

* Solve: $(2x+4y+3)dy = (x+2y+1)dx$

→ Solution: $\frac{dy}{dx} = \frac{x+2y+1}{2(x+2y)+3}$

Let $x+2y = v$

$\Rightarrow 1 + 2 \frac{dy}{dx} = \frac{dv}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{2} \left(\frac{dv}{dx} - 1 \right)$

∴ Use the above in the given ODE, we get

$\frac{1}{2} \left(\frac{dv}{dx} - 1 \right) = \frac{v+1}{2v+3}$

$\Rightarrow \frac{dv}{dx} = \frac{2v+2}{2v+3} + 1 = \frac{4v+5}{2v+3}$

$\Rightarrow \frac{2v+3}{4v+5} dv = dx$

$\Rightarrow \frac{4v+6}{4v+5} dv = 2dx$

$\Rightarrow 2dx = \left[1 + \frac{1}{4v+5} \right] dv$

$\Rightarrow 8dx = 4dv + \frac{4dv}{4v+5}$

Integrating both sides, we obtain

$8x = 4v + \ln(4v+5) + C_1$

\Rightarrow put $v = x+2y$

$\Rightarrow 8x = 4x+8y + \ln(4x+8y+5) + C_1$

$\Rightarrow \ln(4x+8y+5) = 4x-8y + c$,

where $c = -C_1$ is arbitrary constant.