SAVEETHA University

CSA13 Theory of Computation

Time: **2 Hours** Total marks: **70**

Answer all the questions. Make your answers short and precise.

1. (2.5+2.5 marks) Let the alphabet be $\{a, b\}$. Write regular expressions for the following languages.
   (a) The set of all words w that satisfies the following property: if there is an a in w then there is also a b in w. For example, bb and ba are in the language but not aa.
   (b) The set of all words that contain at least two a's.

2. (5 marks) Define what is Chomsky Normal Form (CNF). Convert the following grammar to CNF.
$$S \to \epsilon \mid aSb \mid bSa \mid SSSS$$

3. (12 marks) Let $L \subseteq \{0, 1\}^*$ be the set of words of length atleast 2 whose second last letter is 0. For instance $101 \in L$ but neither 10 nor 1. Draw an NFA for the language L. Then determinize it and draw an equivalent DFA.

4. (6+6 marks)
   (a) Show that if $L \subseteq \Sigma^*$ is regular, then so is the language
$$double(L) = \{ a_1 a_1 a_2 a_2 \cdots a_n a_n \mid a_1 a_2 \cdots a_n \in L, a_i \in \Sigma \}.$$
   for example, if $L = \{a, aab\}$ then $double(L) = \{aa, aaaabb\}$.
   (b) How do you check that all the words accepted by a given NFA are of even length?

5. (6+6 marks)
   (a) Write a context-free grammar for the language $\{a^m b^n \mid m \leq n\}$.
   (b) Are context-free languages closed under intersection? Justify.

6. (12 marks) Is the language $\{a^n b^n c^n \mid n \in \mathbb{N}\}$ accepted by a deterministic Turing machine? If not, prove it. Otherwise give an informal definition of such a machine..

7. (6+6 marks)
   (a) Let L_1 and L_2 be recursive. Is $L_1 \cdot L_2$ recursive? Justify.
   (b) Without using Rice's theorem, show that it is undecidable to check if a Turing machine accepts an odd length word.