# Lab 2: Electrostatics

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***Abstract*—The main objective of this lab was to study the electrostatics between two charged plates, creating a capacitor, to help measure the electrostatic force between two objects. Over ANSYS Maxwell, the data was used to collect data on how energy was transferred and the total amount of energy transferred. The capacitor developed with a $5V$ differential. With a different material between the charged plates of a air or ceramic dielectric respectively gave a capacitance of 0.1645 pF and 1.1398 pF. Errors in the experiment were measured respectively to 25.67 % and 47.43 % with the theoretical values of an ideal capacitor with proper permittivity at 0.2213 pF and 2.1683 pF. This shows an increase in capacitance as the permittivity of a material between a plate increases.**

## I. Introduction and Objectives

The main objective of this lab was to study the properties of electrostatics between two charged plates and the dielectric properties of different materials that are inserted between each plate. This study is able to create a relationship between the permittivity of different dielectric materials between a capacitor assuming the same size and separation distance. Objects placed within a vacuum between the capacitors are also measured to find an electric force at a point of interest in the center of the capacitor.

## II. Background and Relevant Theory

### A. Electrostatics

Electrostatics study the electric charges at rest. To understand an electric charge, the electric field, Gaussian law, and electrostatic potential energy and voltage must be understood.

The electric field ($\vec{E}$) is the vector field of the forces acting on an object divided by the unit charge. Field lines generated in ANYSYS Maxwell are useful for visualizing the electric field from an object. These electric field lines travel from a positive to negative charge and are parallel to the direction of the electric field. The density of these field lines represents the magnitude of the electric field [1].

Electric potential, represented by Equation 1, is proportional to the charge of the object. This proportionality explains that energy flows from a high to low gradient. As the charge moves across this distance, one can represent this as work (force over distance) in the unit of Joules.

$$E = QV \tag{1}$$

Equation 1 is represented where electric charge (Q) , measured in coulombs, is represented by one coulomb that has a charge of $6.242 * 10^{-18}$ electrons[1].

### B. Plate Capacitance

Capacitors are electrical circuit elements that store charge. These elements consist of two plates of separated conductors or semiconductors. Voltage across a capacitor is equal to the charge stored on it divided by its capacitance, measured in Farads or 1 coulomb per volt.

$$C = \frac{Q}{V} \tag{2}$$

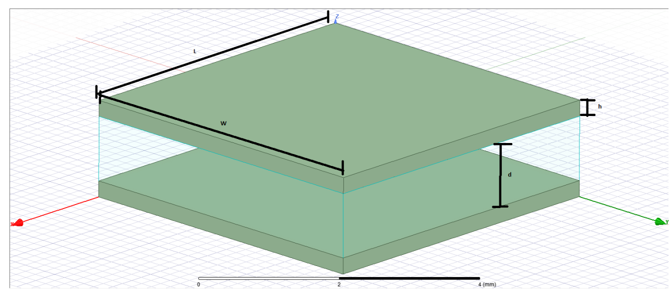


Fig. 1. Parallel-Plate Capacitor with a plate area of W by L by h with a 'd' distance between plates.

For the parallel-plate capacitor, given in the experiment by Figure 1, the capacitance is given by Equation 3,

$$C = \frac{\epsilon_R A}{d} \tag{3}$$

where $\epsilon_R$ is the relative permittivity of the material between the charged plates, $A$ is the area of each plate surface (length (L) * width (W)), and $d$ is the separation distance between the parallel plates. The lowest possible permittivity is that of a vacuum. Vacuum permittivity, is represented by $\epsilon_0$ and has a value approximately equal to $8.85 * 10^{12} \frac{F}{m}$. Equation 3 was used to determine the theoretical capacitance of the parallel-plate capacitor given by Figure 2

### C. Dielectrics

A dielectric is an electrical insulator that can be polarized by an applied electric field. When dielectrics are placed in an electric field, ideally, no current flows between the material due to no free electrons able move through the material. Instead, electric polarization occurs, producing a positive and negative charge between the material [1]. Capacitors filled with dielectric material increases is capacitance by reducing the electric field between the capacitor's plates. Dielectrics possess different permittivity ($\epsilon$) and can be represented by its electric flux density over the electric field strength. A lager permittivity determines a higher electric flux density over the same electric field strength.

