# Fourier Transform

## Decoding the Hidden Language of Signals

Md junaid ahmmad, Gourove Roy, Sadrul Islam Faysal

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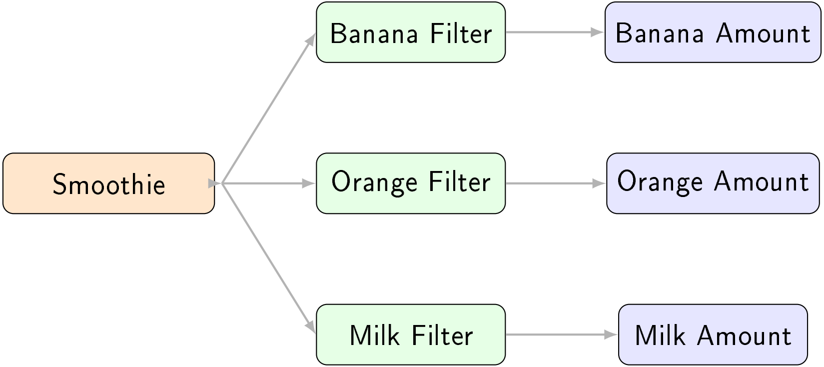
Given a Smoothie , lets find its Recipe

Given a Smoothie , lets find its Recipe

Analysis

# How Can We Do That?

Extracting the recipe using filters



## Analogy

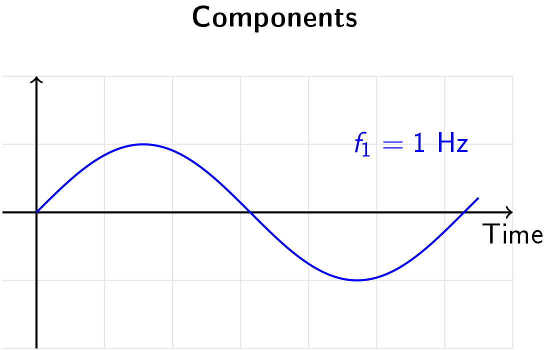
Smoothie → Signal

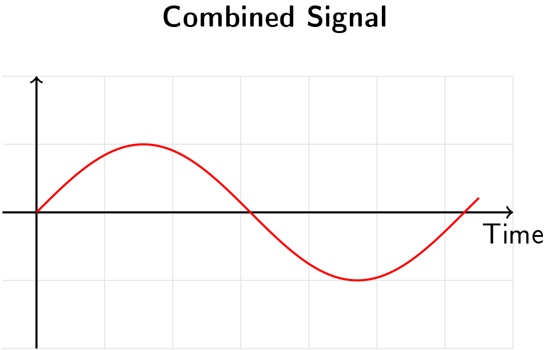
Ingredient → Frequency

Amount → Amplitude

And the whole transformation process is identical to Fourier transform.

# Building a Signal: Fourier Components

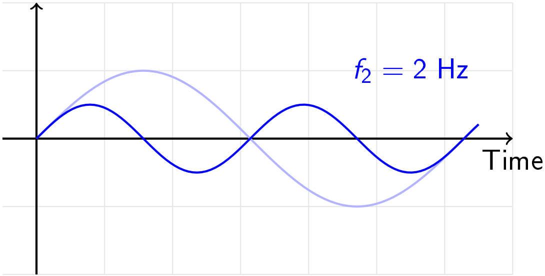



Starting with the fundamental frequency

# Building a Signal: Fourier Components

**Components**

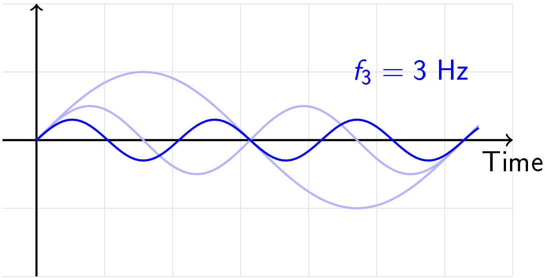



**Combined Signal**

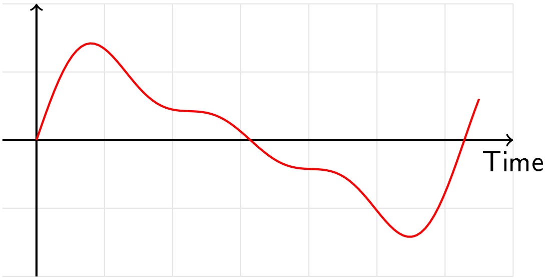

Adding the second harmonic

# Building a Signal: Fourier Components

**Components**




**Combined Signal**




Adding the third harmonic

# Building a Signal: Fourier Components

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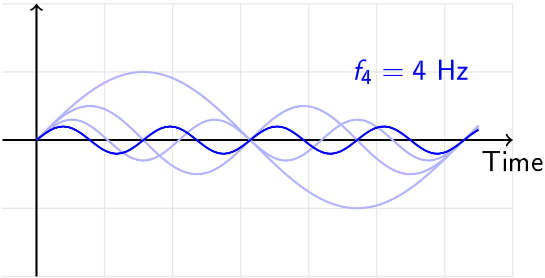


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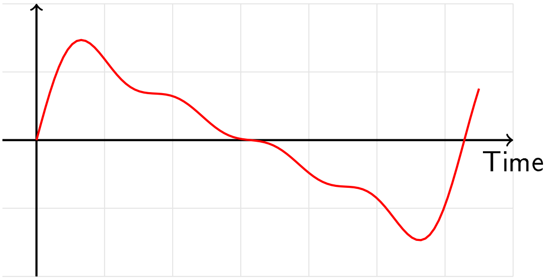
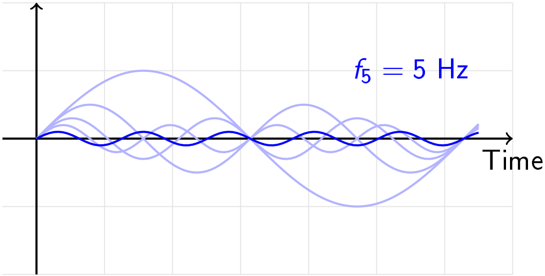


