REAL NUMBERS

1.Introduction:

In Class IX, you began your exploration of the world of real numbers and encountered irrational numbers. We continue our discussion on real numbers in this chapter. We begin with two very important properties of positive integers in Sections 1.2 and 1.3, namely the Euclid's division algorithm and the Fundamental Theorem of Arithmetic.

2.Notes:

- If p is a prime and p divides a2 , then p divides q, where a is a positive integer.
- . The Fundamental Theorem of Arithmetic : Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.
- Let x = p q be a rational number, such that the prime factorisation of q is of the form 2n 5m, where n, m are non-negative integers. Then x has a decimal expansion which terminates.

3.Example Sums:

*Show that every positive even integer is of the form 2q, and that every positive odd integer is of the form 2q + 1, where q is some integer.

Solution : Let a be any positive integer and b = 2. Then, by Euclid's algorithm, a = 2q + r, for some integer q ≥ 0, and r = 0 or r = 1, because 0 ≤ r < 2. So, a = 2q or 2q + 1. If a is of the form 2q, then a is an even integer. Also, a positive integer can be either even or odd. Therefore, any positive odd integer is of the form 2q + 1.

* Find the LCM and HCF of 6 and 20 by the prime factorisation method.

Solution : We have : 6 = 21 × 31 and 20 = 2 × 2 × 5 = 22 × 51 . You can find HCF(6, 20) = 2 and LCM(6, 20) = 2 × 2 × 3 × 5 = 60, as done in your earlier classes. Note that HCF(6, 20) = 21 = Product of the smallest power of each common prime factor in the numbers. LCM (6, 20) = 22 × 31 × 51 = Product of the greatest power of each prime factor, involved in the numbers. From the example above, you might have noticed that HCF(6, 20) × LCM(6, 20) = 6 × 20. In fact, we can verify that for any two positive integers a and b, HCF (a, b) × LCM (a, b) = a × b. We can use this result to find the LCM of two positive integers, if we have already found the HCF of the two positive integers.

4.Practice Sums:

*Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where q is some integer.

*Given that HCF (306, 657) = 9, find LCM (306, 657).