PROBLEM SET 2

GOURPRABH

QUESTION 1

Construct the equal-weighted bond market return, value-weighted bond market return, and 1 lagged total bond market capitalization using CRSP Bond data. Your output should be from January 1926 to December 2019, at a monthly frequency.

The dataset that I downloaded from CRSP contained the required input variables (KYCRSPID, MCALDT, TMRETNUA, TMTOTOUT) from *December 1925 to December 2019.*

The data clean-up steps that I followed are -

1. **TMTOTOUT** - I set all data rows of column containing *NA* values to *0*.

2. **TMRETNUA** - Similarly,I also set all data rows containing *NA* values to *0*.

3. **Lagged market value** - I first shifted amount outstanding of each bond, filling *0* in the rows where this operation created NA values. Then I did a sum of the total market value of the bonds at each month giving me the Lagged Bond Market Value.

4. **Equally-weighted return** - I took mean of returns of the bonds at each month.

$$Bond\ Ew\ Ret = \frac{sum(Return_t)}{N_t}, where\ \ t\ \ stands\ \ for\ \ the\ \ months\ \ of\ \ data$$

5. **Value-weighted return** - I first created market weight for each bond each month based on its market value last month. The formula I used is -

$$Market\ \ weight\ \ of\ \ bond_i = \frac{lagged\ \ market\ \ value\ \ of\ \ bond_i}{Total\ \ lagged\ \ market\ \ value\ \ of\ \ all\ \ bonds}$$

   Then, I took a weighted sum of the bonds return where the weights were from the formula I calcualted above.

$$(Bond\ VW\ \ Ret)_t = sum(Return_{it} * weight_i)$$

   where t stands for the month,
   and i stands for the specific bond

QUESTION 2

Aggregate stock, bond, and riskless datatables. For each year-month, calculate the lagged market value and excess value-weighted returns for both stocks and bonds. Your output should be from January 1926 to December 2019, at a monthly frequency.

Risk-free rate - I am considering the 30-day T-bill rate from the Treasury data of CRSP as my risk-free rate. My motivation for choosing the 30-day T-bill rate is because the Leverage Aversion and Risk parity paper considers it to be its basic risk-free rate. In Table B1 of the paper, the authors also consider other risk-free rates like swaps and LIBOR in comparison with their basic 30-day T-bill risk-free rate. Logically, it makes sense to use 30-day T-bill rate instead of the 90-day T-bill rate as the T-bill with the longer maturity will have risk associated with it due to its longer maturity.

After deciding the risk-free rate, the next step is to calcualted Excess stock and bond returns using the below formula -

$$(Excess \quad Return)_{i,t} = Return_{i,t} - t30ret_t$$

QUESTION 3

Calculate the monthly unlevered and levered risk-parity portfolio returns as defined by Asness, Frazzinin and Peterson. For the levered risk-parity portfolio, match the value-weighted portfolio's σ over the longest matched holding period of both. Your output should be from January 1926 to December 2019, at a monthly frequency.

Volatility of the stock -
As defined by Asness, Frazzinin and Peterson, I calculated volatility at every month in the data by taking the volatility of the past 36 months (excluding the current month whose volatility is being calculated). This gives us data from January 1929 to December 2018.

Volatility of the bond -
To calculate this, I repeated the procedure for volatility calculation for stock.

Inverse volatility -
To calculate the inverse of volatility, we just took $1/volatility$ in our data.

Market Value of bond -
We notice that in our Monthly CRSP universe data from Question 2, the Bond lagged Market value for the end of December 2018 was 15,224,083. A quick search showed the actual US bond market value to be in trillions. Hence, I multiplied Market Value of each month in our data by 10^6.

Market Value of stock -
We notice that in our Monthly CRSP universe data from Question 2, the Stock lagged Market value for the end of December 2018 was 27,586,215,520. A quick search showed the actual US stock market value to be in trillions. Hence, I multiplied Market Value of each month in our data by 10^3.

