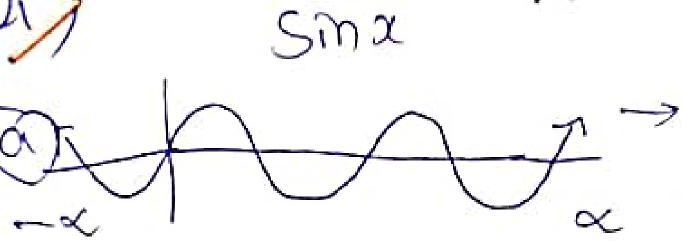
Q1) Tutorial 3

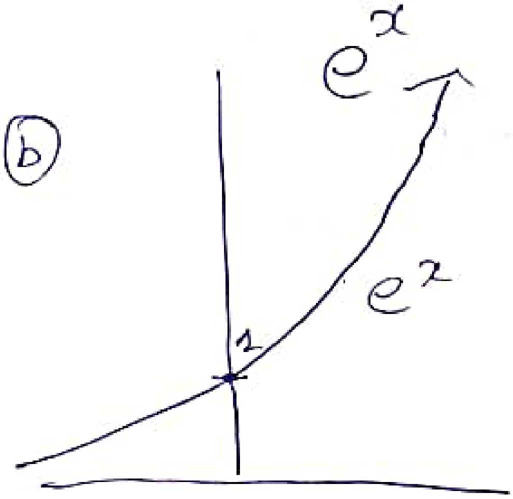
(a)



✗

never converges to zero

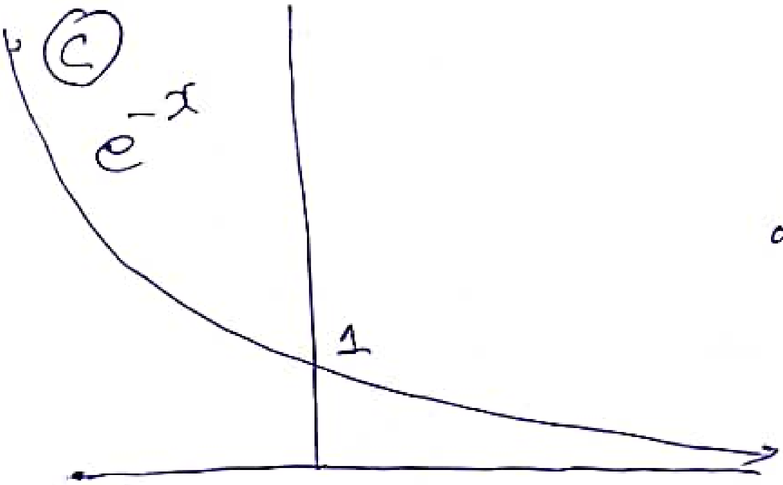
(b)



✗

diverges at $x = \infty$

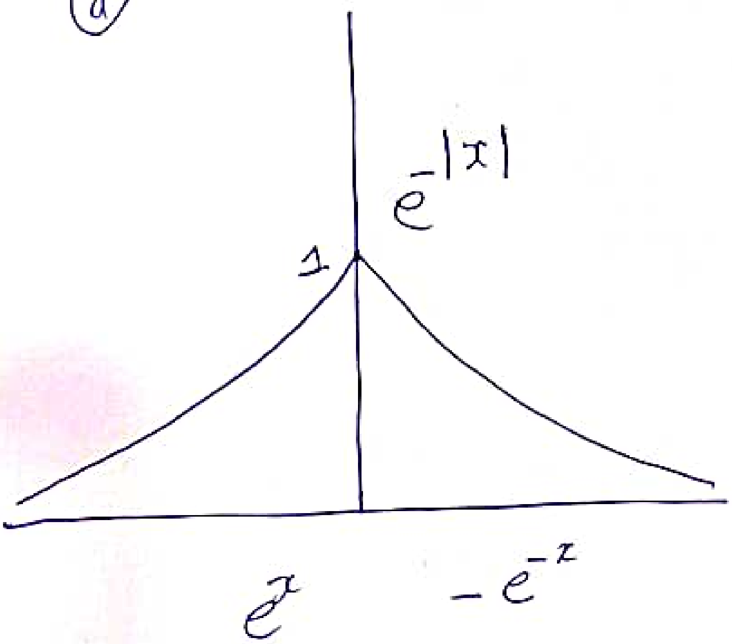
(c)



