FINANCE & ECONOMICS CLUB

GAME THEORY

Gouri & Rethyam

INDEX

ABOUT

This project involves fundamentals of any research work, experimentation and of course, game theory.

"What distinguishes Economics from other disciplines is not its subject matter but its approach.

The economic approach is applicable to all human behaviour."
-Gary S. Becker (Nobel in Economics in 1991)

WHAT IS AN AUCTION?

Auction is a mechanism of allocating a particular object at a certain price.

The underlying assumption we make when modeling auctions is that each bidder has an intrinsic value for the item being auctioned, she is willing to purchase the item for a price up to this value, but not for any higher price.

THE TYPES DISCUSSED

1. **The First-Price, Sealed-Bid Auction**
2. **The Second-Price, Sealed-Bid (Vickrey) Auction**

THEORY

First-Price, Sealed-Bid Auction

In this kind of auction, bidders submit simultaneous "sealed bids" to the seller.

The terminology comes from the original format for such auctions, in which bids were written down and provided in sealed envelopes to the seller, who would then open them all together.

The highest bidder wins the object and pays the value of her bid.

THEORY

The Second-Price, Sealed-Bid (Vickrey) Auction

Second-price sealed-bid auctions, also called Vickrey auctions. Bidders submit simultaneous sealed bids to the sellers.

The highest bidder wins the object and pays the value of the second-highest bid.

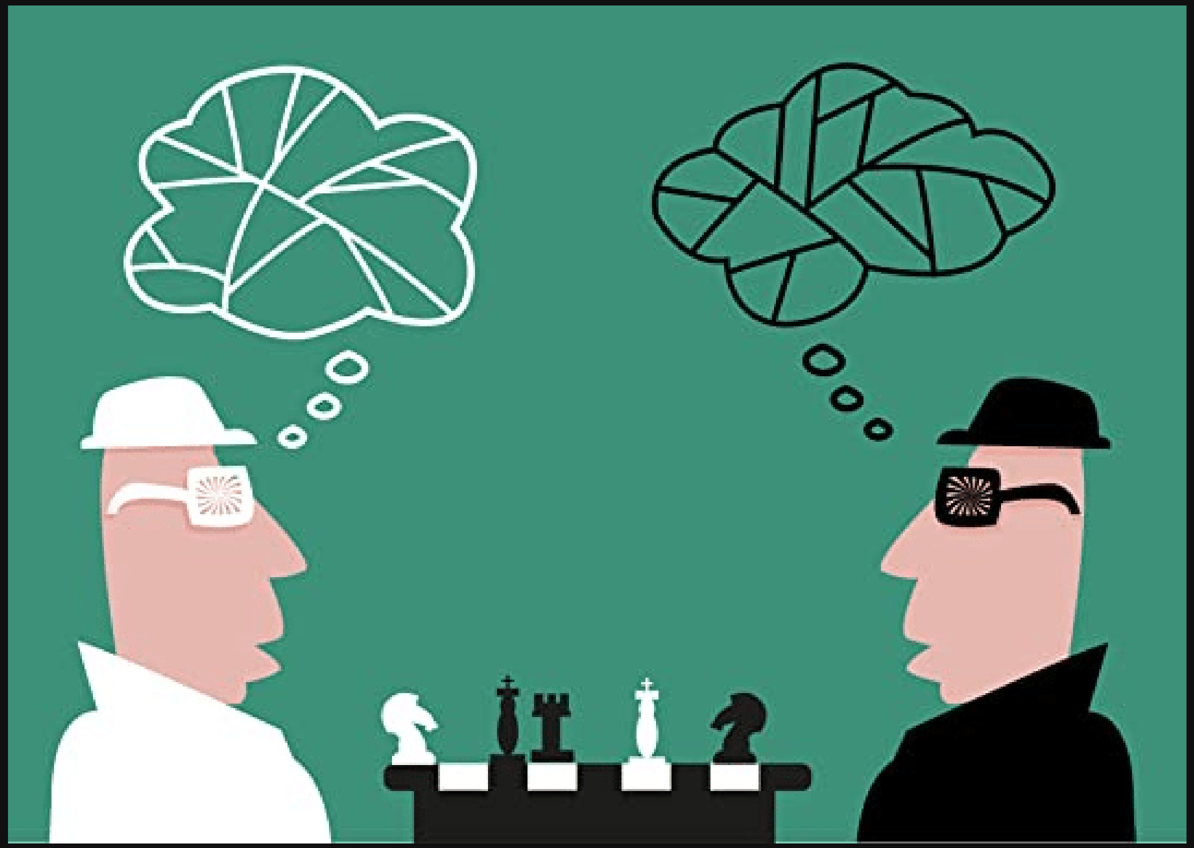
These auctions are called Vickrey auctions in honor of William Vickrey, who wrote the first game-theoretic analysis of auctions.

