ECE 09.303 Fall 2019
Homework 1 Solutions

1.

Given the vectors $\mathbf{M} = -10\mathbf{a}_x + 4\mathbf{a}_y - 8\mathbf{a}_z$ and $\mathbf{N} = 8\mathbf{a}_x + 7\mathbf{a}_y - 2\mathbf{a}_z$, find:

a) a unit vector in the direction of $-\mathbf{M} + 2\mathbf{N}$.

$-\mathbf{M} + 2\mathbf{N} = 10\mathbf{a}_x - 4\mathbf{a}_y + 8\mathbf{a}_z + 16\mathbf{a}_x + 14\mathbf{a}_y - 4\mathbf{a}_z = (26, 10, 4)$

Thus

$$\mathbf{a} = \frac{(26, 10, 4)}{|(26, 10, 4)|} = \underline{(0.92, 0.36, 0.14)}$$

b) the magnitude of $5\mathbf{a}_x + \mathbf{N} - 3\mathbf{M}$:

$(5, 0, 0) + (8, 7, -2) - (-30, 12, -24) = (43, -5, 22)$, and $|(43, -5, 22)| = \underline{48.6}$.

c) $|\mathbf{M}||2\mathbf{N}|(\mathbf{M} + \mathbf{N})$:

$|(-10, 4, -8)||(16, 14, -4)|(-2, 11, -10) = (13.4)(21.6)(-2, 11, -10)$

$= \underline{(-580.5, 3193, -2902)}$

2.

The three vertices of a triangle are located at $A(-1, 2, 5)$, $B(-4, -2, -3)$, and $C(1, 3, -2)$.

a) Find the length of the perimeter of the triangle: Begin with $\mathbf{AB} = (-3, -4, -8)$, $\mathbf{BC} = (5, 5, 1)$, and $\mathbf{CA} = (-2, -1, 7)$. Then the perimeter will be $\ell = |\mathbf{AB}| + |\mathbf{BC}| + |\mathbf{CA}| = \sqrt{9 + 16 + 64} + \sqrt{25 + 25 + 1} + \sqrt{4 + 1 + 49} = \underline{23.9}$.

b) Find a unit vector that is directed from the midpoint of the side AB to the midpoint of side BC: The vector from the origin to the midpoint of AB is $\mathbf{M}_{AB} = \frac{1}{2}(\mathbf{A} + \mathbf{B}) = \frac{1}{2}(-5\mathbf{a}_x + 2\mathbf{a}_z)$. The vector from the origin to the midpoint of BC is $\mathbf{M}_{BC} = \frac{1}{2}(\mathbf{B} + \mathbf{C}) = \frac{1}{2}(-3\mathbf{a}_x + \mathbf{a}_y - 5\mathbf{a}_z)$. The vector from midpoint to midpoint is now $\mathbf{M}_{AB} - \mathbf{M}_{BC} = \frac{1}{2}(-2\mathbf{a}_x - \mathbf{a}_y + 7\mathbf{a}_z)$. The unit vector is therefore

$$\mathbf{a}_{MM} = \frac{\mathbf{M}_{AB} - \mathbf{M}_{BC}}{|\mathbf{M}_{AB} - \mathbf{M}_{BC}|} = \frac{(-2\mathbf{a}_x - \mathbf{a}_y + 7\mathbf{a}_z)}{7.35} = \underline{-0.27\mathbf{a}_x - 0.14\mathbf{a}_y + 0.95\mathbf{a}_z}$$

where factors of 1/2 have cancelled.

c) Show that this unit vector multiplied by a scalar is equal to the vector from A to C and that the unit vector is therefore parallel to AC. First we find $\mathbf{AC} = 2\mathbf{a}_x + \mathbf{a}_y - 7\mathbf{a}_z$, which we recognize as $-7.35\,\mathbf{a}_{MM}$. The vectors are thus parallel (but oppositely-directed).

3.

Express in cylindrical components:

a) the vector from $C(3,2,-7)$ to $D(-1,-4,2)$:
$C(3,2,-7) \rightarrow C(\rho = 3.61, \phi = 33.7^\circ, z = -7)$ and
$D(-1,-4,2) \rightarrow D(\rho = 4.12, \phi = -104.0^\circ, z = 2)$.
Now $\mathbf{R}_{CD} = (-4,-6,9)$ and $R_\rho = \mathbf{R}_{CD} \cdot \mathbf{a}_\rho = -4\cos(33.7) - 6\sin(33.7) = -6.66$. Then $R_\phi = \mathbf{R}_{CD} \cdot \mathbf{a}_\phi = 4\sin(33.7) - 6\cos(33.7) = -2.77$. So $\mathbf{R}_{CD} = \underline{-6.66\mathbf{a}_\rho - 2.77\mathbf{a}_\phi + 9\mathbf{a}_z}$

b) a unit vector at D directed toward C:
$\mathbf{R}_{CD} = (4,6,-9)$ and $R_\rho = \mathbf{R}_{DC} \cdot \mathbf{a}_\rho = 4\cos(-104.0) + 6\sin(-104.0) = -6.79$. Then $R_\phi = \mathbf{R}_{DC} \cdot \mathbf{a}_\phi = 4[-\sin(-104.0)] + 6\cos(-104.0) = 2.43$. So $\mathbf{R}_{DC} = -6.79\mathbf{a}_\rho + 2.43\mathbf{a}_\phi - 9\mathbf{a}_z$
Thus $\mathbf{a}_{DC} = \underline{-0.59\mathbf{a}_\rho + 0.21\mathbf{a}_\phi - 0.78\mathbf{a}_z}$

c) a unit vector at D directed toward the origin: Start with $\mathbf{r}_D = (-1,-4,2)$, and so the vector toward the origin will be $-\mathbf{r}_D = (1,4,-2)$. Thus in cartesian the unit vector is $\mathbf{a} = (0.22, 0.87, -0.44)$. Convert to cylindrical:
$a_\rho = (0.22, 0.87, -0.44) \cdot \mathbf{a}_\rho = 0.22\cos(-104.0) + 0.87\sin(-104.0) = -0.90$, and
$a_\phi = (0.22, 0.87, -0.44) \cdot \mathbf{a}_\phi = 0.22[-\sin(-104.0)] + 0.87\cos(-104.0) = 0$, so that finally, $\mathbf{a} = \underline{-0.90\mathbf{a}_\rho - 0.44\mathbf{a}_z}$.

4. Give the result of a·b for each of the following:

a) a = [1,2], b = [2,5].

b) a = [1,2,3], b = [2,5,−7].

Solution:

a) a·b = 1·2 + 2·5 = 12.

b) −9.

5. Give the result of a×b for each of the following:

a) a = [1,2,3], b = [3,2,1].

b) a = i−j + k, b = [3,2,1].

Solution:

a) a×b = [2·1−3·2,3·3−1·1,1·2−2·3] = [−4,8,−4].

b) [−3,2,5].