✗

diverges at $x = -\infty$

(d)

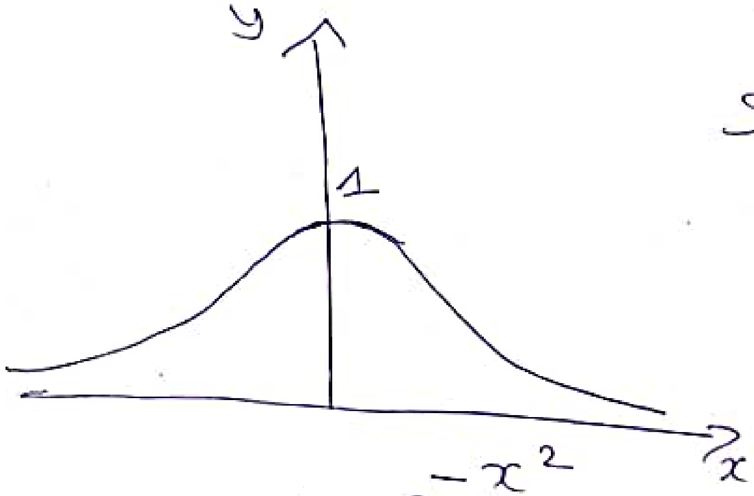


✗

derivative does not exist at $x = 0$

(e)

e^{-x^2}



$-2x e^{-x^2}$

✓

Suitable.
for $-\infty$ to ∞

derivative exist at all point
Smooth (continuous)
Sigle valued.
at ∞ & $-\infty$
Converges to zero.

(f) $e^{ikx} = \cos x + i \sin x$

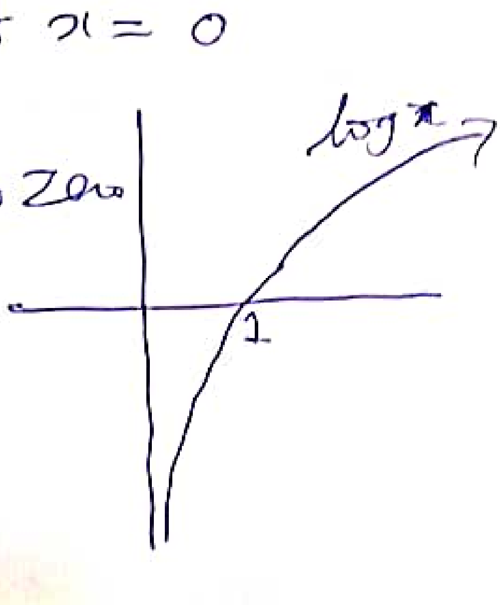
never converges to zero.

(g) x^n

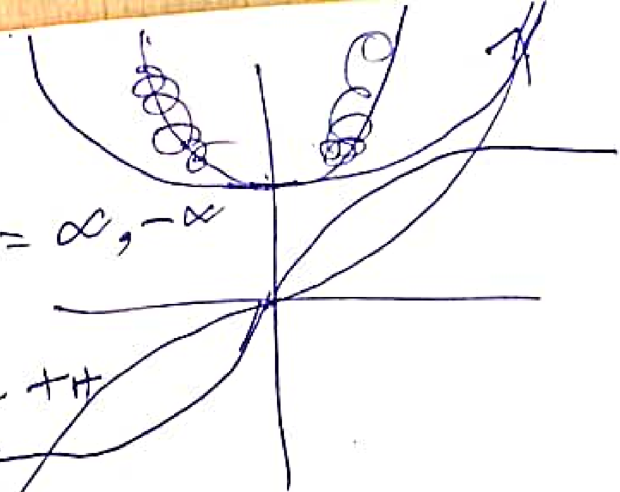
diverges at $x = \infty$

(h) $\frac{1}{x^n}$ diverges at $x = 0$

(i) $\log x$ diverges at $x = 0$
$\log(0) = -\infty$
& never converges to zero
at $x = \infty$



