cV} is the volume clutter radar cross section,

η is the volumetric backscatter coefficient (average reflectivity per unit volume), and

V is the volume of the illuminated clutter cell.

The volume V of the cell illuminated by the radar, depends on the azimuthal beamwidth of the antenna, the elevation beamwidth of the antenna and the pulse width as depicted in Figure 2.8. Chapter 5 includes a thorough discussion of the volume calculation, which depends on the radar scenario. Figure 2.8 depicts the volume of a clutter cell illuminated in the atmosphere. The clutter consists of a multitude of individual reflecting objects (rain, fog droplets, etc.). The resultant of these many *scatterers* is a single net received signal back at the radar receiver.

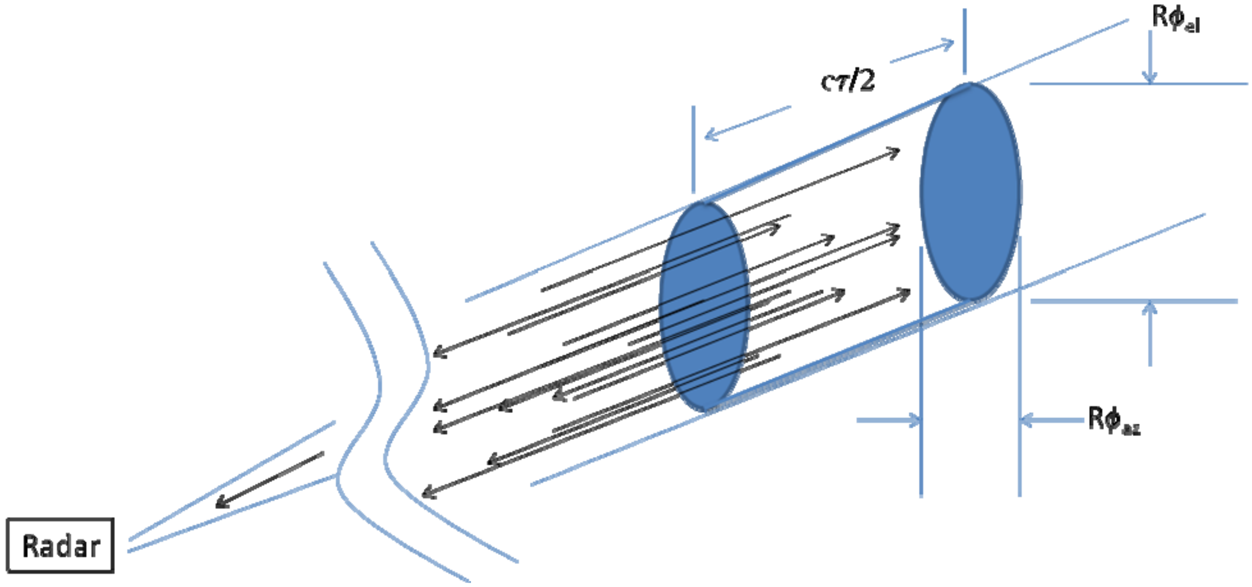


Figure 2.8. Volumetric (atmospheric) clutter.

2.14. One-way (link) equation

To this point, the interfering signals discussed have been receiver noise, and clutter. In many cases, intentional jamming is the most limiting interference. Jamming signals are one of two varieties, noise and false targets. The effect of noise jamming is to degrade the SNR of target signals in the radar receiver to delay initial detection, or degrade tracking performance. The intent of false target jamming is to present so many target-like signals that the radar processor is overloaded, or create false track files. Volume 2 has a complete discussion of the details of the jamming environment. In any case, the power received from a jammer is required to determine its effect on radar performance. Since the signal from the jammer to the radar has to propagate in only one direction, a simplification of the radar equation for one-way propagation is valuable.

