

Tutorial Sheet 1  
Modern Physics – Unit 2

**Q1)** Choose acceptable wave functions for the interval  $-\infty$  to  $+\infty$

(a)  $\sin(x)$

(b)  $e^x$

(c)  $e^{-x}$

(d)  $e^{-|x|}$

(e)  $e^{-x^2}$

(f)  $e^{ikx}$

(g)  $x^n$

(h)  $1/x^n$

(i)  $\log(x)$

(j)  $\sinh(x)$

(k)  $\cosh(x)$

(l)  $\tanh(x)$

(m)  $\sec(x)$

(n)  $\csc(x)$

**Q2)** Find the normalization constants of the wave functions for the given intervals

(i)

$$\phi = A \sin(n\pi x/L)$$

$$0 < x < L$$

$$\phi = 0$$

else where

(ii)

$$\phi = A \cos(n\pi x/L)$$

$$0 < x < L$$

$$\phi = 0$$

else where

(iii)

$$\phi = Ae^{-sx^2}$$

$$-\infty < x < \infty$$

here s is a constant

**Q3)** A particle of mass m is trapped inside the 1D box of length L. Calculate the probability of finding the particle between 0 to  $L/q$  where  $1 < q < \infty$ .

**Q4)** Find the average position  $\langle x \rangle$  and average momentum of the particle  $\langle p \rangle$  using the wavefunctions given below.

(i)

$$\begin{aligned} \phi &= A \sin(n\pi x/L) & 0 < x < L \\ \phi &= 0 & \text{else where} \end{aligned}$$

(ii)

$$\phi = A e^{-sx^2} \quad -\infty < x < \infty \quad \text{here } s = m\omega/2\hbar \text{ is a constant}$$

**5. Find the value of the normalization constant A for the wave function  $\phi = A e^{-x^2}$**

**6. Normalise the following wave function in one dimension**

$$\Psi(x) = A e^{-\alpha x} \text{ for } x > 0$$

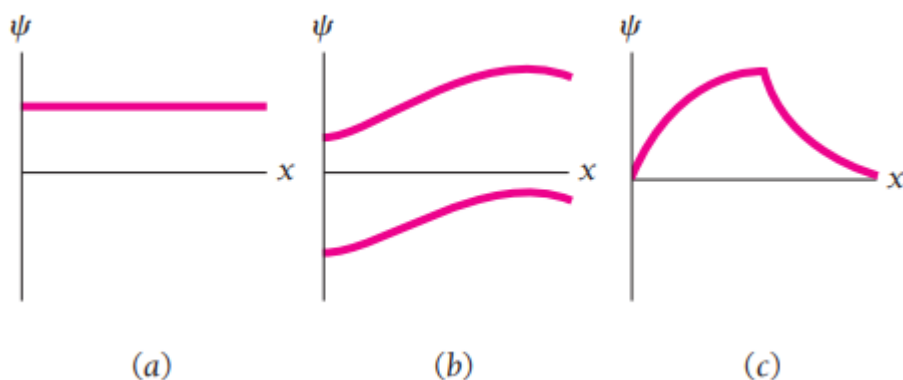
**7. A particle limited to the  $x$  axis has the wave function  $\Psi(x) = ax$  between  $x = 0$  and  $x = 1$ ;  $\Psi(x) = 0$  elsewhere.**

- (a) Find the probability that particle can be found between  $x = 0.45$  and  $x = 0.55$ .
- (b) Find the expectation value  $\langle x \rangle$  of the particle's position.

**8. The wave function of a certain particle is  $\Psi(x) = A \cos^2 x$  for  $-\frac{\pi}{4} < x < \frac{\pi}{2}$**

- (a) Find the value of A
- (b) Find the probability that particle can be found between  $x = 0$  and  $x = \pi/4$ .

9. Which of the following wave functions shown in the figure below cannot have physical significance in the interval shown? Why not?



10. Why are the following wave functions not physically possible for all values of  $x$ ?

(a)  $\Psi(x) = A e^x$

(b)  $\Psi(x) = A \tan x$

11. Calculate the expectation value of ' $p$ ' for the wave-function

$$\Psi(x) = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L} \quad \text{in the region } 0 < x < L$$