(j) ✗ $\sinh x$



diverges at $x = \infty, -\infty$

x very large +ve

$\sinh x = e^{x}$

~~very~~ large --

$\sinh x = - e^{x}$

$\sinh x = \dfrac{e^{x} - e^{-x}}{2}$

(k) ✗ $\cosh x$

diverges at $x = \infty, -\infty$

$\cosh x = \dfrac{e^{x} + e^{-x}}{2}$

(l) ✗ $\tanh x$ never converged to zero

(m) ✗ $\sec(x) = \dfrac{1}{\cos x}$

diverges at $x = \pi/2, 3\pi/2 \ldots \dfrac{(2n+1)\pi}{2}$

(n) ✗ ~~csc(x)~~ $\csc(x) = \dfrac{1}{\sin x}$

diverges at $x = 0, \pi, 2\pi .. n\pi$

(1)

$$\int_{-\infty}^{\infty} \psi^* \psi \, dx = 1$$

$$\int_{-\infty}^{0} \psi^* \psi \, dx + \int_{0}^{L} \psi^* \psi \, dx + \int_{L}^{\infty} \psi^* \psi \, dx = 0$$

$$\underbrace{\int_{-\infty}^{0} \psi^* \psi \, dx}_{=0} \qquad \underbrace{\int_{L}^{\infty} \psi^* \psi \, dx}_{=0}$$

$$\int_{0}^{L} A^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

$$A^2\left[\frac{1}{2}\int_0^L dx - \frac{1}{2}\int_0^L \cos\frac{2n\pi x}{L}\,dx\right] = 1$$

$$A^2\left[\frac{1}{2}[L] - \frac{1}{2}\left[\sin\frac{2n\pi x}{L} \times \frac{L}{2\pi n}\right]_0^L\right] = 1$$

$$A^2\left[\frac{L}{2} - \frac{1}{2}[0-0]\right] = 1$$

$$A^2\left[\frac{L}{2}\right] = 1$$

$$A = \sqrt{\frac{2}{L}}$$

$$\int_{-\infty}^{0} \psi^* \psi \, dx + \int_{0}^{L} \psi^* \psi \, dx + \int_{L}^{\infty} \psi^* \psi \, dx = 1$$

$$\Downarrow \quad 0 \qquad\qquad \Downarrow \quad 0$$

$$A^2 \int_0^L \cos^2\left(\frac{n\pi x}{L}\right) dx = 1$$

$$A^2 \int_0^L \frac{1}{2}\left[1 + \cos\left(\frac{2n\pi x}{L}\right)\right] dx = 1$$

$$A^2 \int_0^L \frac{1}{2}\, dx + A^2 \int_0^L \frac{1}{2} \cos\left(\frac{2n\pi x}{L}\right) dx = 1$$

$$A^2 \left[\frac{L}{2}\right] + A^2 \frac{1}{2}\left[\sin\left(\frac{2n\pi x}{L}\right) \times \frac{L}{2n\pi}\right]_0^L = 1$$

$$A^2 \left[\frac{L}{2}\right] + A^2 \frac{1}{2}\left[0 - 0\right] = 1$$

$$\boxed{A = \sqrt{\frac{2}{L}}}$$

Q2

$$A^2\int_{-\infty}^{\infty} e^{-sx^2} e^{-sx^2}\, dx = 1$$

$$A^2\int_{-\infty}^{\infty} e^{-2sx^2}\, dx = 1$$

$$A^2\sqrt{\frac{\pi}{2s}} = 1 \qquad \int_{-\infty}^{\infty} e^{-ax^2}\, dx = \sqrt{\frac{\pi}{a}}$$

