

Trees  $\rightarrow$  Pointers



- $\rightarrow$  Binary Search Trees (BST)
- $\rightarrow$  AVL (Balanced BSTs)
- $\rightarrow$  Funken Reading.

## Why BST?

- $\rightarrow$  Search
- $\rightarrow$  Insert

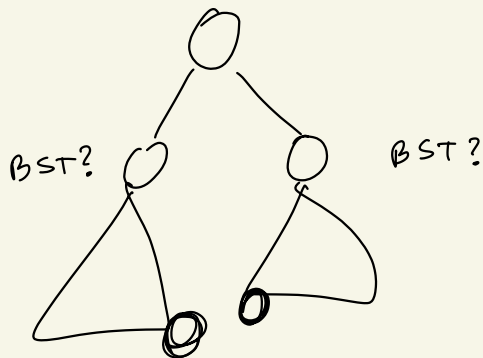
- $\rightarrow$  Delete.
- $\rightarrow$  Traverse Sorted.

- $\rightarrow$  Closest.
- $\rightarrow$  Rank

	Search	Insert	Deletes	Closest Value.	Sorted Traversal
Array (unsorted)	$O(n)$	$O(1)$	$O(n)$	$O(n)$	$O(n \log n)$
Array (sorted)	$O(\log n)$	$O(n)$	$O(n)$	$O(\log n)$	$O(n)$
Linked List	$O(n)$	$O(1)/O(n)$	$O(1)/O(n)$	$O(n)$	$O(n \log n)$
Hash Table	$O(1)$	$O(1)$	$O(1)$	$O(n)$	$O(n \log n)$
BST	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(n)$

→ keys on the left subtree  $<$  root  
key on the right subtree  $>$  root } recursively for all nodes.

→ rightmost node of the left.



When in an interview / coding round → Don't code yourself.



How are these implemented?



Balanced BSTs  
(Red-Black) } → AVL.

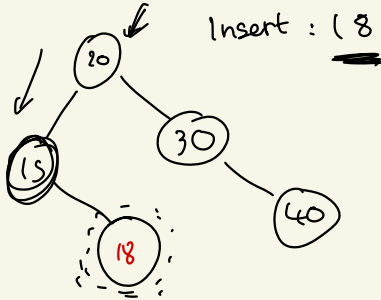
C++ → sets, maps, ordered sets  
multimaps.

Java → TreeSet, TreeMap.

Search: Iteratively -  
 $O(1)$

$O(h)$   $\leftarrow$  stack space.  
Recursive.

Insert: 20, 15, 30, 40

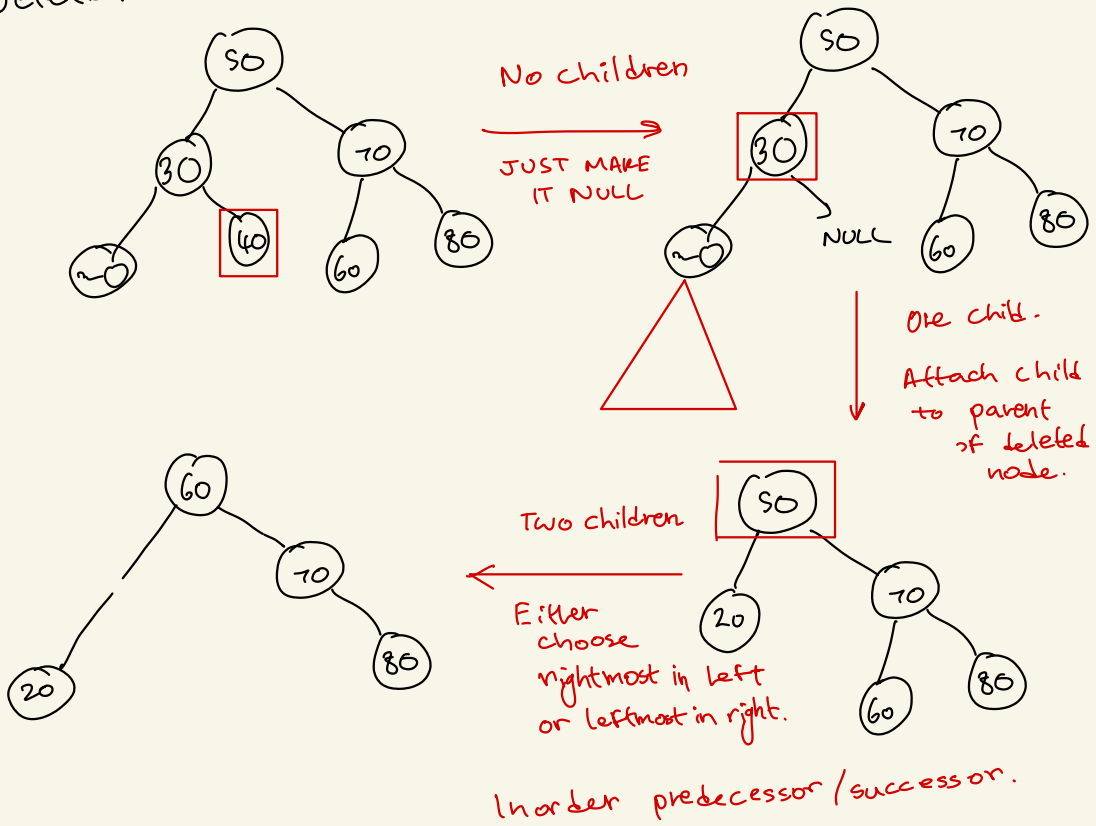


```
bool search (Node* root,
             int x) {
    Node* cur = root;
    while (cur != NULL) {
        if (cur->key == x)
            return true;

        if (cur->key > x)
            cur = cur->left;
        else
            cur = cur->right;
    }
    return false;
}
```