AUCTION THEORY

Modeling an auction as a game

In all games, we have players who selfishly maximize their payoffs using all the available information. They choose their best responses to other players' strategies.

Any game starts with rules. Rules of the game include players, strategies available for each player, payoffs for all possible combinations of strategies of all players. Asymmetric information adds one more element: each player can be of more than one type and each player's type is his private information.



TO BE NOTED

Assumptions while analysing the data

- Each buyer knows for sure his own valuation and the number of buyers who participate in the auction, but not the exact valuations of the other buyers.
- Each buyer knows the distribution of valuations, the range of all possible valuations for the object and the probability with which each valuation happens.
- Out of all possible bids the buyer will **choose the one that will maximize his expected payoff** given the expectations about strategies (bids) of the other buyers (remember in equilibrium all players play best responses to each other strategies, same here).

TO BE NOTED

Assumptions while analysing the data

- Auctioneer has no clue whether actual valuations of all Buyers who came to his auction are high, low, one high and others down, two high and a bunch of ordinary, bunch of high and bunch of joint. All valuations are precisely average, etc.

- Unlike normal sealed bid auctions which occur in one round only our modified games consist of multiple rounds with the same rules, **thus the human factor to maximize one's payoff we have assumed that the bidder will improve his bid to maximize his chance to win.**

MATHEMATICS

Mathematics of First price auction

We will look for an equilibrium where each bidder uses a bid strategy that is a strictly increasing, continuous, and differentiable function of their value. To do this, suppose that bidders j not equal to I use identical bidding strategies $b_j = b(s_j)$ with these properties and consider the problem facing bidder i.

Bidder i's expected payoff, as a function of his bid b_i and signal s_i, is:

$$U(b_i, s_i) = (s_i - b_i) \cdot \Pr[b_j = b(S_j) \leqslant b_i, \ \forall j \text{ not equal to } i]$$

COMPARISON

Mathematics of Sealed bid auction

Basically states that in this general symmetric setting, the more information on which the winner's payment is based, the higher will be the expected revenue.

Thus, the first price auction will have lower expected revenue than the second price auction.

The winner's payment in the first price auction is based only on her own signal, while in the second price auction it is based on her own signal and the second-highest signal.