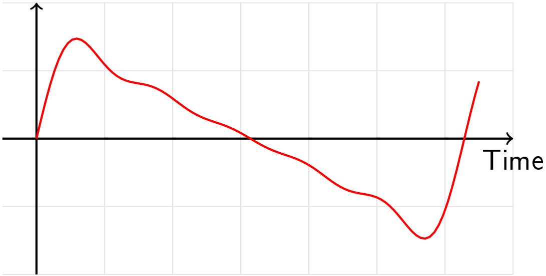

Adding the fourth harmonic

# Building a Signal: Fourier Components

**Components**

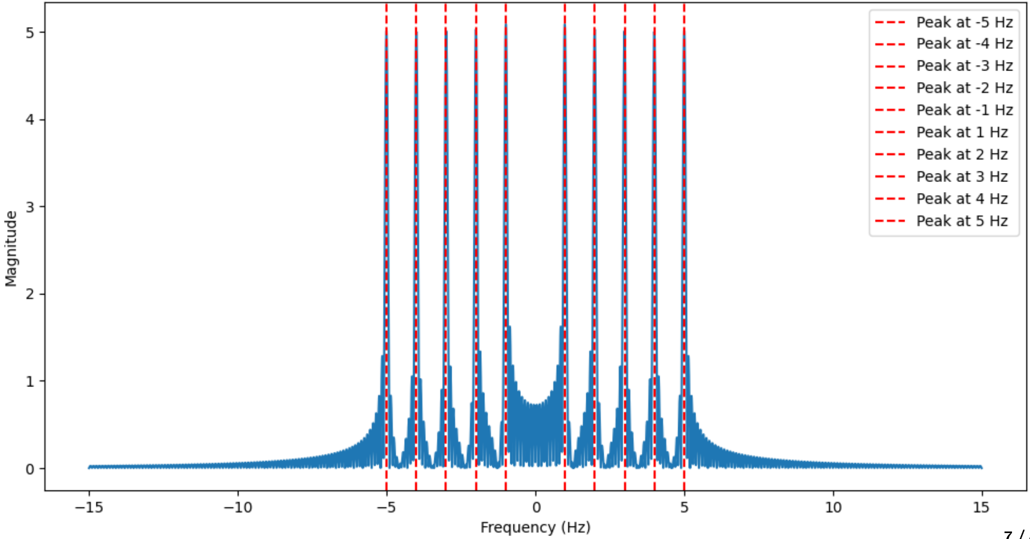



**Combined Signal**

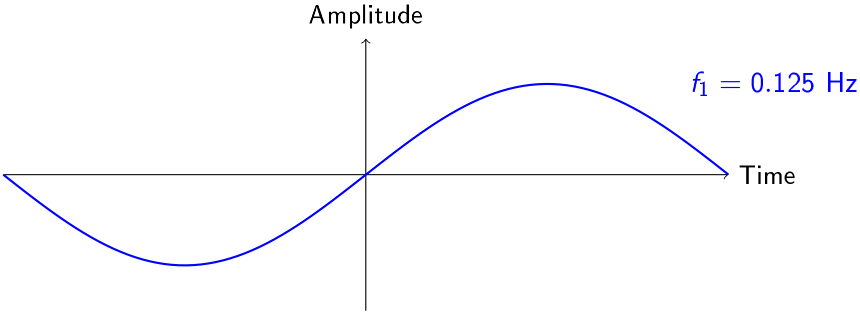



Final signal with five components

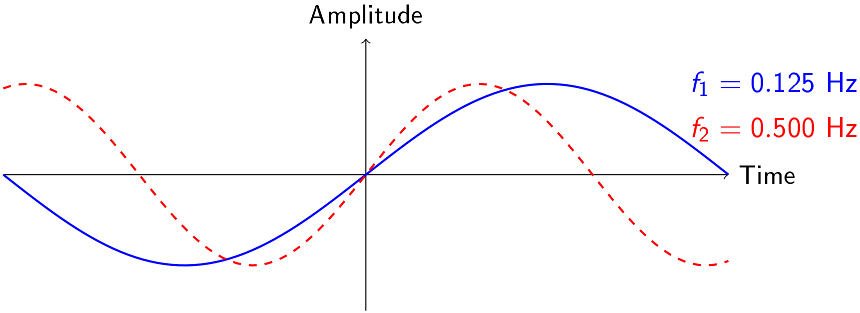
# Fourier Transform Analysis



# How Fourier Transform Detects Frequencies



# How Fourier Transform Detects Frequencies



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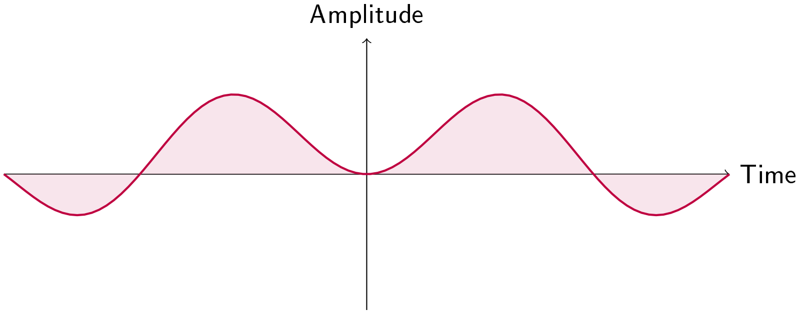
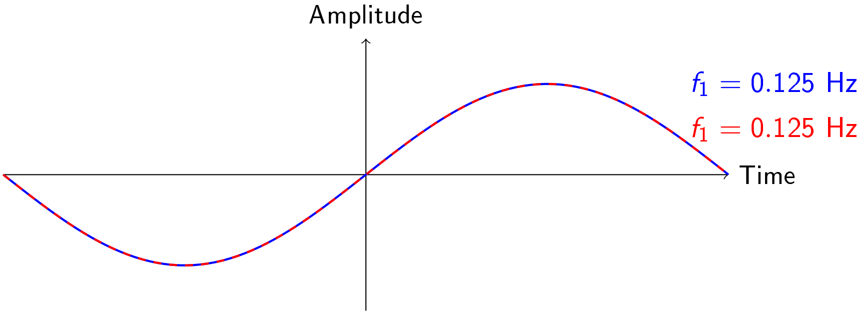



**Area under curve = 0**

# How Fourier Transform Detects Frequencies



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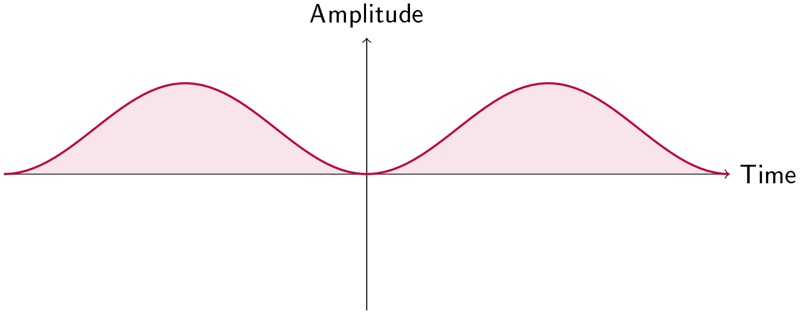
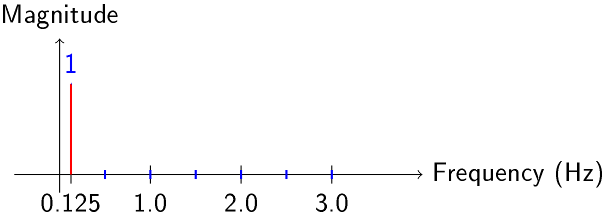


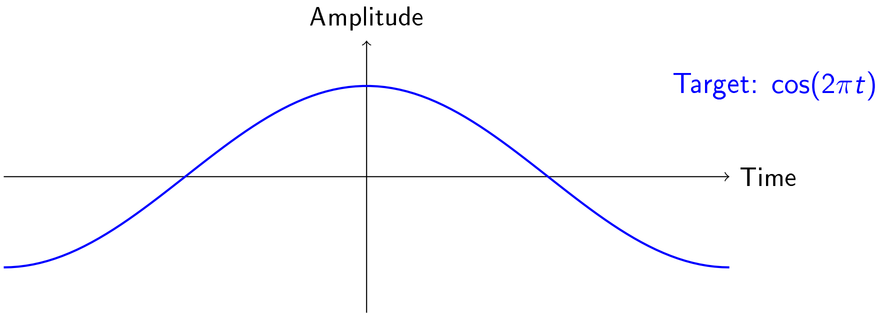

**Area under curve > 0 (Correlation)**

# Frequency Domain Representation




**Single frequency component at $f = 0.125$ Hz**

# Why We Need Both Sine and Cosine Terms



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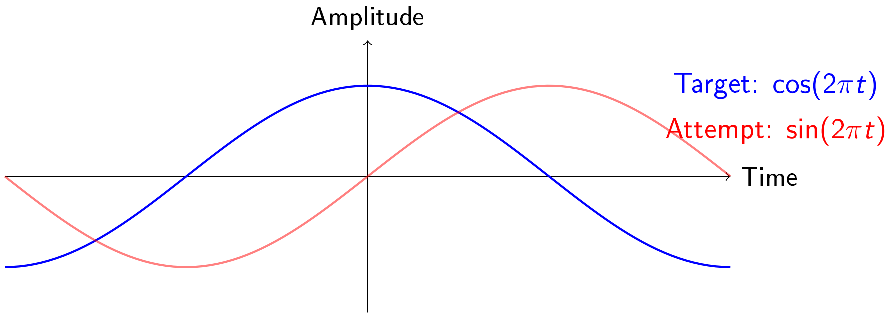