### D. Percent Error

The error produced in this experiment is between the ANSYS Maxwell simulations and the theoretical capacitance of the parallel-plates in question. Error is variable by the difference in electric potential determined by the simulation. The Equation 4 represents the percent error between the capacitance of the experimental simulation and the theoretical capacitance determined by Equation 3.

$$\%Error = |\frac{(C_{Theoretical} - C_{Experimental})}{C_{Theoretical}}| * 100 \quad (4)$$

## III. Procedure

In this experiment, ANSYS Maxwell was used to create two plates with an air dielectric material between the plates separated at 1mm with 5mm surface square plates. The thickness, represented by h in Figure 1, is valued at 250 $\mu$m in this experiment. Electric fields and electric flux density in the dielectric were shown with fringing fields near the edge of the plate. The capacitance of the structure was determined by reading the results of energy from the Maxwell simulation and measured with Equation 2. Capacitance obtained from the Maxwell simulation was compared with with the theoretical capacitance given by Equation 3 neglecting the fringing fields. Results explained were repeated with a dielectric other than air,free space, or a vacuum. A relative permittivity in the range of $\epsilon_R$ $9 - 100$ was selected to be AI2_O3 ceramic with a $\epsilon_R$ equal to 9.8.

Electrostatic fields where then studied to interact with perfect conductors and dielectrics. Perfect conductor materials and dielectrics were placed between the plates of the capacitor in the shape of a cube and a small sphere at the center between the plates. The electric fields were shown inside the capacitor in the region of the cube and sphere. Finally, a spherical object, orbited by a grounded disk was analyzed and visualized with respect to the electrostatic fields.

## IV. Results and Discussion

### A. Parallel-Plate Air E-field

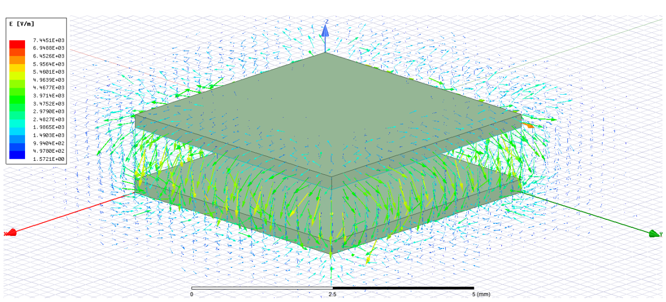

Fig. 2. Electric field line plot of parallel-plate capacitor with a plate volume of 5mm by 5mm by 250 $\upsilon$m with a 1mm distance between plates of air separation. E-field legend is included in the top left displaying the strength of each vector.

Field distributions are generated between the two charged plates when an electric potential difference is applied to them as shown in Figure 2. When the dielectric material is air, there is a large field that extends from the opposing faces of the electrode plates to the region surrounding the capacitor.

### B. Parallel-Plate Air Capacitor Calculations

Energy of the structure was given to be $4.1127pJ$ with an error of 0.58 %. Calculations to find the experimental capacitance of the simulated structure of Figure 2 is provided in Equation 6

$$Q = \frac{E}{V} = \frac{4.1127pJ}{5v} = 0.8225pC \quad (5)$$

$$C = \frac{Q}{V} = \frac{0.8225pC}{5V} = 0.1645pF \quad (6)$$

Using the energy obtained from Maxwell, the charge, measured in Coulombs, could be found using Equation 1. This charge, in Equation 2, can be used with the electric potential of 5 V between the charged plates to find the capacitance of 0.1645 pF.

### C. Parallel-Plate Air Capacitor Numerical Comparisons

The parallel-plate capacitance of Equation 6 is used to compare with the theoretical capacitance of an ideal capacitor with a air between the charged plates with Equation 3.

$$C = \frac{\epsilon_0 A}{d} = \frac{8.85 * 10^{-12} * 0.000025}{0.001} = 0.2213pF \quad (7)$$

where $\epsilon_0$ is approximately the permittivity of air measured at $8.85 * 10^{-12}\frac{F}{M}$, $d$ is the distance of $1mm$ between the plates, and $A$ is the area of the plate surface valued and calculated at $5mm * 5mm = 0.000025mm^2$. Equation 6 reports a number to compare with the theoretical capacitance given by Equation 7. The percent error between these two values is given by Equation 8

$$\%Err = |\frac{(0.2213 - 0.1645)}{0.2213}| = 0.2567 * 100 = 25.67\% \quad (8)$$

### D. Parallel-Plate with Ceramic Dielectric

A parallel-plate structure with the identical size area was given a ceramic dielectric of AI2_O3 ceramic with a $\epsilon_R$ of 9.8 $\frac{F}{M}$. Respectively shown by Figure 3, the overall electrostatic field increased when compared to the field from Figure 2 with air separation. To prove this, the resulting energy from this simulation was produced to be 28.493 pJ, compared to the 4.1127 pJ of the air separation.