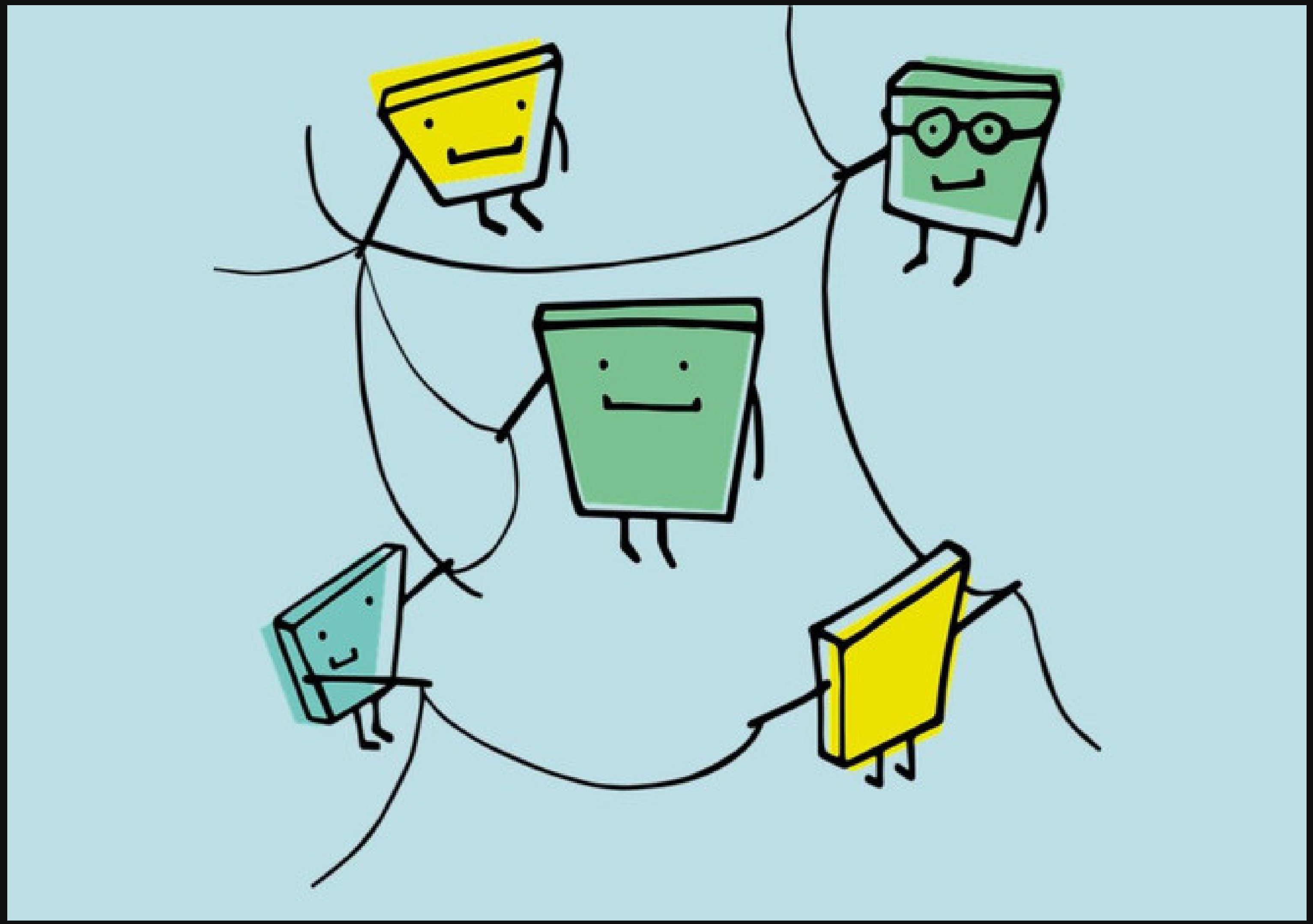
GAME 1 :

RMS 7

A game based on First price Auction

Let's begin!

COMPLETE STUDY



Explanation of the game

Consider a **“Best of X",(where X is a variable),** a game in which the bidder with the most points after X rounds is declared as the winner of the auction.

He has to pay a sum equal to the **root mean square** of all the bids he placed.

Note that no deduction from bidders amount takes place from 1st to (X-1)th round, the only deduction that takes place is after Xth round i.e. the final round.

If after Xth round, there is a tie, the winner is randomly decided.

FINANCE & ECONOMICS CLUB

Turn	A Bid	B Bid	C Bid	D Bid	Highest Bid
Game 1					
1	100	110	80	90	B
2	110	110	90	100	Random Toss B Won.
3	115	115	100	105	Random Toss B Won.
4	125	120	110	120	A
5	125	130	120	125	B
6	135	135	125	135	Random Toss D Won
7	140	140	135	145	D
8	145	150	140	150	Random Toss D Won

Details of the Game.

