SAVEETHA University

CSA13 Theory of Computation

Time: **2 Hours** Total marks: **70**

Answer all the questions. Make your answers short and precise.

1. (2.5+2.5 marks) Let the alphabet be $\{a, b\}$. Write regular expressions for the following languages.
   (a) Set of all words that contain no ab. For instance the words b and baa are in the language but not abb or aab.
   (b) Set of all words that contain an a as well as a b.
2. (5 marks) Construct a grammar for the set of words given by the regular expression $(ab)^*a$.
3. (12 marks) Let $L \subseteq \{a, b\}^*$ be the set of all words of length at least 2 whose last two letters are the same. For example abb is in L but not aba. Construct an NFA for the language and determinize it and draw the resulting DFA.
4. (6+6 marks)
   (a) Show that if $L \subseteq \Sigma^*$ is regular then the language $w^{-1}L = \{u \mid wu \in L\}$ is also regular.
   (b) Is the language $\{a^n b^{2n} \mid n \geq 0\}$ regular? Justify.
5. (6+6 marks)
   (a) Simplify the following grammar (remove all the useless productions, unit productions and epsilon productions).
   $$\begin{aligned} S &\to aA \mid aBB \\ A &\to aaA \mid \epsilon \\ B &\to bB \mid bbC \\ C &\to B \end{aligned}$$
   (b) Show that the language $\{a^n b^m c^n d^m \mid m \geq 0, n \geq 0\}$ is not context-free.
6. (6+6 marks)
   (a) Give the informal description of a push down automaton that accepts the language $\{a^m b^{2m} \mid m \geq 0\}$.
   (b) Show that if L_1 and L_2 are languages accepted by Turing machines then $L_1 \cup L_2$ is also accepted by a Turing machine.
7. (6+6 marks)
   (a) Is it decidable to check if the language of a Turing machine is finite? Justify.
   (b) Show that there is a language $L \subseteq \{1\}^*$ that is not-recursively enumerable.