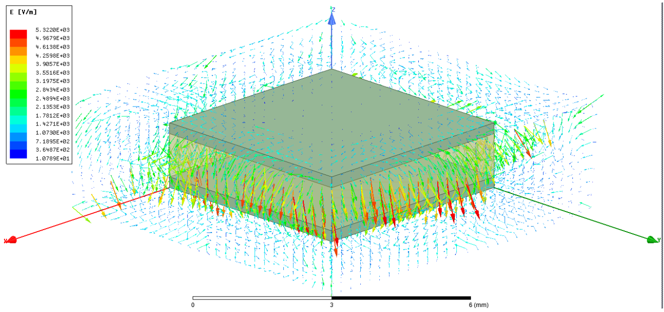

Fig. 3. Electric field line plot of parallel-plate capacitor with a plate volume of 5mm by 5mm by 250 $\mu$m with a 1mm distance between plates of AI2_O3 ceramic separation. E-field legend is included in the top left displaying the strength of each vector.

Due to this, a trend is set displaying that the relative permittivity of a plate separation material increases proportionally with the energy of a capacitor structure.

### *E. Parallel-Plate Ceramic Capacitor Numerical Comparisons*

Similarly to the air capacitor calculations, the values of the ceramic capacitor is shown in Equation 10.

$$Q = \frac{E}{V} = \frac{28.493pJ}{5V} = 5.699pC \tag{9}$$

$$C = \frac{Q}{V} = \frac{5.699pC}{5V} = 1.1398pF \tag{10}$$

Notably, the experimental capacitance has increased between the dielectric of ceramic and air of the capacitor structure. Equation 7 displays the theoretical capacitance of a capacitor structure of similar size with the relative permittivity equal to that of ceramic.

$$C = \frac{\epsilon_0 A}{d} = \frac{8.673 * 10^{-11} * 0.000025}{0.001} = 2.1683pF \tag{11}$$

Where the relative permittivity of a vacuum is $8.85 * 10{-}12$, ceramic used as this dielectric has a permittivity 9.8 times higher at $8.673 * 10^{-11} \frac{F}{m}$ Equation 10 reports a number to compare with the theoretical capacitance given by Equation 11. The percent error between these two values is given by Equation 12

$$\%Err = |\frac{(2.1683 - 1.1398)}{2.1683}| = 0.4743 * 100 = 47.43\% \tag{12}$$

### *F. Capacitor Electrostatics with Cube Obstruction*

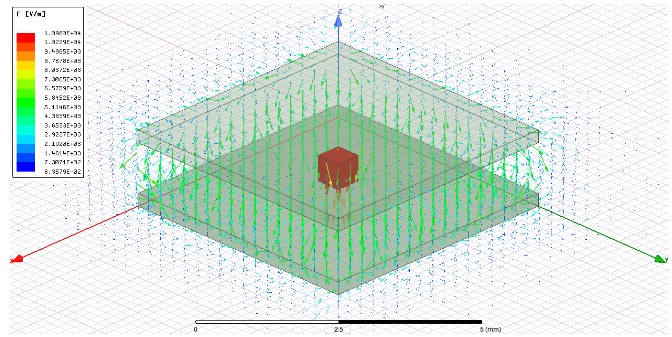

Fig. 4. Electric field line plot of parallel-plate capacitor with a plate volume of 5mm by 5mm by 250 $\mu$m with a 1mm distance between plates of vacuum separation used to display the field effected by a small cube at the center of the structure with perfect conductive material material. E-field legend is included in the top left displaying the strength of each vector.

With a perfect conducting cube placed at the center of the capacitor structure, the total electric force is shown through the entire structure. At the center of the capacitor, the cube takes a larger electric force as represented by the darker vector at the point of interest in comparison to the ceramic cube.

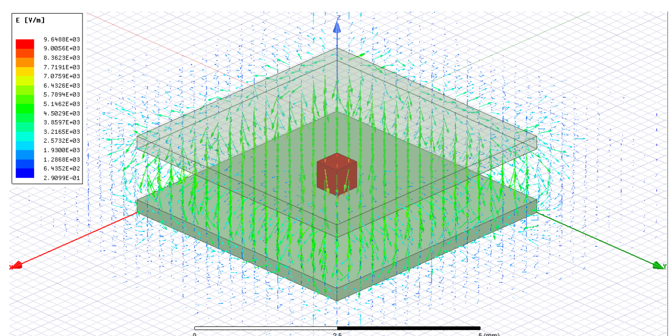

Fig. 5. Electric field line plot of parallel-plate capacitor with a plate volume of 5mm by 5mm by 250 $\mu$m with a 1mm distance between plates of vacuum separation used to display the field effected by a small cube at the center of the structure with ceramic material. E-field legend is included in the top left displaying the strength of each vector.