- No. of times the games were played out: **4**
- Sample size of participants: **4** (can be played with any number of bidders.)
  (In one Game)
- Sample size Total: **16**

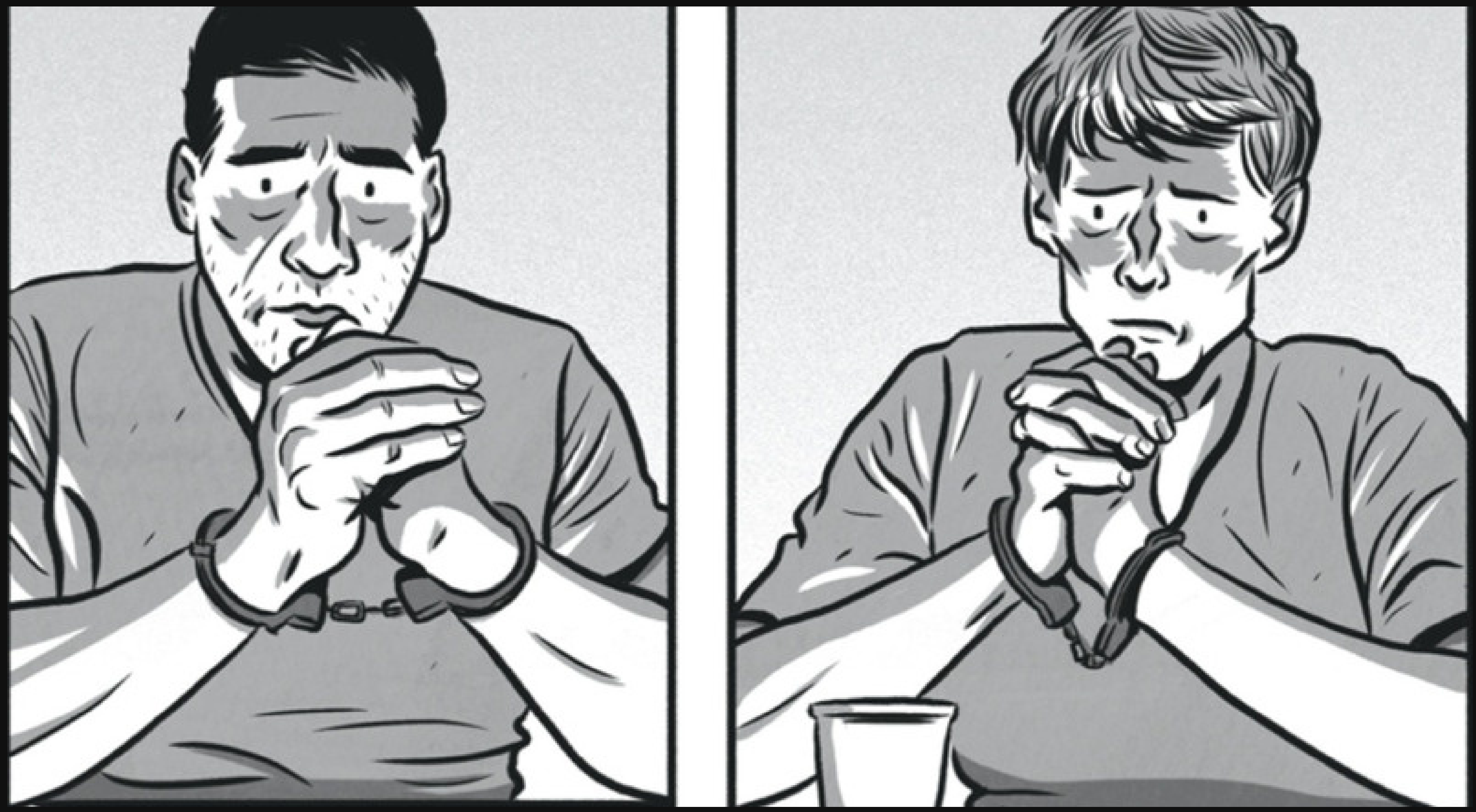
As we can see the bidder B has the most number of points after 8 rounds i.e

The bidder B wins the overall auction and has to pay an **amount equal to the RMS of all the bids he has placed.**

A LOOK AT THE DATA

Link to the spreadsheet

https://docs.google.com/spreadsheets/d/1x-bzPkQGVj5uwJj2-_hBbC_esPH0Iwpf/edit?usp=sharing&ouid=109598794526299000385&rtpof=true&sd=true



OUTCOMES

“What if we don’t change at all ...
and something magical just happens?”

