Maxwell Tutorial in Electrostatics

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1 Abstract

In this report, the overall objective was to progress understanding of the ANSYS Maxwell electromagnetic analysis tool to find the electrostatic properties and electromagnetic field of a cylinder and a lightning rod. Cylindrical analysis found the electromagnetic field at the surface area of the sides of the object showing the properties of a conductive object in a vacuum. The lightning rod system explained how a rod behaves with a highly conductive atmosphere around the point of the rod. These methods explain the basic properties of electromagnetic force in a given space.

2 Introduction

The goals of this simulation are to develop an understanding of Maxwell software and to improve on electrostatic concepts through an analysis of the electric fields surrounding certain systems. Electrostatic phenomena arise from the forces that electric charges exert on each other. Such forces are described by Coulomb's law. Additionally, with the aim of analyzing a cylindrical electrostatic force, Gauss's law is used to calculate the fields on the surface while understanding the reasoning behind the calculations.

3 Background

3.1 Electrostatics

Electrostatics is the study of the electric fields and forces created by static arrangements of electric charge in a given space. In this report, the behavior of electrostatics display how an object behaves in a conductive material or in an electromagnetic field.

To visualize the vector ($\vec{E}$) in a space, field lines are drawn in the direction of the force. ANSYS Maxwell is capable of detailing these field lines of an object.[1]

3.2 Coulomb's Law

Coulomb's Law details the electronic force between two charges of distance r away in a plane.

$$E = k\frac{q_1 q_2}{|r^2|}\hat{r} \tag{1}$$

The Coulomb constant k is defined as

$$k = \frac{1}{4\pi\epsilon_0} \tag{2}$$

where $\epsilon_0 = 8.85 * 10^{-12}\frac{C^2}{Nm^2}$ is the permittivity of free space. This specifies the strength of the electrostatic force. In this experiment the total electromagnetic force was measured between a charged object, such as a conductive cylinder, and a grounded vacuum.

3.3 Gauss's Law

In this report, Gauss's Law is detailed as the electric field multiplied by the area of the surface projected in a plane perpendicular to the field. With symmetry around the Gaussian surface, Gauss' law can be used to find charge distributions.

$$\oint \vec{E} * d\vec{A} = \frac{Q}{\epsilon_0} \tag{3}$$

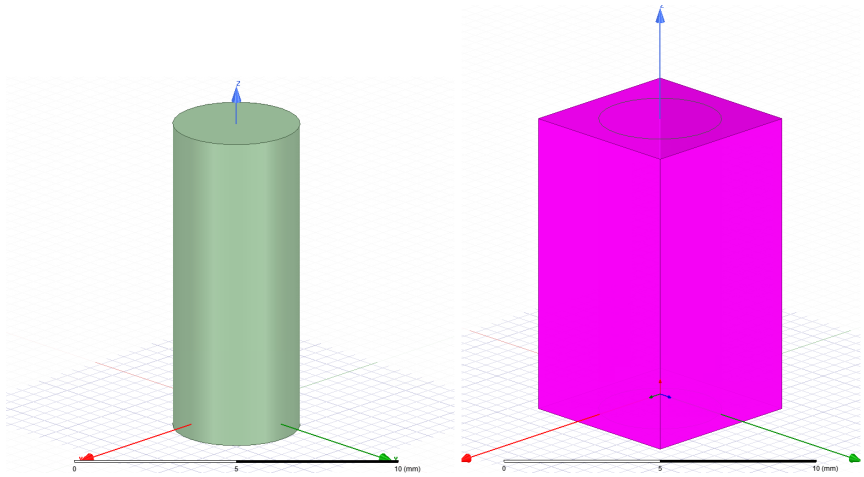
Equation 3 defines the electric charge over an area or free space. This equation is the first step in analyzing a cylinder where R, defined as the radius of the cylindrical object, is compared with an imaginary cylinder with radius r, and compared with the charge of the original cylinder to find the electric field.

4 Evaluation and Results

4.1 Cylindrical Electrostatics

A 3D model of the cylindrical structure was created for analysis along with the field to generate the electromagnetic vector. Figure 1(a) details the figure in question and the size field around the object extending twenty percent in the x-y plane shown in Figure 1(b).

