ARITHMETIC PROGRESSIONS

1.Introduction:

You must have observed that in nature, many things follow a certain pattern, such as the petals of a sunflower, the holes of a honeycomb, the grains on a maize cob, the spirals on a pineapple and on a pine cone etc. We now look for some patterns which occur in our day-to-day life. Some such examples are : (i) Reena applied for a job and got selected. She has been offered a job with a starting monthly salary of Rs 8000, with an annual increment of Rs 500 in her salary. Her salary (in Rs) for the 1st, 2nd, 3rd, . . . years will be, respectively 8000, 8500, 9000, ...

2.Notes:

- An arithmetic progression (AP) is a list of numbers in which each term is obtained by adding a fixed number d to the preceding term, except the first term. The fixed number d is called the common difference. The general form of an AP is a, a + d, a + 2d, a + 3d, . . .
- In an AP with first term a and common difference d, the nth term (or the general term) is given by an = a + (n – 1) d.
- The sum of the first n terms of an AP is given by : S = n/2[2a+(n-1)d].
- If l is the last term of the finite AP, say the nth term, then the sum of all terms of the AP is given by : S = n/2(a+l).

3.Example Sums:

*How many two-digit numbers are divisible by 3?

Solution : The list of two-digit numbers divisible by 3 is : 12, 15, 18, . . . , 99 Is this an AP? Yes it is. Here, a = 12, d = 3, an = 99. As an = a + (n – 1) d, we have 99 = 12 + (n – 1) × 3 i.e., 87 = (n – 1) × 3 i.e., n – 1 = 87 3 = 29 i.e., n = 29 + 1 = 30.

*Find the sum of the first 22 terms of the AP : 8, 3, –2, . . .

Solution : Here, a = 8, d = 3 – 8 = –5, n = 22. We know that S = [ ] 2 (1) 2 n an d + − Therefore, S = [ ] 22 16 21(5) 2 + − = 11(16 – 105) = 11(–89) = – 979 So, the sum of the first 22 terms of the AP is – 979.

4.Practice Sums:

*An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

*Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.