

Time: 2 hrs.

Total marks: 60

1. Briefly explain the failures of classical physics with one example. (4 Marks)
2. Explain how does photoelectric effect prove particle-like nature of light? Also, discuss about the effect of intensity and frequency of incident light on the photo-current and threshold potential. (5 Marks)
3. Consider the Compton scattering of an X-rays with wavelength $\lambda = 1.0 \text{ \AA}$ by an electron at rest. The scattered X-ray radiations are viewed at 90° to the incident ray direction. Evaluate the change in (a) wavelength of scattered X-rays, and (b) momentum of scattered electron. (c) Draw the direction of scattered electron with respect to incident ray. (2+2+3 Marks)
4. A bullet of mass 0.03 Kg is moving with a speed of 500 m/s. The speed is measured with an accuracy of 0.02%. Calculate the uncertainty in position (x). (3 Marks)
5. If Davisson and Germer had used 100 volts to accelerate their electron beam instead of 54 volts, at which scattering angle ϕ would they have found a peak in the distribution of scattered electrons (the intensity)? 0.091 nm is the distance between adjacent crystal planes in Ni crystal. (4 Marks)
6. Let's assume a lion (144 Kg weight) is somehow captured in a long tunnel. The lion is running with a speed of 72 Km/hour to escape from the tunnel. Assume, the size of tunnel entrance is equivalent to the size (width) of the line, say 1.8 m. (a) What's the de-Broglie wavelength of the lion? (b) Will you be able to see diffraction like a wave? Explain. (c) What will be the door size to see the diffraction pattern, if you are allowed to change the entrance width? Is it practically possible? Explain. (2+2+3 Marks)
7. Prove that the function $\phi = \sin(k_1x) \sin(k_2y) \sin(k_3z)$ is an eigen function of kinetic energy operator in 3D Cartesian coordinate and determine the eigen value. (5 Marks)
8. The ground state wave function of a particle of mass "m" is given by $\phi = \exp(-a^2x^4/4)$ with energy eigen value $\hbar^2 a^2/m$. What is the potential in which the particle moves? $-\frac{\hbar^2 a^2}{m} x^2$ (5 Marks)
9. Find the normalized wave function and energy eigen values of a free particle confined in a rectangular box of length L_x and breadth L_y . Fix one corner of the rectangular box at (0, 0). (5 Marks)
- Or
10. What is the degeneracy of an energy level? Find the degree of degeneracy of the energy level $E_n = 14 \left(\frac{\hbar^2}{8mL^2} \right)$ of the particle in a cubical potential box of side L. (2+3 Marks)
11. (a) Sketch the wave function $\Psi(x)$ and probability $|\Psi(x)|^2$ density of a particle (in the first excited state) confined to a 1D box of width L with infinitely high barrier potential at each end. (2+2 Marks)
- (b) State the difference between the average position and most likely position of the particle. Mark the most likely position of particle in first excited state in the sketch (mentioned in part a). (2+1 Marks)
12. Show that the expectation values $\langle p_x x \rangle$ and $\langle x p_x \rangle$ satisfy the following relation:

$$\langle p_x x \rangle - \langle x p_x \rangle = -i\hbar$$
 Where p_x and x represent the operators associated with position and linear momentum (3 Marks)
13. Electrons with energies of 2.0 eV are incident on a barrier 10.0 eV high and 0.50 nm wide.
 (a) Find the probability that the electron tunnels through the barrier. (3 Marks)
- (b) Comment on the transmission probability (calculated in part (a)) if the electrons are replaced by protons of same energy. (2 Marks)

Useful Constants:

- Plank Constant $h = 6.626 \times 10^{-34}$ Js,
- rest mass of electron $m_e = 9.1 \times 10^{-31}$ Kg,
- rest mass of proton $m_p = 1.67 \times 10^{-27}$ Kg

Useful formulas

- Uncertainty : $\Delta x \Delta p \geq \frac{h}{4\pi}$

- De-Broglie wavelength, $\lambda = h/p$ where $p =$ momentum

- Compton scattering effect: $\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_0 c} (1 - \cos\theta)$

- Bragg diffraction: $2d\sin\theta = n\lambda$, for n -th order diffraction between two planes of inter-planer distance " d ". θ is the Bragg angle

$$K_{\max} = h(\nu - \nu_0) \quad \bullet \quad \text{Photoelectric effect:}$$

Where ν_0 is the threshold frequency

$$u(\nu) d\nu = \frac{8\pi h \nu^3}{c^3} \frac{1}{\left(e^{\frac{h\nu}{kT}} - 1 \right)} d\nu \quad \bullet \text{ Plank black body radiation:}$$

$$\psi = e^{-\frac{\kappa^2 x^2}{4}}$$

$$E_T = \frac{h^2 \kappa^2}{2m} e^{-\frac{\kappa^2 x^2}{4}}$$

$$\kappa \cdot E = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2}$$

$$\kappa E + U\psi = \cdot E +$$

$$U\psi = E_t - KE$$