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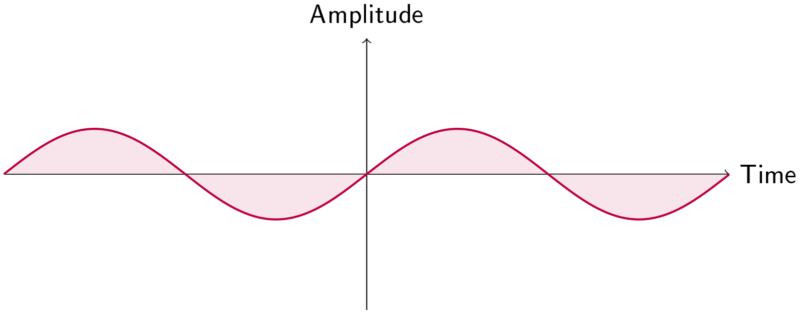
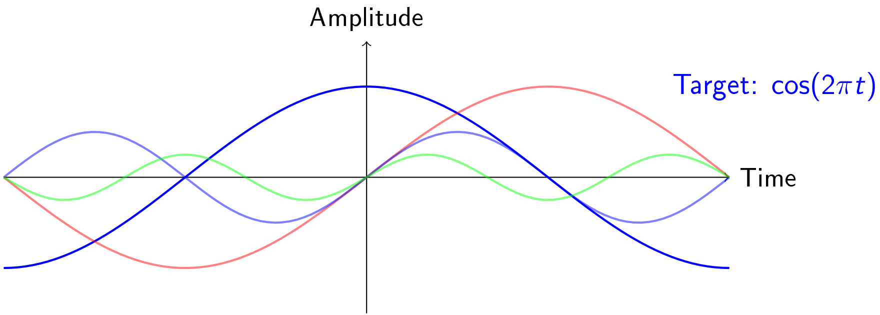


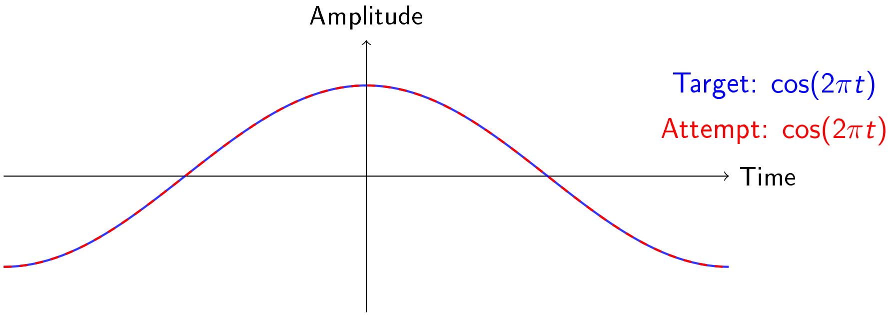

**Area under curve = 0**

# Why We Need Both Sine and Cosine Terms




**Cannot match cosine with any combination of sines**

## Why We Need Both Sine and Cosine Terms




$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(2\pi nt) + b_n \sin(2\pi nt)\right)$$

# Why Fourier Transform?

Analyze and simplify complex signals like audio, images, and communication data.

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Analyze and simplify complex signals like audio, images, and communication data.

Break signals into fundamental sine and cosine components.

# Why Fourier Transform?

- Analyze and simplify complex signals like audio, images, and communication data.
- Break signals into fundamental sine and cosine components.
- Applications in:
  - Audio processing (e.g., equalizers, compression).
  - Medical imaging (e.g., MRIs).
  - Image compression (e.g., JPEGs).

## Fourier transform and it's properties

# What is Fourier Transformation?(Recap)

The generalized form of the complex Fourier series is referred to as the Fourier transform. It helps to expand the non-periodic functions and convert them into easy sinusoid functions.

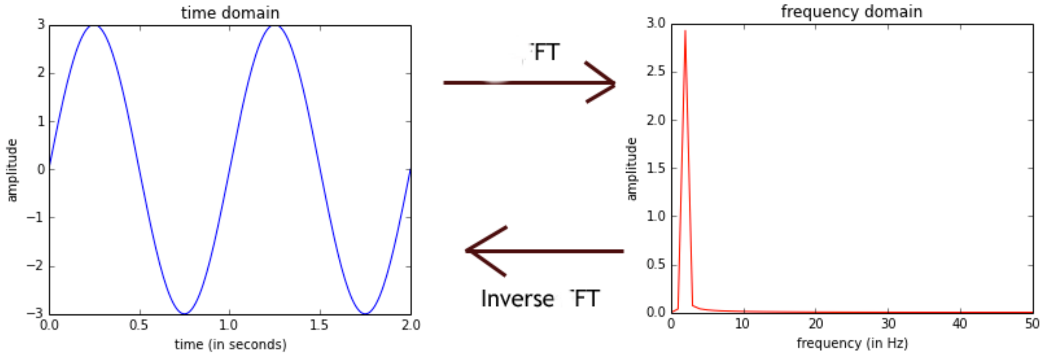


Figure: Fourier Transformation

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# Fourier Transform Type

There are two types of Fourier transform i.e., forward Fourier transform and inverse Fourier transform.
The forward and inverse Fourier transform is used to decompose a function or a signal into its constituent frequencies and times respectively.

# Forward Fourier Transform

The forward Fourier transform is a mathematical technique used to transform a time-domain signal into its frequency-domain representation. The forward Fourier transform of a continuous-time signal $x(t)$ is given by

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

where $X(\omega)$ is the Fourier transform of $x(t)$, $\omega$ is the angular frequency, $j$ is the imaginary unit($\sqrt{-1}$), and $t$ is the time.

# Inverse Fourier Transform

The inverse Fourier transform is the process of converting a frequency-domain representation of a signal back into its time-domain form. The inverse Fourier transform is given by

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

where $x(t)$ is the time-domain signal and $X(\omega)$ is the Fourier transform of $x(t)$, $\omega$ is the angular frequency, $j$ is the imaginary unit($\sqrt{-1}$), and $t$ is the time.

# Fourier Transform Notation

For convenience, we will write the Fourier transform of a signal $x(t)$ as

$$X(f) = \mathcal{F}\{x(t)\}$$

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and the inverse Fourier transform of $X(f)$ as

$$x(t) = \mathcal{F}^{-1}\{X(f)\}.$$

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$$X(f) = \mathcal{F}\{x(t)\}$$

and the inverse Fourier transform of $X(f)$ as

$$x(t) = \mathcal{F}^{-1}\{X(f)\}.$$

Note that

$$\mathcal{F}^{-1}\{\mathcal{F}\{x(t)\}\} = x(t)$$

at points of continuity of $x(t)$.

# Example of Fourier Transform

Let $x(t) = rect(t)$ where rect(t) is the rectangular pulse function defined as

$$rect(t) = \begin{cases} 1 & \text{if } |t| < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

# Example of Fourier Transform

Let $x(t) = rect(t)$ where rect(t) is the rectangular pulse function defined as

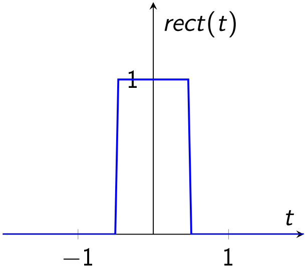
$$rect(t) = \begin{cases} 1 & \text{if } |t| < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

and $sinc(\omega)$ is the sinc function defined as

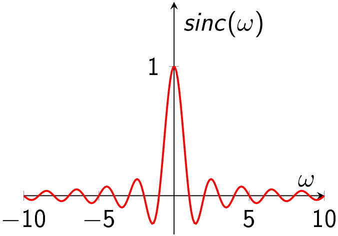
$$sinc(\omega) = \begin{cases} \frac{sin(\omega/2)}{\omega/2} & \text{if } \omega \neq 0 \\ 1 & \text{if } \omega = 0 \end{cases}$$

# Example of Fourier Transform

The forward Fourier transform of $rect(t)$ is $sinc(\omega)$ and the inverse Fourier transform of $sinc(\omega)$ is $rect(t)$.