The first step is to determine the power density at a distance R_{jr} from the jammer radiating antenna to the radar. The jammer signal power density Q_j in watts per square meter at a distance (range) R from a transmitting source has been presented in equation 2.1. It depends on the source's transmitted power, in the case of a jammer P_j, the range, in this case R_{jr}, the gain of the jammer transmitting antenna, G_j, and the losses through the propagation medium (L_{atm}). Usually, the antenna gain includes the losses between the antenna input terminal and the aperture. The power density, Q_j from an isotropic radiating jammer source Q_j is the total power, reduced by losses and distributed uniformly over the surface area of a sphere of radius R_{jr},

$$Q_j = \frac{P_j}{4\pi R_{jr}{}^2 L_{atm}} \tag{2.33}$$

The power density given in 2.33 is increased by the effects of a jammer antenna which concentrates the energy in a given direction. The power density Q_j, at the center of the beam for a radiating source with an attached jammer antenna of gain G_j is

$$Q_j = \frac{P_j G_j}{4\pi R_{jr}{}^2 L_{atm}} \tag{2.34}$$

The jammer antenna peak gain, G_j, accounts for the fact the transmitted radio waves are "focused" by the antenna in a given direction, thus increasing the power density in that direction.

Next, consider the radar receiving system at a distance R_{jr} from the jammer to the radar. Such a receiving system will have a directive antenna having an effective area of A_e. The total power received at the radar from the jammer (P_{rj}) is:

$$P_{rj} = Q_j A_e = \frac{P_j G_j A_e}{4\pi R_{jr}{}^2 L_{atm}} \tag{2.35}$$

Equation 2.35 is the *one-way link equation*. It is very useful in predicting the performance of a one-way system such as a communications system or, for a radar, a jammer.

Often, the antenna gain is known instead of the effective area. In this case, using equation 2.7 the area can be replaced by:

$$A_e = \frac{G_{rj}\lambda^2}{4\pi} \tag{2.36}$$

resulting in:

$$P_{rj} = \frac{P_j G_j G_{rj} \lambda^2}{(4\pi)^2 R_{jr}{}^2 L_{atm}} \tag{2.37}$$

where G_{rj} is the gain of the radar antenna in the direction of the jammer. This is important, since the radar antenna is not necessarily pointed directly at the jammer. In this case, the main beam gain is not appropriate, but the sidelobe gain is. The sidelobes are not easily determined until the antenna is tested in a high quality environment.

2.15 Search Form of the Equation

Section 1.8.1 (Chapter 1) and Figure 1.23 depict a scanning antenna being used to scan a volume. Analysis of such a system designed to search a given solid angular volume (Ω) in a given search frame time (T_{fs}) is often made easier by using the so-called "search form" of the radar equation. The predominant figure of merit for such as system is the *power-aperture product* of the system. This section derives and describes this form of the radar equation.

The total time required to search a volume (T_{fs}), is easily determined from the number of beam positions required (M), multiplied by the dwell time required at each of these positions (T_d):

$$T_{fs} = M \cdot T_d \tag{2.38}$$

The number of beam positions required is the solid angular volume to be searched, Ω, divided by the solid angular volume of the antenna beam, which is approximately the product of the azimuth and elevation beamwidths:[8]

$$M = \Omega / \theta_{az} \theta_{el} \tag{2.39}$$

[8] There are $(180/\pi)^2 \cong 3282.8$ square degrees in a steradian.

If the estimated beamwidth is about $1.22\lambda/D$, then the solid angle $\theta_{az}\theta_{el}$ is about $1.5\lambda^2/D^2$. The area A of a circular aperture is $\pi D^2/4$, and, from Eq. (2.8) the effective aperture, depending on the weighting function and shape, is about $0.6\pi D^2/4$, leading to

$$\theta_{az}\theta_{el} \approx \lambda^2/A_e \tag{2.40}$$

The antenna gain G is related to the effective area by:

$$G = 4\pi A_e/\lambda^2 \tag{2.41}$$

Using the above substitutions into (2.15), it can be shown that the resulting SNR can be expressed as

$$SNR = (P_{avg}A_e)\frac{1}{4\pi k T_0 F L_s}\left(\frac{\sigma}{R^4}\right)\left(\frac{T_{fs}}{\Omega}\right) \tag{2.42}$$

By substituting the minimum SNR required for reliable detection, SNR_{min}, for SNR and arranging the terms differently, the equation can be repartitioned to place the "user" terms on the right side, and the system designer terms on the left side.

