ECE 09.303 Fall 2019
Homework 2 Solutions
Chapter 2/20 - Electric charge, Force and Field

1.

The electron and proton in a hydrogen atom are 52.9 pm apart. Find the magnitude of the electric force between them.

INTERPRET The electron and proton each carry one unit of electric charge, but the sign of the charge is opposite for the two particles. Given the distance between them, we are to find the force (magnitude and direction) between these particles.

DEVELOP Coulomb's law (Equation 20.1) gives the force between two particles. For this problem, $|q_1| = |q_2| = e = 1.6\times10^{19}\ \text{C}$ and $r = 52.9 \times 10^{-12}$ m. Because the charges have opposite signs, the force will be attractive.

EVALUATE The force between these particles is attractive and has the magnitude

$$F = \frac{ke^2}{r^2} = \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2}{\left(52.9\times10^{-12}\ \text{m}\right)^2} = 8.2\times10^{-8}\ \text{N}$$

ASSESS Because the electron is much less massive than the proton, the electron does most of the accelerating in this two-particle system.

2.

A 68-nC charge experiences a 150-mN force in a certain electric field. Find (a) the field strength and (b) the force that a 35-μC charge would experience in the same field.

INTERPRET This problem involves calculating the electric field needed to produce the given force on the given charge, and then find the force experienced by a second charge is the same electric field.

DEVELOP Equation 20.2a shows that the electric field strength (magnitude of the field) at a point is equal to the force per unit charge that would be experienced by a charge at that point: $E = F / q$. The equation allows us to calculate E give that F = 150 mN and q = 68 nC. For part **(b)**, the force experienced by another charge q' in the same field is given by Equation 20.2b: $F' = q'E$.

EVALUATE **(a)** With $q = 68\ \text{nC}$, we find the field strength to be

$$E = \frac{F}{|e|} = \frac{150\ \text{mN}}{68\ \text{nC}} = 2.2\times10^6\ \text{N/C}$$

(b) The force experienced by a charge q' = 35 μC in the same field is

$$F' = q'E = \left(35\ \mu\text{C}\right)\left(2.21\times10^6\ \text{N/C}\right) = 77\ \text{N}$$

ASSESS The force a test charge particle experiences is proportional to the magnitude of the test charge. In our problem, since $q' = 35\ \mu\text{C} > q = 68\ \text{nC}$, we find $F' > F$.

3.

In Fig. 20.28, point P is midway between the two charges. Find the electric field in the plane of the page (a) 5.0 cm to the left of P, (b) 5.0 cm directly above P, and (c) at P.

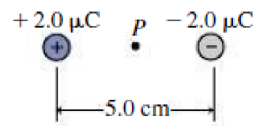
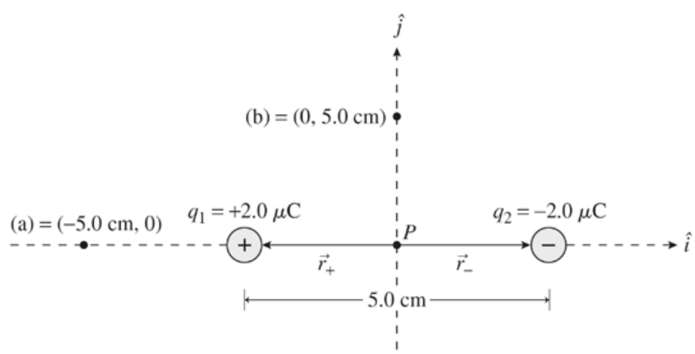


FIGURE 20.28 Exercise 29

INTERPRET For this problem, we are to find the electric field at several locations in the vicinity of two given point charges. We can apply the principle of superposition to solve this problem.

DEVELOP Take the origin of the x-y coordinate system to be at the midpoint between the two charges, as indicated in the figure below, with unit vectors $\hat{i}$ pointing to the right, and $\hat{j}$ pointing up. We use Equation 20.4 to find the electric field at the given points. Let $q_1 = +2.0\ \mu\text{C}$ and $q_2 = -2.0\ \mu\text{C}$. Let $\vec{r}_\pm = \pm(2.5\text{ cm})\hat{j}$ denote the positions of the charges and $\vec{r}$ denote that of the field point (i.e., the point at which we are calculating the electric field). A unit vector from charge i to the field point is $(\vec{r}-\vec{r}_\pm)/|\vec{r}-\vec{r}_\pm|$ [where the plus (minus) sign corresponds to the positive (negative) charge)]. Thus, the spatial factors in Coulomb's law are $\hat{r}_i/r_i^2 = \vec{r}_i/r_i^3 = (\vec{r}-\vec{r}_\pm)/|\vec{r}-\vec{r}_\pm|^3$. By the principle of superposition (Equation 20.4), the total electric field at any point is

$$\vec{E} = k\left(\frac{q_1\vec{r}_1}{r_1^3} + \frac{q_2\vec{r}_2}{r_2^3}\right)$$



EVALUATE **(a)** For the point at 5.0 cm to the left of P, we have

$$\vec{r} = (-5.0\text{ cm})\hat{i}$$
$$\vec{r}_1 = \vec{r} - \vec{r}_+ = (-5.0\text{ cm})\hat{i} - (-2.5\text{ cm})\hat{i} = (-2.5\text{ cm})\hat{i}$$
$$\vec{r}_2 = \vec{r} - \vec{r}_- = (-5.0\text{ cm})\hat{i} - (2.5\text{ cm})\hat{i} = (-7.5\text{ cm})\hat{i}$$

so the electric field is

$$\vec{E} = k\left(\frac{q_1\vec{r}_1}{r_1^3} + \frac{q_2\vec{r}_2}{r_2^3}\right) = k\left(-\frac{q_1\hat{\imath}}{r_1^2} - \frac{q_2\hat{\imath}}{r_2^2}\right)$$

$$= \left(9.0\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[-\frac{\left(2.0\times10^{-6}\ \text{C}\right)\hat{\imath}}{\left(0.025\ \text{m}\right)^2} - \frac{\left(-2.0\times10^{-6}\ \text{C}\right)\hat{\imath}}{\left(0.075\ \text{m}\right)^2}\right] = \left(-26\ \text{MN/C}\right)\hat{\imath}$$

or 26 MN/C, pointing to the left.