Outcomes of First price auction

In first-price sealed bid auction each buyer’s optimal bid is equal to some fraction of the buyer’s true valuation.

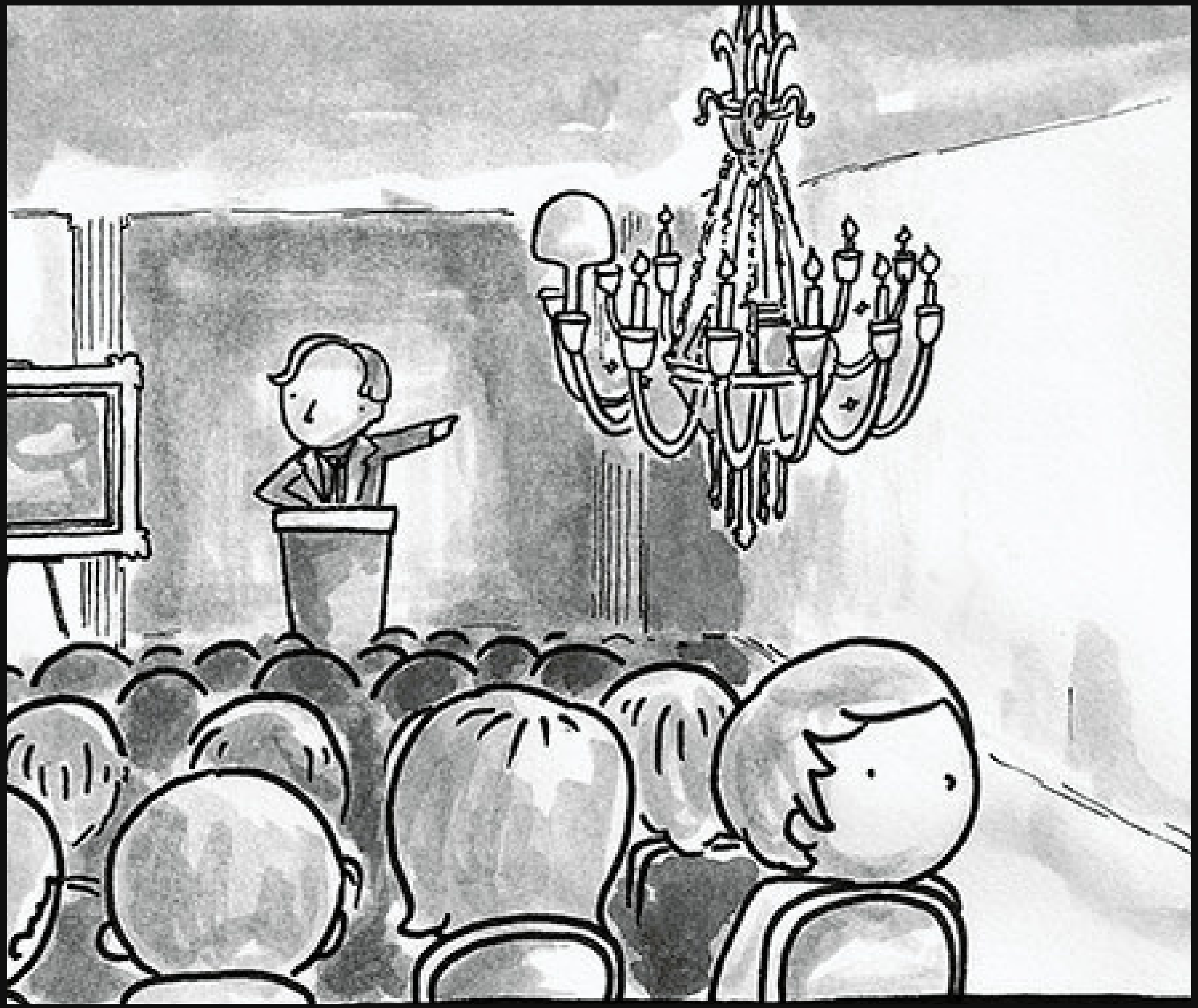
The winner is the buyer with the highest valuation, price is equal to some fraction of the highest valuation.

