Department of Mathematical Sciences

Rajiv Gandhi Institute Of Petroleum Technology, Jais

DIFFERENTIAL EQUATIONS (MA 121)

Week 1 / April 2022 | Problem Set 1 | G.R.

■ **Notations :** ODE $\equiv$ Ordinary Differential Equation, $y' \equiv \frac{dy}{dx}$, $y'' \equiv \frac{d^2y}{dx^2}$, $y^{(i)} \equiv \frac{d^iy}{dx^i}$ for $i = 3, 4, 5, \ldots$.

■ Introduction of ODE, Solutions of First order and First degree ODE

1. Determine the order and degree of the following ODEs.
   (a) $y' + 2x = 0$,
   (b) $y'' + 7x = 0$,
   (c) $(y'')^2 + 7x = 0$,
   (d) $\sqrt{y + (y')^2} = 1 + x$,
   (e) $y^{(4)} + 2k^2y'' + k^4y = 0$.

2. Show that every function f defined by
$$f(x) = \left(x^3 + c\right) e^{-3x},$$
where c is an arbitrary constant, is a solution of the differential equation
$$\frac{dy}{dx} + 3y = 3x^2e^{-3x}.$$

3. Show that $x^3 + 3xy^2 = 1$ is an implicit solution of the differential equation $2xy\frac{dy}{dx} + x^2 + y^2 = 0$ on the interval $0 < x < 1$.

4. Show that the differential equation $|\frac{dy}{dx}| + |y| + 1 = 0$ has no solution.

5. Obtain the differential equation of the family of conics
$$\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1,$$
in which λ is the arbitrary parameter and a, b are given constants.

6. Show that all circles of radius r are represented by the ODE
$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = r\frac{d^2y}{dx^2}.$$

7. Find the differential equation of all circles, which pass through the origin and whose centres are on the x-axis.

8. Show that the substitution $z = \sin x$ transforms the equation

$$\frac{d^2y}{dx^2} + \tan x\,\frac{dy}{dx} + y\,cos^2 x = 0$$

into

$$\frac{d^2y}{dz^2} + y = 0.$$

9. Consider the differential equation of the form

$$\left[y + x\,f\left(x^2 + y^2\right)\right] dx + \left[y\,f\left(x^2 + y^2\right) - x\right] dy = 0.$$

(a) Show that an equation of this form is not exact.

(b) Show that $\frac{1}{x^2+y^2}$ is an integrating factor of an equation of this form.

10. (a) Prove that if f and g are two different solutions of

$$\frac{dy}{dx} + P(x)\,y = Q(x), \tag{1}$$

then $f - g$ is a solution of the equation $\dfrac{dy}{dx} + P(x)\,y = 0.$

(b) Thus show that if f and g are two different solutions of equation (1) and c is an arbitrary constant, then

$$c\,(f - g) + f$$

is a one-parameter solutions of (1).

11. Solve the following ODEs.

(a) $x\frac{dy}{dx} - y = x\sqrt{x^2 + y^2}$

(b) $\frac{dy}{dx} + 2xy = x^2 + y^2$

(c) $y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}$

(d) $x^3\frac{dy}{dx} = y^3 + y^2\sqrt{y^2 - x^2}$

(e) $(5x + 4y - 4)\,dx + (4x + 5y - 5)\,dy = 0$

(f) $(2x + 3y + 4)\,dx = (4x + 6y + 5)\,dy$

(g) $(a^2 - 2xy - y^2)\,dx - (x + y)^2\,dy = 0$, where a is fixed constant.

(h) $x\,dy + (xy + y - 1)\,dy = 0$

(i) $y\,dx + (xy^2 + x - y)\,dy = 0$

(j) $(\cos^2 x - y\ \cos x)\,dx - (1 + \sin x)\,dy = 0$

(k) $dy + (4y - 8y^3)\,x\,dx = 0$

(l) $dy + (x\ \sin 2y - x^3 \cos^2 y)\ dx = 0$

(m) $\frac{dy}{dx} + \frac{y}{x} \ln y = \frac{y}{x^2} (\ln y)^2.$

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