ECE 09.303 Fall 2019
Homework 4 Solutions
Chapter 4/22 – Electric Potential

1.

An electron passes point A moving at 6.5 Mm/s. At point B it comes to a stop. Find the potential difference ΔV_{AB}.

INTERPRET This problem involves the work-energy theorem (Equation 6.14), which we can use to find the potential energy difference given the change in kinetic energy. Once we know the potential energy difference, we can divide it by the charge to find the potential difference.

DEVELOP The work-energy theorem (Equation 6.14) says that the change in an objects kinetic energy is equal to the work done on the object, or $\Delta K_{AB} = W_{AB}$. We also know from the discussion preceding Equation 22.1a that the work done on the charge by the electric field is the negative of the potential energy change, or

$$\Delta U_{AB} = -W_{AB} = -\Delta K_{AB}$$

For this problem, the kinetic energy of the electron decreases, so $\Delta K_{AB} = -m_e v^2/2$.

EVALUATE Using Equation 22.1b, $\Delta V_{AB} = \Delta U_{AB}/q$, we find

$$\Delta V_{AB} = -\frac{\Delta K_{AB}}{q} = -\frac{-m_e v_A^2}{-2e} = \frac{\left(9.11\times10^{-31}\ \text{kg}\right)\left(6.5\times10^{6}\ \text{m/s}\right)^2}{2\left(-1.6\times10^{-19}\ \text{C}\right)} = -120\ \text{V}$$

ASSESS There potential difference is negative because we are dealing with an electron, which has negative charge. In other words, to stop an electron, a negative potential difference must be applied.

2.

Find the potential as a function of position in the electric field $\vec{E} = ax\hat{\imath}$, where a is a constant and where you're taking $V = 0$ at $x = 0$.

INTERPRET For this problem, we are given the electric field as a function of position, and we are to find the electric potential as a function of position. We are also given the electric potential at a given point, so we will define our electric potential with respect to this point.

DEVELOP Apply Equation 22.1a,

$$\Delta V_{AB} = -\int_A^B \vec{E}\cdot d\vec{r}$$

where $\vec{E} = ax\left(\hat{i}\right)$ and $d\vec{r} = dx\left(\hat{i}\right)$. Furthermore, we take point A to be x = 0, so $V_A = V(x = 0) = 0$, and point B to be an arbitrary point x.

EVALUATE Evaluating the integral gives

$$\Delta V_{AB}(x) = -\int_A^B ax'dx'(\hat{i}\cdot\hat{i}) = -\int_0^x ax'dx' = -\frac{a}{2}x^2$$

so

$$V(x) = V(0) + \Delta V_{AB}(x) = -\frac{a}{2}x^2$$

ASSESS This potential increases quadratically with position, whereas the electric field is linear in position.

3.

The electric potential in a region is given by $V = -V_0(r/R)$, where V_0 and R are constants and r is the radial distance from the origin. Find expressions for the magnitude and direction of the electric field in this region.

INTERPRET We are given the electric potential and asked to find the corresponding electric field.

DEVELOP We first note that the potential $V(r) = -V_0 r/R$ depends only on *r*. This implies that the electric field is spherically symmetric and points in the radial direction. The field can be calculated using Equation 22.9,

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

In spherical coordinates, this is

$$\vec{E} = -\left[\left(\frac{\partial V}{\partial r}\right)\hat{r} + \frac{1}{r\sin\theta}\left(\frac{\partial V}{\partial \phi}\right)\hat{\phi} + \frac{1}{r}\left(\frac{\partial V}{\partial \theta}\right)\hat{\theta}\right]$$

Because the potential depends only on *r*, the second two terms in this expression will give zero.

EVALUATE The electric field is

$$\vec{E} = -\frac{dV}{dr}\hat{r} = \frac{V_0}{R}\hat{r}$$

where $\hat{r}$ is a unit vector that points radially outward.

ASSESS The electric field is uniform, but the potential is linear in *r*. The difference of one power in *r* is because the potential is an integral of the field over distance.

4.

A 2.0-cm-radius metal sphere carries 75 nC and is surrounded by a concentric spherical conducting shell of radius 10 cm carrying −75 nC. (a) Find the potential difference between shell and sphere. (b) How would your answer change if the shell's charge were +150 nC?

INTERPRET This problem involves a pair of concentric spheres, the inner one solid and the outer hollow. Both are conducting and carry the given charge. We are to find the potential between the spheres. The principle of superposition will be of use for this problem.

DEVELOP The potential inside a conducting spherical shell is constant and the same as the potential at the surface of the sphere, which we can find using Equation 22.3. The potential difference between the two spheres is the difference in the potential of the each sphere individually.

EVALUATE **(a)** The potential at the surface of the outer spheres is

$$V_1 = \frac{kQ_1}{R_1}$$

and the potential due to the inner sphere is

$$V_2 = \frac{kQ_2}{R_2}$$

The potential difference between the spheres is

$$\Delta V = V_2 - V_1 = k\left(\frac{Q_2}{R_2} - \frac{Q_1}{R_1}\right) = \left(9.0\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(\frac{75\ \mathrm{nC}}{0.020\ \mathrm{m}} - \frac{-75\ \mathrm{nC}}{0.10\ \mathrm{m}}\right) = 41\ \mathrm{kV}$$

(b) Adding more charge to the outer shell will change V_1, but this will become the new constant potential inside the sphere to which is added the potential of the inner sphere. Thus, the potential difference will not change. Only the potential difference outside the outer sphere will change with respect to the potential at infinity.

ASSESS Adding the charge to the outer sphere raises the potential of the entire space inside it, so the potential of the inner sphere simply "floats" up on that of the outer sphere. Therefore, the potential difference does not change.