How do we know? Because bids are proportional to buyers’ valuations and the higher is the valuation, the higher is the bid. Therefore, buyer with the highest valuation will submit the highest bid and will win.

Price is equal to the highest bid and it is some proportion of the highest valuation.

OUTCOMES

Analysis of outcomes from the experiment



Most of the outcomes from the experiment were in line with the existing First price auction theory.

But as expected there were some deviations because of "Increase in the number of bidding rounds per auction".

Unlike the existing First price auction theory, here the bidder with the highest bid is announced after each round thus giving his competitors a chance to improve their bids in the successive rounds and their total score, thus giving them another chance to win.

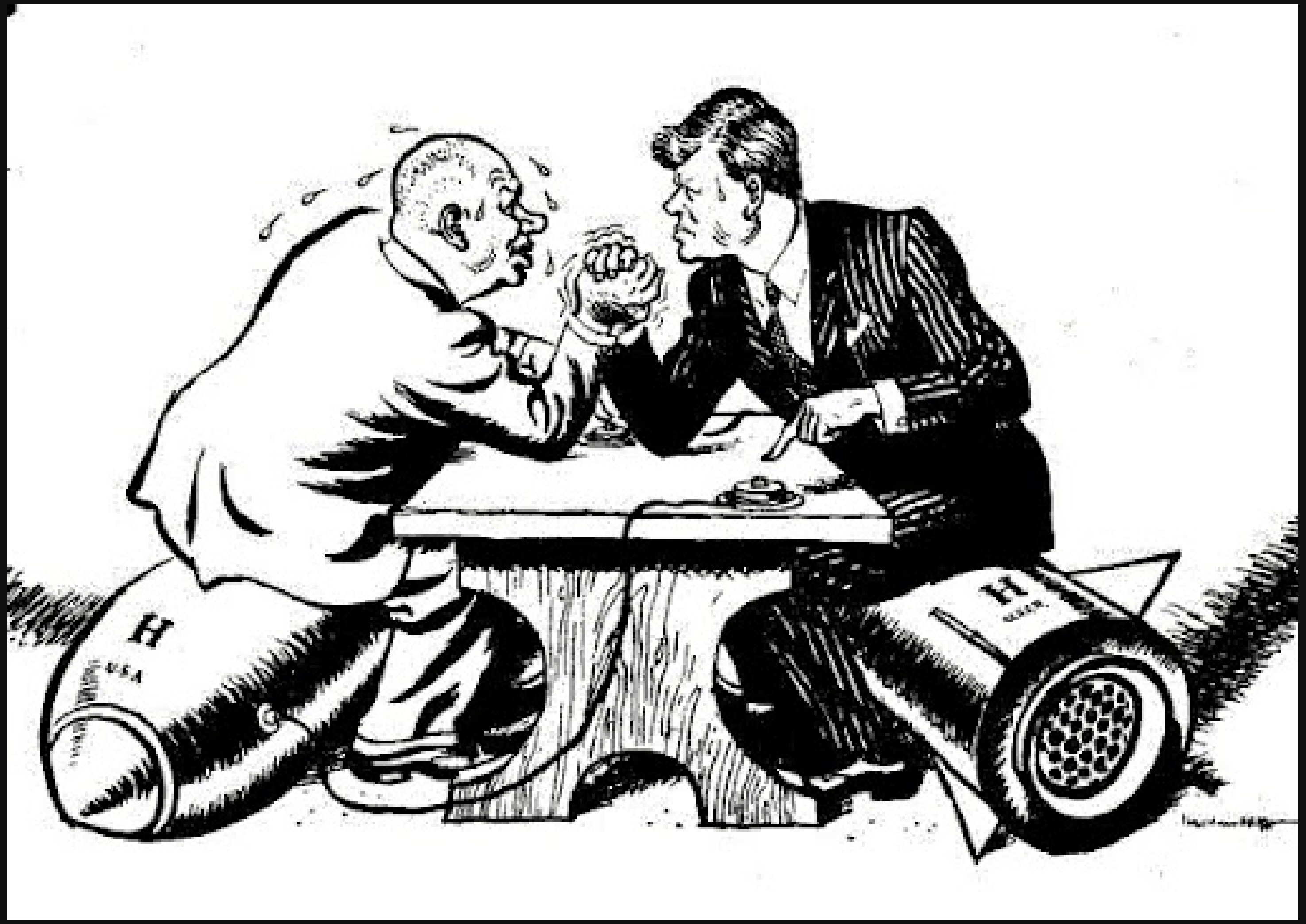
One more deviation that exists is that the winner has to Pay an amount equalling the RMS of all the bids placed by the him.

