$\nabla f \equiv \text{grad } f$

$\nabla \cdot f \equiv \text{div } f$

$\nabla \times f \equiv \text{curl } f$

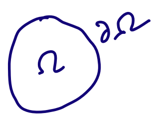
$\nabla^2 f \equiv \text{div}(\text{grad}(f))$

Goal of Identities is to make Integral symmetric

Most commonly used Operator : $\nabla^2(\cdot)$

Identity 1

$$\int_{\Omega} g \, \nabla^2 f \, d\vec{x} = \int_{\partial\Omega} g \, \nabla f \cdot \hat{n} \, ds - \int_{\Omega} \nabla g \cdot \nabla f \, d\vec{x}$$



used for

i) Diffusion Equation

$$\frac{\partial c}{\partial t} = D \nabla^2 c$$

ii) Wave Equation

$$\frac{1}{c^2} \frac{\partial^2 c}{\partial t^2} = \nabla^2 c$$

iii) Helmholtz Equation

$$\nabla^2 \vec{A} = k^2 \vec{A}$$

iv) Euler - Bernoulli

$$\nabla^2 \quad D \, \nabla^2 u(\vec{x}) = f$$

Identity 2

$\bar{\sigma}$ is a tensor - generalisations of vectors (Like a matrix)

$a = [a_{ij}]$ $\quad i = 1, 2, 3$

$b = [b_{ij}]$ $\quad j = 1, 2, 3$

$$a : b = \sum_i \sum_j a_{ij} b_{ji}$$

$\vec{v}$ is a vector

$\bar{\sigma}$ & $\nabla\vec{v}$ are tensors

$$\bar{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$\nabla\vec{v} = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} & \frac{\partial v_x}{\partial z} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z} \end{bmatrix}$$

$$\int_\Omega (\nabla\cdot\bar{\sigma})\cdot\vec{v}\, d\vec{u} = \int_{\partial\Omega} (\bar{\sigma}\cdot\hat{n})\, ds\, \vec{v} - \int_\Omega \bar{\sigma} : \nabla\vec{v}\, d\vec{u}$$

This is used when

$\bar{\sigma}$ is a gradient

as in $\bar{\sigma} = \nabla\vec{u}$

then the integral becomes

symmetric

$$\left(\int_\Omega (\nabla\vec{u} : \nabla\vec{v})\, d\vec{u} \right)$$

Elasticity.

$\bar{\sigma} \rightarrow$ stress tensor

$\bar{\varepsilon} \rightarrow$ strain tensor

$\bar{\varepsilon} = \nabla \vec{u} \rightarrow$ gradient of displacement vector

Identity 3

For $\rightarrow$ $\nabla \times \nabla \times \vec{A} \rightarrow$ usually Maxwell equations

$$\int_{\Omega} \nabla \times (\nabla \times \vec{A}) \cdot \vec{B} \, d\vec{n}$$

$$= \int_{\Omega} (\nabla \times \vec{A}) \cdot (\nabla \times \vec{B}) \, d\vec{n}$$

$$- \int_{\partial\Omega} \vec{B} \cdot (\nabla \times \vec{A}) \times \hat{n} \, dS$$