IE590

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Q-1 Arrange the following functions in increasing order of their growth rate

$0.1n^2, 1000n, \log n, \sqrt{n}, 2^n, n!$

$$\lim_{n\to\infty} \frac{\log n}{1000n} = \lim_{n\to\infty} \frac{1}{1000n} = 0$$

$$\lim_{n\to\infty} \frac{2^n}{n!} = \frac{2 \times 2 \times \cdots (n \text{ times})}{n(n-1) \times \cdots 1}$$

$$= \underline{\underline{0}}$$

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\log n} = \lim_{n\to\infty} = \frac{\frac{1}{2}n^{-1/2}}{1/n} = \infty$$

ascending order: $\log n < \sqrt{n} < 1000n < 0.1n^2 < 2^n < n!$

Q.2 Show $T(n) = O(\log_2 n)$

$T(n) = T(n/2) + 1$

$T(n) = (T(n/4) + 1) + 1$

$T(2^m) = T(2^{m-2}) + 2$

$T(2^m) = T(2^{m-3}) + 3$

$T(2^m) = T(2^{m-m}) + m$

$T(2^m) = 1 + m$

let $n = 2^m$

$\therefore T(n) = 1 + m$

$\therefore T(n) = 1 + \log_{m_2} n$

$\therefore T(n) = O(\log_2 n)$

(Q-3) State max flow min cut thm:-

The max flow min cut thm states that in a flow network, the amount of max. flow is equal to capacity of the min cut

Let $N = (V, E)$ be a directed graph where V denotes the set of vertices & E is the set of edges

The capacity of an edge is mapping $c: E \to R^+$, denoted by $c(u,v)$

An s-t cut is a division of the vertices of the network in two parts, with the source in one part & sink in other

Therefore the max value of an s-t flow is equal to the minimum capacity over all the cuts
(Source: Wikipedia)

Q-4 Can you formulate the assignment problem as a linear program?

⇒ Let 'n' men to whom 'n' women are to be matched

The compatibility of man 'i' with woman 'j' is c_{ij}

4. ~~Analyze the time complexity of merge sort~~

let $x_{ij} = \begin{cases} 1 & \text{if man 'i' is matched with woman 'j'} \\ 0 & \text{if man 'i' is not matched with woman 'j'} \end{cases}$

maximize $\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_{ij}$

s.t

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \text{for } j = 1, \ldots, m$$

$$\sum_{j=1}^{n} x_{ij} = 1 \quad \text{for } i = 1, 2 \ldots, m$$

$$x_{ij} = 0 \text{ or } 1$$

If the problem, $\min cx$
s.t $Ax = b$
$x \geq 0$

such that all the components of 'b' are integers, has at least one feasible solution, then it has integer valued feasible sol^n.

If it has an optimal sol^n, then it has an integer valued optimal sol^n.

(Q-5)

Prove the following statement

A^k: If $(A^k)_{ij} \neq 0 \rightarrow$ a path of length $\leq k$ betn i & j

If $(A^k)_{ij} = 0$, there is no path of length $\leq k$,

Let us prove the above using proof by induction,

$(A^k) = A^{k-1} \cdot A$

$(A^k)_{ij}$ = i^{th} row of A^{k-1} & j^{th} col of A

If $(A^k)_{ij} \neq 0$ for some 'r' $a_r \neq 0$ & $b_r \neq 0$

$a_r \neq 0$ tells us that there exist a path $\leq k-1$ from $i \rightarrow r$

$b_r \neq 0$ $\exists$ a path of length 1 from $r \rightarrow j$

Thus, we ~~can~~ proved that diagonal elements of adjacency matrix are ~~w~~ raise to power k non zero while all other elements are zero.