

Q1

$$-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} A \left( \frac{x}{x_0} \right) e^{-x/x_0} + V(x) \frac{Ax}{x_0} e^{-x/x_0} = E \frac{x}{x_0} e^{-x/x_0}$$

$$-\frac{\hbar^2}{2m} A \frac{d}{dx} \left[ \frac{1}{x_0} e^{-x/x_0} - \frac{x}{x_0^2} e^{-x/x_0} \right] + V(x) \frac{Ax}{x_0} e^{-x/x_0} = E \frac{x}{x_0} e^{-x/x_0}$$

$$-\frac{\hbar^2}{2m} A \left[ -\frac{1}{x_0^2} e^{-x/x_0} - \frac{1}{x_0^2} e^{-x/x_0} + \frac{x}{x_0^3} e^{-x/x_0} \right] + V(x) \frac{Ax}{x_0} e^{-x/x_0} = E \frac{x}{x_0} e^{-x/x_0}$$

$\div \frac{Ax}{x_0} e^{-x/x_0}$  on both side

$$-\frac{\hbar^2}{2m} \left[ -\frac{2}{xx_0} + \frac{1}{x_0^2} \right] + V(x) = E \quad \text{--- (1)}$$

at  $x \rightarrow \infty$   $V(x) = 0$

$$\therefore -\frac{\hbar^2}{2m x_0^2} = E \rightarrow \text{use this in eqn (1)}$$

$$\therefore V(x) = \frac{-2 \hbar^2}{2m x x_0} = \frac{-\hbar^2}{2x x_0}$$

$$-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} A e^{-\frac{\sqrt{km} x^2}{2\hbar}} + 0.5 k x^2 A e^{-\frac{\sqrt{km} x^2}{2\hbar}} = E A e^{-\frac{\sqrt{km} x^2}{2\hbar}}$$

$$\frac{\hbar^2}{2m} A \frac{d}{dx} \left[ \frac{-2x\sqrt{km}}{2\hbar} e^{-\frac{\sqrt{km} x^2}{2\hbar}} \right] + \frac{1}{2} k x^2 A e^{-\frac{\sqrt{km} x^2}{2\hbar}} = E A e^{-\frac{\sqrt{km} x^2}{2\hbar}}$$

$$-\frac{\hbar^2}{2m} A \frac{\sqrt{km}}{\hbar} \left[ -e^{-\frac{\sqrt{km} x^2}{2\hbar}} + 2x \frac{\sqrt{km}}{2\hbar} e^{-\frac{\sqrt{km} x^2}{2\hbar}} \right] + \frac{1}{2} k x^2 A e^{-\frac{\sqrt{km} x^2}{2\hbar}} = E A e^{-\frac{\sqrt{km} x^2}{2\hbar}}$$

$$+\frac{\hbar^2}{2m} A \frac{\sqrt{km}}{\hbar} e^{-\frac{\sqrt{km} x^2}{2\hbar}} - \frac{\hbar^2}{2m} \frac{k x^2}{\hbar} A e^{-\frac{\sqrt{km} x^2}{2\hbar}} + \frac{1}{2} k x^2 A e^{-\frac{\sqrt{km} x^2}{2\hbar}} = E A e^{-\frac{\sqrt{km} x^2}{2\hbar}}$$

$$+\frac{\hbar^2}{2m} A \frac{\sqrt{km}}{\hbar} e^{-\frac{\sqrt{km} x^2}{2\hbar}} - \frac{1}{2} k x^2 A e^{-\frac{\sqrt{km} x^2}{2\hbar}} + \frac{1}{2} k x^2 A e^{-\frac{\sqrt{km} x^2}{2\hbar}} = E A e^{-\frac{\sqrt{km} x^2}{2\hbar}}$$

$$+\frac{\hbar^2}{2} \frac{\sqrt{k}}{\sqrt{m}} A e^{-\frac{\sqrt{km} x^2}{2\hbar}} = E A e^{-\frac{\sqrt{km} x^2}{2\hbar}}$$

$$\therefore E = \frac{1}{2} \hbar \sqrt{\frac{k}{m}}$$

$$T = e^{-2k_2 L}$$

$$\ln T = -2k_2 L$$

Width  
of the  
barrier

$$L = - \frac{\ln T}{2k_2}$$

$$k_2 = \frac{\sqrt{2m(U_0 - E)}}{\hbar}$$

$$= - \frac{2.303 \log T}{2k_2}$$

$$L = \frac{-2.303 \times \hbar \times \log T}{\sqrt{2m(U_0 - E)}}$$

$$= \frac{-2.303 \times \hbar \times \log_{10} 10^{-3}}{\sqrt{2 \times 9.1 \times 10^{-31} \times (2-1) \times 1.602 \times 10^{-19}}}$$