(a) Inverse fourier transform of $sinc(\omega)$




(b) forward fourier transform of $rect(t)$

# Properties of Fourier Transform

- ⋆ Linearity
- ⋆ Time Shifting
- ⋆ Time Scaling
- ⋆ Time Reversal
- ⋆ Differentiation
- ⋆ Integration
- ⋆ Convolution

# Properties of Fourier Transform

- ⋆ Linearity
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- ⋆ Integration
- ⋆ Convolution

## Linearity

If $x_1(t)$ and $x_2(t)$ are two signals with fourier transform $X_1(\omega)$ and $X_2(\omega)$ respectively, then the Fourier transform of a linear combination of the signals is linear:

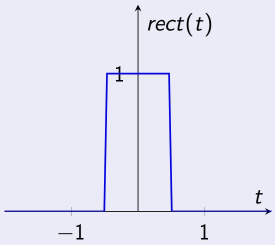
$$\mathcal{F}\{ax_1(t) + bx_2(t)\} = aX_1(\omega) + bX_2(\omega)$$

where $a$ and $b$ are constants.

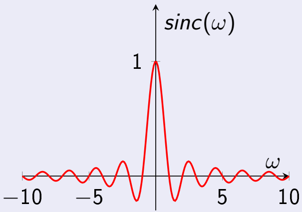
# Properties of Fourier Transform

## Linearity

Let $x_1(t) = rect(t)$ and $x_2(t) = rect(t)$ be two signals and let $X_1(\omega) = \mathcal{F}\{x_1(t)\} = sinc(\omega)$ and $X_2(\omega) = \mathcal{F}\{x_2(t)\} = sinc(\omega)$ be their fourier transforms respectively.



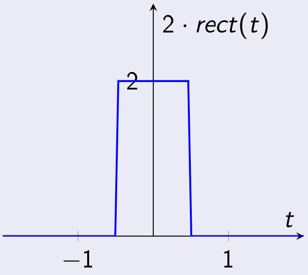

(a) $x_1(t)$ and $x_2(t)$



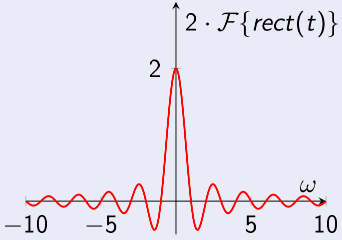

(b) $\mathcal{F}\{x_1(t)\}$ and $\mathcal{F}\{x_2(t)\}$

# Properties of Fourier Transform

## Linearity




(a) $2 \cdot x_1(t)$



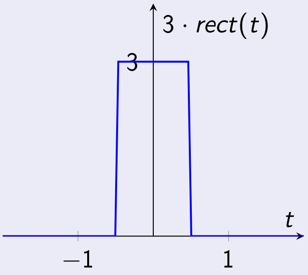

(b) $\mathcal{F}\{2 \cdot x_1(t)\}$

Here,

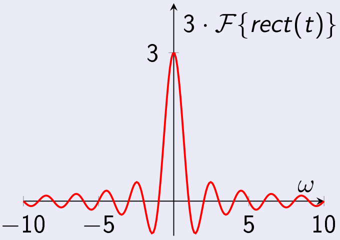
$$\mathcal{F}\{2 \cdot x_1(t)\} = 2 \cdot \mathcal{F}\{x_1(t)\}$$

# Properties of Fourier Transform

## Linearity




(a) $3 \cdot x_2(t)$

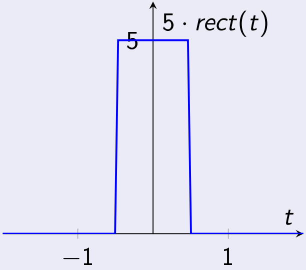



(b) $\mathcal{F}\{3 \cdot x_2(t)\}$

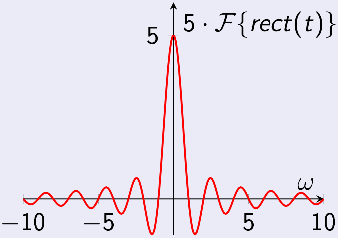
Here,

$$\mathcal{F}\{3 \cdot x_2(t)\} = 3 \cdot \mathcal{F}\{x_2(t)\}$$

## Linearity




(a) $2 \cdot x_1(t) + 3 \cdot x_2(t)$




(b) $\mathcal{F}\{2 \cdot x_1(t)\} + \mathcal{F}\{3 \cdot x_2(t)\}$

Here,

$$\mathcal{F}\{2 \cdot x_1(t) + 3 \cdot x_2(t)\} = 2 \cdot \mathcal{F}\{x_1(t)\} + 3 \cdot \mathcal{F}\{x_2(t)\}$$

# Properties of Fourier Transform

- Linearity
- Time Shifting
- Time Scaling
- Time Reversal
- Differentiation
- Integration
- Convolution

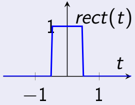
### Time Shifting

If $x(t)$ is a signal with Fourier transform $X(\omega)$, then shifting the signal in time corresponds to a phase shift in its Fourier transform:

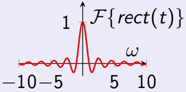
$$x(t - t_0) \xrightarrow{\text{FT}} X(\omega)e^{-j\omega t_0}$$

## Time Shifting

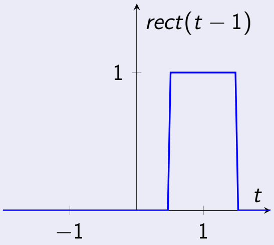
Let $x(t) = rect(t)$ be a signal with Fourier transform $X(\omega) = \mathcal{F}\{rect(t)\}$.



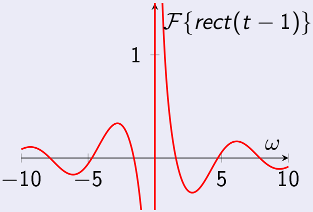

(a) $rect(t)$




(b) $\mathcal{F}\{rect(t)\}$




(a) $rect(t-1)$




(b) $\mathcal{F}\{rect(t-1)\}$

### Time Shifting

Here,

$$rect(t-1) \xrightarrow{\text{FT}} X(\omega)e^{-j\omega 1}$$

# Properties of Fourier Transform

- ⋆ Linearity
- ⋆ Time Shifting
- ⋆ Time Scaling
- ⋆ Time Reversal
- ⋆ Differentiation
- ⋆ Integration
- ⋆ Convolution

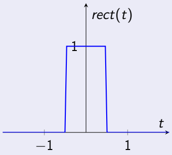
## Time Scaling

If $x(t)$ is a signal with Fourier transform $X(\omega)$, then stretching or compressing a signal in time inversely scales its frequency spectrum:

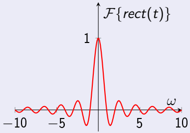
$$x(at) \xrightarrow{\text{FT}} \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$$

## Time Scaling

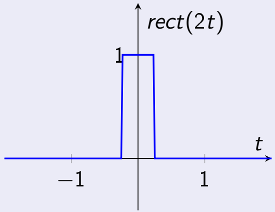
Let $x(t) = rect(t)$ be a signal with Fourier transform $X(\omega) = \mathcal{F}\{rect(t)\}$.



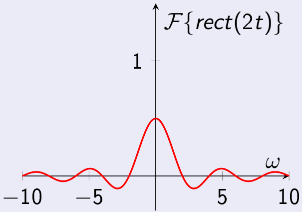

(a) $x(t)$




(b) $\mathcal{F}\{x(t)\}$




(a) $x(2t)$




(b) $\mathcal{F}\{x(2t)\}$

# Properties of Fourier Transform

## Time Scaling

Here,

$$rect(2t) \xrightarrow{\text{FT}} \frac{1}{2} sinc(\frac{\omega}{2})$$

# Properties of Fourier Transform

- ⋆ Linearity
- ⋆ Time Shifting
- ⋆ Time Scaling
- ⋆ **Time Reversal**
- ⋆ Differentiation
- ⋆ Integration
- ⋆ Convolution

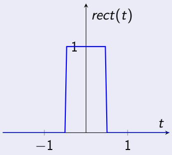
## Time Reversal

If $x(t)$ is a signal with Fourier transform $X(\omega)$, then time reversal of the signal in the time domain corresponds to frequency reversal in the frequency domain:

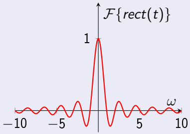
$$x(-t) \xrightarrow{\text{FT}} X(-\omega)$$

## Time Reversal

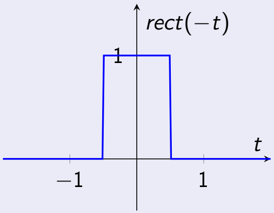
Let $x(t) = rect(t)$ be a signal with Fourier transform $X(\omega) = \mathcal{F}\{rect(t)\}$.