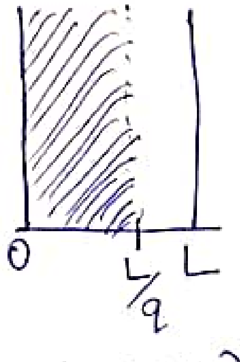
$\boxed{\text{Here } a = 2s}$

$$A^2 = \sqrt{\frac{2s}{\pi}}$$

$$A = \left(\frac{2s}{\pi}\right)^{1/4}$$

Q3

$$\psi_{1D-box} = \sqrt{\frac{2}{L}}\, \sin\left(\frac{n\pi x}{L}\right)$$



Prob. distribution of the particle is given by $\psi^*\psi = \frac{2}{L}\sin^2\left(\frac{n\pi x}{L}\right)$

$\therefore$ Prob. to find the particle between 0 to $L/4$ is

$$\frac{2}{L}\int_0^{L/4} \sin^2\left(\frac{n\pi x}{L}\right) dx = \frac{2}{L}\int_0^{L/4} \frac{1}{2}\left(1 - \cos\frac{2n\pi x}{L}\right) dx$$

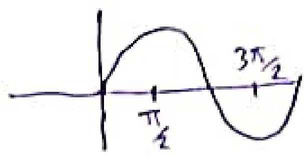
Q3

$$= \frac{1}{L}\int_0^{L/q} dx - \frac{1}{L}\int_0^{L/q} \cos\frac{2n\pi x}{L}\,dx$$

$$= \frac{1}{\cancel{L}}\left(\frac{\cancel{L}}{q}\right) - \frac{1}{\cancel{L}} \sin\left(\frac{2n\pi x}{L}\right)\Bigg|_0^{L/q} \times \frac{\cancel{L}}{2n\pi}$$

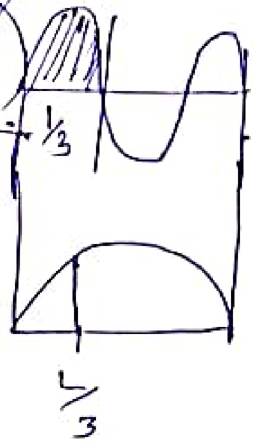
$$= \frac{1}{q} - \frac{\sin\left(\frac{2n\pi \cancel{L}}{\cancel{L}\, q}\right)}{2n\pi} = \frac{1}{q} - \frac{\sin\left(\frac{2n\pi}{q}\right)}{2n\pi}$$

if $n = 1$ $\quad q = 2$



$q = 3$

$$\text{Prob} = \frac{1}{2} - 0 \quad \cancel{= \frac{1}{3}} - \frac{\sin\left(\frac{2\pi}{3}\right)}{2\pi}$$



if $n = 1$ $\quad q = 3$

$$\text{Prob} = \frac{1}{3} - \frac{\sin\left(\frac{2\pi}{3}\right)}{2\pi}$$

$$\text{Prob} = \frac{1}{3} + \frac{1}{2\pi}$$

if $n = 3$ and $q = 3$	$n = 3$ $q = 2$ $\text{Prob} = \frac{1}{2}$
if $n = q$ $\text{Prob} = \frac{1}{q}$	if $q = 2$ $\text{Prob} = \frac{1}{2}$

$$\text{Prob} = \frac{1}{3}$$

$q = 3/2$ $\quad n = 3$

$\text{Prob} = \frac{2}{3}$.