$$\frac{P_{avg}A_e}{L_s T_0 F} \geq SNR_{\min} 4\pi k\left(\frac{R^4}{\sigma}\right)\left(\frac{\Omega}{T_{fs}}\right) \tag{2.43}$$

This *power-aperture form of the RRE* provides a convenient way to partition the salient radar parameters (P_{avg}, A_e, L_s, and F) given the requirement to search for a target of RCS σ at range R over a solid angle volume Ω in time T_{fs}. Since it is derived from the average power form of the basic RRE (2.15), it is applicable for any waveform, whether pulse compression is used or not, and for any arbitrary length dwell time. It does assume that the entire radar resources are used for search; that is, if the system is a multifunction radar then a loss must be included for the time the radar is in the track mode, or performing some function other than search.

2.16 Track Form of the Equation

With modern radar technology moving toward *electronically scanned arrays* (ESAs, or phased arrays), target tracking systems can track multiple targets simultaneously. As with the search form of the radar equation, analysis of a system designed to track multiple targets with a given precision is often described in terms of the power-aperture-cubed, or equivalently, the power-aperture-gain form of the radar range equation. This variation is also called the track form of the RRE. This section derives and describes this form of the radar range equation.

Recalling (2.7), the relationship between an antenna's gain G and its effective area A_e is

$$G \cong \frac{4\pi A_e}{\lambda^2} \tag{2.44}$$

The approximate beamwidth (BW) of an antenna in degrees is [6]:

$$\theta \quad (\text{degrees}) = k\frac{\lambda}{D} \cong 70\frac{\lambda}{D} \tag{2.45}$$

Since there are 180/π degrees in a radian, this is equivalent to:

$$\theta \quad (\text{radians}) \cong 1.22\frac{\lambda}{D} \tag{2.46}$$

Of course, there is some variation in this estimate due to specific design parameters and their effects, however, this approximation serves as a good estimate for now. Also, the effective area for a circular aperture of diameter D is [1]

$$A_e \cong \frac{0.6\pi D^2}{4} \tag{2.47}$$

For a more general elliptical aperture, it is:

$$A_e \cong \frac{0.6\pi D_{major} D_{\min or}}{4} \tag{2.48}$$

From the above, the solid angle beamwidth is

$$\theta^2 \cong \frac{\lambda^2 \pi}{4A_e} \tag{2.49}$$

As described in Chapter 18, a common expression for the estimated tracking noise (standard deviation, σ_θ) is:

$$\sigma_\theta \cong \frac{\theta}{k_m \sqrt{2SNR}} \tag{2.50}$$

where k_m is a tracking system parameter. Substituting (2.49) into (2.50) and solving for SNR gives

$$SNR \cong \frac{\pi \lambda^2}{8A_e {k_m}^2 {\sigma_\theta}^2} \tag{2.51}$$

Given a requirement to track N_t targets, each at an update rate of r measurements per second, the dwell time T_d per target is

$$T_d = \frac{1}{r \cdot N_t} \tag{2.52}$$

Finally, substituting Eqs. (2.44), (2.51) and (2.52) into (2.15) and rearranging terms gives

$$\frac{\pi\lambda^2}{8k_m{}^2\sigma_\theta^2} = \frac{P_{avg}A_e{}^3\sigma}{4\pi r N_t \lambda^2 k T_0 F L_s R^4} \tag{2.53}$$

As with the search form of the RRE, the terms are arranged so that the "user" terms are on the right and the "designer" terms are on the left, providing:

$$\frac{P_{avg}A_e{}^3 k_m^2}{\lambda^4 L_s T_0 F} = \left(\frac{\pi^2}{2}\right)\left(\frac{krN_t R^4}{\sigma\cdot\sigma_\theta{}^2}\right)\left(\frac{1}{\cos^5(\theta_{scan})}\right) \tag{2.54}$$

This form of the RRE shows that, given N_t targets of RCS σ at range *R* to track, each at rate *r* and with a precision σ_θ, the power–aperture-cubed of the radar becomes the salient determinant of performance.

