Quiz Sheet

Thursday, October 4, 2018 10:41 PM

The three vertices of a triangle are located at $A(-1, 2, 5)$, $B(-4, -2, -3)$, and $C(1, 3, -2)$.

a) Find the length of the perimeter of the triangle: Begin with $\mathbf{AB} = (-3, -4, -8)$, $\mathbf{BC} = (5, 5, 1)$, and $\mathbf{CA} = (-2, -1, 7)$. Then the perimeter will be $\ell = |\mathbf{AB}| + |\mathbf{BC}| + |\mathbf{CA}| = \sqrt{9 + 16 + 64} + \sqrt{25 + 25 + 1} + \sqrt{4 + 1 + 49} = \underline{23.9}$.

b) Find a unit vector that is directed from the midpoint of the side AB to the midpoint of side BC: The vector from the origin to the midpoint of AB is $\mathbf{M}_{AB} = \frac{1}{2}(\mathbf{A} + \mathbf{B}) = \frac{1}{2}(-5\mathbf{a}_x + 2\mathbf{a}_z)$. The vector from the origin to the midpoint of BC is $\mathbf{M}_{BC} = \frac{1}{2}(\mathbf{B} + \mathbf{C}) = \frac{1}{2}(-3\mathbf{a}_x + \mathbf{a}_y - 5\mathbf{a}_z)$. The vector from midpoint to midpoint is now $\mathbf{M}_{AB} - \mathbf{M}_{BC} = \frac{1}{2}(-2\mathbf{a}_x - \mathbf{a}_y + 7\mathbf{a}_z)$. The unit vector is therefore

$$\mathbf{a}_{MM} = \frac{\mathbf{M}_{AB} - \mathbf{M}_{BC}}{|\mathbf{M}_{AB} - \mathbf{M}_{BC}|} = \frac{(-2\mathbf{a}_x - \mathbf{a}_y + 7\mathbf{a}_z)}{7.35} = \underline{-0.27\mathbf{a}_x - 0.14\mathbf{a}_y + 0.95\mathbf{a}_z}$$

where factors of 1/2 have cancelled.

c) Show that this unit vector multiplied by a scalar is equal to the vector from A to C and that the unit vector is therefore parallel to AC. First we find $\mathbf{AC} = 2\mathbf{a}_x + \mathbf{a}_y - 7\mathbf{a}_z$, which we recognize as $-7.35\,\mathbf{a}_{MM}$. The vectors are thus parallel (but oppositely-directed).

Vector Rules -

$|\bar{a}| = \sqrt{x^2 + y^2 + z^2}$

$\hat{a} = \frac{x}{|\bar{a}|} + \frac{y}{|\bar{a}|} + \frac{z}{|\bar{a}|}$

$R_{CD} = D - C$

Cylindrical

$\rho = \sqrt{x^2 + y^2}$

$\Phi = \tan^{-1}\frac{y}{x}$

$z = z$

$R_C + R_D = R_{CD}$

$R_{CD,\rho} = x\cos(\phi) + y\sin(\phi)$

$R_{CD,\phi} = -x\sin(\phi) + y\cos(\phi)$

$R_{CD,z} = z$

Dot Product

$a \cdot b = [x_a * x_b + y_a * y_b] = N_{ab}$

Cross Product

$a = x_1, y_1, z_1$

$b = x_2, y_2, z_2$

$$a \times b = \begin{array}{ccc} i & j & k \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{array} = (y_1 z_2 - z_1 y_2)i + (z_1 x_2 - x_1 z_2)j + (x_1 y_2 - y_1 x_2)k$$

Electromagnetics

$e = 1.6 * 10^{-19}\ C$

$k = 9.0 * 10^9\ N * \frac{m^2}{C^2}$

$E = \frac{F}{e}$

$\bar{F} = \frac{kq^2}{r^2}\hat{r}$

$E_{pt} + E_{shell} = \frac{k(-2Q)}{r^2} + E_{out}$

If r < R then $E_{out} = 0$