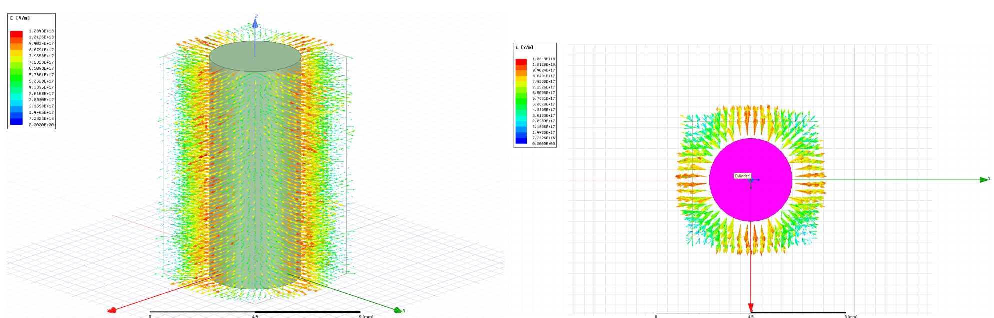
These objects required a charge and material to operate in the electromagnetic field. As such, each object, cylinder and region, was given a charge of 1000 Coulombs and 0 Coulombs respectively. The material of the cylinder was chosen to be a conductive element to show a strong electrostatic field in the vacuum of the region surrounding the cylinder. Additionally, the voltage of the surrounding region had to be set



(a) (b)

Figure 1: 3D Ansys Maxwell Cylinder model displaying the generated region with 20% extensions used for electromagnetic field boundaries

to zero volts in order for the electromagnetic field to flow to the region as a voltage potential.



(a) (b)

Figure 2: Cylindrical model displaying the electromagnetic field lines flowing from the perfect conductor model to the grounded vacuum of the region surrounding the object with (b) top view of cylindical model of radius R.

After allowing the state of the model to enter an analysis setup and check, the Electronic Vector field was generated to should the electromagnetic field (E Vector) lines from the cylinder. Such a model is provided in Figure 2(a) along with the magnitude table of the field lines.

4.2 Lighting Cone Electrostatics

This model simulates how a lightning rod behaves in a highly conductive model. The electrostatics of the cone are simulated in Figure 4. In Figure 3, the system displays a conductive iron cone displaying the conductive properties a the top of a common lightning rod. Where the box above the cone simulates a highly conductive water cloud to simulate a humid atmosphere. A vacuum region is surrounding the lightning rod to display the field between the two objects.

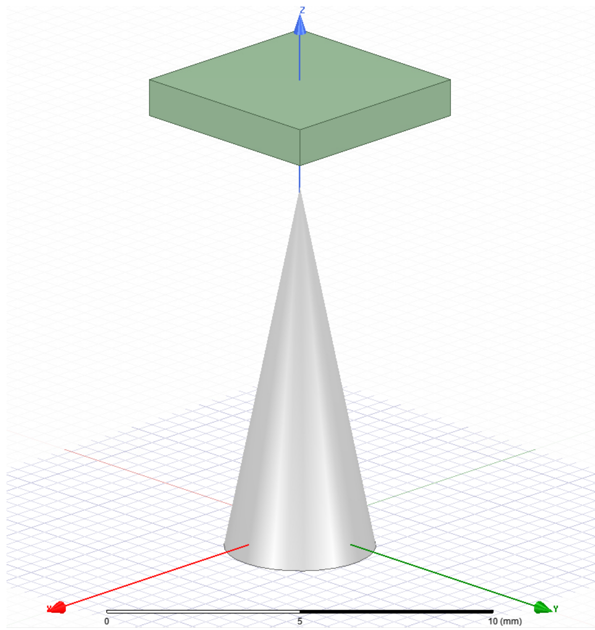


Figure 3: Conductive cone model simulating the top of a lightning rod against a charged ”cloud” box above the rod.

A similar process was followed as for Section 4.1 of the cylindrical electrostatic model. Each model was generated and given a material and electric property. In this example, the cloud above the cone was given a charge of 1000 C while the cone was grounded along with the vacuum surrounding it. In this, the field collected around the top of the cone and attracted to the sides of the surface.

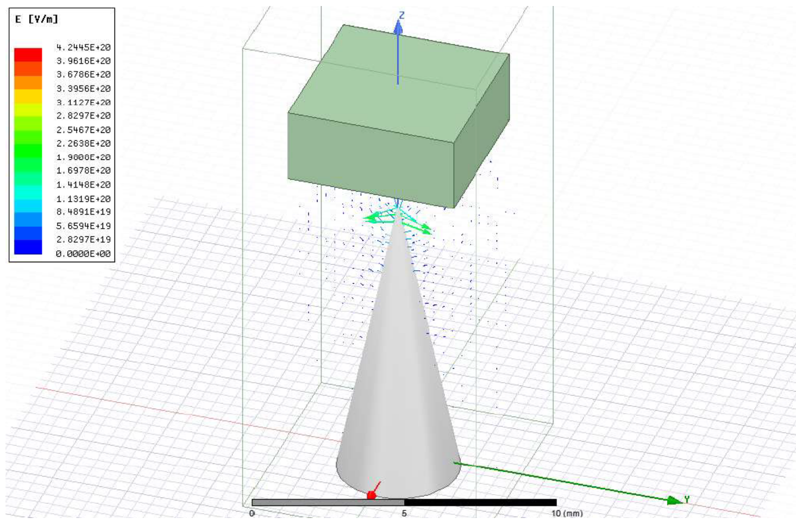


Figure 4: Electrostatic forces and field lines exerted on a grounded iron cone with a charged surface above the object

5 Discussion/Conclusions

5.1 Electric Field in a Cylinder

The field of the cylinder, in this model, is expected to be radial and perpendicular to the sides of the conductive object. The volume charge density of the object is covered and displayed by Figure 2(a) with the electric field line strength shown on the sides of the object perpendicular to the surface. As shown, the top of the cylinder, reflective of the equation explanation from Equation 5, possesses no field lines due to the radial cancellation and set region estimation in the z-direction.