Equation (2.54) also introduces a cosine^5 term that has not been seen so far. This term accounts for the beam broadening effect and the gain reduction that accompanies the scanning of an electronically scanned antenna to an angle of θ_{scan} from array normal. To a first order, the beamwidth increases as the beam is scanned away from array normal by the cosine of the scan angle due to the reduced effective aperture along the beam pointing direction. The gain is also reduced by the cosine of the scan angle due to the reduced effective aperture, and by another cosine factor due to the off-axis gain reduction in the individual element pattern, resulting in a net antenna gain reduction by a factor of $\cos^2(\theta_{\text{scan}})$. SNR is reduced by the product of the transmit and receive antenna gains, thus squaring the reduction to a factor of $\cos^4(\theta_{\text{scan}})$. Therefore, to maintain a constant angle precision, the radar sensitivity needs to increase by $\cos^5(\theta_{\text{scan}})$: $\cos^4$ due to gain effects, and another cosine factor due to beam broadening. This term is an approximation, because the individual element pattern is not strictly a cosine function. However, it is a good approximation, particularly at angles beyond about 30 degrees. Additional details on the effect of scanning on the gain and beamwidth of ESAs is given in Chapter 9.

Figure 2-9 is a plot of the loss associated with scanning an electronically scanned beam. Because of the wider antenna beam and lower gain, the radar energy on target must increase by this factor. Compared to a target located broadside to the array, a target at a 45 degree scan angle requires increased energy on the order of 7 dB to maintain the same tracking precision; at 60 degrees, the required increase is 15 dB! Often, once a target is in track, it will be close to the antenna normal (for an airborne interceptor, for example) and for a fixed or ship-based system, the scan loss is partially offset by using longer dwell times at the wider scan angles. Also, for a target in track, straddle losses (see Chapters 14, 17, and 18) are reduced because the target is likely to be near the center of the beam, and also to be nearly centered in the range and Doppler bins. Nonetheless, the system power and aperture must be robust enough to allow for shorter than average dwell times at near-normal scan angles, so as to allow time for longer dwells at the extreme angles to offset large scan losses.

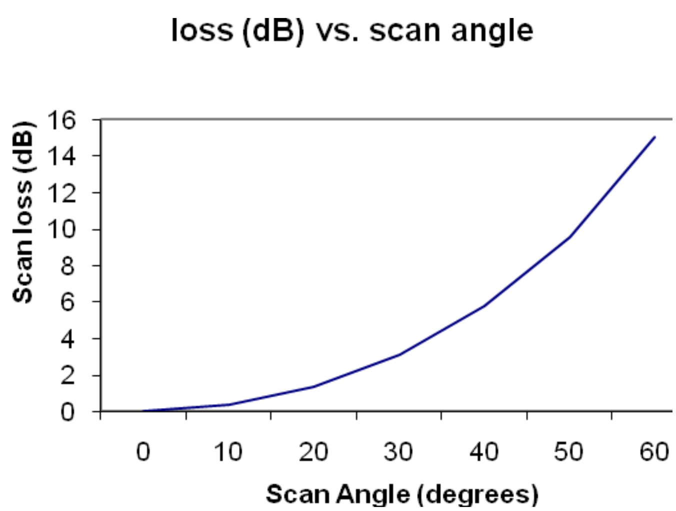


Figure 2-9. Scan loss vs. angle for an electronically scanned antenna beam.

Occasionally, another form of the radar equation as it relates to a tracking system is encountered. Beginning with Eq. (2.10), repeated here for convenience,

$$SNR = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 k T_0 B F L_s R^4} \tag{2.55}$$

making the substitution P_{avg} = average power = $P_t \cdot \tau \cdot PRF$ and solving for P_{avg} gives

$$P_{avg} = \frac{SNR(4\pi)^3 R^4 kT_0 BFL_s \tau PRF}{G_t G_r \sigma \lambda^2} \tag{2.56}$$

Substituting from Eq. (2.44) of the antenna gain gives

$$P_{avg} = \frac{4\pi SNR \cdot R^4 kT_0 FL_s PRF \lambda^2}{\sigma A_e^{\,2}} \tag{2.57}$$

or, rearranged,

$$\frac{P_{avg} A_e^{\,2}}{L_s F \lambda^2} = \frac{4\pi SNR \cdot R^4 kT_0 PRF}{\sigma} \tag{2.58}$$

Equation (2.58) is the power-aperture-squared form of the radar range equation. This form is most useful when the required SNR is known, whereas Eq. (2.54) is used when the required tracking precision (σ_θ) is known. The two forms (2.54) and (2.58) are equivalent when the substitutions associated with the relationship between aperture, SNR, and tracking precision are incorporated.

