INTRODUCTION TO EUCLID'S GEOMETRY

1.Introduction:

The word 'geometry' comes form the Greek words 'geo', meaning the 'earth', and 'metrein', meaning 'to measure'. Geometry appears to have originated from the need for measuring land. This branch of mathematics was studied in various forms in every ancient civilisation, be it in Egypt, Babylonia, China, India, Greece, the Incas, etc. The people of these civilisations faced several practical problems which required the development of geometry in various ways.

2.Notes:

- Postulate 1 : A straight line may be drawn from any one point to any other point.
- Postulate 2 : A terminated line can be produced indefinitely.
- Postulate 3 : A circle can be drawn with any centre and any radius.
- Postulate 4 : All right angles are equal to one another.
- Postulate 5 : If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

3.Example sums:

*If A, B and C are three points on a line, and B lies between A and C then prove that AB + BC = AC.

Solution : In the figure given above, AC coincides with AB + BC. Also, Euclid's Axiom (4) says that things which coincide with one another are equal to one another. So, it can be deduced that AB + BC = AC Note that in this solution, it has been assumed that there is a unique line passing through two points.

*Consider the following statement : There exists a pair of straight lines that are everywhere equidistant from one another. Is this statement a direct consequence of Euclid's fifth postulate? Explain.

Solution : Take any line l and a point P not on l. Then, by Playfair's axiom, which is equivalent to the fifth postulate, we know that there is a unique line m through P which is parallel to l. Now, the distance of a point from a line is the length of the perpendicular from the point to the line. This distance will be the same for any point on m from l and any point on l from m. So, these two lines are everywhere equidistant from one another.

4.Practice sums:

*Consider two 'postulates' given below: (i) Given any two distinct points A and B, there exists a third point C which is in between A and B. (ii) There exist at least three points that are not on the same line. Do these postulates contain any undefined terms? Are these postulates consistent? Do they follow from Euclid's postulates? Explain.

*How would you rewrite Euclid's fifth postulate so that it would be easier to understand?