(b) For the point at 5.0 cm directly above P, we have

$$\vec{r} = \left(5.0\ \text{cm}\right)\hat{j}$$

$$\frac{\vec{r}_1}{r_1^3} = \frac{\vec{r}-\vec{r}_+}{\left|\vec{r}-\vec{r}_+\right|^3} = \frac{-\left(-2.5\ \text{cm}\right)\hat{\imath} + \left(5.0\ \text{cm}\right)\hat{j}}{\left[\left(5.0\ \text{cm}\right)^2 + \left(-2.5\ \text{cm}\right)^2\right]^{3/2}}$$

$$\frac{\vec{r}_2}{r_2^3} = \frac{\vec{r}-\vec{r}_-}{\left|\vec{r}-\vec{r}_-\right|^3} = \frac{-\left(2.5\ \text{cm}\right)\hat{\imath} + \left(5.0\ \text{cm}\right)\hat{j}}{\left[\left(5.0\ \text{cm}\right)^2 + \left(2.5\ \text{cm}\right)^2\right]^{3/2}}$$

so the electric field is

$$\vec{E} = \left(9.0\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{2.0\times10^{-6}\ \text{C}}{\text{m}^2}\right)\left\{\frac{-\left(-0.025\right)\hat{\imath} + 0.050\hat{j}}{\left[\left(-0.025\right)^2 + \left(0.050\right)^2\right]^{3/2}} - \frac{-\left(0.025\right)\hat{\imath} + 0.050\hat{j}}{\left[\left(0.025\right)^2 + \left(0.050\right)^2\right]^{3/2}}\right\}$$

$$= \left(9.0\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{2.0\times10^{-6}\ \text{C}}{\left[\left(0.025\right)^2 + \left(0.050\right)^2\right]^{3/2}\ \text{m}^2}\right)\left(0.050\hat{\imath}\right) = (5.2\ \text{MN/C})\hat{\imath}$$

or 5.2 MN/C to the right.

(c) For $\vec{r} = 0$, we have

$$\frac{\vec{r}_1}{r_1^3} = \frac{\vec{r}-\vec{r}_+}{\left|\vec{r}-\vec{r}_+\right|^3} = \frac{-\left(2.5\ \text{cm}\right)\hat{\imath}}{\left(2.5\ \text{cm}\right)^3} = \frac{-\hat{\imath}}{\left(2.5\ \text{cm}\right)^2}$$

$$\frac{\vec{r}_2}{r_2^3} = \frac{\vec{r}-\vec{r}_-}{\left|\vec{r}-\vec{r}_-\right|^3} = \frac{\left(2.5\ \text{cm}\right)\hat{\imath}}{\left(2.5\ \text{cm}\right)^3} = \frac{\hat{\imath}}{\left(2.5\ \text{cm}\right)^2}$$

so the electric field is

$$\vec{E} = \left(9.0\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{2.0\times10^{-6}\ \text{C}}{\text{m}^2}\right)\left[\frac{\hat{\imath}}{(0.025)^2} - \frac{-\hat{\imath}}{(0.025)^2}\right] = \left(+58\ \text{MN/C}\right)\hat{\imath}$$

or 58 MN/C to the right.

ASSESS The electric field for part **(b)** is much weaker because the fields from the two charges largely cancel.

4.

Show that the field on the x-axis for the dipole of Example 20.5 is given by

$$\vec{E} = \frac{2kp}{|x|^3}\hat{\imath} \qquad \left(\begin{array}{c}\text{dipole field}\\ \text{for } |x| \gg a\text{, on axis}\end{array}\right)$$

INTERPRET We find the electric field on the axis of a dipole, and show that Equation 20.6b is correct. To do this we will use the equation for electric field.

DEVELOP The spacing between the + and – charges is $2a$. We will use $E = k\dfrac{q}{r^2}$ for each charge to find the total field at a point $x \gg a$.

EVALUATE

$$\vec{E} = k\frac{+q}{(x-a)^2}\hat{i} + k\frac{-q}{(x+a)^2}\hat{i} = kq[(x-a)^{-2} - (x+a)^{-2}]\hat{i}$$

$$\rightarrow \vec{E} = \frac{kq}{x^2}\hat{i}\left[\left(1-\frac{a}{x}\right)^{-2} - \left(1+\frac{a}{x}\right)^{-2}\right]$$

For $x \gg a$, $\left(1 \pm \dfrac{a}{x}\right)^{-2} \approx 1 \mp 2\dfrac{a}{x}$, so

$$\vec{E} \approx \frac{kq}{x^2}\hat{i}\left[\left(1+2\frac{a}{x}\right) - \left(1-2\frac{a}{x}\right)\right] = \frac{kq}{x^2}\left(4\frac{a}{x}\right) = 2\frac{k(2qa)}{x^3}\hat{i}.$$

But $p = qd = 2qa$, so $\vec{E} = \dfrac{2kp}{x^3}\hat{i}$.

ASSESS We have shown what was required.