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ECOMMS Homework 1 Joshua Gould

Problem 1 Textbook 1-2

$f = 96.9\ MHz$, $h = 1000\ ft$

To be recieved 55 miles

How high to mount?

$d_1 = \sqrt{2h} = \sqrt{2(1000)} = 44.72$ miles

$d_2 = (55 - 44.72) = 10.28$

$\left(\frac{10.28^2}{2}\right) = 52.84\ ft$

Problem 2 Textbook 1-13

12 digits, two levels

$(1/2)^{13}$ $3 \times (1/2)^{13}$ $\sum P_j = 2^{11}\left[(1/2)^{13} + 3\times(1/2)^{13}\right]$

Entropy?

$$H = \sum_{j=1}^{m} P_j I_j = \sum_{j=1}^{m} P_j \log_2\left(\frac{1}{P_j}\right) \text{ bits}$$

$(1/2)^{13}$ $\sum (1/2)^{13} \cdot 13 \log_2(2) = 0.019043$

$\frac{(1/2)^{13}}{2} \log(2^{13}) + 3\times(1/2)^{13} \cdot \log\left(\frac{2}{3}^{13}\right)$

$\frac{2^{12}}{2}(0.5)^3 \log(2^{13}) + \frac{2^{12}}{2}\left(3\times(0.5)^{13}\right) \log\left(\frac{2}{3}^{13}\right)$

$3.25 + 8.56 = 11.81$

Problem 3 Textbook number 1-14

300 Hz bandwidth

SNR 30 db → W

$30 \text{ db} = 10 \log_{10}(x) = 1000$

$10^{x0}$

$C = B \log_2(1 + S/N)$ bits/sec

$10^{x}$/sec

$300 \log_2(1 + S/N) = 2{,}900$ bits/sec

Problem 4 Textbook 2-6

a) $w(t) = \Pi(t/T_0)$

Energy $\lim_{T\to} \int (\Pi\, t/T_0)^2$

$E_0 = T_0$ Joules

$E = \int_{-\infty}^{\infty} w^2(t)\,dt = \int_{-T_0/2}^{T_0/2} 1^2\,dt = T_0$

b) $w(t) = \Pi(t/T_0)\cos \omega_0 t = \Pi(t/T_0)\cos\omega_0 t$

$\cancel{\lim_{T\to\infty} \int_{T/2}^{T_0/2} (\Pi(t/T_0)\cos\omega_0 t)^2}$

$\cos\omega_0 t \to \dfrac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$ Energy: $T_0$ Joules

$\dfrac{\Pi\, t/T_0\, e^{j\omega_0 t}}{2} + \dfrac{\Pi\, t/T_0\, e^{-j\omega_0 t}}{2}$

$E = \int_{-\infty}^{\infty} w^2(t)\,dt = \int_{-T_0/2}^{T_0/2} \frac{1}{2}(\cos(\sin t) + 1)$

$E = \frac{1}{2} T_0$

c) $w(t) = \cos^2 \omega_0 t$

Power

$P = \langle w^2(t)\rangle = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} w^2(t)\,dt$

$w(t) = \dfrac{1 + \cos 2\omega_0 t}{2}$ $E = \infty$ $\cos^2(\omega_0 t)$

$= \frac{1}{2} + \dfrac{\cos 2\omega_0 t}{2} =$

$w(t) = \cos^2 \omega_0 t$

$P = \lim_{T\to\infty} \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} w^2(t)\,dt$

$P = 3/8\,W$

$-1/2 < 2t - 5 < 1/2$

$9/4 < t < 11/4$

d) $\Pi(2t-5)$

Energy $= \lim_{T\to\infty} \int_{-T_0/2}^{T_0/2} (2t-5)^2 \int_{-9/4}^{11/4} w^2(t)$

$4t^2 - 20t + 25$

$= \int_{-9/4}^{11/4} 1$

$E = 1/2\ J$

$E = \cancel{25}\ w$

5) Convolution property

$w(t) = \sin(2\pi f_1 t)\cos(2\pi f_2 t)$

$$\frac{1}{4j}\left[\delta(f-f_1-f_2) + \delta(f-f_1+f_2) - \delta(f+f_1-f_2) - \delta(f+f_1+f_2)\right].$$