Q11 (i)

$$\psi = A \sin\left(\frac{n\pi x}{L}\right).$$

$$A = \sqrt{\frac{2}{L}} \text{ from Q7 (i)}$$

$$\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

$$\langle x \rangle = \int_{-\infty}^{\infty} \psi^* x \psi \, dx$$

$$= \underbrace{\int_{-\infty}^{0} \psi^* x \psi \, dx}_{=0} + \int_{0}^{L} \psi^* x \psi \, dx + \underbrace{\int_{L}^{\infty} \psi^* x \psi \, dx}_{=0}$$

$$= \left(\frac{2}{L}\right) \int_{0}^{L} x \sin^2\left(\frac{n\pi x}{L}\right) dx$$

$u = x \qquad dv = \sin^2\left(\frac{n\pi x}{L}\right) dx$

$du = dx \qquad v = \left[\frac{x}{2} - \frac{\sin 2x}{4}\right]$

$$\frac{n\pi x}{L} = y \qquad x = 0 \quad y = 0$$

$$x = y\left(\frac{L}{n\pi}\right) \qquad x = L \quad y = n\pi$$

$$dx = dy\left(\frac{L}{n\pi}\right)$$

$$= \left(\frac{2}{L}\right)\int_0^{n\pi} y\left(\frac{L}{n\pi}\right)\sin^2 y\, dy\left(\frac{L}{n\pi}\right)$$

$$= \frac{2}{L}\left(\frac{L}{n\pi}\right)^2\int_0^{n\pi} y\sin^2 y\, dy$$

$u = y \qquad dv = \sin^2 y\, dy$

$du = dy \qquad v = \left[\frac{y}{2} - \frac{\sin 2y}{4}\right]$

$$= \frac{2}{L}\left(\frac{L^2}{n^2\pi^2}\right)\left\{\left[\frac{y}{2}\times y\right]_0^{n\pi} - \int_0^{n\pi}\left[\frac{y}{2} - \frac{\sin 2y}{4}\right]dx\right\}$$

$$= \frac{2}{L}\left(\frac{L^2}{n^2\pi^2}\right)\left\{\frac{n^2\pi^2}{2} - \left[\frac{y^2}{4}\right]_0^{n\pi} + \int_0^{n\pi}\left[\frac{\cos 2y}{8}\right]_0^{n\pi}\right\}$$

$$\Downarrow$$

$$0$$

$$= \frac{2L^2}{Ln^2\pi^2}\left\{\frac{n^2\pi^2}{2} - \frac{n^2\pi^2}{4}\right\} + \cancel{\left[\frac{\sin 2y}{8}\right]_0^{n\pi}}$$

$\cos 2n\pi = 1$

$\cos 0 = 1$

$$= \frac{\cancel{2}L^{\cancel{2}}}{\cancel{L}\cancel{n^2\pi^2}}\left\{\frac{\cancel{n^2\pi^2}}{\cancel{4}\,2}\right\}$$

$$= \frac{L}{2}$$

Q4) (i)

$$\langle p \rangle = \left(\frac{2}{L}\right)\int_0^L \sin\left(\frac{n\pi x}{L}\right)\left(-i\hbar\frac{\partial}{\partial x}\right)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{2}{L}\int_0^L \sin\left(\frac{n\pi x}{L}\right) - i\hbar\cos\left(\frac{n\pi x}{L}\right)dx\left(\frac{n\pi}{L}\right)$$

$$= \frac{-2}{L}i\hbar\left(\frac{n\pi}{L}\right)\int_0^L \sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{-\cancel{2}i\hbar n\pi}{L^2}\frac{1}{\cancel{2}}\int_0^L \sin\frac{2n\pi x}{L}\,dx$$

$$= \frac{-i\hbar n\pi}{L^2}\left[\frac{-\cos\frac{2n\pi x}{L}}{2n\pi/L}\right]_0^L$$

