SAVEETHA University

CSA13 Theory of Computation

Time: **2 Hours** Total marks: **70**

Answer all the questions. Make your answers short and precise.

1. (2.5+2.5 marks) Let the alphabet be $\{a, b\}$. Write regular expressions for the following languages.
   (a) Set of words of length at most 2.
   (b) Set of all words beginning and ending with the same letter.

2. (1+2+2 marks) Consider the following grammar.

$$\begin{aligned} S &\to T \mid U \\ T &\to aT \mid X \\ U &\to Uc \mid Y \\ X &\to \epsilon \mid bXc \\ Y &\to \epsilon \mid aYb \end{aligned}$$

   (a) What is the language defined by the grammar?
   (b) Give the derivation tree for the word *aaabbbc*.
   (c) A grammar is ambigous if there exists two distinct derivations for a word. Is the above grammar ambigous? Justify.

3. (12 marks) Let $L \subseteq \{a, b\}^*$ be the language of words w such that the last letter of w also appears somewhere else in w. For example *abaab* is in L but not *bba*. Construct an NFA for L. Then determinize it and draw the equivalent DFA.

4. (6+6 marks)
   (a) Write a context-free grammar for the language $\{a^p b^q c^q d^p \mid p \geq 0, q \geq 0\}$.
   (b) Show that if L is a context-free language then $L^r = \{w^r \mid w \in L\}$ is context-free, where w^r denotes the reverse of the word w. For example if $w = abaab$ then $w^r = baaba$.

5. (6+6 marks)
   (a) The symmetric difference of two languages L_1 and L_2 is the set of all words that is in exactly one of them. Show that if L_1 and L_2 are regular, their symmetric difference is also regular.
   (b) Is the language $\{w \in \{a, b\}^* \mid w$ contains the same number of a's and b's$\}$ regular? Justify.

6. (12 marks) Is the language $\{ww \mid w \in \Sigma^*\}$ accepted by a deterministic Turing machine? If not, prove it. Otherwise give an informal definition of such a machine..

7. (6+6 marks)

   (a) Is the set of all recursively-enumerable languages countable? Justify.

   (b) Without using Rice's theorem, show that it is undecidable to check if the language of a given Turing machine is regular.