GAME 2 :
BIDDING FROLIC

A game based on Second price Auction

Let's begin!

COMPLETE STUDY



Explanation of the game

Consider a **“Best of 5"** game in which the first player to make three moves wins. Each player begins with **100 chips.**

Note that updating each player’s chip count only depends on the difference between their bids, not the bid values themselves.

In the event of tied bids, the player with more chips wins. One player is arbitrarily designated to win ties when chip counts are equal.

Details of the Game.

- No. of times the games were played out : 9
- Sample size of participants : 2 (In one Game)
- Sample size Total : 18

GAME : 2

TURN	A	B	A's bid	B's bid	Score
1	100	100	20	50	0-1
2	130	70	45	30	1-1
3	115	85	55	60	1-2
4	120	80	100	50	2-2
5	70	130	70	130	2-3

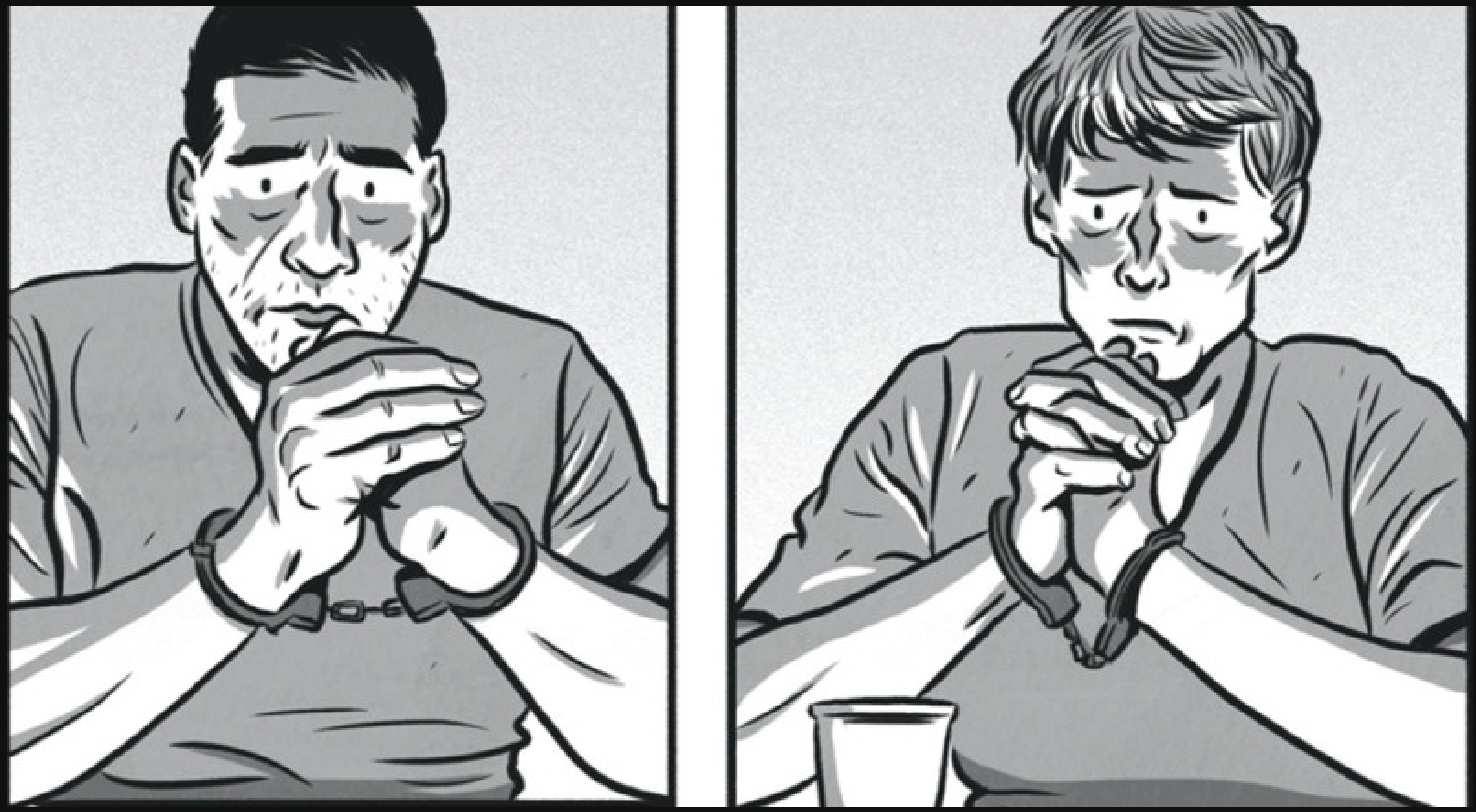
As the final score stands in favour of B after 5 rounds with a score of 2-3.

