POLYNOMIALS

1. Introduction:

You have studied algebraic expressions, their addition, subtraction, multiplication and division in earlier classes. You also have studied how to factorise some algebraic expressions. You may recall the algebraic identities : $(x + y)2 = x2 + 2xy + y2$ $(x – y) 2 = x2 – 2xy + y2$ and $x2 – y2 =(x + y) (x – y)$ and their use in factorisation. In this chapter, we shall start our study with a particular type of algebraic expression, called polynomial, and the terminology related to it. We shall also study the Remainder Theorem and Factor Theorem and their use in the factorisation of polynomials. In addition to the above, we shall study some more algebraic identities and their use in factorisation and in evaluating some given expressions.

2.Notes:

- The degree of a non-zero constant polynomial is zero.
- a linear polynomial has one and only one zero.
- Dividend = (Divisor × Quotient) + Remainder

3.Example sums:

*Example 1 : Find the degree of each of the polynomials given below: (i) $x5 – x4 + 3$ (ii) $2 – y2 – y3 + 2y8$ (iii) 2

Solution : (i) The highest power of the variable is 5. So, the degree of the polynomial is 5. (ii) The highest power of the variable is 8. So, the degree of the polynomial is 8. (iii)The only term here is 2 which can be written as $2x0$. So the exponent of x is 0. Therefore, the degree of the polynomial is 0.

*Example 2 : Check whether –2 and 2 are zeroes of the polynomial $x + 2$.
Solution : Let $p(x) = x + 2$. Then $p(2) = 2 + 2 = 4$, $p(–2) = –2 + 2 = 0$ Therefore, –2 is a zero of the polynomial $x + 2$, but 2 is not.

*Example 7 : Divide the polynomial 3x4 – 4x3 – 3x –1 by x – 1.

Solution : By long division, we have: 3x3 – x2 – x – 4 x – 1 3x4 – 4x3 – 3x – 1 – 3x4 –+ 3x3 – x3 – 3x – 1 –+ x3 + – x2 – x2 – 3x – 1 – +x2 + – x – 4x – 1 – + 4x + – 4 – 5 Here, the remainder is – 5. Now, the zero of x – 1 is 1. So, putting x = 1 in p(x), we see that p(1) = 3(1)4 – 4(1)3 – 3(1) – 1 = 3 – 4 – 3 – 1 = – 5, which is the remainder.

4.Practice sums:

*Write the coefficients of x2 in each of the following:

(i) 2 + X2 + X (ii) 2 – X2 + X3 (iii) 2 2 x X π + (iv) 2 1

*Find the value of the polynomial 5x – 4x2 + 3 at:

(i) x = 0 (ii) x = –1 (iii) x = 2

* Find the remainder obtained on dividing p(x) = x3 + 1 by x + 1.