$\hookrightarrow$  HW: Change  
for insert.

Deletion:



HW: Implement these iteratively

Q: Given  $x$  output the greatest element in the tree that is  $\leq x$ .

Node\* floor ( Node\* root, int  $x$  ) {

Node\* ans = NULL;

Node\* cur = root;

while (cur != NULL) {

if (cur->key ==  $x$ ) {

ans = cur;

break;

}

if (cur->key >  $x$ ) {

cur = cur->left;

}

else {

ans = cur;

cur = cur->right; ↙

}

}

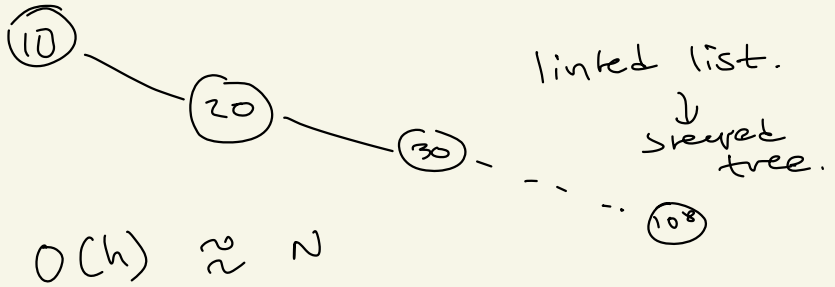
return ans;

}

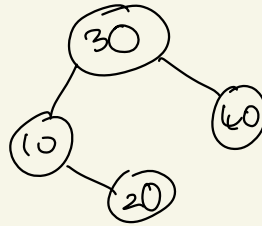
HW: Implement  
ceil( $x$ ).

# Balanced BST

10, 20, 30, 40, ...  $10^8$



→ 30, 10, 40, 20  
We don't know  
what order the  
keys arrive.



↓  
Balanced BSTs

↓  
Self-Balancing BSTs

# AVL Tree

For each node.

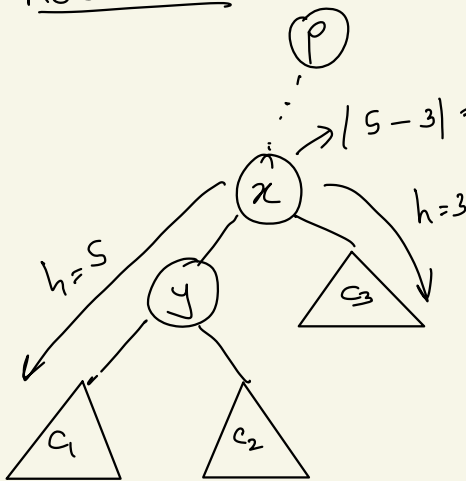
$lh$  = height of left subtree

$rh$  = " right "

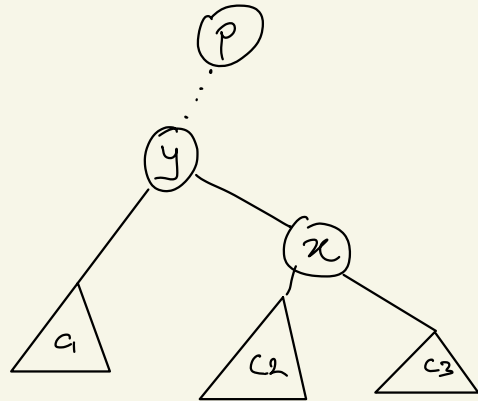
AVL Property -

$$|lh - rh| \leq 1.$$

## Rotations



$|5-3|=2$  Right Rotation.