$$= \frac{-2.303 \times 6.626 \times 10^{-34} \times -3 \times 1}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1 \times 1.602 \times 10^{-19}}}$$

$$L = 1.4526 \times 10^{-8} \text{ m}$$

$$\boxed{L = 14.526 \text{ nm}}$$

$$T = e^{-2k_2 L}$$

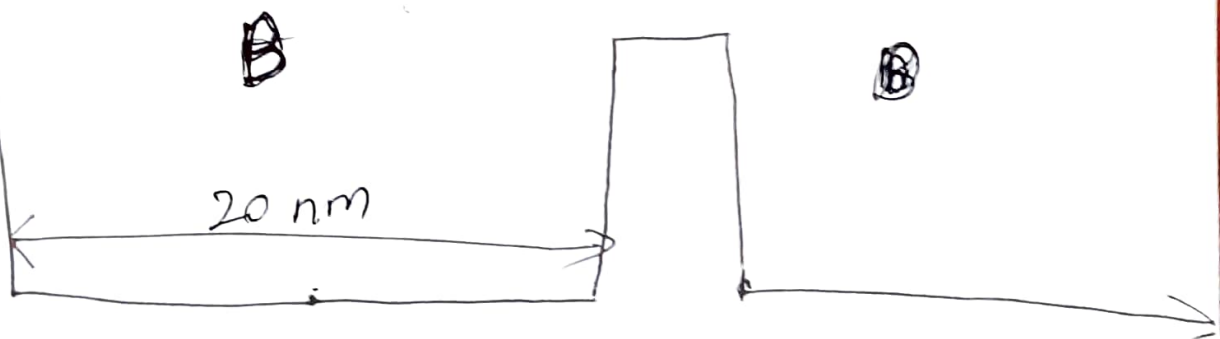
$$= e^{-2 \frac{\sqrt{2m(V_0 - E)}}{\hbar} \times L}$$

$$= e^{-2 \frac{\sqrt{2 \times 9.1 \times 10^{-31} \times (6-1) \times 1.602 \times 10^{-19}}}{6.626 \times 10^{-34}} \times 5 \times 10^{-9}}$$

$$T = \cancel{4.022 \times 10^{-50}} 1.8 \times 10^{-50}$$

$\propto \uparrow$

Q5



Q5) Transmission probability  $T = e^{-2k_2 L}$

Here  $L = 5 \times 10^{-9} \text{ m}$

$$k_2 = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$

$$= \frac{\sqrt{2 \times 9.1 \times 10^{-31} \times (6-1) \times 1.602 \times 10^{-19}}}{\frac{6.62 \times 10^{-34}}{2 \times 3.14}}$$

to convert  
eV to J

$$= \frac{2 \times 3.14 \times \sqrt{9.1 \times 10^{-31} \times 1.602 \times 10^{-19}}}{6.626 \times 10^{-34}}$$

$$k_2 = 1.145 \times 10^{10}$$

$$T = e^{-2 \times 1.145 \times 10^{10} \times 5 \times 10^{-9}}$$

$$= e^{-2 \times 5 \times 1.145 \times 10}$$

$$T = e^{-114.5}$$

$$\boxed{T = 1.876 \times 10^{-50}} \Rightarrow \frac{1}{T} = f = 5.3 \times 10^{49}$$

So,  $\frac{1}{T}$  gives the minimum frequency of hitting the barrier before <sup>it is</sup> escaping from the region B

the particle is having energy 1 eV  
 velocity of the particle is given by

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \times 1 \times 1.602 \times 10^{-19}}{9.1 \times 10^{-31}}}$$

to convert eV to J

$$v = 5.93 \times 10^5 \text{ m/s}$$

As given in the problem, the particle is at  $x=0$  moving toward  $-x$  direction at  $t=0$ .

$\therefore$  the particle has to travel  $(2 \times 20 \text{ nm}) = 40 \text{ nm}$  before it hit the barrier 1 time.

~~So, it will be hitting the~~

To hit the barrier  $5 \times 10^4$  times, it has to travel the distance of  $\underbrace{5 \times 10^4 \times 40 \times 10^{-9} \text{ m}}_{200 \times 10^4 \text{ m}}$

So, time taken by the particle to cover this distance

$$t = \frac{d}{v} = \frac{200 \times 10^4}{5.93 \times 10^5} = 3.37 \times 10^{-1} \text{ s}$$

to read