A final form, also sometimes encountered is found by replacing one of the A_e terms on the left side of (2.58) with its equivalent in terms of gain,

$$A_e = \frac{G\lambda^2}{4\pi} \tag{2.59}$$

resulting in

$$\frac{P_{avg} A_e G}{L_s F} = \frac{(4\pi)^2 SNR \cdot R^4 kT_0 PRF}{\sigma} \tag{2.60}$$

which does not include wavelength (λ). Clearly some of the terms are dependent on λ, such as L and F. Equation (2.60) is often called the power-aperture-gain form of the RRE.

2.17 Some Implications of the Radar Range Equation

Now that the reader is somewhat familiar with the basic RRE, its use in evaluating radar performance can be explored.

2.17.1 Average Power and Dwell Time.

Considering Eq. (2.29), the average power form of the RRE, it can be seen that for a given hardware configuration which "freezes" the P_{avg}, G, λ, F, and L_s, the dwell time T_d can easily be changed without affecting the hardware design. This directly affects the SNR. For example, doubling the dwell time increases the SNR by 3 dB. As will be seen in Chapter 3, this increase in SNR improves the detection statistics, that is, the probability of detection P_D for a given probability of false alarm P_{FA} will improve.

For an electronically scanned antenna beam, the radar received signal power, and therefore the SNR degrades as the beam is scanned away from normal to the antenna surface, as was seen in the context of Eq. (2.54) and Figure 2.9

. This reduction in SNR can be recovered by adapting the dwell time to the antenna beam position. For example, whereas a mechanically scanned antenna beam might have a constant 2 msec dwell time, for the radar system using an ESA, the dwell can be adaptable. It might be 1 msec near normal (say, 0 to 30 degrees), 2 msec from 30 to 40 degrees scan angle, and 4 msec from 40 to 45 degrees.

2.17.2 Target RCS Effects

Much is being done today to reduce the radar cross section of radar targets, such as missiles, aircraft, land vehicles (tanks and trucks) and ships. The use of radar absorbing material and target shape modifications can produce a significantly lower RCS, compared to conventional designs. This technology is intended to make the target "invisible" to radar. In fact, the change in radar performance is subtle for modest changes in target RCS. For example, if the RCS is reduced by 3 dB, the detection range decreases by only about 16 percent, to 84% of the baseline value. In order to reduce the effective radar range performance by half, the RCS must be reduced by a factor of 16, or 12 dB. Thus, an aggressive RCS reduction effort is required to create significant reductions in radar detection range. Basic concepts of RCS reduction are introduced in Chapter 6.

2.18 Further Reading

Most standard radar text books have a section which develops the radar range equation, each with a similar, but different approach. A somewhat more detailed approach to development of the peak power form of the RRE can be found in Chapter 4 of Barton [4], Difranco and Rubin [6], and Sullivan [8]. A further discussion of the energy (average power) form is found in the same references. It is sometimes appropriate to present the results of RRE analysis in a tabular form. One form which has been in use since about 1969 is the so-called Blake Chart [9]. A more complete discussion of the various sources of system noise is presented in Blake [3], Chapter 4.

A comprehensive discussion of the various RRE loss terms is also presented in many of the above-referenced texts, as well as in Nathanson [5] and Barton, Cook, and Hamilton [4].

2.19 References

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[9] L. V. Blake, Radar Range-Performance Analysis, page 383. (Artech House, Dedham, MA., 1986.)

2.20 Problems

1) Target received power: What is the single-pulse received power level from a target for a radar system having the following characteristics?