$$= \frac{-i\hbar n\pi}{L^2}\left[-1+1\right]$$

$$\langle p \rangle = 0$$

Q4 (ii) $A^2\int_{-\infty}^{\infty} x e^{-2sx^2}\,dx$

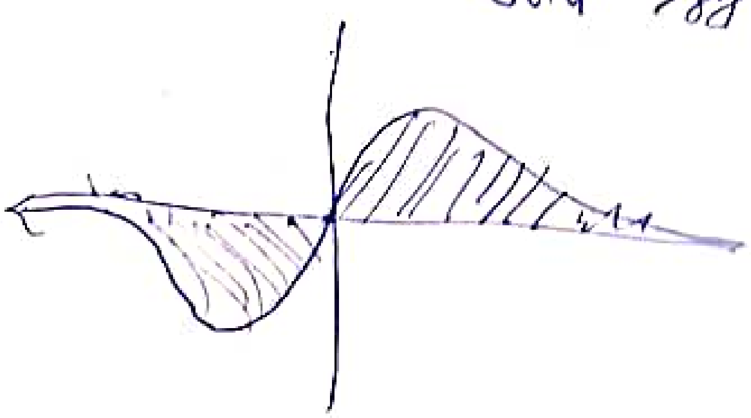
from Q2 (iii) $A = \left(\frac{2s}{\pi}\right)^{1/4}$

$\therefore \left(\frac{2s}{\pi}\right)^{1/2}\int_{-\infty}^{\infty} x\, e^{-2sx^2}\,dx$

$s = \frac{m\omega}{2\hbar}$

$\left(\frac{m\omega}{\pi\hbar}\right)^{1/2}\int_{-\infty}^{\infty} x\, e^{-\cancel{2}\left(\frac{m\omega}{\cancel{2}\hbar}\right)x^2}\,dx$

$\left(\frac{m\omega}{\pi\hbar}\right)^{1/2}\int_{-\infty}^{\infty} \underbrace{x\, e^{-\frac{m\omega x^2}{\hbar}}}_{\text{odd symmetric}}\,dx = 0$



Q4 (ii) $\cancel{\langle p \rangle} = \left(\frac{m\omega}{\pi\hbar}\right)^{1/2} \int_{-\alpha}^{\alpha} e^{-\frac{m\omega}{2\hbar}x^2} \left(-i\hbar\frac{\partial}{\partial x}\right) e^{-\frac{m\omega}{2\hbar}x^2} dx$

$$= \left(\frac{m\omega}{\pi\hbar}\right)^{1/2} \int_{-\alpha}^{\alpha} e^{-\frac{m\omega}{2\hbar}x^2} \left(+i\hbar(\cancel{+2}x)\right) e^{-\frac{m\omega}{2\hbar}} \left(\frac{m\omega}{\cancel{2}\hbar}\right) dx$$

$$= \left(\frac{m\omega}{\pi\hbar}\right)^{1/2} \int_{-\alpha}^{\alpha} \frac{i\hbar m\omega}{\hbar} e^{-\frac{m\omega}{2\hbar}x^2} x\, e^{-\frac{m\omega}{2\hbar}x^2} dx$$

$$= \left(\frac{m\omega}{\pi\hbar}\right)^{1/2} \left(\frac{i\hbar m\omega}{\hbar}\right) \underbrace{\int_{-\alpha}^{\alpha} x\, e^{-\frac{m\omega x^2}{\hbar}} dx}_{0}$$

$$\langle p \rangle = 0$$

$$A^2\int_0^\infty e^{-2\alpha x}\,dx = 1 \qquad \int_0^\infty e^{-ax}\,dx = \frac{1}{a}$$