Valid BST

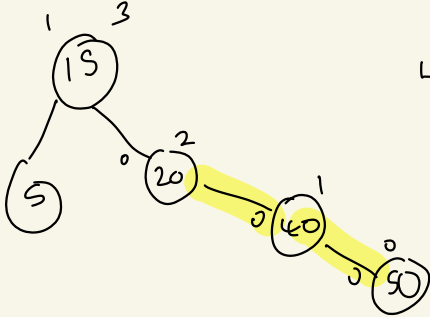
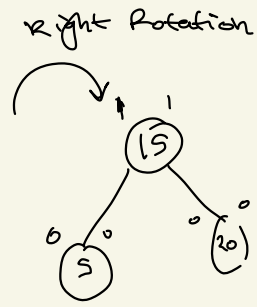
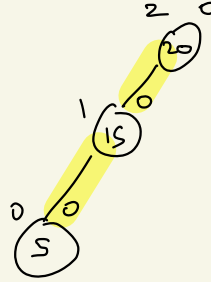
$$y < x < \text{all } c_3$$

$$\text{all } c_1 < x$$

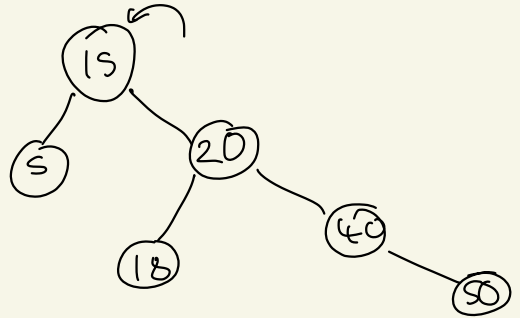
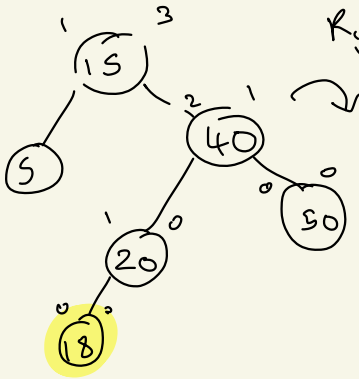
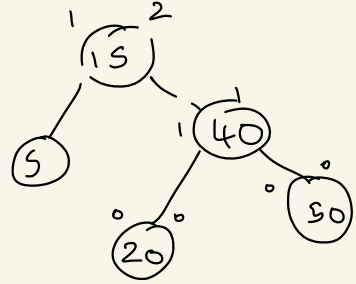
$$\text{all } c_2 < x //$$



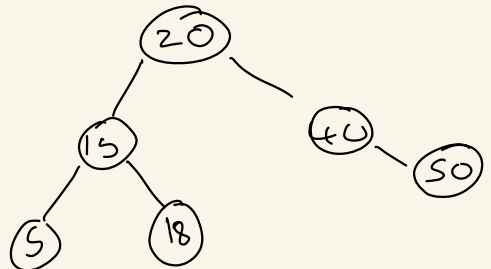
Insert: 20, 15, 5, 40, 50, 18



Left Rotation



left rotate.



$LL \rightarrow \text{Right Rotate}$   
 $RR \rightarrow \text{Left Rotate.}$  } Single Rotation

$LR \rightarrow L, R$   
 $RL \rightarrow R, L$  } Double Rotation

Insert } coding  $\rightarrow$  HW  
Delete.

## Further Reading

Assignment. , problems on BSTs  $\rightarrow \geq 4-5$   
 $\hookrightarrow$  Try solving without IDE.  
 $\hookrightarrow$  code iterative versions also.

AVL.

Extra ordered-set.  $\rightarrow$  Policy based DS.  
 $\hookrightarrow$  coding rounds. (know how to use, sgtax).

optional Red-Black Trees.