Shortest Path Problem / Algorithm

$G(V, E) \rightarrow$ weighted graph

Weight of a path P, $w(P) \rightarrow$ is the sum of weights of its constituent edges

$P = \langle v_0, v_1, \ldots, v_K \rangle$

$$w(P) = \sum_{i=1}^{K} w(v_{i-1}, v_i)$$

Shortest path weight $\delta(u, v)$ from u to v is defined as

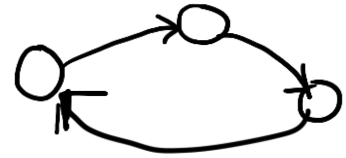
$$\delta(u, v) = \begin{cases} \min\{w(P) : u \overset{P}{\rightsquigarrow} v\}, & \text{if there is a path from } u \text{ to } v \\ \infty, & \text{otherwise} \end{cases}$$

Edge weights can represent cost, time, ...

Variants of shortest path problem:

1. Single source shortest-path problem $s \to u, v, x$
2. Single destination " " " $u, v, x \to t$
3. Single pair " " " u — v
4. All-pairs " " "

If a graph is having a cycle, then what will happen to its shortest path?



cycle → ② Positive-weighted, ① Negative-weighted

① Negative-weighted cycle:

· Source vertex, S

If a graph contains negative weighted cycle in its shortest path, then the shortest path weight is not well-defined.

i.e. the graph will not have negative weighted cycle in its shortest path.

② Positive-weighted cycle: A shortest path can't contain a positive-weighted cycle also. Removing the cycle from the path produces a path with same source and destination vertices and a lower path weight.

Conclusion: A graph can't contain a cycle (positive/negative-weighted) in its shortest-path.