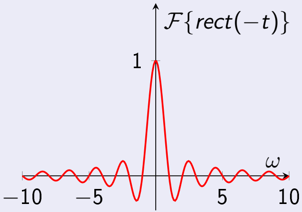

(a) $x(t)$




(b) $\mathcal{F}\{x(t)\}$




(a) $x(-t)$




(b) $\mathcal{F}\{x(-t)\}$

### Time Reversal

Both functions are even hence they remain same. Here,

$$rect(-t) \xrightarrow{\text{FT}} \mathcal{F}\{x(-t)\}$$

# Properties of Fourier Transform

- ⋆ Linearity
- ⋆ Time Shifting
- ⋆ Time Scaling
- ⋆ Time Reversal
- ⋆ Differentiation
- ⋆ Integration
- ⋆ Convolution

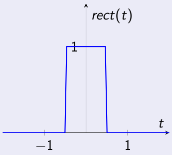
## Differentiation

If $x(t)$ is a signal with Fourier transform $X(\omega)$, then differentiation in the time domain corresponds to multiplication by $j\omega$ in the frequency domain:

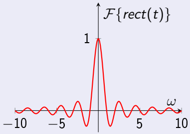
$$\frac{d}{dt}x(t) \xrightarrow{\text{FT}} j\omega X(\omega)$$

## Differentiation

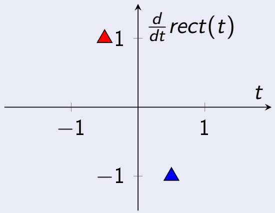
Let $x(t) = rect(t)$ be a signal with Fourier transform $X(\omega) = \mathcal{F}\{rect(t)\}$.



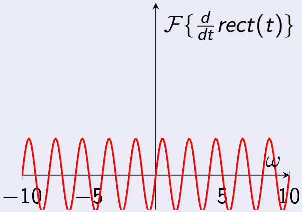

(a) $x(t)$




(b) $\mathcal{F}\{x(t)\}$




(a) $\frac{d}{dt}x(t)$




(b) $\mathcal{F}\{\frac{d}{dt}x(t)\}$

# Properties of Fourier Transform

- ⋆ Linearity
- ⋆ Time Shifting
- ⋆ Time Scaling
- ⋆ Time Reversal
- ⋆ Differentiation
- ⋆ **Integration**
- ⋆ Convolution

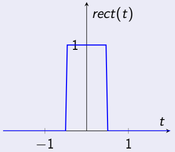
## Integration

If $x(t)$ is a signal with fourier transform $X(\omega)$, then integration in the time domain corresponds to multiplication by $\frac{1}{j\omega}$ in the frequency domain:

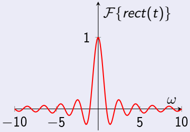
$$\int x(t)dt \xrightarrow{\text{FT}} \frac{1}{j\omega}X(\omega)$$

## Integration

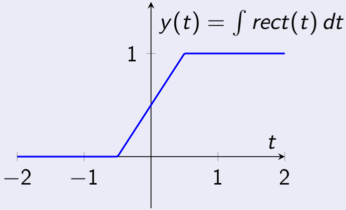
Let $x(t) = rect(t)$ be a signal with Fourier transform $X(\omega) = \mathcal{F}\{rect(t)\}$.



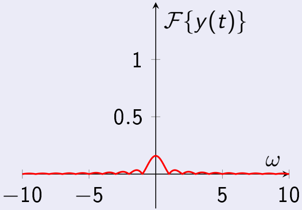

(a) Signal




(b) $\mathcal{F}\{rect(t)\}$




(a) $y(t) = \int x(t)dt$




(b) $\mathcal{F}\{y(t)\}$

# Properties of Fourier Transform

- ⋆ Linearity
- ⋆ Time Shifting
- ⋆ Time Scaling
- ⋆ Time Reversal
- ⋆ Differentiation
- ⋆ Integration
- ⋆ Convolution

# Properties of Fourier Transform

## Convolution

If $x(t)$ and $y(t)$ are two signals with fourier transform $X(\omega)$ and $Y(\omega)$, then convolution of the two signals in the time domain corresponds to multiplication in the frequency domain:

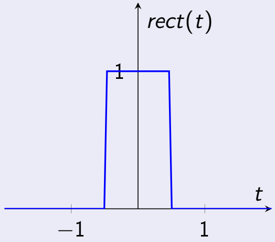
$$x(t) * y(t) \xrightarrow{\text{FT}} X(\omega) \cdot Y(\omega)$$

# Properties of Fourier Transform

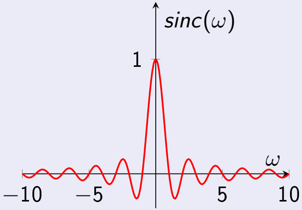
## Convolution

Let $x(t) = rect(t)$ and $y(t) = rect(t)$ be signals with Fourier transforms $X(\omega) = \mathcal{F}\{rect(t)\}$ and $Y(\omega) = \mathcal{F}\{rect(t)\}$. The convolution in the time domain corresponds to the product in the frequency domain:

$$z(t) = x(t) * y(t) \quad \leftrightarrow \quad Z(\omega) = X(\omega) \cdot Y(\omega).$$

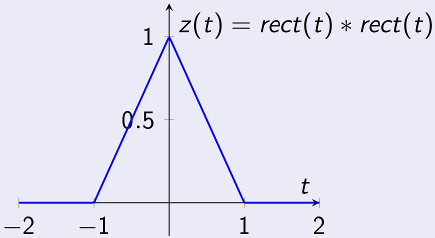



(a) $rect(t)$

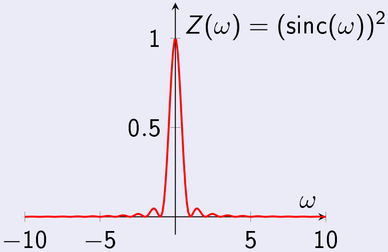



(b) $\mathcal{F}\{rect(t)\}$

## Convolution




(a) $z(t) = rect(t) * rect(t)$




(b) $Z(\omega) = \mathcal{F}\{z(t)\}$

Here,

$$rect(t) * rect(t) = X(\omega) \cdot Y(\omega)$$

# Fourier Transform Properties (Table 1)

| Property | Time Domain | Fourier Transform |
|---|---|---|
| Linearity | $x(t) = Ax_1(t) + Bx_2(t)$ | $X(j\omega) = AX_1(j\omega) + BX_2(j\omega)$ |
| Time Shifting | $x(t - t_0)$ | $e^{-j\omega t_0} X(j\omega)$ |
| Conjugation | $x^*(t)$ | $X^*(-j\omega)$ |
| Differentiation in Time | $\frac{d^n x(t)}{dt^n}$ | $(j\omega)^n X(j\omega)$ |
| Differentiation in Frequency | $-jtx(t)$ | $\frac{dX(j\omega)}{d\omega}$ |
| Time Integration | $\int_{-\infty}^{t} x(\tau) d\tau$ | $\frac{1}{j\omega} X(j\omega) + \pi X(0)\delta(\omega)$ |

# Fourier Transform Properties (Table 2)

| Property | Time Domain | Fourier Transform |
|---|---|---|
| Time Scaling | $x(at)$ | $\frac{1}{\|a\|} X\left(\frac{j\omega}{a}\right)$ |
| Time Reversal | $x(-t)$ | $X(-j\omega)$ |
| Frequency Shifting | $x(t)e^{j\omega_0 t}$ | $X(j(\omega - \omega_0))$ |
| Duality | $X(t)$ | $2\pi x(-j\omega)$ |
| Time Convolution | $x(t) * h(t)$ | $X(j\omega)H(j\omega)$ |
| Parseval's Theorem | $\int_{-\infty}^{\infty} \|x(t)\|^2 dt$ | $\frac{1}{2\pi} \int_{-\infty}^{\infty} \|X(j\omega)\|^2 d\omega$ |
| Modulation | $z(t) = x(t)y(t)$ | $Z(\omega) = \frac{1}{2\pi} X(j\omega) * Y(j\omega)$ |