Value-weighted portfolio return -
To calcualte value-weighted return, at each month from Jan 1929, I first calculated weight of stock and bond in the market portfolio based on their previous month's market value The formula used is -

$$(Stock\ \ weight)_t = \frac{(Total\ \ Market\ \ value\ \ of\ \ stock)_{t-1}}{(Total\ \ Market\ \ Value)_{t-1}}$$

Simialrly,

$$(Bond\ \ weight)_t = \frac{(Total\ \ Market\ \ value\ \ of\ \ Bond)_{t-1}}{(Total\ \ Market\ \ Value)_{t-1}}$$

The total market value at each time t is calculated as -

$$(Total\ \ market\ \ value)_t = (Total\ \ Stock\ \ Market\ \ Value)_t + (Total\ \ Bond\ \ Market\ \ Value)_t$$

Hence our value-weighted market return at time t would be -

$$(Stock\ \ weight)_t * (Stock\ \ excess\ \ return)_t + (Bond\ \ weight)_t * (Bond\ \ excess\ \ return)_t$$

60-40 portfolio return -

In this case, the weight in stock market portfolio at each time t was 0.6 and the weight in bond market portfolio was 0.4. Hence at each month t, the return of the 60-40 portfolio was -

$$0.6 * (Stock\ \ excess\ \ return)_t + 0.4 * (Bond\ \ excess\ \ return)_t$$

Unlevered Portfolio weights (Unlevered_k) -

According to Asness, Frazzinin and Peterson, the unlevered Risk Parity portfolio weights of each asset class is equal to inverse of its volatility. But, these weights are rescaled to sum to 1 at each rebalancing. The way to do that would be to set the rescaling term(Unleverd_k) at time t to be-

$$Unlevered\ \ k = \frac{1}{(Inverse\ \ volatility\ \ of\ \ Total\ \ Stock\ \ portfolio)_t + (Inverse\ \ volatility\ \ of\ \ Total\ \ Bond\ \ portfolio)_t}$$

Hence, the unlevered stock portfolio weight at time t is -

$$(stock\ \ weight)_t = \frac{(Inverse\ \ volatility\ \ of\ \ Total\ \ Stock\ \ portfolio)_t}{(Inverse\ \ volatility\ \ of\ \ Total\ \ Stock\ \ portfolio)_t + (Inverse\ \ volatility\ \ of\ \ Total\ \ Bond\ \ portfolio)_t}$$

Similarly, the unlevered bond portfolio weight at time t is -

$$(Bond\ \ weight)_t = \frac{(Inverse\ \ volatility\ \ of\ \ Total\ \ Bond\ \ portfolio)_t}{(Inverse\ \ volatility\ \ of\ \ Total\ \ Stock\ \ portfolio)_t + (Inverse\ \ volatility\ \ of\ \ Total\ \ Bond\ \ portfolio)_t}$$

Excess Unlevered Risk Parity portfolio return -
Once we have the portfolio weights of each asset, we can calculate the Excess Unlevered Risk Parity portfolio return at time t as -

$$(Stock\ \ weight)_t * (Stock\ \ excess\ \ return)_t + (Bond\ \ weight)_t * (Bond\ \ excess\ \ return)_t$$

Putting the formulae used in the previous heading back, we get the return calcualtion as defined by Asness, Frazzinin and Peterson -

$$(Unlevered\ \ k)_t * (Inverse\ \ volatility\ \ of\ \ Total\ \ Stock\ \ portfolio)_t * (Stock\ \ excess\ \ return)_t +$$

$$(Unlevered\ \ k)_t * (Inverse\ \ volatility\ \ of\ \ Total\ \ Bond\ \ portfolio)_t * (Bond\ \ excess\ \ return)_t$$

Levered K-
According to Asness, Frazzinin and Peterson, for the levered RP portfolio, the weights of the assets are constant and is defined as to match the post realized volatility of the value-weighted portfolio. The formula used is -

$$\frac{SD(Value - weighted\ \ portfolio\ \ return\ \ (1929 - 2010))}{SD((Inverse\ \ vol\ \ of\ \ stock)_t * (Stock\ \ excess\ \ return)_t + (Inverse\ \ vol\ \ of\ \ bond)_t * (Bond\ \ excess\ \ return)_t)}$$

where SD stands for Standard Devaition.
This gives us a constant Levered_k value of 0.0254.