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$P = \lim_{T\to\infty} \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \cos^4(\omega_0 t)\,dt$

$= 1/8(3 + 4\cos 2\omega_0 t + \cos 4\omega_0 t)$

$\frac{1}{T_0} \cdot 3/8\, T_0 = 3/8$

$1/8(3) = 3/8$

5 $w(f) = w_1(f) * w_2(f_0) = \left[\frac{j}{2}\delta(f+f_1) - \frac{j}{2}\delta(f-f_0)\right]$

$* \frac{1}{2}\delta(f+f_2) + \delta(f_1 - f_2$

$\delta(f+f_1) * \delta(f+f_2)$

$= f_{-\infty}^{\infty}$

# Homework 2 – Ecomms

Joshua Gould

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**Problem 1 Textbook number 4-2**

An AM signal is modulated by a waveform such that the complex envelop is

$$g(t) = A_c\{1 + a[0.2\cos(\pi 250t) + 0.5\sin(\pi 2500t)]\}$$

Where Ac=10. Find the value of a such that the AM signal has a positive modulation percentage of 90%. Hint: Look at Ex.4-3 and Eq. (5-5a), you can use MATLAB to find out the max and min.

$0.2 + 0.5 = 0.7a \rightarrow 90\% \rightarrow 0.7a = 0.9$

```
A_c = 10;
a = 4.5;
t = 0:0.0001:1;
gt = A_c*(1 + a*(0.2*cos(0.2*(pi*250*t)+ 0.5*sin(pi*2500*t))));
maxgt = max(gt)
mingt = min(gt)
plot(t,gt);

maxgt =  19
mingt =  1.0000
```

$a = 9/7 = 1.285$

$.9 = \frac{A_{max} - A_{min}}{2A_c}$

X – 1

positive modulation percentage $= \frac{A\max - A_c}{A_c} * 100 = \frac{19-10}{10} * 100 = \mathbf{90\%}$

**Problem 2 Textbook number 5-5**

A 50,000-W AM broadcast transmitter is being evaluated by means of a two-tone test. The transmitter is connected to a 50-Ω load, and m(t)= A1cosω1t + A1cos2ω1t, where ω1=2πf1, f1= 500Hz. Assume that a perfect AM signal is generated.

a. Evaluate the complex envelope for the AM signal in terms of A1 and ω1.
b. Determine the value of A1 for 90% modulation.
c. Find the values for the peak current and average current into the 50-Ω load for the 90% modulation case (assume ωc>> ω1, where ωc is the carrier frequency).

a. $50000 = \frac{A_C^2}{2(50\Omega)} \rightarrow A_c = 2236\ V$

$g(t) = A_C[1 + m(t)] = 2236[1 + A_1(\cos(\omega t) + \cos(2\omega t)]$

b. $\frac{A\max - A_{min}}{2A_c} * 100 = \frac{\max[m(t)] - \min[m(t)]}{2} = \frac{3.125}{2} A_i * 100 = \mathbf{90\%}$

$A_1 = 0.576$

$\frac{3.125 A_1}{2} = 0.9$

$\frac{1.8}{3.125}$

$x(\theta) = \cos\theta + \cos 2\theta$

$\frac{dx(\theta)}{d\theta} = -\sin\theta - 2\sin 2\theta = 0$

$-\sin\theta = 4\sin\theta\cos\theta$

$\cos\theta = -1/4$

$\theta = 104.5°$

c. $A_{max} = 2236[1 + 2(0.576)] = 4811.9\ V$

$I_{max} = \frac{A_{max}}{50} = 96.238\ A$

$V_{avg} \leftrightarrow I_{avg} = 0\ A\ if\ \omega_c \gg \omega_1$

$\langle s(t)\rangle = \langle 2.236\rangle[1 + .576(\cos\omega_1 t) + \cos 2\omega_1 t]\cos\omega_c t$

**Problem 3 Textbook number 5-12**

A DSB-SC signal can be generated from two AM signal as shown in Fig. 5-12. Using mathematics to describe signals at each point on the figure, prove that the output is a DSB-SC signal. (AM signal has complex envelope Ac(1+m(t))). (The positive input of the op amp is grounded)