# Fourier Transform Table

| Signal in Time Domain | Fourier Transform |
|---|---|
| $\delta(t)$ | $1$ |
| $u(t)$ | $\frac{1}{j\omega} + \pi\delta(\omega)$ |
| $\delta(t - t_0)$ | $e^{-j\omega t_0}$ |
| $te^{-at}u(t)$ | $\frac{1}{(a+j\omega)^2}$ |
| $u(-t)$ | $\pi\delta(\omega) - \frac{1}{j\omega}$ |
| $e^{at}u(-t)$ | $\frac{1}{a-j\omega}$ |

# Fourier Transform Table

| Signal in Time Domain | Fourier Transform |
|---|---|
| $e^{-a\|t\|}$ | $\frac{2a}{a^2+\omega^2}$ |
| $\cos(\omega_0 t)$ | $\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$ |
| $\sin(\omega_0 t)$ | $-j\pi[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$ |
| $\frac{1}{a^2+t^2}$ | $e^{-a\|\omega\|}$ |
| $\text{Sgn}(t)$ | $\frac{2}{j\omega}$ |
| 1 (for all t) | $2\pi\delta(\omega)$ |

[1]

[1] https://www.philadelphia.edu.jo/academics/qhamarsheh/uploads/Lecture

## Application of Fourier transform

# Application of Fourier Transform include

- ⋆ Signal Processing

# Application of Fourier Transform include

- $\star$ Signal Processing
- $\star$ Image Processing

# Application of Fourier Transform include

- ⋆ Signal Processing
- ⋆ Image Processing
- ⋆ Filtering:

# Application of Fourier Transform include

- ⋆ Signal Processing
- ⋆ Image Processing
- ⋆ Filtering:

  High Pass Filter

# Application of Fourier Transform include

- ⋆ Signal Processing
- ⋆ Image Processing
- ⋆ Filtering:
  - High Pass Filter
  - Low Pass Filter

# Application of Fourier Transform include

- ⋆ Signal Processing
- ⋆ Image Processing
- ⋆ Filtering:

  High Pass Filter
  Low Pass Filter
  Band Pass Filter

## Application of Fourier Transform include

- ⋆ Signal Processing
- ⋆ Image Processing
- ⋆ Filtering:

  High Pass Filter
  Low Pass Filter
  Band Pass Filter
  Band Reject Filter

# Application of Fourier Transform include

- ⋆ Signal Processing
- ⋆ Image Processing
- ⋆ Filtering:

  High Pass Filter
  Low Pass Filter
  Band Pass Filter
  Band Reject Filter

- ⋆ Audio Processing

# Application of Fourier Transform include

- ⋆ Signal Processing
- ⋆ Image Processing
- ⋆ Filtering:

  High Pass Filter
  Low Pass Filter
  Band Pass Filter
  Band Reject Filter

- ⋆ Audio Processing
- ⋆ Reducing Noise

# Application of Fourier Transform include

- ⋆ Signal Processing
- ⋆ Image Processing
- ⋆ Filtering:

  High Pass Filter
  Low Pass Filter
  Band Pass Filter
  Band Reject Filter

- ⋆ Audio Processing
- ⋆ Reducing Noise
- ⋆ Telecommunications

# Application of Fourier Transform include

- ⋆ Signal Processing
- ⋆ Image Processing
- ⋆ Filtering:

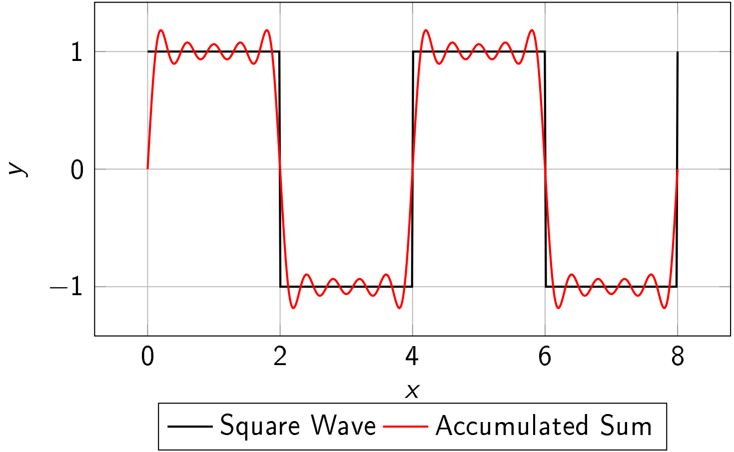
  High Pass Filter
  Low Pass Filter
  Band Pass Filter
  Band Reject Filter
- ⋆ Audio Processing
- ⋆ Reducing Noise
- ⋆ Telecommunications

# Application of Fourier Transform include

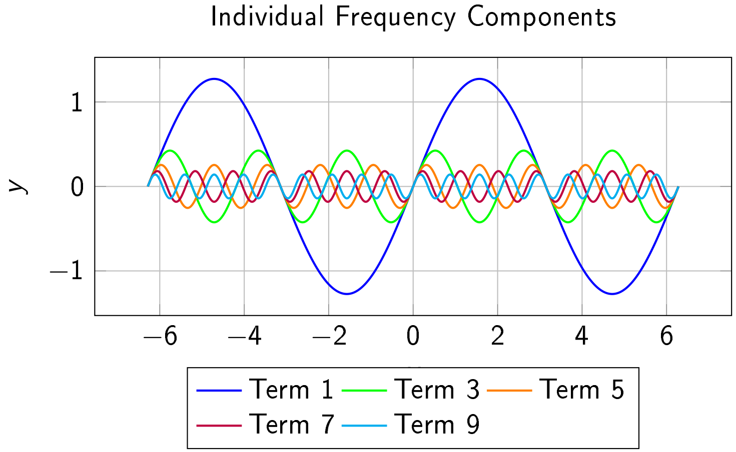
- ⋆ Signal Processing
- ⋆ Image Processing
- ⋆ Filtering:
  - High Pass Filter
  - Low Pass Filter
  - Band Pass Filter
  - Band Reject Filter
- ⋆ Audio Processing
- ⋆ Reducing Noise
- ⋆ Telecommunications
- ⋆ Representing wave propagation