5.1.1 Hand Calculations

The electric field of the cylinder of Figure 1(a) can be calculated by looking at the regions of radii provided. At the first region, the electromagnetic field at the gaussian cylinder at $r < R$ is

$$\oint E * dA = \int_A E * dA_1 + \int_A E * dA_2 + \int_A E * dA_3 \tag{4}$$

where A_1 and A_3 are perpendicular to the field and the top and bottom of the cylinder and therefor cancel and sum to zero. This is shown in Figure 2(a) as there are no field lines are the top of the cylindrical object. Therefore, the field at the sides of the object (A_2) are the only ones of concern where E is the surface of the cylinder.[2]

$$\int_A E * dA_2 = E(r)2\pi rL \tag{5}$$

Where r is the smaller radius of the cylinder and L is the length of the cylinder. This multiplies the electromagnetic field by the area of the cylindrical surface. This equals the flux of the cylinder shown by Equation 6.

$$\frac{Q_{enc}}{\epsilon_0} = \frac{pV}{\epsilon_0} = \frac{\rho \pi r^2 L}{\epsilon_0} = E(r) = 2\pi L = \frac{\rho \pi r^2 L}{\epsilon_0} = E(r) = \frac{\rho r}{2\epsilon_0} \tag{6}$$

Where the flux on the surface of the smaller radius r within the cylinder increases as the electromagnetic field increases and $V = \pi r^2 L$. Finding these results, the electromagnetic field at $r = R$ is

$$\vec{E} = \frac{\rho R}{2\epsilon_0} \tag{7}$$

Finally, and similarly, at $r > R$,

$$\vec{E} = \frac{\rho R^2}{2\epsilon_0 r} \tag{8}$$

The electric field, therefore, increases within the cylinder with increasing radius as shown in the Figure 5 below. Opposing this, the electric field decreases outside the cylinder with increasing radius.

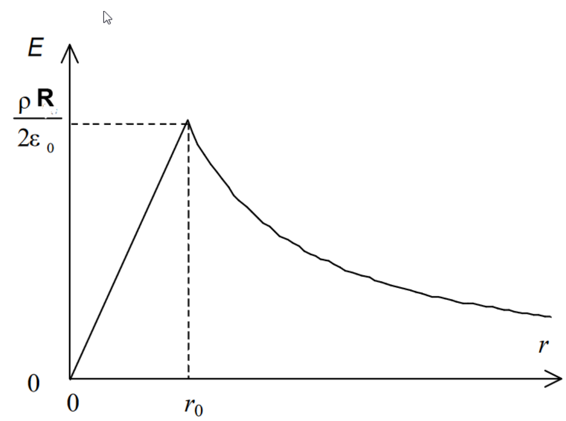


Figure 5: Electric field chart with respect to radius of a cylinder showing that electric field increases with radius up until $r = R$, then decreases logarithmicly

With an R of value 5mm, r of 1mm, and charge density (ρ) $100000 C/m$, the electric fields are provided in Equation 9

$$\vec{E} = \frac{\rho * .001}{2 * \pi * 8.85 * 10^-12 * 0.005^2} = 7.19 * 10^17 N/C \tag{9}$$

5.2 Electric Field in a Lightning Rod

According to Figure 4, the charged surface above the iron cone attracts the surrounding electromagnetic forces to the sides of the cone. In this, the point of the cone exerts a field outwards. The simulation shows how the fields of the lightning rod attract electric forces and divert lightning towards a cone tip. As a charge searches for the fastest route to ground, the cone shape of a lightning rod is able to attract the electrostatic force.

5.3 Conclusion

When specifying the electrostatic properties through ANSYS Maxwell, it is possible to both understand and calculate the forces exerted from and on the object as well as the field lines generated between charged objects. Additionally, a charged electromagnetic field can be calculated on a conductive cylinder by understanding the Gaussian equation along the surface area of the cylinder in question.

References

[1] Hayt, W.H., Buck, J.A.: Engineering electromagnetics. McGraw-Hill Education, New York, NY (2019).

[2] Gauss's Law - Infinite charged cylinder, https://www.youtube.com/watch?v=5W0-onMkhIM.