Fourier Transformation

Using Beamer

Gourove Roy

Department of CSE, BUET

November 22, 2024

Introduction to Fourier Transforms

- ⋆ Fourier transform
- ⋆ Inverse Fourier transform: The Fourier integral theorem
- ⋆ The rect and sinc functions
- ⋆ Cosine and Sine Transforms
- ⋆ Symmetry properties
- ⋆ Periodic signals and δ functions

Fourier Transforms

Given a continuous time signal $x(t)$, de ne its Fourier transform as the function of a real $f, X(f)$[1] is :

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

This is similar to the expression for the Fourier series coefficients.

[1] Usually $X(f)$ is written as $X(i2\pi f)$ or $X(i\omega)$. This corresponds to the Laplace transform notation which we encountered when discussing transfer functions $H(s)$.

Inverse Fourier Transforms

Continuous-time Fourier Transform yields the *inversion formula* for the Fourier transform, the *Fourier integral theorem*:

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df$$

The Fourier transform and its inverse are symmetric in the sense that the Fourier transform of $x(t)$ is $X(f)$ and the Fourier transform of $X(f)$ is $x(-t)$. This is a consequence of the symmetry of the complex exponential.

Cosine and Sine Transforms

Assume x(t) is a possibly complex signal.

$$
\begin{aligned}
X(f) &= \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \\
&= \int_{-\infty}^{\infty} x(t) \cos(2\pi ft) dt - j \int_{-\infty}^{\infty} x(t) \sin(2\pi ft) dt \\
&= \int_{-\infty}^{\infty} x(t) \cos(\omega t) dt - j \int_{-\infty}^{\infty} x(t) \sin(\omega t) dt
\end{aligned}
$$

Fourier Transform Notation

For convenience, we will write the Fourier transform of a signal $x(t)$ as

$$X(f) = \mathcal{F}\{x(t)\}$$

and the inverse Fourier transform of $X(f)$ as

$$x(t) = \mathcal{F}^{-1}\{X(f)\}$$

Note that

$$\mathcal{F}^{-1}\{\mathcal{F}\{x(t)\}\} = x(t)$$

at points of continuity of x(t).

The rect and sinc functions

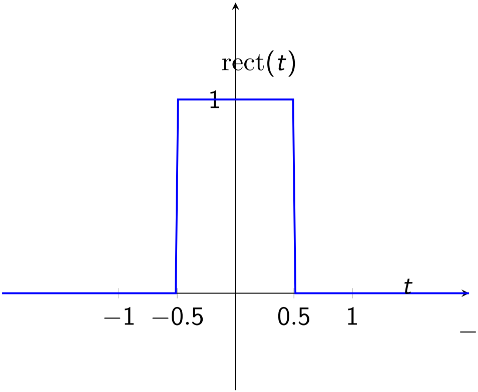
The Fourier transform of a rectangular pulse is a sinc function and the Fourier transform of a sinc function is a rectangular pulse.

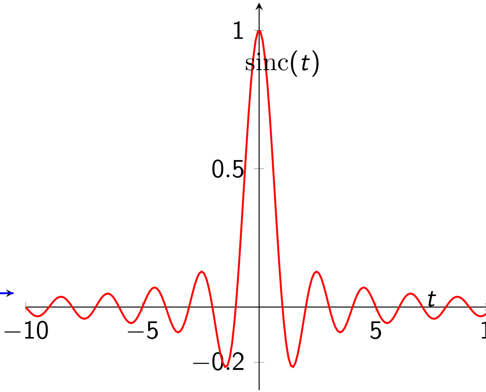
$$x(t) = \text{rect}(t) = \begin{cases} 1 & \text{if } |t| < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$X(f) = \text{sinc}(f) = \begin{cases} 1 & \text{if } |f| < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

Note that the sinc function is defined as $\text{sinc}(f) = \frac{\sin(\pi f)}{\pi f}$.

rect and sinc function graphs





Duality

Notice that the Fourier transform F and the inverse Fourier transform F are almost the same.