

\* Solve:  $\frac{dy}{dx} = \frac{x+y+1}{3(x+y)+1}$

→ Solution:

Let  $x+y = v$

$\Rightarrow 1 + \frac{dy}{dx} = \frac{dv}{dx}$

∴ The given ODE becomes

$$\frac{dv}{dx} - 1 = \frac{v+1}{3v+1}$$

$$\Rightarrow \frac{dv}{dx} = \frac{v+1}{3v+1} + 1 = \frac{4v+2}{3v+1}$$

$$\Rightarrow \frac{3v+1}{2v+1} dv = 2 dx$$

$$\Rightarrow \left( \frac{3}{2} - \frac{1}{2} \cdot \frac{1}{2v+1} \right) dv = 2 dx$$

Integrating both sides, we get

$$\frac{3}{2} v - \frac{1}{4} \ln(2v+1) = 2x + c_1$$

$$\Rightarrow \frac{3}{2} (x+y) - \frac{1}{4} \ln(2x+2y+1) = 2x + c_1 \quad \left[ \begin{array}{l} \text{putting} \\ v = x+y \end{array} \right]$$

$$\Rightarrow \ln(2x+2y+1) + 2x - 6y = c,$$

where  $c = -8c_1$  is arbitrary constant. □