The bidder B wins the overall auction and has to pay an **amount equal to the average of all the bids Bidder A placed.**

A LOOK AT THE DATA

Link to the spreadsheet

https://docs.google.com/spreadsheets/d/1Bbpwj9F5ppTunq9_mHgB40bbaakA5mlA/edit?usp=sharing&ouid=109598794526299000385&rtpof=true&sd=true



OUTCOMES

"This really is an innovative approach, but I'm afraid we can't consider it. It's never been done before."

Outcomes of Second price auction

The winner is the buyer with the highest valuation, price is equal to the second-highest valuation.

How do we know? Because buyers have a dominant strategy, we know that if buyers are smart, then the bids that auctioneer actually collected are the same as the true actual valuations of the buyers. **Since the highest bid is equal to the highest valuation, then the object goes to the person with the highest valuation**.

Since the price is equal to the second highest bid and second-highest bid is equal to the second highest valuation, by simple logic price is equal to the actual true second-highest valuation.

OUTCOMES

Analysis of outcomes from the experiment

"We need to innovate! Buck the status quo! Blaze a new trail! Here's how everyone else is doing it..."

Most of the outcomes from the experiment were in line with the existing Second price auction theory.

But as expected there were some deviations because of "Increase in the number of bidding rounds per auction".

We have also given them a fixed amount of initial chips i.e. 100 chips, which directly indicates that the auctioneer has instructed the bidders that the valuation of the to be auctioned object is between 0 to 100 chips.

One more deviation that exists is that the winner has to Pay an amount equalling the average of all the bids placed by the other bidder.

BEHAVIORAL ANALYSIS

Differences in the behavior of people found between the 2 games.

Thus far, we have assumed that bidders' values for the item being auctioned are independent.

Each bidder knows his/her own value for the item and is not concerned with how much it is worth to anyone else.

A purely superficial comparison of the first-price and second-price sealed-bid auctions might suggest that the seller would get more money for the item if he ran a first price auction.

After all, he'll get paid the highest bid rather than the second-highest bid.

BEHAVIORAL ANALYSIS

Differences in the behavior of people found between the 2 games.

Here, the point is that bidders in a the first-price auction will tend to bid lower than they do in a second-price auction, and in fact this lowering of bids will tend to offset what would otherwise look like a difference in the size of the winning bid.

One more difference between the two games was that in the modified first price auction the bidder was bidding his own valuation of the object in consecutive rounds and has to pay the same at the last whereas, in the modified second price auction game even though someone knows prior to the last round that he is not going to win but still he will purposefully increase his bid so that the winner has to pay a larger amount

FINANCE & ECONOMICS CLUB

THE END