⋆ **Signal Decomposing into Frequencies Components**

Square Wave and Accumulated Sum of Frequency Components



⋆ **Signal Decomposing into Frequencies Components**



⋆ **Signal Decomposing into Frequencies Components**

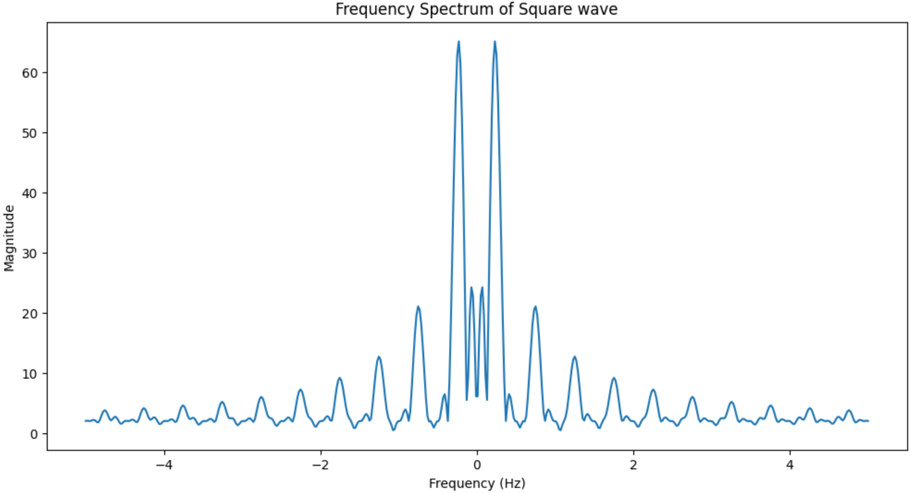
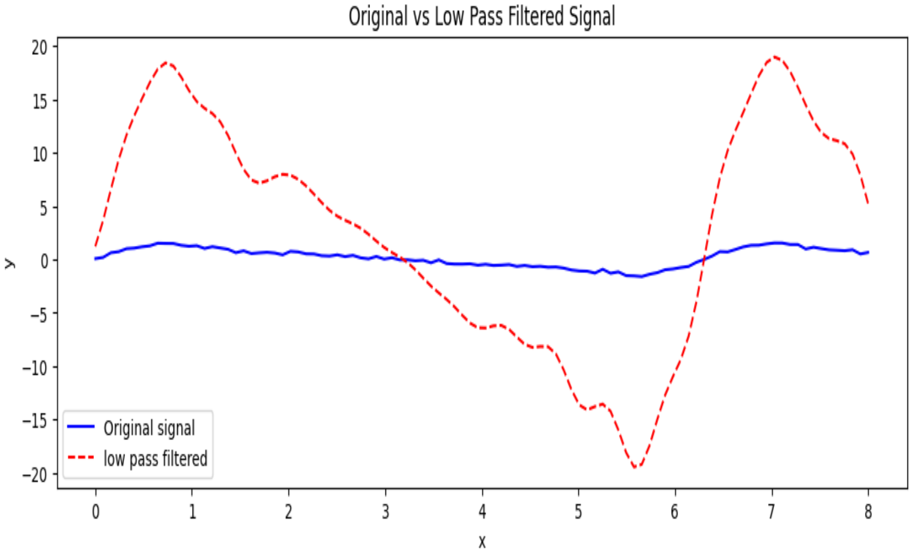


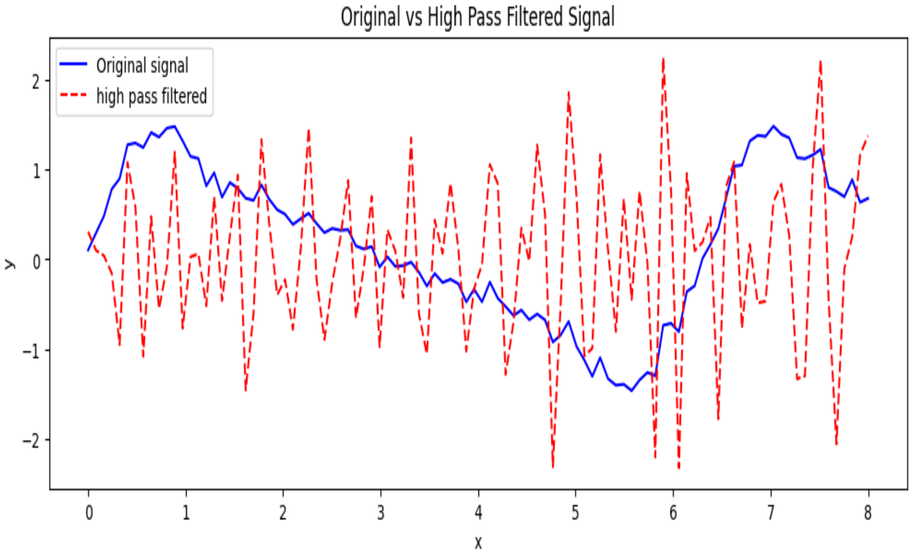

Figure: Frequency spectrum of square signal

# Application of Fourier Transform: Filtering

- ⋆ Filter: **Low Pass Filter**

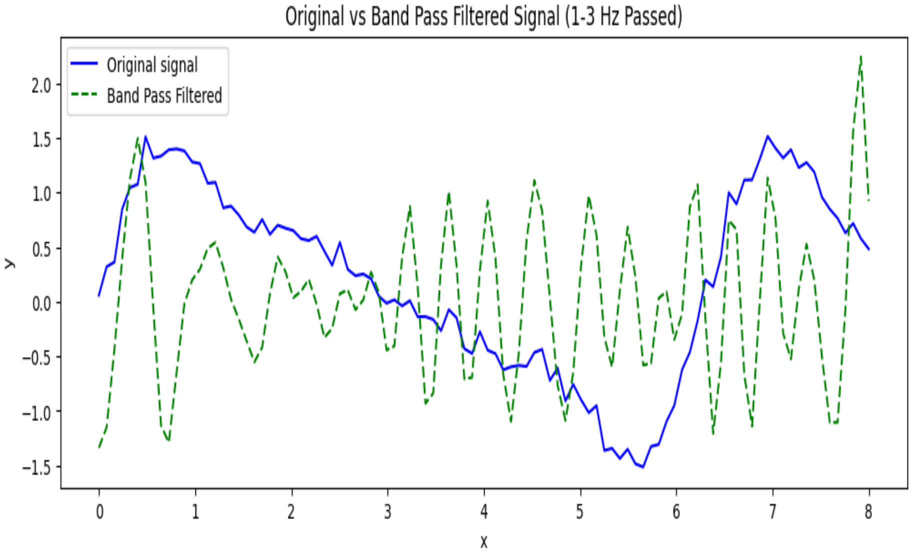


⋆ Filter: **High Pass Filter**



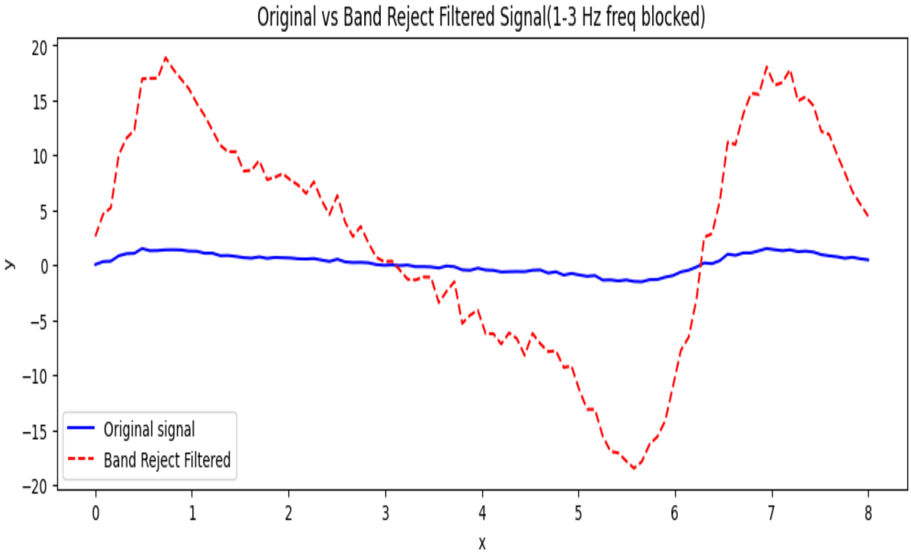
# Application of Fourier Transform: Filtering

⋆ Filter: **Band Pass Filter**



# Application of Fourier Transform: Filtering

⋆ Filter: **Band Reject Filter**



* **Frequency Analysis**

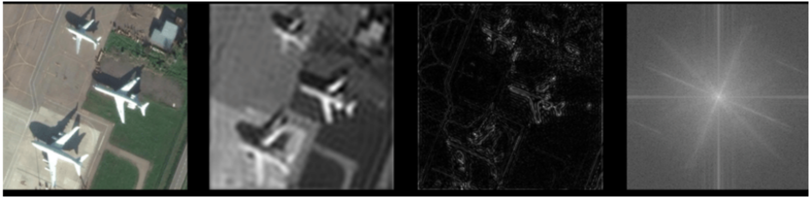


Figure: From right to left: Original image, Low-frequency components, High frequency components,Fourier transform

*Image source:* `https://www.researchgate.net/figure/sualization-of-low-frequency-components-high-frequency-components-and-Fourier-spectrums_fig2_347515196`

⋆ **Frequency Analysis**

- FT separates high-frequency components from low-frequency components. This helps in analyzing image characteristics