Transmitter:	100 kilowatt peak
Frequency:	9.4 GHz
Pulse width:	2 microsecond
PRF:	10 kilohertz
Antenna:	2 meter diameter circular aperture
Target RCS:	-10 dBsm
Processing dwell time	18.3 milliseconds
Receiver Noise Figure:	1.8 dB
Transmit Losses:	2.9 dB
Receive Losses:	2.8 dB
Signal Processing Losses:	4.2 dB
Atmospheric Losses:	negligible
Target Range:	200 km

2) What is the single-pulse SNR for the target described in question 1?

3) What is the receiver noise power (in dBm) for a receiver having a noise figure of 2.7 dB and an instantaneous bandwidth of 10 MHz.?

4) Assume that the SNR required for reliable detection is 18 dB. The system is designed to provide the required SNR for a 1 square meter target, at a range of 60 km. What would be the equivalent detection range if the target RCS is reduced to 0.1 square meters, (Ignore the effects of propagation loss.)

5) A system SNR can be calculated on the basis of its hardware parameters, and the following loss terms.

Beam Shape Loss:	1.6 dB
Range Straddle Loss:	1.0 dB
Doppler Straddle Loss:	1.0 dB
CFAR loss:	1.2 dB
Transmit path loss:	3.2 dB
Receiver path loss:	2.8 dB

These parameters provide a SNR of 18 dB in the search mode.
In the tracking mode, all of the straddle losses can be made to zero, because of a priori knowledge of the target position and Doppler frequency. In this case, what is the tracking SNR?

6) For the preceding problem, if the antenna is an electronically scanned antenna, then the on-axis parameters are as given above. At approximately what scan angle will the track SNR drop to the on-axis search SNR?

7) What is the coherent integrated SNR for the case described in 2)?

8) The SNR can be increased by extending the dwell time. If the original dwell time is 1.75 msec., what new dwell time is required to make up for the loss in target RCS from 1 square meter to 0.5 square meters?17) A search radar system being designed by an engineering staff has to search the following solid angle volume in the stated amount of time.

Azimuth angle:	90 degrees
Elevation angle:	6 degrees
Full scan time:	1.2 seconds
Maximum Range:	50 km
Target RCS:	-10 dBsm

What is the required power aperture product of the system if, independent of wavelength, the system has the following characteristics?

Noise figure:	2.5 dB
System losses:	9.7 dB
Required SNR:	18 dB

9) Given the RRE for a target, and the same RRE for a surface clutter cell, with the appropriate substitutions, show the simplest way to express the ratio of the received signal from a target to the received signal from the clutter.

10) If the radar system in problem 1 is looking at volume clutter having a value of η = -70 dBm^2/m^3, what is the clutter RCS and the resulting target-to-clutter ratio (TCR)?

11) How much power is received by a radar receiver located 100 km from a jammer with the following characteristics? Assume that the radar antenna has an effective area of 1.2 square meters, and the main beam is pointed in the direction of the jammer. Consider only atmospheric attenuation, excluding the effects of component loss, etc. Provide the answer in terms of watts, and dBm (dB relative to a milliwatt.)

Jammer Peak Power	100 watts
Jammer Antenna gain	15 dB
Atmospheric loss	0.4 dB per km (one-way)
Radar average sidelobe level	-30 dB (relative to the main beam)

12) Using the answers of problems 2) and 11) what is the jammer-to-noise ratio?

13) Since the jammer signal reduces as the radar system gets farther away from the jammer, how far would the radar have to be from the jammer to reduce the jammer signal to match the internal receiver noise?

14) If the receiver antenna is not pointed directly at the jammer, but a -30 dB sidelobe is, then what would the answer to 11) be?

15) What would the resulting JNR be for the sidelobe jamming signal?

16) How much closer can the radar be to the jammer in this case for the jammer signal to match the receiver noise?

17) A search system being designed by an engineering staff has to search the following solid angle volume in the stated amount of time.

Azimuth angle:	90 degrees
Elevation angle:	6 degrees
Full scan time:	1.2 seconds
Maximum Range:	50 km
Target RCS:	-10 dBsm

What is the required power aperture product of the system if, independent of wavelength, the system has the following characteristics?

Noise figure:	2.5 dB
System losses:	9.7 dB
Required SNR:	18 dB

18) For the radar system in question 17, if the antenna has an effective aperture of 0.5 square meters, and the transmit duty cycle is 1%, what is the peak power required?