Reasonably, with the ceramic cube, the permittivity of the object is lowered in comparison to the

### G. Capacitor Electrostatics with Sphere Obstruction

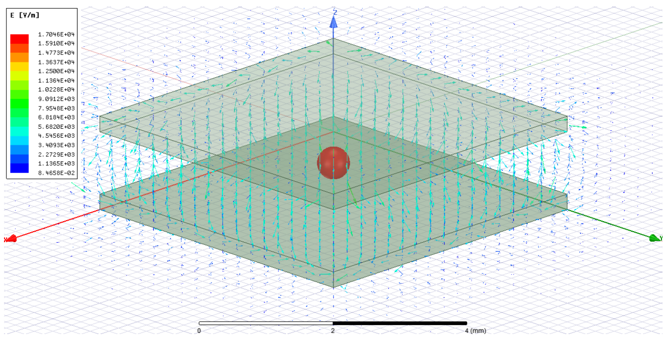

Fig. 6. Electric field line plot of parallel-plate capacitor with a plate volume of 5mm by 5mm by 250 $\mu$m with a 1mm distance between plates of vacuum separation used to display the field effected by a small sphere of radius 0.25 mm at the center of the structure with perfect conductive material. E-field legend is included in the top left displaying the strength of each vector.

With a perfect conducting sphere placed at the center of the capacitor structure, the total electric force is shown through the entire structure. At the center of the capacitor, the cube takes a larger electric force as represented by the darker vector at the point of interest in comparison to the ceramic sphere. Against the cube, the sphere allows a smaller volume to be taken up between the plates and therefore has a smaller increase in electric force.

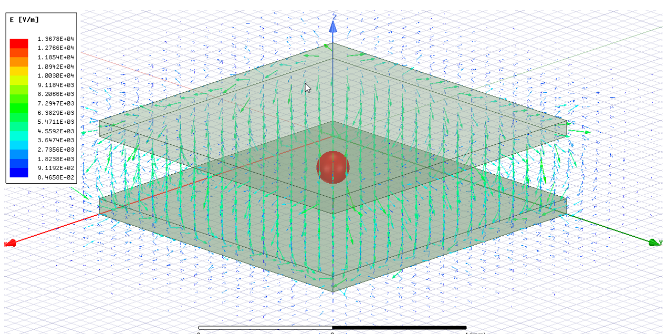

Fig. 7. Electric field line plot of parallel-plate capacitor with a plate volume of 5mm by 5mm by 250 $\mu$m with a 1mm distance between plates of vacuum material separation used to display the field effected by a small sphere of radius 0.25 mm at the center of the structure with ceramic material. E-field legend is included in the top left displaying the strength of each vector.

According to Figure 8, the charged sphere behaves very similarly to the plate capacitor as the force flows from a high to low voltage potential of 5V. In the spherical capacitor however, the overall energy from the object under the same relative positions, has a stronger capacitance. The capacitance produced by Maxwell is 7.5024 pJ compared to the parallel-plate capacitor of the same material and conditions with 4.1127 pJ. Despite this, the spherical capacitor is a still a weaker candidate for a capacitor in electrical design without even considering the packaging of this capacitor or the radius of the center sphere having a notable role in the charge on the capacitor. If the spherical surface was enclosed, no electric force would be carried between the voltage potential.

### H. Electrostatics of a Spherical Capacitor

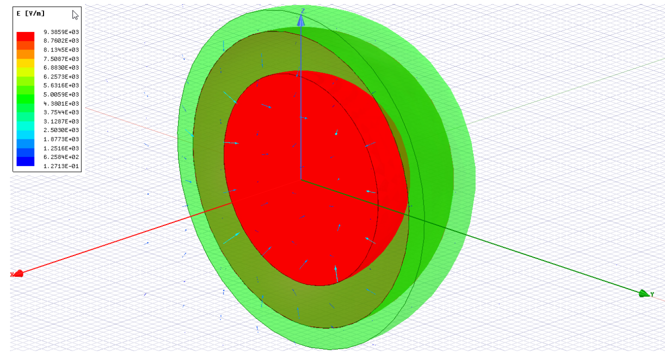

Fig. 8. Electric field line plot of spherical capacitor structure with a sphere volume of 5mm radius with a 1mm distanced vacuum between torus plates used to display the electric field of a spherical capacitor. E-field legend is included in the top left displaying the strength of each vector.

$$Q = \frac{E}{V} = \frac{7.5024pJ}{5v} = 1.5005pC \tag{13}$$

$$C = \frac{Q}{V} = \frac{1.5005pC}{5V} = 0.3001pF \tag{14}$$

The overall capacitance for a spherical capacitor is provided for Equation 14.

## V. Conclusions

When specifying the electrostatic properties through ANSYS Maxwell, it is possible to both understand and calculate the capacitance exerted from an object with charged parallel-plates over a dielectric. The field lines generated between charged objects are displayed in this experiment to show either, how the electric force goes around a dielectric to a grounded plate in a capacitor, or how it flows through a small object between charged parallel plates. Additionally, a charged electrostatic field can be calculated on a conductive sphere by understanding the properties of electric force along the surface area of the sphere and dielectric between it and the grounded surface.

## References


[1] Hayt, W.H., Buck, J.A., *Engineering Electromagnetics. McGraw-Hill Education*, New York, NY (2019).