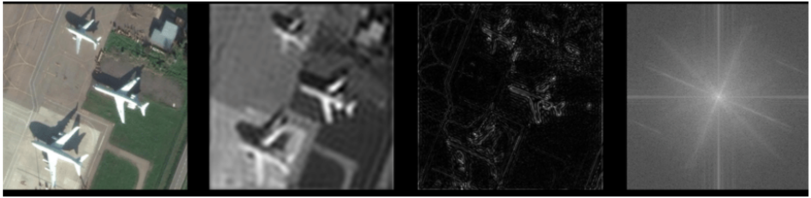


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*Image source:* https://www.researchgate.net/figure/sualization-of-low-frequency-components-high-frequency-components-and-Fourier-spectrums_fig2_347515196

⋆ **Compression**

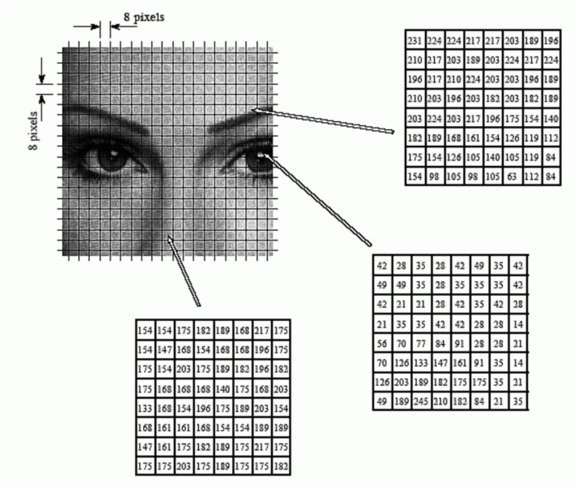


Figure: JPEG Transform compression

*Image source:* `https://www.dspguide.com/ch27/6.htm`

⋆ **Image Reconstruction**

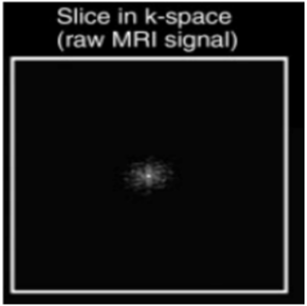


Figure: Fourier transform in Medical Imaging

*Image source:* https://www.researchgate.net/figure/A-a-Fourier-transform-is-applied-to-the-q-space-images-to-reconstruct-the-image-in-the_fig12_281047295

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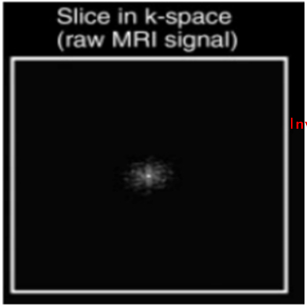


Inverse Fourier Transform

Figure: Fourier transform in Medical Imaging

*Image source:* https://www.researchgate.net/figure/A-a-Fourier-transform-is-applied-to-the-q-space-images-to-reconstruct-the-image-in-the_fig12_281047295

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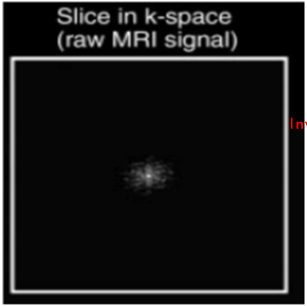


Inverse Fourier Transform
$\longrightarrow$

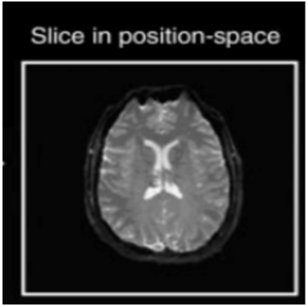


Figure: Fourier transform in Medical Imaging

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⋆ **Image Reconstruction**

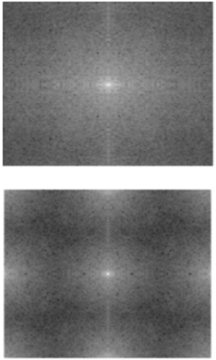
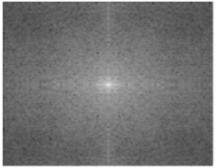


Figure: Fourier Reconstruction in MRI

*Image source:* `https://link.springer.com/chapter/10.1007/978-3-030-30511-6_9`

# Application of Fourier Transformation: Image Processing

## ⋆ Image Reconstruction



2D Inverse Fourier Transform

$\longrightarrow$

Inverse Fourier Transform

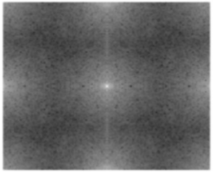


Figure: Fourier Reconstruction in MRI

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⋆ **Image Reconstruction**

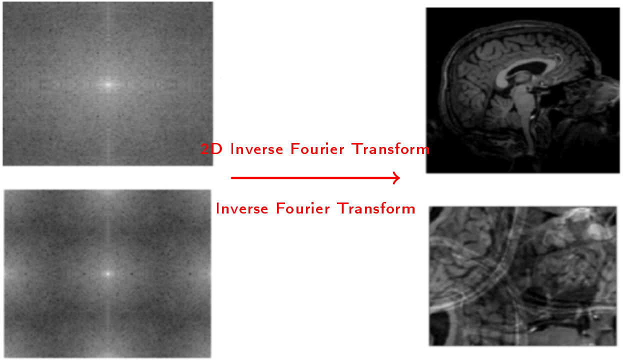


Figure: Fourier Reconstruction in MRI

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⋆ **Removing Noise**

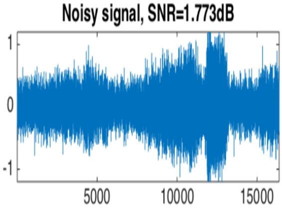


Figure: Comparison of Noisy and Denoised Signal Using Fourier Transform

***Image source:*** `https://www.numerical-tours.com/matlab/audio_1_processing/`

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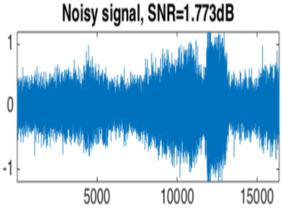



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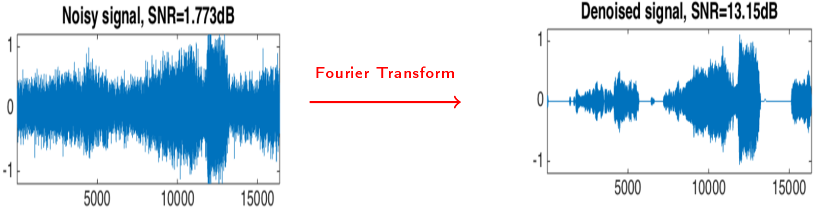


Figure: Comparison of Noisy and Denoised Signal Using Fourier Transform

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# Application of Fourier Transformation: Audio Processing

- ⋆ Equalization

# Application of Fourier Transformation: Audio Processing

- ⋆ Equalization
- ⋆ Pitch shifting and time stretching

## Application of Fourier Transformation: Audio Processing

- ⋆ Equalization
- ⋆ Pitch shifting and time stretching
- ⋆ Sound synthesis

# Application of Fourier Transformation: Audio Processing

- ⋆ Equalization
- ⋆ Pitch shifting and time stretching
- ⋆ Sound synthesis
- ⋆ Audio effects

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- ⋆ Real time audio processing:

# Application of Fourier Transformation: Audio Processing

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- ⋆ Pitch shifting and time stretching
- ⋆ Sound synthesis
- ⋆ Audio effects
- ⋆ Real time audio processing:
    - Live Sound Enhancement

# Application of Fourier Transformation: Audio Processing

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- ⋆ Sound synthesis
- ⋆ Audio effects
- ⋆ Real time audio processing:
    - Live Sound Enhancement
    - Interactive Audio

# Application of Fourier Transformation: Audio Processing

- ⋆ Equalization
- ⋆ Pitch shifting and time stretching
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- ⋆ Audio effects
- ⋆ Real time audio processing:
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- ⋆ Audio Fingerprinting