$$A^2\left(\frac{1}{2\alpha}\right) = 1$$

$$A^2 = 2\alpha$$

$$A = \sqrt{2\alpha}$$

$$\phi = \sqrt{2\alpha}\, e^{-\alpha x} \qquad x > 0$$

7

$$\psi(x) = ax \quad (0 \le x \le 1) \qquad A^2\int_0^1 (ax)^2\,dx = 1$$

$$\psi(x) = 0 \text{ elsewhere}$$

$$A^2 a^2 \left[\frac{x^3}{3}\right]_0^1 = 1$$

$$\psi(x) = \sqrt{\frac{3}{a^2}}\, ax$$

$$= \frac{\sqrt{3}}{a}\, ax \qquad A^2 a^2\left(\frac{1}{3}\right) = 1$$

$$\boxed{\psi(x) = \sqrt{3}\, x} \qquad A^2 = \frac{3}{a^2}$$

$$A = \sqrt{\frac{3}{a^2}} = \frac{\sqrt{3}}{a}$$

$$3\int_{0.45}^{0.55}\left[\frac{x^2}{0}\right]$$

0.45

0.55

$$3\int_{0.45}^{0.55} x^2\,dx = 3\left[\frac{x^3}{3}\right]_{0.45}^{0.55}$$

$$= \left[0.55^3 - 0.45^3\right]$$

$$=$$

7 b

$$\langle x\rangle = \sqrt{3}\sqrt{3}\int_0^1 x\ x\ x\,dx$$

$$= 3\int_0^1 x^3\,dx$$

$$= 3\left[\frac{x^4}{4}\right]_0^1$$

$$\langle x\rangle = \frac{3}{4}$$

$$A^2\int_{-\pi/4}^{\pi/2} \cos^4 x \, dx = 1$$

$$A^2\int_{-\pi/4}^{\pi/2} \frac{1}{2}(1+\cos 2x)\frac{1}{2}(1+\cos 2x)\,dx = 1$$

$$A^2\int_{-\pi/4}^{\pi/2} \frac{1}{4}\left(1+\cos^2 2x + 2\cos 2x\right) dx = 1$$

$$\frac{A^2}{4}\int_{-\pi/4}^{\pi/2} \left(1+\left(\frac{1+\cos 4x}{2}\right)+2\cos 2x\right) dx = 1$$

$$\frac{A^2}{4}\int_{-\pi/4}^{\pi/2} \left(1+\frac{1}{2}+\frac{\cos 4x}{2}+2\cos 2x\right) dx = 1$$

$$A^2\left\{\int_{-\pi/4}^{\pi/4} 3\left(\frac{dx}{8}\right) + \int_{-\pi/4}^{\pi/2} \frac{\cos 4x}{8}\,dx + \frac{1}{2}\int_{-\pi/4}^{\pi/2} \cos 2x\,dx = 1\right.$$

$$A^2\left\{\left[\frac{3x}{8}\right]_{-\pi/4}^{\pi/2} + \left[\frac{\sin 4x}{32}\right]_{-\pi/4}^{\pi/2} + \left[\frac{\sin 2x}{4}\right]_{-\pi/4}^{\pi/2}\right\} = 1$$

$$A^2\left\{\frac{3}{8}\left(\frac{\pi}{2}+\frac{\pi}{4}\right)+0+0-\frac{\sin\left(-\frac{\pi}{2}\right)}{4}\right\}=1$$

$$A^2\left\{\frac{3}{8}\left(\frac{3\pi}{4}\right)+\frac{\sin\left(\frac{\pi}{2}\right)}{4}\right\}=1 \qquad \underline{7.1A^{\circ}}$$

$$A^2\left\{\frac{9}{32}\pi+\frac{1}{4}\right\}=1$$

$$A^2\left(\frac{9\pi+8}{32}\right)=1$$

$$A^2=\frac{32}{9\pi+8}$$

$$A=\frac{4\sqrt{2}}{\sqrt{9\pi+8}}$$

For $-\pi/2$ to $\pi/2$

$$A^2\left[\frac{3}{8}\left(\frac{\pi}{2}+\frac{\pi}{2}\right)\right]+0+0=1$$

$$A^2\frac{3\pi}{8}=1$$

$$A=\frac{2\sqrt{2}}{\sqrt{3\pi}}=\sqrt{\frac{8}{3\pi}}$$

$$\text{Prob} = \frac{32}{9\pi+8}\int^{\pi/4} \cos^4 x \, dx$$

$$= \frac{32}{9\pi+8}\left\{\left[\frac{3x}{8}\right]_0^{\pi/4} + \left[\frac{\sin 4x}{32}\right]_0^{\pi/4} + \left[\frac{\sin 2x}{4}\right]_0^{\pi/4}\right\}$$