Excess Levered Risk Parity portfolio return -
Once we have the Levered K constant, we can calculate the Excess levered Risk Parity portfolio return as -

$$(levered\ \ k) * (Inverse\ \ volatility\ \ of\ \ Total\ \ Stock\ \ portfolio)_t * (Stock\ \ excess\ \ return)_t +$$

$$(levered\ \ k) * (Inverse\ \ volatility\ \ of\ \ Total\ \ Bond\ \ portfolio)_t * (Bond\ \ excess\ \ return)_t$$

QUESTION 4

Replicate and report Panel A of Table 2 in Asness, Frazzini, and Pedersen (2012), except for Alpha and t-stat of Alpha columns. Specifically, for all strategies considered, report the annualized average excess returns, t-statistic of the average excess returns, annualized volatility, annualized Sharpe Ratio, skewness, and excess kurtosis. Your sample should be from January 1930 to June 2010, at monthly frequency. Match the format of the table to the extent possible. Discuss the difference between your table and the table reported in the paper. It is zero? If not, justify whether the difference is economically negligible or not. What are the reasons a nonzero difference?

The replicated Panel A of Table 2 in Asness, Frazzinin and Peterson is shown below -

	Annualized Mean	t-stat of Annualized Mean
CRSP stocks	6.993424	3.310075
CRSP bonds	1.555907	4.330391
Value-weighted portfolio	4.009786	2.460239
60/40 portfolio	4.818417	3.711752
unlevered RP	2.254258	4.779675
levered RP	7.979394	4.873193

	Annualized Standard Deviation	Annualized Sharpe Ratio	Skewness	Excess Kurtosis
CRSP stocks	18.946328	0.3691176	0.2796323	7.889078
CRSP bonds	3.222031	0.4828965	-0.0515250	4.835679
Value-weighted portfolio	14.615609	0.2743495	0.6737666	15.358394
60/40 portfolio	11.641219	0.4139100	0.2737404	7.814646
unlevered RP	4.229396	0.5329977	0.0669428	4.787124
levered RP	14.683493	0.5434262	-0.3502214	1.993186

Annualized Mean -

The formula used is -

$$\left(\frac{\sum_{t=1}^{N}(Excess\ \ Return)_t}{N}\right) * 12$$

t-statistics of Annualized Mean

We know that the t-statistics stands for-

$$t = \frac{b_1 - \beta_1^*}{s_{b_1}} = \frac{estimate - hypothesized\ \ value}{Standard\ \ error}$$

In our case, we are testing for significance of our excess returns, i.e are they statistically differentfrom 0. Hence our Hypothesized value is 0.
The standard error of our estimate is given by -

$$Standard\ \ error = \frac{Standard\ \ Deviation}{N-1}$$

where N is the length of the sample.
Hence, we get our t-stat as -

$$t = \frac{b_1 * \sqrt{(N-1)}}{sd(b_1)}$$

Annualized Volatility

It is calculated as the standard deviation of the returns. The formula is -

$$(Monthly\ \ Standard\ \ deviation\ \ of\ \ results) * \sqrt{12}$$

Annualized Sharpe Ratio

The formula used is -
$$\frac{Annualized\ \ Mean\ \ of\ \ Excess\ \ Return}{Annualized\ \ Volatility}$$

Skewness

The formula used by the skewness function is -

$$\frac{\sum_{i=1}^{N}(Y_i - \bar{Y})^3/N}{s^3}$$

where $\bar{Y}$ is the mean of the sample
and s is the standard devaition
We don't see high skewness for any of our return data, signifying that the return distribution is symmetric around it's mean.

Excess Kurtosis The formula used by the kurtosis function is -

$$\frac{\sum_{i=1}^{N}(Y_i - \bar{Y})^4/N}{s^4}$$

To calcualte Excess kurtosis, I have subtracted 3 from the outptut of the kurtosis function above.

We have only taken sample from January 1930 to June 2010. We see some difference between the replicated table and the table in the paper where the authors take the sample from 1926 to 2010.

We see higher returns in our replicated methods in CRSP stocks, value-weighted portfolio and 60/40 portfolio. One reason could be that we did not include part of the financial crisis of 1929, thus giving us higher average returns. The same reason could be for the lower returns of levered RP portfolio which would have given better risk-adjusted returns compared to the valie-weighted portfolio during the 1929 crisis. Hence, I don't believe the difference to be economically significant.