$$= \frac{32}{9\pi+8}\left\{\frac{3}{8}\times\frac{\pi}{4} + 0 - 0 + \frac{1}{4} - 0\right\}$$

$$= \frac{32}{9\pi+8}\left\{\frac{3\pi}{32} + \frac{1}{4}\right\}$$

$$= \frac{\cancel{32}}{9\pi+8}\left\{\frac{3\pi+8}{\cancel{32}}\right\}$$

$$= \frac{1}{9\pi+8}\left\{\frac{3\pi+8}{1}\right\}$$

$$\text{Prob.} = \frac{3\pi+8}{(9\pi+8)} = 0.48.$$

$$\text{For } \phi = \sqrt{\frac{8}{3\pi}} \cos^2 x$$

$$\text{Prob} = \frac{8}{3\pi}\left(\frac{3\pi}{32} + \frac{1}{4}\right) = \frac{\cancel{8}}{3\pi}\,\frac{(3\pi+8)}{\cancel{32}\,4}$$

$$= \frac{3\pi+8}{12\pi} = 0.46$$

Solution :- 5

$$\phi = A e^{-x^2}$$

Normalization condition

$$\int_{-\infty}^{\infty} |\phi|^2 dx = 1 \Rightarrow \underbrace{\int_{-\infty}^{\infty} A^2 e^{-2x^2} dx = 1}$$

$$\int_{-\infty}^{\infty} e^{-\alpha x^2 + \beta x} dx = \sqrt{\frac{\pi}{\alpha}} \exp\left[\frac{\beta^2}{4\alpha}\right]$$ Formula

$$\int_{-\infty}^{\infty} e^{-2x^2} dx = \sqrt{\frac{\pi}{2}}$$

$$\Rightarrow A^2 \left(\sqrt{\frac{\pi}{2}}\right) = 1 \Rightarrow A = \left(\frac{2}{\pi}\right)^{1/4}$$

Sol:→ 6

$$\psi(x) = A e^{-\alpha x} \quad \text{for } x > 0$$

Normalization condition

$$\int_{-\infty}^{\infty} \psi^*(x)\, \psi(x)\, dx = 1$$

In the question it is given for $x > 0$ $\Rightarrow$

$$\int_{0}^{\infty} A^2 e^{-2\alpha x} dx = 1$$

$$A^2 \int_{0}^{\infty} e^{-2\alpha x} dx = 1 \Rightarrow A^2 \left(\frac{1}{2\alpha}\right) = 1$$

$$\Rightarrow \boxed{A = [2\alpha]^{1/2}}$$

Solⁿ 7 → (a) Probability is $\int_{x_1}^{x_2} \psi^*(x)\, \psi(x)\, dx$

$$= \int^{0.55} ax \times ax\, dx \Rightarrow \underline{0.025192}$$

$$\langle x \rangle = \int_0^1 \psi^* x \psi \, dx$$

$$= \int_0^1 (\alpha x) x (\alpha x) dx \quad \text{solving}$$

$$\langle x \rangle = \frac{a^2}{4}$$

(8) question (4) can be solved as question (3)

$$\psi(x) = A \cos^2 x$$

(4)

$$\int_{-\pi/4}^{\pi/4} A^2 (\cos^2 x)^2 dx = 1$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$(\cos^2 x)^2 = \left(\frac{1+\cos 2x}{2}\right)^2 = \frac{1 + \cos^2 2x + 2\cos 2x}{4}$$

$$\text{Again } \cos^2 2x = \frac{1+\cos 4x}{2}$$

$$A^2 \left(\frac{3}{8}\pi\right) = 1 \quad \Rightarrow A = \sqrt{\frac{8}{3\pi}}$$

(5)

$$P = \int_0^{\pi/4} (\psi)^2 dx$$

$$= \int_0^{\pi/4} A^2 (\cos^2 x)^2 dx \quad \Rightarrow \underline{0.46}$$